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Designing a Hybrid GEO-LEO Constellation Pattern for Regional Satellite Navigation in Iran

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ABSTRACT

Keywords: *Simulation, Coverage analysis, DOP, Skyplot, Iran.*

In this paper, a few hybrid satellite constellations including combinations of LEO and GEO satellites for providing satellite navigation and positioning services for users in Iran have been designed and proposed. The performance of the constellations has been analyzed based on DOP values variations. It is shown that theoretically, it is possible to provide satellite positioning and navigation service with acceptable DOP values based on the introduced hybrid pattern including three GEO satellites and a constellation of about 30 to 60 LEO satellites in 3 or 4 orbit planes. The design has been performed based on studying the skyplot of the Iranian territory considering the GEO satellites as fixed points, and then determining the effect of the instantaneous position of the LEO satellites on the DOP values. A few LEO constellations have been designed to provide best DOP values based on the skyplot analysis results. Then, scenarios including similar GEO satellites and different patterns for LEO satellites have been simulated for half a sidereal day. The performance of the hybrid constellations provides satisfactory results with the average PDOP values of less than 4 which is acceptable. Optimizing the resulted pattern can lead to more desirable performance. In addition to navigation mission, hybrid constellations can perform other missions. Therefore, the proposed constellations can be operated as multi-mission space platforms.

Introduction

Satellite navigation systems have traditionally been designed and operated to provide global coverage and the term GNSS has been coined consequently. Global coverage has been achieved by incorporating satellites at Middle Earth Orbit (MEO) to form delta walker type patterns with half sidereal day period. Furthermore, Geostationary Orbit (GEO) and Inclined Geosynchronous Orbit (IGSO) satellites have also been incorporated to increase the performance of those systems locally. The layout of such satellites within the constellation as well as their orbital parameters, specifically the altitude, has resulted in the realization of global coverage [1] and [2].
Although the aforementioned satellite navigation services are ubiquitously and reliably available, policymakers have already conceived non-global and independent systems due to geopolitical drives

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and thus, regional satellite navigation systems, called RNSS hereafter, have emerged and put into operation. It is worth mentioning that depending on the type of service, local systems fall into two types. While some local systems provide augmentation to global GNSSs for a confined territory, there are others which perform as a standalone system and the latter concept is regarded in this paper. Among such regional systems are the Indian IRNSS [3] and the Japanese QZSS [4] which incorporate hybrid constellations of GEO (Geostationary orbit) and IGSO (inclined geosynchronous orbit) satellites.

Designed and operated by Indian Research Organization, ISRO, IRNSS provides navigation and time services for two regions including mainland India and the region within 1500 km from its border called Primary Service Area and a wider area limited to the latitudes -30 to +50 degrees and longitudes +30 to +130 degrees called Extended Service Area [3]. IRNSS is the most extensive operational regional satellite navigation system at the moment and nominally incorporates eight satellites in a constellation of GEO and IGSO orbits. The ground track of three GEO satellites along with four nominal IGSO satellites in two orbit planes are as depicted in Fig. 1. As it can be seen, the IGSO orbits of this constellation are circular. Although the performance of this system regarding dilution of precision is less desirable in comparison with GPS, specifically in its secondary service area, it can be relied upon as an independent system [5].

The QZSS is a local navigation satellite system which has become operational to complement GPS service over Japan [4]. However it can also be regarded as a standalone system. It comprises one GEO satellite and three IGSO satellites with almost similar ground track as shown in Fig. 2. The IGSO orbits of this system are elliptical and provide more coverage on the northern hemisphere than the southern one.

In addition to the abovementioned operational RNSSs, some others have been proposed and studied. As stated by [6] and [7], a satellite navigation system has been designed for the coverage of the East Asian region. This system comprises three GEO and four eccentric IGSO satellites. The architecture of this constellation is similar to that of the QZSS.

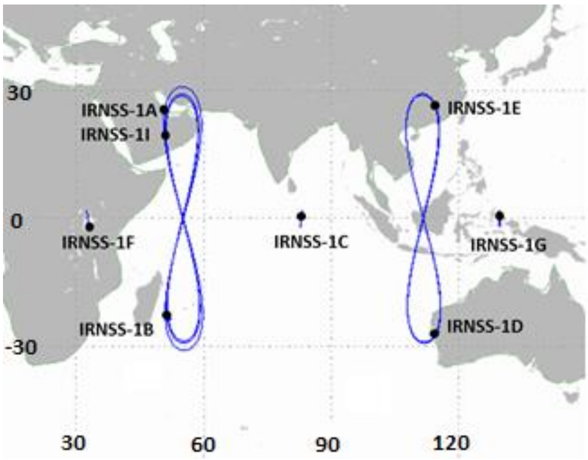

Figure 1: The IRNSS space segment.

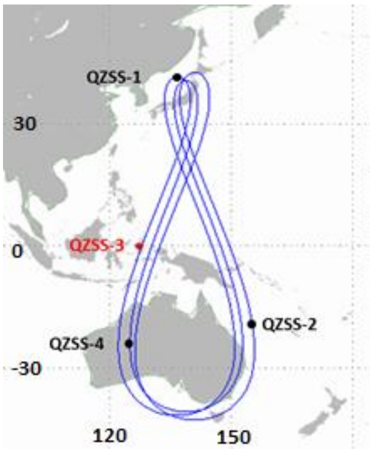

Figure 2: The QZSS space segment.

