

# A Novel Approach to Design Truss-Beams for Natural Frequency

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*Here, we present a novel two-step approach for optimum design of cellular truss-beams based on desired natural frequencies. The proposed approach attempts to decrease the design complexities, based on the so-called Axiomatic Design, before any effort to solve the physical problem itself. It also serves as a generic approach which finds its generality through dimensional analysis, its accuracy from finite element analysis and finally its optimality from the fact that it remains true for all similar frameworks. The applicability as well as the strength of the method is highlighted by some numerical examples.*

## INTRODUCTION

Truss-like beams are common for space applications due to their interesting performance [1]. In fact, most topology optimization studies often suggest truss-like solutions to replace beams [2]. In addition, the amount of research conducted on a special category of truss structures shows that the subject is still an interesting topic for research (such as tension [3], tensegrity [4] and hierarchical [5,6] trusses).

In the process of structural design, various requirements related to the dynamical behavior of a structure could be investigated. Nonetheless, in most cases, the dynamical behavior of a stationary structure is simply defined by its natural frequencies as well as mode shapes, especially the smallest one (associated with the first mode) which is simply referred to as *main natural frequency*. For large and lightly loaded structures, natural frequencies are more likely to be critical in controlling the overall design of the structure. For the sake of simplicity, the value of the first natural frequency ( $f$ ) may be regarded as a metric for identifying the structural rigidity; obviously, a zero or near zero value of  $f$  indicates that a kind of instability exists in the structure.

Design studies regarding vibration of truss struc-

tures vary from pre-design (based on the so-called *back-of-the-envelope* calculations [7]) to the utilization of optimization tools such as topology optimization for natural frequencies [8]. In addition, there have been some attempts to facilitate analysis of trusses. For example, adaptive generalized FEM is introduced for free vibration analysis of straight trusses in Ref. [9], global-local approach is proposed for large trusses in Ref. [10], and continuum modeling is proposed for trusses in Ref. [1]. The last case has been verified in Ref. [11]. Nevertheless, what we present in this work is different in approach and scope as it uses the philosophy of the so-called *Axiomatic Design* [12]. It is more accurate compared to that of *back-of-the-envelope* calculations and it is simpler as opposed to the topology optimization procedure for natural frequency.

In addition to the previous references, in the line of work, Ref. [13] presents a basic theory to determine whether an optimal truss-solution exists if we had to put constraints on natural frequencies. This work is capable of considering any arbitrary higher natural frequencies after the first one while not being concerned with local vibration of links as a bar. Moreover, Ref. [13] works on fixed topologies and materials only. On the other hand, while only constraint of the first natural frequency is of concern, our approach - by extremely lower complexity - is with comparably more efficient, as it covers local vibration. We also provide global sense through design charts (in contrast to intangible mathematical formulations). Our method also works as a tool for material selection and has the capacity to work with various topologies and materials.

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### Design Problem Definition

Here, the objective is to introduce an effective approach to design truss-beams for both minimum required natural frequency and minimum possible material weight, which are both very critical for space applications. The objectives are expressed as:

- Design:** a truss-beam,  
**To minimize:** structural total mass,  
**Subject to:**  $f \geq f_d$

where  $f_d$  is the desired natural frequency ( $f$ ) for the structure at hand.

The approach used here attempts to decrease the design complexity before any effort to solve the design problem. To this end, some relevant theoretical and design backgrounds are discussed, where the problem is stated in a non-dimensional form. Findings in non-dimensional terms are then expanded to FEM. It is shown that the analysis results obtained by link element type are the locus of optimum points where max.  $f$  occurs. On the other hand, a truss (of interested categories) may not exhibit a first natural frequency more than what was predicted by ideal link element. Moreover, this extreme value may be achieved by optimum design of the truss. This finding results in a *material and scale invariant* design graph which relates required natural frequency to possible optimum values of  $n$ . After predicting the possible values of  $n$ , it is an easy task to size and select the cross sections either analytically (by approximation) or numerically (by exact matching).

The proposed approach is further verified by numerical examples in both 2D and 3D space. Comparison of the results with an existing work shows the effectiveness of our approach. Finally, the entire design algorithm is re-investigated in the context of *Axiomatic Design* to clarify the design philosophy behind the proposed approach.

