

Dynamics of Space Free-Flying Robots with Flexible Appendages

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A Space Free-Flying Robot (SFFR) includes an actuated base equipped with one or more manipulators to perform on-orbit missions. Distinct from fixed-based manipulators, the spacecraft (base) of a SFFR responds to dynamic reaction forces due to manipulator motions. In order to control such a system, it is essential to consider the dynamic coupling between the manipulators and the base. Explicit dynamics modeling of such systems with flexible appendages is developed in this paper. The SFFR is divided into two parts, the manipulator(s), and the main base body (spacecraft) that consists of flexible appendages. The recursive Lagrangian approach is used to describe dynamics model of the flexible base system. For modeling the multi-manipulator system, a Recursive Newton-Euler approach is followed. The obtained dynamics model can be employed either numerically or symbolically. Interacting forces and torques acting between the manipulators and the main body are also modeled, which could be used for simulation studies of controller design.

NOMENCLATURE

n_b	Number of total appendages	$\theta_{x_i}^{(l)}, \theta_{y_i}^{(l)}, \theta_{z_i}^{(l)}$	The $x_i^{(l)}, y_i^{(l)}, z_i^{(l)}$ rotation components of link i , of appendage l
k	Vibration in the x, y , or θ_z direction	$\dot{\theta}_p^{(l)}$	Joint angle rate of the l^{th} single arm manipulator corresponding to unit vector $\hat{i}, \hat{j}, \hat{k}$
$m_{ik}^{(l)}$	Number of assumed modes for the k^{th} direction of vibration of the i^{th} link of l^{th} appendage	$\mathbf{F}_{IF}, \boldsymbol{\tau}_{IF}$	Interaction forces and torques between the rigid manipulators and their flexible base
$r_i^{(l)}$	Position of an arbitrary point on each link	M, J, C, K	Mass, mass moment of inertia, translational damping and translational stiffness matrices
n_l	Number of l^{th} appendage links	\mathbf{Q}	Vector of generalized forces and torques applied to the flexible base
$L_i^{(l)}$	The length of link i of l^{th} appendage	\mathbf{a}_{c_0}	Acceleration vector of the base
ρ	Mass density	$\boldsymbol{\omega}_1^{(l)}$	Angular velocity vector of the l^{th} single arm manipulator
A	Cross sectional area	$\mathbf{r}_{c_1}^{(l)}$	Position vector from $O_0^{(l)}$ to the center of mass of the l^{th} single arm manipulator
E	Modulus of elasticity	δ_0	Coordinates governing the rotational motion of the base
I_x, I_y	Area moment of inertia computed about the x or y axis	$\dot{\delta}_0$	Angular acceleration vector of the base due to flexibility
G	Modulus of rigidity	δ_{ps}	Kronecker delta
J	Polar area moment computed about the neutral axis		

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$I_1^{(l)}$	Moment of inertia matrix of the l^{th} single arm manipulator
\mathbf{q}_m	Rigid manipulator variable
\mathbf{q}_b	Flexible base generalized coordinates
$B_f, B_{\tau 0}$	Controllable rigid manipulator inertia effects
$C_f, C_{\tau 0}$	Inertia effects of the flexible base
r_{c0}	Coordinates governing the translational motion of the flexible base
N_{Rf}, N_{Cf}	Rigid manipulator coriolis and centrifugal interaction force matrices
$N_{R\tau f}, N_{C\tau f}$	Rigid manipulator coriolis and torque matrices

INTRODUCTION

Dynamics and control of a Space Free-Flying Robot (SFFR) is a highly challenging subject because of complicated nonlinear dynamics and dynamics coupling between different parts, [1-3]. Papadopoulos and Dubowsky have employed a barycentric vector approach to study kinematics and dynamics of a single-arm rigid SFFR in free-floating mode, [4]. Taking the center of mass of the whole system as a representative point for the translational motion and using barycentric vectors, which reflect both geometric configuration and mass distribution of the system, results in decoupling the total linear and angular motion from the rest of the equations. This approach was compared to another one proposed as Direct Path Method in [5]. It was shown that this new approach results in a larger number of dynamics equations with simpler terms with clearer physical meaning. It was also applied by Moosavian and Papadopoulos, [6], to develop explicit dynamics of a multiple manipulator space free-flying robot SFFR with rigid Links based on Lagrangian formulation. This model has been successfully employed for model-based control, and simulation studies of such complicated systems, [7-9].