LEO satellites for satellite navigation

Although the aforementioned combination of satellites including GEO and IGSO satellites has proven to be efficient for both GNSS and RNSS, the idea of employing LEO (low altitude earth orbit) satellites for navigation has come to the fore and the benefits of such constellation, specifically, employing current communication LEO constellations for navigation mission, has been investigated and evaluated. In [8] it has been shown that LEO satellites can be used either as a compliment to traditional GNSSs, and act as a platform to enhance performance regarding accessibility and precision, or as a standalone independent system. Furthermore, it has been shown that LEO satellites can improve functions like PPP (precise point positioning) due to rapid movement of satellites from users' vantage point in comparison with slow movement of the MEO (medium altitude earth orbit) satellites of traditional GNSSs which seem almost stationary [9], [10] and [11]. However, to date, no operational

GNSS or RNSS have incorporated LEO satellites into their constellation. Considering the fact that commercial LEO mega constellations have already become globally operational, it is probable that the concept of such practice, as stated in [8], will become realized in future. In this paper, the concept of hybrid satellite constellation including GEO and LEO satellites has been conceived and six constellations including GEO and LEO satellites have been demonstrated to be capable of providing the desired accuracy for regional navigation and positioning for users in Iran.

Hybrid constellation for navigation in Iran

The idea of proposing a constellation for providing regional satellite navigation service for users within Iranian territory has already been addressed. Several constellation patterns have been proposed in this regard. In [12] and [13], combinations of GEO and IGSO satellites, similar to the Indian and Japanese satellite navigation system have been proposed. In [14], the idea of a combination of GEO and LEO satellites has been proposed for providing satellite navigation and positioning in Iran. In that research, it has been conceived that one to three GEO satellites in combination with several LEO satellites can constitute a hybrid constellation to deliver satellite navigation and positioning service.

Iranian territory is confined between longitudes of 44 to 63 deg and latitudes of 25 to 40 deg north. Therefore, the visible section of GEO for all users in this region constitutes an arc in the southern celestial hemisphere. To demonstrate this, three locations, A, B and C, in skyplots north eastern, central and south eastern Iran have been chosen as depicted in Fig 3. Then the skyplots for these locations including three GEO satellites stationed at orbital positions of 0, 55 and 105 deg have been determined and are shown in Fig. 4. It is observed that the GEO satellites appear in the vicinity of each other in the celestial hemisphere. Together with the three GEO satellites, at least one more satellite is required to meet the minimum 4 in-sight reference points for satellite positioning as described in [2]. However, the precision of satellite positioning is highly dependent on the geometrical orientation of the reference points from user's view. For the case of three reference points allocated on the GEO, the fourth point needs to be placed somewhere far from GEO. To evaluate the effect of the fourth reference point on the precision of positioning based on the aforementioned combination of satellites, DOP (dilution of precision) values have been determined considering the three GEO satellites to be stationary in the skyplot and the fourth satellites to be at all the azimuth and elevation angles in the skyplot.

DOP parameters are briefly expressed in Eq. (1) to Eq. (7). Comprehensive explanation of these parameters and equations are available in [1] and [2]. In Eq. (1), E_n and A_n denote the elevation and azimuth angles corresponding to the reference point *n* as seen by the user respectively. The DOP values in Eq. (3) to Eq. (7) are the combination of the elements of the matrix in Eq. (2). DOPs are geometrical scalar parameters, which show to what extent the errors in measured pseudoranges affect the accuracy of the user's position and time estimation [1] and [2]. The less the DOP values, the higher the precision. The result of the analysis of PDOP, HDOP and VDOP for points A, B and C are shown in Fig. 5 to 7 respectively.

$$G = \begin{bmatrix} \cos E_1 \sin A_1 & \cos E_1 \cos A_1 & \sin E_1 & 1 \\ \cos E_2 \sin A_2 & \cos E_2 \cos A_2 & \sin E_2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \cos E_n \sin A_n & \cos E_n \cos A_n & \sin E_n & 1 \end{bmatrix} \quad (1)$$

$$(G^T G)^{-1} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \\ D_{31} & D_{32} & D_{33} & D_{34} \\ D_{41} & D_{42} & D_{43} & D_{44} \end{bmatrix} \quad (2)$$

$$GDOP = \sqrt{D_{11} + D_{22} + D_{33} + D_{44}} \quad (3)$$

$$PDOP = \sqrt{D_{11} + D_{22} + D_{33}} \quad (4)$$

$$HDOP = \sqrt{D_{11} + D_{22}} \quad (5)$$

$$VDOP = \sqrt{D_{33}} \quad (6)$$

$$TDOP = \sqrt{D_{44}} \quad (7)$$

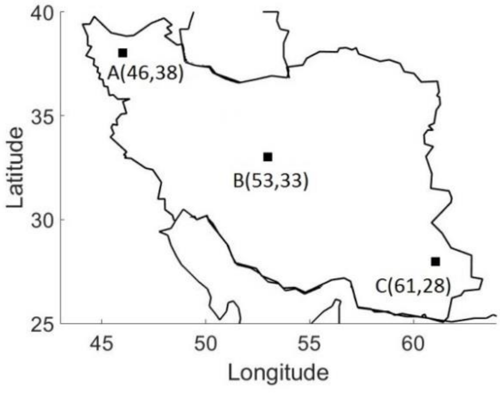

Figure 3: Locations of A, B and C.