### THEORETICAL BACKGROUND

To give a general understanding of the process and to reduce the number of governing parameters, dimensional analysis is used and the vibration of uniform beams is then stated in the non-dimensional terms. The result is further extended to truss-beams using FEM.

#### Dimensional Analysis

Natural frequency ( $f$ ) of a typical beam has a functional dependency as:

$$f = f(L, E, \rho, shape, B.C.) \quad (1)$$

where  $L$  is the overall length of the beam,  $E$  is the modulus of elasticity,  $\rho$  is the material density, *shape* and *B.C.* are the collection of all non-dimensional

quantities that define the configuration as well as boundary conditions of the beam. Therefore, only four items remain to be converted to non-dimensional parameter(s):

$$f[T^{-1}], L[L], E[ML^{-1}T^{-2}], \rho[ML^{-3}]$$

Here, T, L and M represent the basic dimensions for Time, Length and Mass, respectively. According to Buckingham's theorem, there is only one ( $4 - 3 = 1$ )  $\pi$ -term (non-dimensional parameter) which could describe all four parameters. Let  $\mathcal{F}$  denote this  $\pi$ -term which we refer to as '*non-dimensional natural frequency*'.  $\mathcal{F}$  is found to be:

$$\mathcal{F} = \frac{2\pi Lf}{\sqrt{E/\rho}} \quad (2)$$

In Eq. (2), coefficient  $2\pi$  is added for convenience,  $\omega = 2\pi f$ , with,  $\omega$  being the circular natural frequency. Therefore, Eq. (1) is reduced to:

$$\mathcal{F} = \mathcal{F}(shape, B.C.) \quad (3)$$

One could conclude that  $f$  is proportional to  $\sqrt{E/\rho}/L$ . This fact will be used throughout this work while developing some design graphs.

Total structural mass ( $m$ ) is another important state variable which may serve either as a design objective or a constraint. For a typical beam;

$$m = m(L, \rho, shape) \quad (4)$$

In Eq. (4),  $m$ ,  $L$  and  $\rho$  are based on two basic dimensions (M and L). Therefore, only one ( $3 - 2 = 1$ )  $\pi$ -term could describe all of these parameters. Let  $\mathcal{M}$  denote this  $\pi$ -term and let us refer to it as '*non-dimensional mass*'.  $\mathcal{M}$  is found to be:

$$\mathcal{M} = \frac{m}{\rho L^3} \quad (5)$$

Now, Eq. (4) is reduced to:

$$\mathcal{M} = \mathcal{M}(shape) \quad (6)$$

The next section shows how we can use the introduced equations for a uniform beam.

#### A uniform beam

Natural frequency ( $f$ ) of a uniform beam follows from:

$$f = \frac{c}{2\pi} \sqrt{\frac{EI}{\rho AL^4}} \quad (7)$$

where  $I$  is the second modulus of area, and  $c$  is a non-dimensional constant which represents the effect of boundary conditions. As an example,  $c =$

3.52,  $\pi^2$ , 15.42 and 22.37 are for clamped-free, pinned-pinned, clamped-pinned, and clamped-clamped B.C.'s, respectively [14].

Eq. (7) would be rewritten as:

$$\frac{2\pi Lf}{\sqrt{E/\rho}} = c\sqrt{\left(\frac{I}{A^2}\right)\left(\frac{A}{L^2}\right)} \quad (8)$$

Considering Eqs. (2) and (3) implies that;

$$\mathcal{F} = \mathcal{F}(\text{shape}, B.C.) = \underbrace{c}_{B.C.} \underbrace{\sqrt{\iota A}}_{\text{shape}} \quad (9)$$

where  $\iota = I/A^2$  and  $A = A/L^2$  are non-dimensional forms of  $I$  and  $A$ . As an immediate conclusion,  $\iota$  must be increased to its largest possible value (upper limit is restricted with local phenomena).