Yoshida *et al.* have studied the problem of impact dynamics of space robotic systems that consist of a rigid manipulator supported by a flexible deployable structure, [10]. Dynamics and control of such space robotic systems in the presence of joint flexibility and with closed kinematic constraints has been developed in [11]. Ma and Wang have introduced a technique for impact-contact dynamics simulations of flexible manipulators by order reduction, [12]. The proposed method first linearizes the contact force model on the right-hand side of the dynamics equations periodically, and then determines the linear “stiffness” and “damping” terms from the linearized contact force model. Finally, these are combined with the existing structural stiffness and damping matrices of the associated multibody system on the left-hand side of the equations. After such a process, the traditional modal analysis and

reduction techniques for linear dynamic systems can be applied to reduce the order of the resulting dynamic system. Martin *et al.* have approximated the dynamics of a flexible joint manipulator on a free-flying base, [13]. This model has been employed to minimize undesirable dynamics and fuel consumption. Similar approaches have been proposed for optimal and near-optimal control of flexible spacecraft, [14-18].

Book has developed a recursive Lagrangian approach for modeling flexible link manipulators, [19]. Describing the position of a point on a flexible link requires both rigid and elastic coordinates. Since flexible manipulators are distributed parameter systems, their motion is described by partial differential equations instead of ordinary differential equations, hence dynamics modeling can become very challenging. On the other hand, rigid manipulators are usually modeled using the same method or Newton-Euler formulation introduced by Sciavicco and Siciliano, [20], and Craig, [21]. First, acceleration of the center of mass of each link is computed from the base to the end-effector, then forces and torques acting on each link and the required joint torques can be calculated backward.

The main focus of this paper is to develop a dynamics model of SFFR with flexible appendages. The system is divided into two parts, the manipulators and the base (spacecraft), which consists of flexible appendages. All rigid manipulators and flexible appendages are attached to a common rigid base. The very large antennas, solar arrays and multi-link flexible manipulators can be assumed as flexible appendages. To develop dynamics model of such systems, a recursive Lagrangian approach and the Newton-Euler formulation are applied. Next, the interaction forces and torques acting between the rigid manipulators and its base are studied.

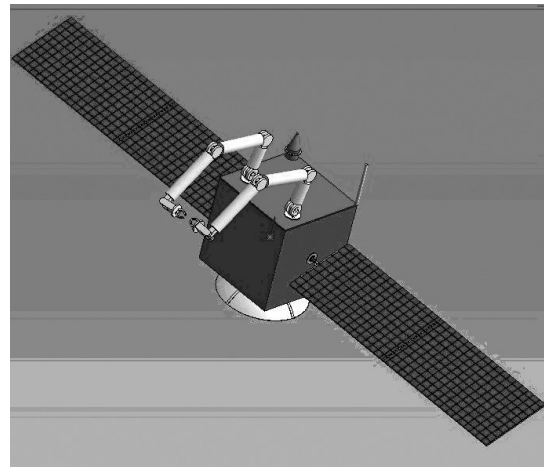


Figure 1. Rigid Body Manipulators Mounted on a Flying Base with Flexible Appendages.

DYNAMICS MODELING

The system shown in Figure 1 is a typical rigid manipulating system mounted on a free-flying base with flexible appendages. It is assumed that the origin of an inertial coordinate system is located at the base of the rigid manipulators; the elastic states of the flexible appendages affect the rigid manipulators system by moving its base in Cartesian space. In developing the equations of motion, these become boundary conditions on the rotational and translational motion of the first link of each rigid manipulator that are then propagated to the other forward links. Then, a backward recursive procedure is employed to compute interaction forces and torques.

Basic assumptions in this research work are:

1. Since the manipulators are stationary at the initial instant, the nonlinear velocity-dependent terms approximate zero.
2. The elastic deflections are assumed to be small; hence all inertia submatrices approximate to functions of the joint variables only.