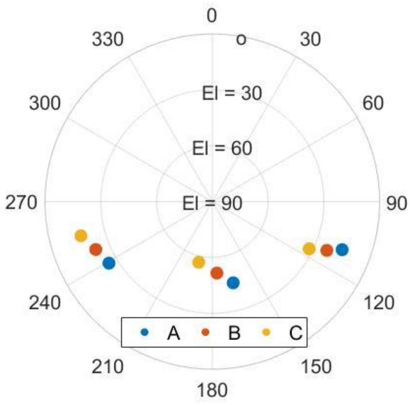

Figure 4: The skyplot of GEO satellites for users in Iranian territory

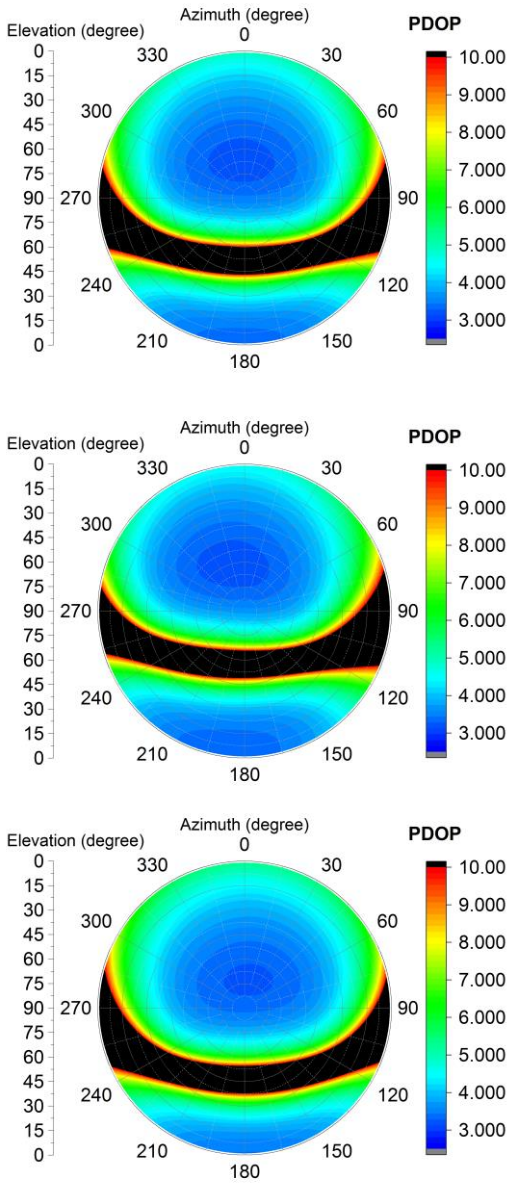

Figure 5: PDOP values for users at location A (above), B (middle), C (bottom), corresponding to different locations of the fourth reference point.

In Fig. 5, the variation of PDOP for the three locations is shown. It is observed that the minimum DOP values correspond to the fourth reference point located at two regions. The first region is confined to azimuth angles between -60 (330) to +60 deg and elevation angles of 30 to 90 deg. The second region is confined to azimuth angles of 140 to 200 deg and elevation angles of 0 to 30 deg. Provided that the fourth reference point is located

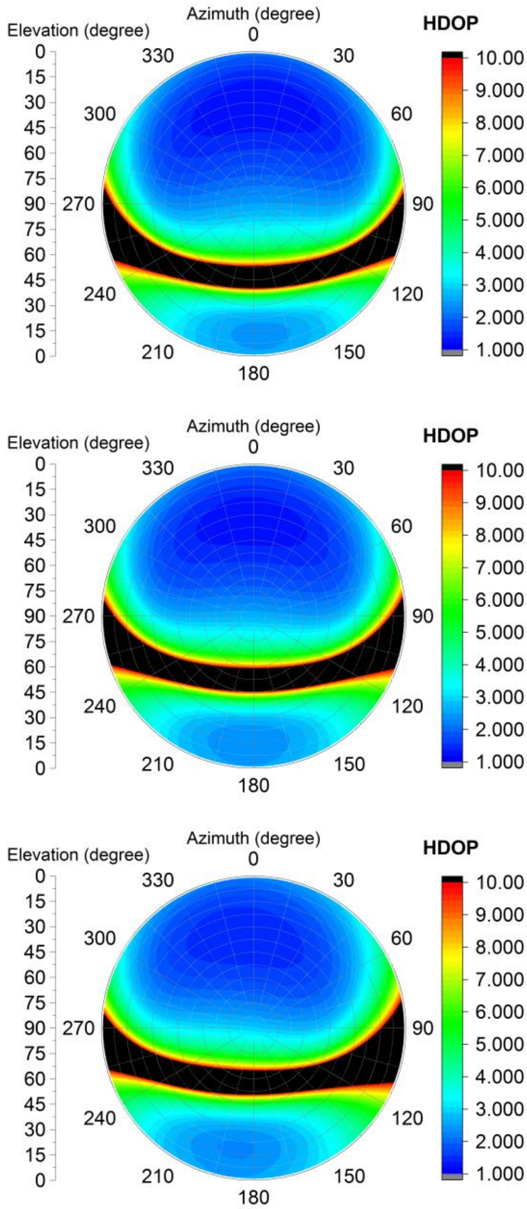

Figure 6: HDOP values for users at location A (above), B (middle), C (bottom), corresponding to different locations of the fourth reference point.

in either of the aforementioned regions, PDOP would be less than 4. This is less than the nominal value of GPS PDOP, which is 6 [15]. Considering the upper acceptable limit of PDOP to be 6, the two regions would even become more extensive. It should also be expressed that the sky plot analysis for the three locations shows almost similar result which is due to the fact that the positions of the three GEO satellites in the skyplot of all the three locations are in the vicinity of each other as already shown in Fig. 4. Furthermore, it is evident that the worst PDOP values correspond to the fourth point being positioned near the GEO satellites in the skyplot. The black strip in Fig. 5 shows the positions in the skyplot for which PDOP values exceed 10. It should also be mentioned that within this strip, PDOP values vary from 10 to infinity.