One could also show that:

$$\mathcal{M} = \frac{m}{\rho L^3} = \frac{\rho AL}{\rho L^3} = \frac{A}{L^2} = \mathcal{A} \quad (10)$$

According to Eq. (9), the value of  $\iota$  remains independent of scale and is, in fact, only a function of shape. Figure 1 shows values of  $\iota$  for all possible prisms with polygonal cross sections. When higher values of  $\iota$  are required, a hollow polygon would be the choice. To identify a hollow polygon, let us define  $d_m/t$  as thickness ratio where

$$d_m = \frac{d_o + d_i}{2} \quad \text{and} \quad t = \frac{d_o - d_i}{2} \quad (11)$$

in which  $d_i$  and  $d_o$  are reference lengths of the inner and outer boundaries (*i.e.* inner and outer diameters or side lengths). Using these parameters, higher possible values of  $\iota$  will be explored according to Figure 2, which provides a glance on possible values of  $\iota$  for a single bar.

### Evaluation of $f$ Using FEM

Eq. (9) is useful only for a single bar. To evaluate  $f$  for a full truss structure, utilization of finite element method (FEM) is a common choice with  $[\mathbf{K}]$  and  $[\mathbf{M}]$  being the stiffness and mass matrix of the structure and  $f$  the smallest eigenvalue of:

$$|-\omega^2 [\mathbf{M}] + [\mathbf{K}]| = 0 \quad (12)$$

where  $\omega = 2\pi f$ .

With Eq. (2), it is possible to rewrite Eq. (12) as follows:

$$|-\mathcal{F}^2 [\bar{\mathbf{M}}] + [\bar{\mathbf{K}}]| = 0 \quad (13)$$

Here,  $\bar{\mathbf{K}}$  and  $\bar{\mathbf{M}}$  are independent of  $E$ ,  $\rho$  and the overall beam length ( $L$ ). It should be quite clear that all complexities regarding diversity in materials and lengths will be included in  $\bar{\mathbf{M}}$  and  $\bar{\mathbf{K}}$  via non-dimensional forms of  $l_i/L$ ,  $\rho_i/\rho$ ,  $E_i/E$ , and others as well.

We might add that it is possible to solve the problem with every parameter set to a unit value (for example:  $\rho=1000$ ,  $E=1e11$  and  $L=1$ ) and then convert the results to a non-dimensional form using Eqs. (2) and (5). In that case, final results would be invariant of initial values. Nonetheless, in such an approach, one might not be able to use his engineering judgments and feelings to evaluate the results or to prevent any numerical problems. However, we might add that in this work both link and beam element types, for which the specifications are given in Table 1 are of primary concern.

In summary, link elements are pin-jointed to one-another while beam elements are clamped together. Moreover, beam elements are based on Euler formulation and are capable of modeling axial strains. In addition, to provide a complete representation, torsion DOF could be added to the formula; nevertheless, here, it is not of primary concern.

In the next section, we discuss some background materials which are important to understanding the process.

## DESIGN BACKGROUND MATERIAL

### Modeling Aspects

Here, we are primarily interested in a type of truss beams which are composed of some identical cellular elements. We might freely refer to them as unit cells.

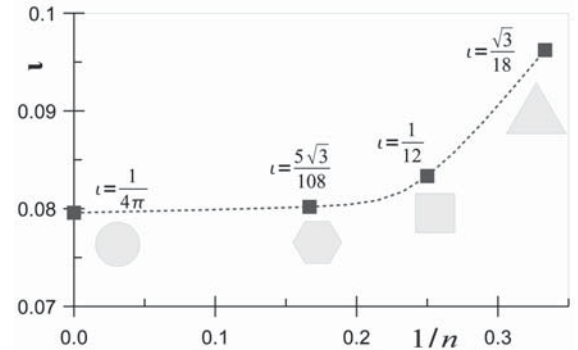


Figure 1. Value of  $\iota$  for polygons ( $n$ : number of sides).

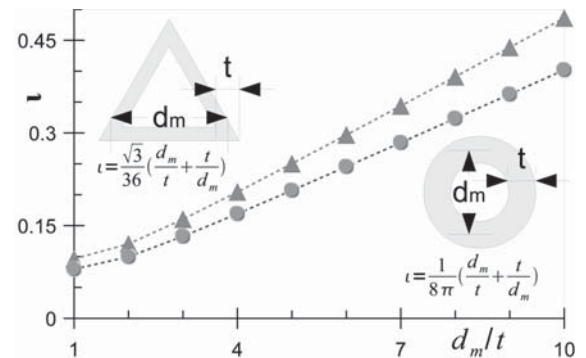


Figure 2. Value of  $\iota$  for hollow polygons.