Dynamics Modeling of Flexible Appendages

The base consists of n_b multi-link flexible appendages with practically passive joints. The method chosen here is a recursive Lagrangian formulation using finite number of assumed modes, which is applicable to a flexible arm. The approach described here begins with assuming an appropriate number of modes to model the flexibility in each link corresponding to each rigid degree of freedom, [22-23]. It is assumed that each link could have a transverse vibration plus torsion about the z -axis and that axial vibration is negligible. Thus, the total number of equations of motion and generalized coordinates, is:

$$N_b = \sum_{l=1}^{n_b} \sum_{i=1}^{n_l} \sum_{k=1}^3 m_{ik}^{(l)} \quad (1)$$

where n_b is the number of total appendages, n_l is the number of l^{th} appendage links, k represents vibration in the x , y , or θ_z direction, and $m_{ik}^{(l)}$ is the number of assumed modes for the k^{th} direction of vibration of the i^{th} link of the l^{th} appendage. The position of an arbitrary point on each link $r_i^{(l)}$ is composed of summations of the assumed mode shapes multiplied by the generalized coordinates. These definitions are used to form the kinetic and potential energy of the system. Then, Lagrange's equations yields the equations of motion of the system. The kinetic energy of a point on the i^{th} link of appendage l , is:

$$dT_i^{(l)} = \frac{1}{2} dm_i^{(l)} Tr\{\dot{r}_i^{(l)} \dot{r}_i^{(l)T}\}, \quad (2)$$

where $dm_i^{(l)} = \rho_i^{(l)} dz_i^{(l)}$ is the mass element that one can integrate over $z_i^{(l)}$ from 0 to $L_i^{(l)}$. $Tr\{\cdot\}$ is the

trace operator. Summing over all $n^{(l)}$ links of all n_b appendages, the system kinetic energy is obtained:

$$T = \sum_{l=1}^{n_b} \sum_{i=0}^{n^{(l)}} \int_0^{L_i^{(l)}} dT_i^{(l)}, \quad (3a)$$

or

$$T = \sum_{l=1}^{n_b} \sum_{i=0}^{n^{(l)}} \left[\frac{1}{2} \rho_i^{(l)} A_i^{(l)} \int_0^{L_i^{(l)}} \dot{r}_i^{(l)T} \dot{r}_i^{(l)} dz_i^{(l)} \right] + \sum_{l=1}^{n_b} \sum_{i=0}^{n^{(l)}} \left[\frac{1}{2} I_i^{(l)} \int_0^{L_i^{(l)}} \dot{\theta}_i^{(l)T} \dot{\theta}_i^{(l)} dz_i^{(l)} \right] \quad (3b)$$

where $L_i^{(l)}$ is the length of link i of l^{th} appendage. The potential energy of the system in a microgravity environment is due to elastic deformation of the links. Therefore, it is obtained by computing the potential energy of an element, then integrating it over the length of the link, and finally summing over links and appendages.

$$V = \sum_{l=1}^{n_b} \sum_{i=0}^{n^{(l)}} \frac{1}{2} E_i^{(l)} I_{xi}^{(l)} \int_0^{L_i^{(l)}} \left(\frac{\partial \theta_{xi}^{(l)}}{\partial z_i^{(l)}} \right)^2 dz_i^{(l)} + \sum_{l=1}^{n_b} \sum_{i=0}^{n^{(l)}} \frac{1}{2} E_i^{(l)} I_{yi}^{(l)} \int_0^{L_i^{(l)}} \left(\frac{\partial \theta_{yi}^{(l)}}{\partial z_i^{(l)}} \right)^2 dz_i^{(l)} + \sum_{l=1}^{n_b} \sum_{i=0}^{n^{(l)}} \frac{1}{2} G_i^{(l)} J_{xi}^{(l)} \int_0^{L_i^{(l)}} \left(\frac{\partial \theta_{xi}^{(l)}}{\partial z_i^{(l)}} \right)^2 dz_i^{(l)}, \quad (4)$$

One of the benefits of this method is that as much detail can be included in the equations of motion as desired. Now Lagrange's equations can be used to derive the equations of motion:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i, \quad (5)$$

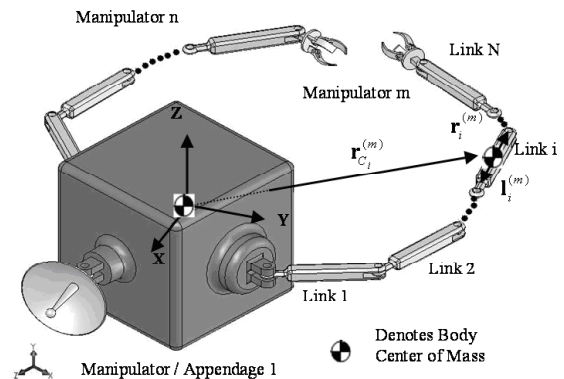


Figure 2. A Space Free-Flying Robot (SFRR) with n_a manipulators.