In Fig. 6 and 7, variation of HDOP and VDOP, with respect to the position of the fourth point in the skyplot is shown. As it is expected, variation of HDOP and VDOP parameters shows contrasting trend. Maximum value for HDOP is achieved with the fourth point being positioned at azimuth and elevation angles of around 330 to 30 deg and 0 to 30 deg, respectively. This is due to the fact that the layout of the three GEO satellites provides considerable coverage of the southern azimuth angles and therefore, the fourth reference point, needs to cover northern azimuth angles to gain lower HDOP values. In Fig. 6, it is observed that minimum values of HDOP are achieved when the fourth reference point is in the north-center section of the northern half of the skyplot. However, for the case of VDOP, as observed in Fig. 7, the minimum values are achieved while the fourth reference point is in the center of the skyplot. The black strip that provides unacceptable values is similarly seen in these shyplots. In positioning the fourth reference point, the black strip should be avoided as much as possible.

Designing hybrid GEO-LEO constellation

Following the skyplot analysis, it is intended to design a LEO satellite constellation to provide the fourth reference point. Conventional LEO constellations fall within two categories of Delta and Star type. Delta pattern includes satellites in orbit planes with considerable inclination. Maximum coverage provided by Delta constellations is near the latitudes that correspond to the inclination of the orbit planes [16]. As it is intended to provide coverage for Iranian territory, Delta pattern is chosen for the LEO constellation.

A LEO constellation is defined with several parameters including altitude and inclination of the satellites, the number of orbit planes, the number of satellites in the orbit plane and phase difference between adjacent orbit planes [16] and [17]. These parameters are related and the performance of a constellation depends on optimal selection of these parameters. As a starting point for the process of determining these parameters, it is suitable to consider the coverage provided by each individual satellite, then expand the analysis to one orbit plane, and finally, determine the features of orbit planes. This procedure is as follows.

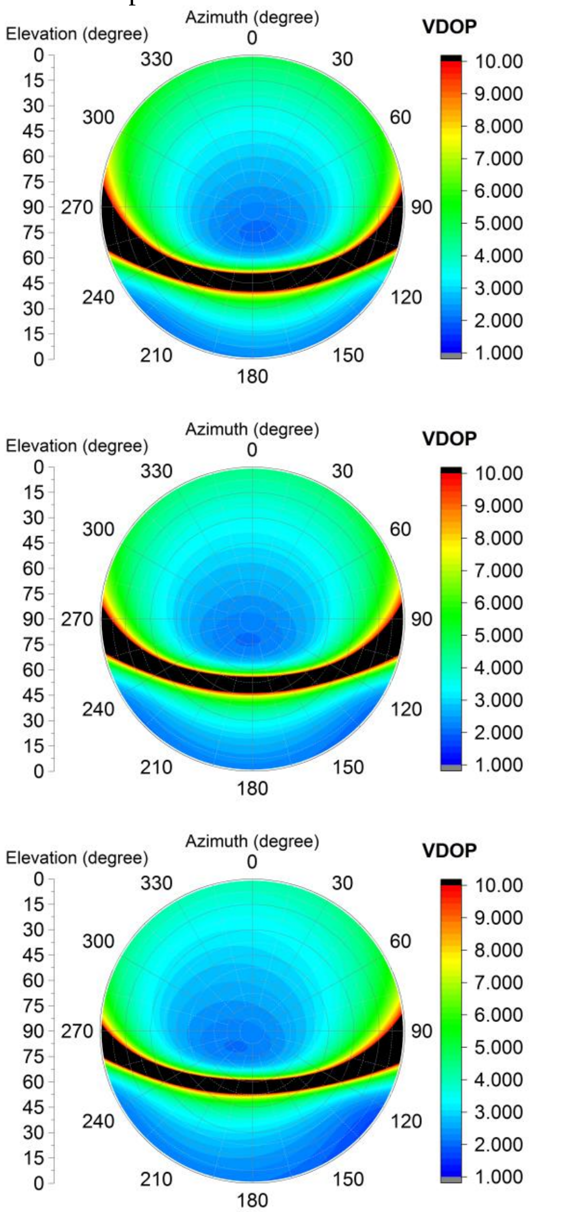

Figure 7: VDOP values for users at location A (above), B (middle), C (bottom), corresponding to different locations of the fourth reference point.

Coverage analysis

The coverage provided by a constellation is the aggregate of the covered regions provided by each individual satellite. The covered area by one satellite is called footprint, denoted by the angle θ, as shown in Fig. 8. This angle depends on altitude, h, minimum elevation angle of the line of sight, ε, and view angle of the satellite sensor, α. These parameters are related to each other in accordance with Eq. (8) and Eq. (9) [16] and [17]. It is obvious that the higher the altitude of the satellite, then the larger its footprint.

Depending on the design strategy, the satellites in an orbit plane can either provide continuous or discontinuous coverage. For continuous coverage, the footprint of successive satellites need to overlap each other to provide a strip called street-of-coverage, as shown in Fig. 9, within which the coverage is continuous. The width of the street-of-coverage, 2λ, is related to satellites separation angle, S, and satellites footprint, θ, as shown by Eq. 11.