Each unit cell is a square in the 2D space or a regular prism in the 3D space. The cells are stacked only along the beam length (Figure 3).

Here, these truss-beams are stated by two parameters, namely number of cells ( $n$ ), and cross sectional areal of individual bars ( $A$ ). Therefore, to design a truss-beam, one simply needs to find the appropriate values for  $n$  and  $A$ . Nonetheless, we still need to make some assumptions to facilitate the process for a Finite Element Model (FEM)

### Assumptions

A geometric model of a truss geometrically consists of some lines or links and joints. For the associated FE model, it is necessary to assign appropriate material, element type, and cross sectional area to the lines. To make the model invariant to any material change, we assume that the material remains linear isotropic and all deformations are small throughout the process.

We notice that dimensional analysis shows that the shape of a cross section may be kept independent of its area. That is, the shape may be selected using Figure 2. This approach helps us keep the results independent of the actual cross sectional shape. We further assume that the sections are locally stable and any local instability is effectively prevented. Now, what remains to be selected is an appropriate element type.

### Hybrid Method for Analysis

As a quite general approach to a truss structure, the candidate element types are links and beams. The link element represented ideally carries axial forces only. The ideal hinges would not allow any shear or bending moments. However, such an element is not able to catch any lateral vibration of individual members. On the other hand, beam element represents actual conditions more realistically; however, they require more computational time and effort.

Here, we compare the results for a given structure. Figure 3 shows a typical family of trusses constructed from solid rods ( $\iota = 1/(4\pi)$ ). Figure 4 represents the results obtained while using links or beams element types for some values of  $n$  and various values of material volume. Here, cross sectional areas of rods are so adjusted to set  $\mathcal{M}$  equal to a desired constant value. It is noted that Figure 4 is invariant of  $\rho$ ,  $E$  and  $L$ . The interesting outline of this figure is that the resulting

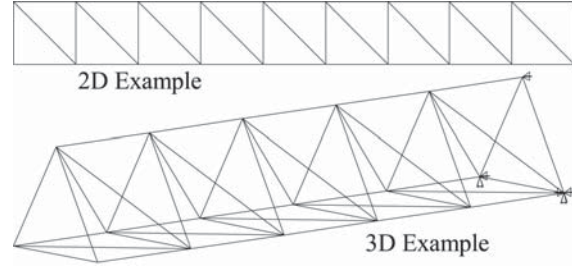


Figure 3. Samples of intended truss-beams

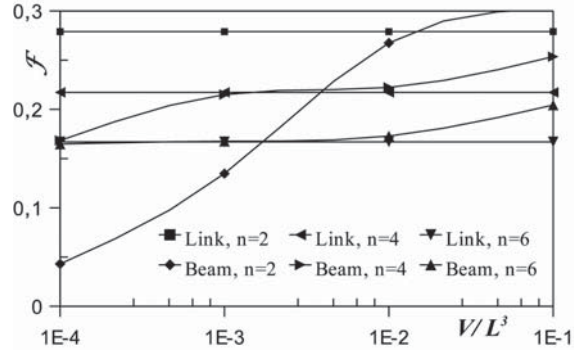


Figure 4. Results of analysis using link and beam element types in various  $V$  and  $n$ .

values of  $f$  are independent of the volume when link element type is utilized. In addition, there is a value of  $n$  (referred to as  $n_{min}$ ) above which the predicted  $\mathcal{F}$  by link element type is equal or greater than the calculated  $\mathcal{F}$  by beam element type which is supposed to be more realistic ( $n = 6$  in Figure 4). This phenomenon, which is based on the same family of trusses and will be discussed subsequently, is also seen in Figure 5.

Figure 5 shows some more interesting features of truss structures. This figure serves as a matching diagram in  $n - \mathcal{F}$  space for three approaches of calculation as follows:

1. Beam element solution (almost exact solution -  $f$ ),
2. Link element solution ( $f_L$ ),
3. Analytical solution based on Eq. (9) for the longest bar in the truss, which are diagonal elements, with pinned-pinned ends condition ( $f_A$ ).