Q_i is the nonconservative generalized force applied to the base corresponding to the generalized coordinate q_i . These are the interaction forces and torques created by the rigid manipulators. The generalized forces are determined based on the virtual work principal considering the rigid manipulators, or:

$$\delta W = \mathbf{F}_{IF} \delta \mathbf{r} + \boldsymbol{\tau}_{IF} \delta \boldsymbol{\theta}, \quad (6)$$

The generalized coordinates \mathbf{q}_b can be written as:

$$\mathbf{q}_b^T = [(q_{b_{ik}}^{(1)}; i = 1, \dots, n_1), (q_{b_{ik}}^{(2)}; i = 1, \dots, n_2), \dots, (q_{b_{ik}}^{(l)}; i = 1, \dots, n_l), \dots, (q_{b_{ik}}^{(n_b)}; i = 1, \dots, n_{n_b})], \quad k = 1, 2, 3 \quad (7)$$

The equations of motion are obtained as:

$$\mathbf{M}(\tilde{\mathbf{q}}_b) \ddot{\tilde{\mathbf{q}}}_b + \mathbf{C}(\tilde{\mathbf{q}}_b) \dot{\tilde{\mathbf{q}}}_b + \mathbf{K}(\tilde{\mathbf{q}}_b) \tilde{\mathbf{q}}_b = \mathbf{Q}, \quad (8)$$

where:

$$\mathbf{Q} = - \begin{bmatrix} F_{IF} \\ \boldsymbol{\tau}_{IF} \end{bmatrix} \quad (9)$$

is the vector of generalized forces and torques applied to the flexible base.

Dynamics Modeling of Multiple Rigid Manipulators and Coupled Equation of Motion
It is assumed that the system consists of n_a rigid manipulators and each manipulator has $m_a^{(l)}$ links (Figure 2). All links and joints are rigid, and each joint has only one DOF. Therefore, the total DOF of the manipulators is obtained as:

$$N_a = \sum_{l=1}^{n_a} m_a^{(l)}, \quad (10)$$

First, assume that n_a rigid single link manipulators are mounted on the base. The acceleration of the center of mass of the l^{th} arm $\mathbf{a}_{c1}^{(l)}$ is obtained as

$$\mathbf{a}_{c1}^{(l)} = \mathbf{a}_{c0} + \dot{\boldsymbol{\omega}}_l^{(l)} \times \mathbf{r}_{c1}^{(l)} + \boldsymbol{\omega}_l^{(l)} \times (\boldsymbol{\omega}_l^{(l)} \times \mathbf{r}_{c1}^{(l)}) \quad (11)$$

where

$$\mathbf{a}_{c0} = \ddot{x}_{c0} \hat{i} + \ddot{y}_{c0} \hat{j} + \ddot{z}_{c0} \hat{k}$$

$$\boldsymbol{\omega}_l^{(l)} = (\dot{\delta}_{x,0} + \dot{\theta}_p^{(l)} \delta_{pi}) \hat{i} + (\dot{\delta}_{y,0} + \dot{\theta}_p^{(l)} \delta_{pj}) \hat{j} + (\dot{\delta}_{z,0} + \dot{\theta}_p^{(l)} \delta_{pk}) \hat{k}$$

$\dot{\boldsymbol{\delta}}_0 = [\dot{\delta}_{x,0}, \dot{\delta}_{y,0}, \dot{\delta}_{z,0}]^T$ = the angular acceleration vector of the base due to flexibility. Also:

$$\delta_{ps} = \begin{cases} 1, & \text{if } p = s \\ 0, & \text{if } p \neq s \end{cases}$$

and is called Kronecker delta.

The interaction forces and torques at the base due to the single link manipulators are given by:

$$F_{IF1} = \sum_{l=1}^{n_b} F_{IF1}^{(l)}$$

$$\boldsymbol{\tau}_{IF1} = - \sum_{l=1}^{n_b} (\mathbf{r}_{c1}^{(l)} \times F_{IF1}^{(l)}) + \sum_{l=1}^{(1)} I_1^{(l)} \dot{\boldsymbol{\omega}}_l^{(l)}$$

$$F_{IF1}^{(l)} = m_1^{(l)} \mathbf{a}_{c1}^{(l)} \quad (12)$$

where $m_1^{(l)}$ is the mass, and $I_1^{(l)}$ is moment of inertia matrix of the l^{th} single arm manipulator. Then, to extend the above procedure for multiple-link rigid manipulators, the Recursive Newton-Euler method is used, [12]. The algorithm uses forward kinematics computations, which propagate the velocities and accelerations of each link from the base to the last link. This is followed by backward kinetics calculations to obtain the forces and torques acting on each link starting with the external forces and torques applied to the end-effector. It is assumed that the end effectors of manipulators are not in contact with any object, so the forces and torques applied to the tip of their last links are zero.