Satellites separation angle equals 2π divided by the number of satellites in the orbit plane. Eq. (10), and 11 are based on the spherical geometry. It should be noted that the street-of-coverage, although shown as a flat surface in Fig. 9, is a spherical surface. The aforementioned equations can be used to determine the required number of satellites to be included in one orbit plane to form the street-of-coverage with the desired width. In Fig. 10, the footprint of one single satellite for different altitudes, considering minimum elevation angle of 10 deg, is shown. In Fig. 11, half-width of street-of-coverage, λ, vs. the number of satellites in the orbit plane for different altitudes is shown. This figure reveals that at every altitude, there is a minimum threshold for the number of satellites in an orbit plane to form the street-of-coverage. The results shown in Fig. 10 and 11 will be used in determining orbital parameters of LEO satellites in next section.

Determining satellites' orbital parameters

Previousely, it was shown that to minimize DOP values, the fourth reference point should be positioned in such a way that it appears in the northern hemisphere of the skyplot of Iranian users.

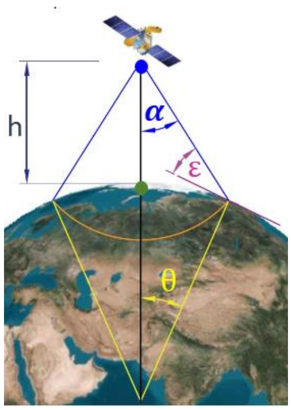

Figure 8: Coverage of a single satellite.

$$\cos(\theta + \varepsilon) = \cos\alpha \quad (8)$$

$$\theta = cos^{-1}\left(\frac{R_e \, cos\, \varepsilon}{R_e + h}\right) - \varepsilon \quad (9)$$

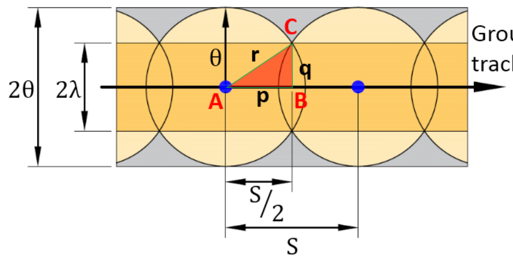

Figure 9: Street-of-coverage of a single orbit plane.

$$\cos q \cdot \cos p = \cos r \quad (10)$$

$$\cos\lambda \cdot \cos {}^{S}\!/_{2} = \cos\theta \quad (11)$$

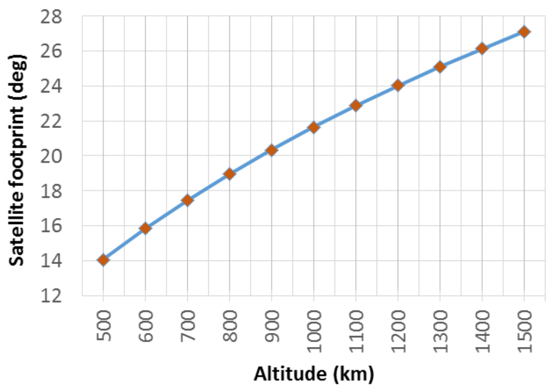

Figure 10: Satellite footprint, θ, vs. altitude.

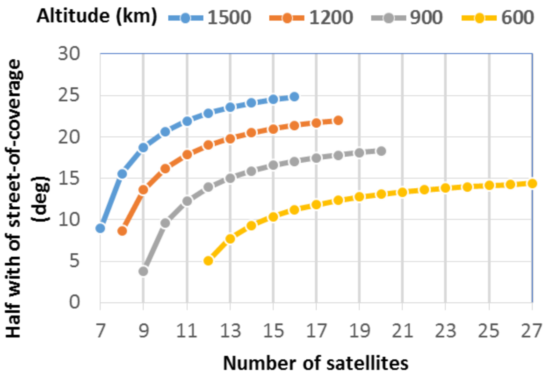

Figure 11: Street-of-coverage of a single orbit plane.

Therefore as a design approach, it seems suitable to adopt the parameters of the LEO planes in such a way that ground tracks of the satellites appear beyond the northern boundary of Iranian territory, in a manner that all the satellites appear in the northern hemisphere of the skyplot. In that case, provided that the satellites of the orbit planes form the street-of-coverage, and half-width of the street-of-coverage, λ, is greater than the extension of the Iranian territory in the lateral direction, then, all the users in Iran will be able to have at least one LEO satellite in their line of sight. However, this depends on the fact that there is an orbit plane in users' view with such ground track at every instant.

The lateral extension of Iranian territory equals 15 deg. Therefore, the altitude and number of satellites in the orbit plane should be determined in such a way that the half-width of the formed street-of-coverage is greater than 15 deg. By taking the altitude to be 1500 km, and the minimum elevation angle of 10 deg, the θ angle would be 27 deg, as shown in Fig. 10, which is greater than 15-deg threshold. Considering 10 satellites in the orbit plane, Eq. (11) determines the half-width of street-of-coverage to be 20 deg which is 33% greater than the lateral extension of Iran. Based on this layout, at least three orbit planes would be required to provide constant coverage. Lower altitudes require more satellites in each orbit plane to provide street-of-coverage with 20-degree half-width. At the altitude of 1200 km, 13 satellites are required while at 900 km, meeting such requirement is not possible. However, precise evaluation of the performance of the LEO constellation requires running a simulation.