With Figure 5, there is a value of  $n$  ( $n_{min}$ ), above which the link element solutions are acceptable. Interestingly, the region for this transition is located where the maximum natural frequency for the truss occurs. It is more exciting when one finds out that the third graph ( $f_A$ ) also passes from the same region. This finding is, in fact, the cornerstone of the design procedure proposed in this work and discussed in the next section.

Table 1. Specification of utilized element types.

	Link Ele.	Beam Ele.
Number of node	2	2
DoF (2D)	$U_x, U_y$	$U_x, U_y, \Theta_z$
DoF (3D)	$U_x, U_y, U_z$	$U_x, U_y, U_z,$ $\Theta_x, \Theta_y, \Theta_z$
Shape Function	Linear	Hermit (bending), linear (other)

$U$ : displacement DoF,  $\Theta$  rotational DoF,  
(x,y): 2D plane, z: third axis.

### DESIGN PROCEDURE

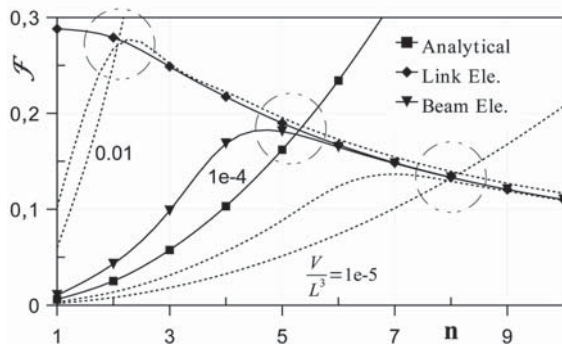
The design objective, as previously outlined, is to find  $n$  and  $A$  to have a truss with  $f \geq f_d$  while the material consumption is kept to a minimum possible. We follow this discussion with these steps:

1. Select the value of  $n$  using the provided design chart of Figure 6. We further discuss the chart in the next section,
2. Calculate the appropriate value of  $A$  using Eq. (9) or an equivalent numerical procedure.

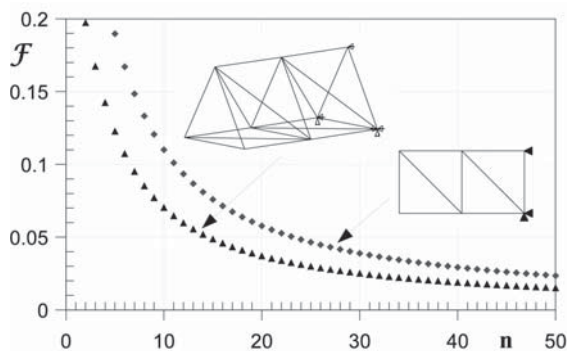
Different case-studies conducted by the authors show that these steps inherently result in a near optimum design point. This, in fact, lies in the approach we use to develop the design chart.

### Design Chart

As stated before, the natural frequency obtained by link element is almost invariant of structural total volume. In addition, there is a value of  $n$  above which the obtained results by the link elements are dependably sound. In fact, with Figure 5, for  $n < 5$ , the beam element shows that the local frequencies dominate the design of the structure while for  $n > 5$ , the resulting truss is so stiff that global frequencies dominate regardless of the type of the elements. It is noted that the transition point of  $n = 5$  is where the



**Figure 5.** Correlation of analysis using link and beam elements and what predicted analytically for single element.



**Figure 6.** Design Graph.

maximum (here, optimum) natural frequency occurs. On the other hand,  $n = 5$  suggests that the resulting structure with link element type is the locus of optimum points. This observation makes it possible to predict an optimum value for  $n$ .

Figure 6 shows values of  $\mathcal{F}$  versus  $n$  while using link element type for two design patterns of Figure 3. Again, the graph is invariant of material properties and structural total volume (*i.e.*  $A$ ), and elements cross-sections. Obviously, for each desired value of  $\mathcal{F}$ , optimum value of  $n$  could directly be obtained from the provided graphs. Each truss pattern will, of course, have its own design chart. It is noted that whence  $\mathcal{F}_d$  falls outside of the normal range (that is,  $\mathcal{F}_d \geq \mathcal{F}_{n=1}$ ), the chart proposes using a material with relatively better performance, that is, a material with a higher value of  $\sqrt{E/\rho}$ . This approach, in fact, lowers the  $\mathcal{F}_d$  to a value less than  $\mathcal{F}_{n=1}$  which, in turn, makes  $n \geq 1$  possible.