Table 1: LEO constellation in each scenario

	inclination (degree)	Altitude (Km)	Number of orbit planes	Number of satellites in each plane	Total number of satellites
Scenario 1	45	1500	3	10	30
Scenario 2	45	1500	4	10	40
Scenario 3	50	1500	4	10	40
Scenario 4	45	1200	3	13	39
Scenario 5	45	1200	4	13	52
Scenario 6	40	900	3	20	60

Simulation of the satellite constellation performance

Six hybrid constellation scenarios, each including three GEO satellites, as previously introduced to be positioned at 0, 55 and 105 deg along with one of the LEO satellite constellation scenarios shown in Table 1, have been considered. In the first three scenarios, the LEO satellites are at the altitude of 1500 km and in the remaining scenarios, altitudes of 1200 and 900 km have been chosen. In scenarios 1, 2, 4 and 5 the inclination is 45 deg and in scenarios 3 and 6, the inclination is 50 and 40 deg respectively. The number of orbit planes is either 3 or 4 in all the scenarios. The number of satellites in each orbit plane corresponds to the minimum street-of-coverage half-width of 20 deg, except for the last scenario which meeting such requirement is not possible.

To evaluate the performance of the hybrid constellations, orbital trajectories of the satellites have been determined based on two-body equation of orbital motion. Corresponding differential equations have been discretized and solved by Runge-Kutta method. The effects of perturbing forces have been excluded in the simulation. The simulation has been performed considering 10-second time steps for a period of half a sidereal day. In each time step, DOP values for the region between longitudes of 43 to 64 deg and latitudes of 25 to 40 deg have been calculated and the results are shown in Fig. 12 to 18.

In Fig. 12, DOP values for scenario 1 are shown. It is observed that maximum values exceed the desired level for areas at latitudes of 33 deg. However, average values seem satisfactory. Contrasting behavior of HDOP and VDOP is evident in the

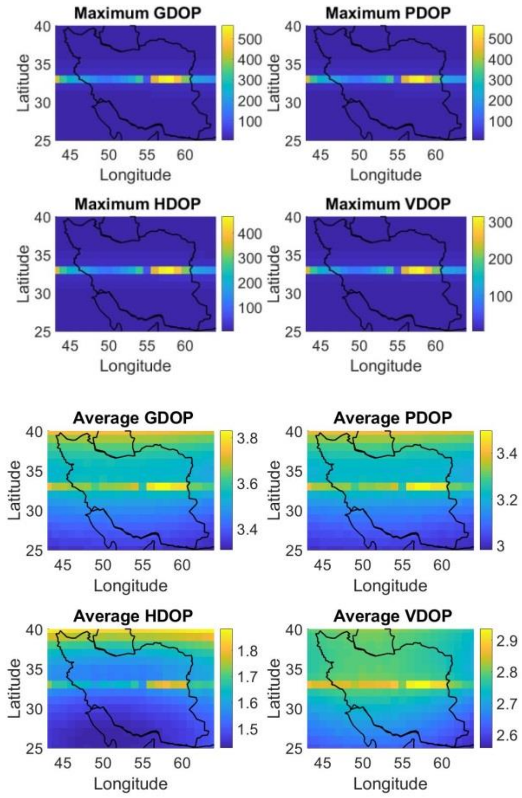

Figure 12: DOP values of scenario 1

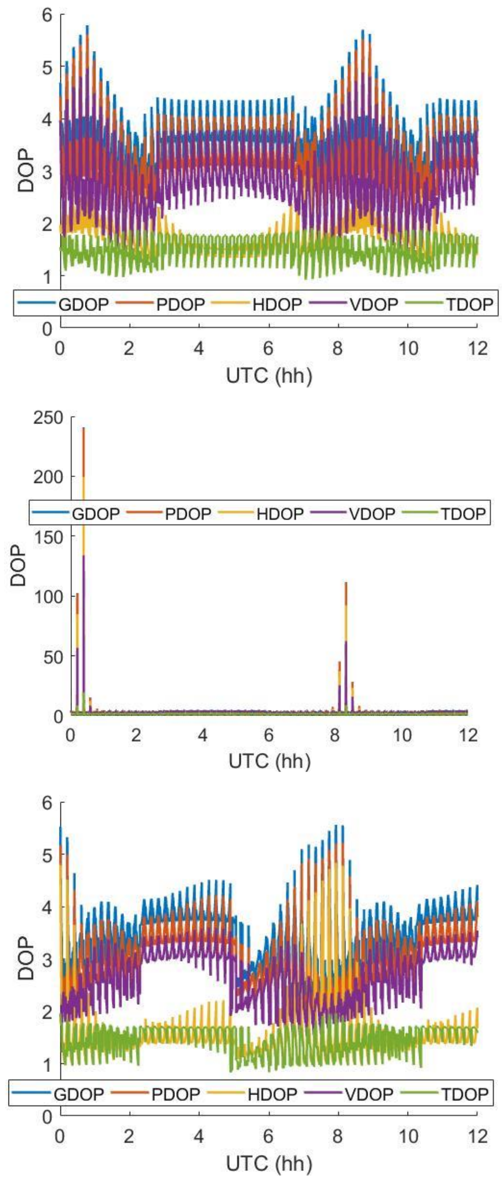

Figure 13: DOP values variation of scenario 1 at location A (above), B (middle) and C (bottom).

average values. To take a closer look at the performance in scenario 1, the variation of DOP values for locations A, B and C, as introduced previously, have also been calculated and are shown in Fig. 13. As expected, the variation of DOP values for the three locations are consistent with the results shown in Fig. 12. The abnormality in maximum values for locations at middle latitude of about 33 deg is evident in DOP values of point B where maximum values of DOPs reach 250 for short periods. However, it is noteworthy that the periods during which the sudden rise in DOP values occurs are much shorter in comparison with the whole simulation timespan. It is probable that fine-tuning the constellation parameters may remove the abnormalities. Specifically, as this scenario includes the lowest number of satellites, it would be extremely desirable to enhance the performance of the LEO constellation with 30 satellites to fulfill operational requirements.

In Fig. 14, DOP values for scenario 2 are shown and it is observed that the maximum DOP values are all below 5 which are highly desirable. It can be seen that DOP values for areas in the southern section of the region are lower than the ones in the northern section.

Fig. 15 demonstrates DOP values for scenario 3 in which it is observed that the results are similar to the previous scenario. However, the trend of variation of maximum values differs to some extent as it can be seen that maximum values are achieved in lower latitudes. The contrasting behavior of HDOP and VDOP values is also seen in this scenario.

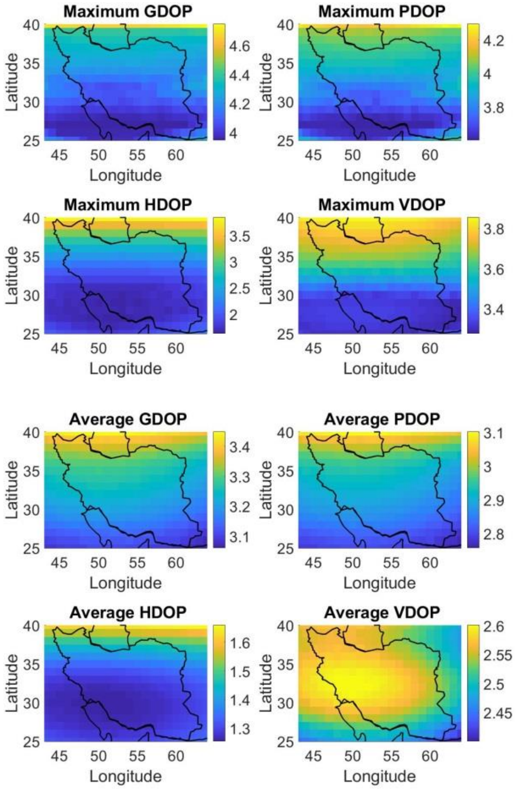

Figure 14: DOP values of scenario 2

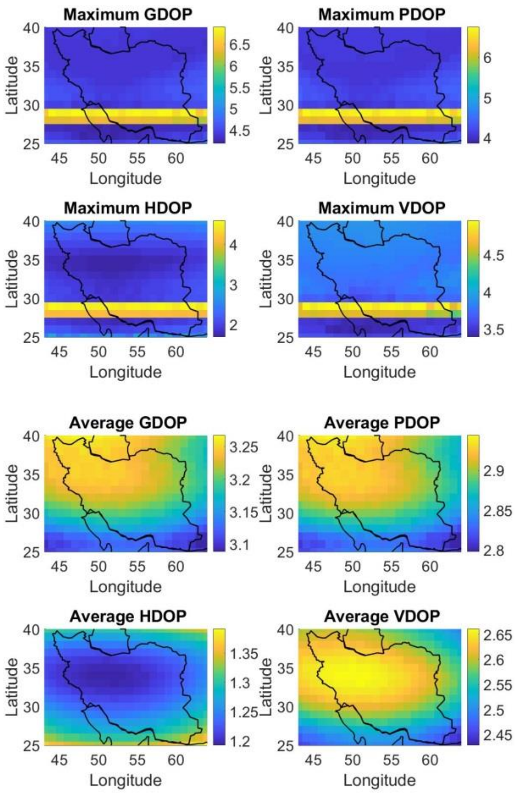

Figure 15: DOP values of scenario 3.

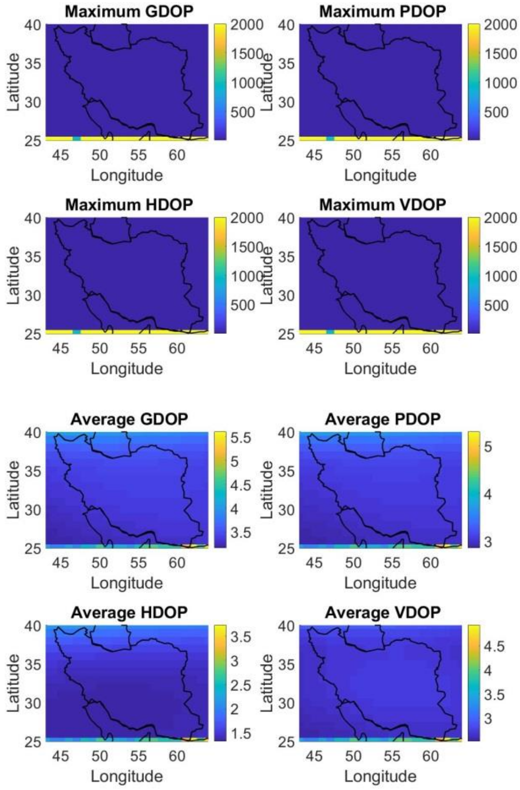

Figure 16: DOP values of scenario 4.