After selecting the optimal  $n$ , it is an easy task to size  $A$  by the existing analytical means, such as Eq. (9), or numerically as is discussed subsequently. In brief, for each value of  $n$ , there is a path in  $\mathcal{F} - \mathcal{M}$  space which describes possible  $\mathcal{F}$  for any available  $\mathcal{M}$ . The desired point in this space is where  $\mathcal{F}(\mathcal{M})$  equals  $\mathcal{F}_d$ . On the other hand,  $\mathcal{M}$  is a function of  $A$ . Therefore, required  $A$  is the root of  $\mathcal{F}(\mathcal{M}(A)) - \mathcal{F}_d$ . Here,  $\mathcal{M}(A)$  is known analytically, while  $\mathcal{F}(\mathcal{M})$  is known numerically and the equation given by  $\mathcal{F}(\mathcal{M}(A)) - \mathcal{F}_d = 0$  may be solved by any available numerical procedures.

### NUMERICAL EXAMPLES

To show the effectiveness of the proposed approach, we seek to design a 100 m cantilever antenna beam the structure of which is expected to have at least 1 Hz as its first natural frequency, while having a minimum possible total weight. We further assume a typical aluminum alloy with  $E = 70$  GPa and  $\rho = 2700$  kg/m<sup>3</sup> is used.

For this design problem,

$$\mathcal{F}_d = \frac{2\pi L f_d}{\sqrt{E/\rho}} = 0.1234$$

and we desire to find both 2D and 3D solutions.

#### 2D Solution

We, first, consider the problem in a 2D space. Based on Figure 6, an optimum number of cells is  $n = 8$  and the value of the cross-section ( $A$ ) follows from:

$$f_A = \frac{c}{2\pi} \sqrt{\frac{EI}{\rho A l_A^4}} \Rightarrow A = \left( \frac{2\pi f_A l_A^2}{c} \right)^2 \left( \frac{\rho}{E} \right)$$

On the other hand,  $l_A = \sqrt{2} L/n$  and  $c = \pi^2$  for pinned end conditions for individual bars. Selecting solid rods with  $\iota = 1/(4\pi)$  leads to  $A = .019$  m<sup>2</sup>.

Evaluating numerically in an almost exact manner with a beam element type, gives a natural frequency of 1.04 Hz which is in excellent agreement with the desired target.

The total structural mass of the resulting truss structure amounts to:

$$m = (3 + \sqrt{2} + \frac{1}{n})\rho AL = 23511 \text{ kg}$$

The numerical approach, however, leads to:

$$A = 0.011 \Rightarrow \text{mass} = 13703 \text{ kg}$$

This represents a structure with exactly 1 Hz natural frequency.

Comparison of the results suggests that a small increase in the first mode natural frequency by only 4% ( $\frac{1.04-1}{1}$ ) could result in a significant increase in the total structural mass up to 72% ( $\frac{23511-13703}{13703}$ ). Therefore, for final sizing of  $A$ , numerical adjustment using the analytical result as a starting point is necessary.

We still need to show that the resulting values are near-optimal enough. For this, we use Figure 7 which is constructed on the basis of careful evaluations. In brief, correlation between  $\mathcal{F}$  and  $\mathcal{M}$  is plotted for three values of  $n$ . The intersecting point of these paths with horizontal line of  $\mathcal{F} = \mathcal{F}_d$  is the exact matching point. For comparison, the result of approximate sizing (using Eq. (9)) is also mapped to space .

As it is seen,  $n = 9$  does not satisfy the requirement of  $f \geq f_d$ . But, if the designer accepts a slightly lower value for  $f_d$ , it would give a light design with lower sensitivity to the material volume. Truss-beams of  $n = 7$  will result in heavier structures, however, if some higher values of  $f$  are required, the optimum value of  $n$  would be switched back to 7. Therefore,  $n = 8$  would be the exact optimum design. Now, what remains is to size  $A$  to achieve the exact optimum point.