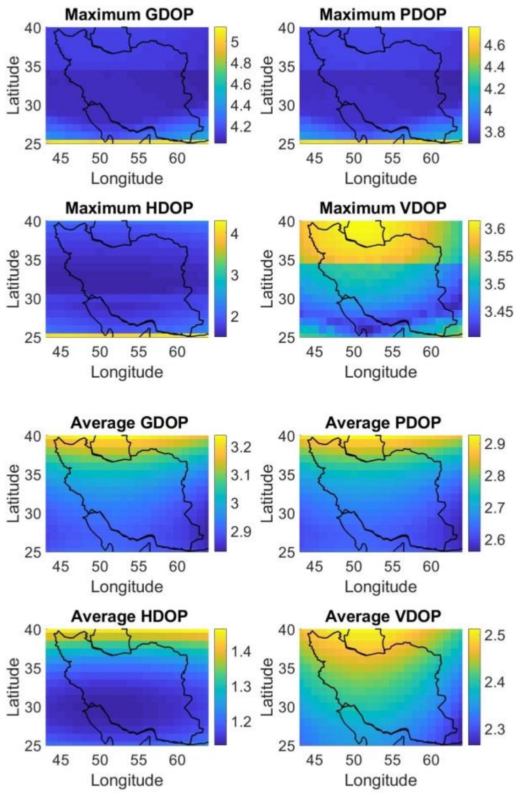

Figure 17: DOP values of scenario 5.

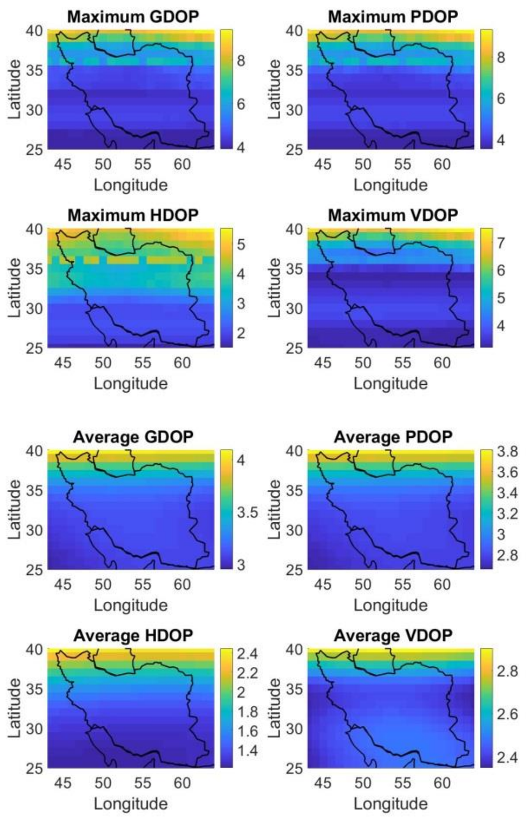

Figure 16: DOP values of scenario 6.

In Fig. 16, DOP values for scenario 4, in which the satellites are at the altitude of 1200 km are demonstrated. It can be seen than the maximum DOP values exceed the desired level in a narrow strip in the south of the intended region, while the average values are satisfactory. However, the results of scenario 5, in which the satellites are at the same altitude but in 4 orbit planes, as shown in Fig. 17, are highly satisfactory.

The last scenario, which includes 60 satellites, in 3 orbit planes, all at the altitude of 900 km, also demonstrates almost satisfactory performance. In Fig. 18, we see that maximum DOP values reach 9 for northern parts of the intended region. But for the rest of the region, maximum values do not exceed 6.

Considering the performance of the six scenarios, it can be concluded that the concept of hybrid GEO-LEO constellation for performing navigation mission is technically feasible by different combination of orbital parameters for the LEO satellites. However, in determining these parameters, many items such as satellite development and launch cost as well as other missions requirement should be considered. Among the orbital parameters, altitude can be regarded as having the pivotal role, on which other parameters are highly dependent.

Conclusion

In this paper, it was demonstrated that a hybrid constellation of GEO-LEO satellites can provide GNSS service for users in Iranian territory with satisfactory precision which can be comparable with currently available global services. Such combination of satellites would also be capable of providing communication services. Therefore, a multi-mission satellite constellation, which has been primarily designed for navigation due to its stringent requirement in comparison with other mission, can meet the performance requirement of other missions. However, the proposed patterns in this paper have been based on a limited range of orbital parameters of both GEO and LEO satellites. It is evident that further research is required for optimal determination of the position of the GEO satellites and parameters of the LEO satellites including altitude, inclination, number of orbit planes, total number of satellites and orbit plane phasing. Methods like genetic algorithm can be utilized for fine-tuning the parameters to optimize the design criterion.

In addition to technical parameters, economic factors in the form of constellation build-up and maintenance cost can also be included as the optimization criterion in the analysis. From cost-benefit perspective, the number of satellites and altitude are inversely related. At lower altitudes, more satellites would be required, which in turn increases the procurement cost. However, satellites designed to operate at lower altitudes would be cheaper and the cost of launch would be more reasonable. However, increasing the altitude, decreases the number of satellites on one hand and increases the associated expense of each satellite, due to subsystems boost-up to function at higher orbits, on the other hand. Needless to say, increasing the altitude would also affect other missions, if already incorporated into the constellation.

Altogether, optimal design of a hybrid satellite constellation is a multi-objective task which depending on the desired cost function, may lead to different patterns. The patterns demonstrated in this paper can be considered as a stepping-stone for further research and optimization.

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