### 3D Solution

Now, let us consider the same problem using the 3D truss pattern of Figure 6 using rods. Figure 6 suggests  $n = 4$  corresponds to the optimum design and results in an *analytical approximation* of:

$$A = 0.307 \text{ m}^2 \quad (\text{let } l_A = \sqrt{2}L/n)$$

$$\Rightarrow \text{mass} = (6 + 3\sqrt{2} + 3/n)\rho AL = 9.11e5 \text{ kg}$$

$$\Rightarrow f = 1.05 \text{ Hz}$$

*Almost Exact* values based on numerical procedure, however, are:

$$A = 0.220 \text{ m}^2, \quad \text{mass} = 6.53e5 \text{ kg}, \quad f = 1 \text{ Hz},$$

These values are in line with that of 2D design but with significantly more total mass which is logical. It

is noted that the number of cells in a 3D truss is lower with respect to that of a 2D design. This means a 3D truss is more influenced by global effects.

Similar to the 2D design, Figure 8 is constructed based on almost exact evaluations which helps us investigate the neighborhood of the obtained design. As it is seen,  $n = 3$  clearly violates the requirement for  $f = 1$  Hz. However,  $n = 5$  approaches the desired  $f$  but it is not capable of reaching  $f = 1$  Hz with a reasonable mass. Therefore,  $n = 4$  remains to be the optimum design.

It would be interesting to use tubular elements instead of solid rods while examining the results. In such a case, there would be a significant reduction in the total structural mass without any change in  $n$ . For example, considering a tube with  $d_m/t = 10$  results in a mass ratio of:

$$\frac{m_{tube}}{m_{rod}} = \frac{l_{rod}}{l_{tube}} = 2\left(\frac{d_m}{t} + \frac{t}{d_m}\right)^{-1} = 0.198$$

which says tubular truss is five times lighter while providing the same first natural frequency.

### Comparison with an Existing Result

In Ref. [7], the natural frequency of a truss supported segmented reflector is analyzed based on the so-called *back-of-the-envelope* calculation. The final result of the

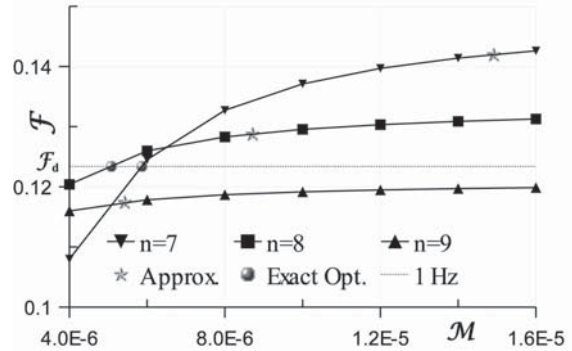


Figure 7. Neighborhood of design point for 2D example.

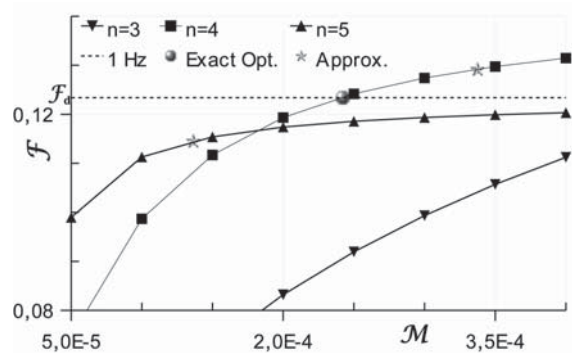


Figure 8. Neighborhood of design point for 3D example.

work gives:

$$f = \frac{0.852}{D}(h/D)\sqrt{\eta E/\rho} \quad (14)$$

where  $D$  is the reflector diameter,  $h$  is its depth (see Figure 9), and  $\eta$  is the ratio of truss (structural) mass to the antenna total mass. This equation may be rewritten as:

$$\mathcal{F} = \frac{2\pi f D}{\sqrt{E/\rho}} = \frac{5.35c_h\sqrt{\eta}}{n} \quad (15)$$

Here,  $h = c_h D/n$  is assumed. Although this approximate equation is only valid for a special case, it shows  $\mathcal{F} = \mathcal{F}(n)$ , which is in agreement with the findings in the current research work.

Eq. (15) encourages us to re-plot Figure 6 with  $\mathcal{F}$  vs.  $1/n$ . The result is shown in Figure 10 which implies that for big values of  $n$ , a linear relationship between  $\mathcal{F}$  and  $1/n$  exists.

### PHILOSOPHIZING THE DESIGN PROCESS BASED ON AD

The design problem, as formulated in this work, revolves around two design parameters of (1)  $n$  and (2)  $A$ . These parameters are used to satisfy two frequency requirements related to (1) FR1 for local natural frequency and (2) FR2 for global natural frequency. Both requirements are affected by design parameters:

$$\begin{bmatrix} f_{Global} \\ f_{Local} \end{bmatrix} = \begin{bmatrix} X_{1,1} & X_{1,2} \\ X_{2,1} & X_{2,2} \end{bmatrix} \begin{bmatrix} n \\ A \end{bmatrix} \quad (16)$$

In Eq. (16), a  $X_{i,j}$  represents a relationship between contributing factors. This implies that the design process is coupled based on the so-called *Axiomatic Design* (AD) [12] and, therefore, hard to solve.

In the current work, we use a different approach, which results in a slightly different design matrix:

$$\begin{bmatrix} \mathcal{F}_{Global} \\ \mathcal{F}_{Local} \end{bmatrix} = \begin{bmatrix} X_{1,1} & 0 \\ X_{2,1} & X_{2,2} \end{bmatrix} \begin{bmatrix} n \\ A/L^2 \end{bmatrix} \quad (17)$$

where the new design matrix is de-coupled as  $X_{1,2} = 0$ . This approach reduces the complexity of the problem at hand [15] and, in turn, means by the design problem could be tackled in two steps using the proposed approach. This is the cornerstone of the Axiomatic Design that examines a design problem and how it is modeled before any attempt to solve the problem [16] and, in this work, we have been able to use the AD approach to design a space structure based on its desired natural frequencies.

### CONCLUSION

Truss-beams are still an interesting topic of research, especially in space applications. Satisfying a minimum required first natural frequency is, in fact, a critical design task for space applications as there is very little if no damping in space environment and we have been able to devise an effective engineering tool for such a task.

The existing practice attempts to solve the problem in its coupled condition and, therefore, requires a great deal of trial and errors, which becomes, very time consuming task and there is no guarantee to give a solution at all. In a mathematical sense, one might recall that any system of equations of the form of Eq. (16) would not necessarily provide a solution. In this work, however, we have been able to convert the system matrix to a lower-triangle one (Eq. (17)). This approach guarantees a solution. In fact, we decrease the design complexity by systematic and proper assumptions. Nonetheless, the proposed process of this work is only useful in cases where the

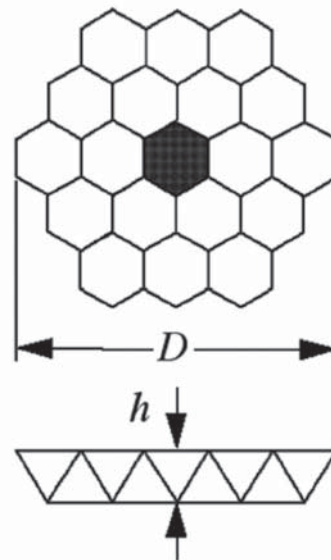


Figure 9. Truss-supported segmented reflector [7].

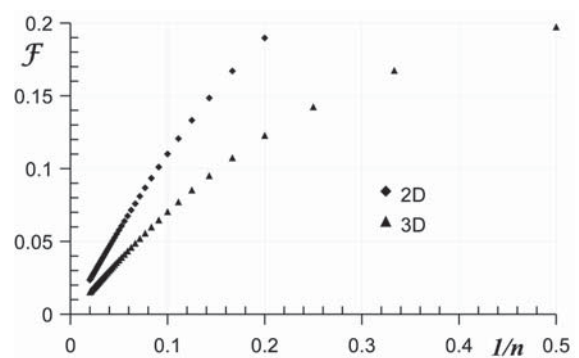


Figure 10. Design graph -  $\mathcal{F}$  vs  $1/n$ .

first natural frequency is of prime importance. For cases where other modes become important, we need some innovative approaches to tackle the problem. Obviously, in most practical applications, the first natural frequency is very important and, therefore, this work remains an efficient design tool.

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