The vector of generalized coordinates for rigid manipulators system is chosen as:

$$\mathbf{q}_m = \left(\mathbf{q}_m^{(0)T}, \mathbf{q}_m^{(1)T}, \dots, \mathbf{q}_m^{(l)T}, \dots, \mathbf{q}_m^{(n_a)T} \right)^T \quad (13)$$

where $\mathbf{q}_m^{(0)} = (\delta_{x,0}, \delta_{y,0}, \delta_{z,0})^T$, and

$$\mathbf{q}_m^{(l)} = \boldsymbol{\theta}^{(l)} = \left(\theta_1^{(l)}, \theta_2^{(l)}, \dots, \theta_{m_a^{(l)}}^{(l)} \right); \quad l \geq 1 \quad (14)$$

As mentioned before, the manipulators are assumed to be at rest at the initial instant, so the nonlinear velocity-dependent terms will be initially zero. The deflections are assumed small, and hence, all inertia submatrices are approximated to be functions of the joint variables only. The interaction forces and torques due to the rigid manipulators motion can be written as:

$$F_{IF} = B_f(q_m) \ddot{q}_m + N_f(q_m, \dot{q}_m)$$

$$+ C_f(q_m) \ddot{q}_b + N_{fc}(q_b, \dot{q}_b, q_m, \dot{q}_m),$$

$$\boldsymbol{\tau}_{IF} = B_{\tau 0}(q_m) \ddot{q}_m + N_{\tau 0}(q_m, \dot{q}_m)$$

$$+ C_{\tau 0}(q_m) \ddot{q}_b + N_{\tau 0c}(q_b, \dot{q}_b, q_m, \dot{q}_m),$$

$$\boldsymbol{\tau} = B_{\tau}(q_m) \ddot{q}_m + N_{\tau}(q_m, \dot{q}_m)$$

$$+ C_{\tau}(q_m) \ddot{q}_b + N_{\tau c}(q_b, \dot{q}_b, q_m, \dot{q}_m), \quad (15)$$

where \mathbf{q}_m represents the rigid manipulator variables and \mathbf{q}_b represents the flexible base generalized coordinates. B_f , $B_{\tau 0}$, C_f and $C_{\tau 0}$ represent inertia effects of

the rigid manipulators and flexible base, respectively. B_f and $B_{\tau 0}$ are particularly important because they represent the controllable rigid manipulator inertia effects. These matrices are not in general symmetric or positively definite (but the inertia matrix for the complete coupled system is, of course). The remaining terms in Eq. (15) represent other nonlinear effects. The third equation is the typical joint torque equation with extra coupling terms. Often actuator dynamics or other effects dominate the manipulator performance, so this equation could take other forms. However, for this work it is assumed that the relationship between the joint actuation torques and joint positions is known and controllable.

The generalized coordinates of the base, \mathbf{q}_b can be written as $q_b^T = [r_{c0}, \delta_0]^T$, then the equation of motion for the base is:

$$\begin{bmatrix} M & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \ddot{r}_{c0} \\ \ddot{\delta}_0 \end{bmatrix} + \begin{bmatrix} C & 0 \\ 0 & C_r \end{bmatrix} \begin{bmatrix} \dot{r}_{c0} \\ \dot{\delta}_0 \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & K_r \end{bmatrix} \begin{bmatrix} r_{c0} \\ \delta_0 \end{bmatrix} = - \begin{bmatrix} F_{IF} \\ \tau_{IF} \end{bmatrix} \quad (16)$$

The dynamics of rigid manipulators, interaction forces and torques, which were introduced in Eq. (9), appear in this equation. Eq. (16) yields a system of coupled nonlinear equations for a SFFR system, where M is mass matrix, J is mass moment of inertia matrix, C is translational damping matrix, C_r is rotational damping matrix, K is translational stiffness matrix, K_r is rotational stiffness matrix, r_{c0} represents the coordinates governing the translational motion of the flexible base, and δ_0 represents the coordinates governing the rotational motion of the base.

The interaction forces and torques exerted by the rigid manipulator can be written as:

$$\begin{aligned} F_{IF} &= B_f(\theta) \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + N_{Rf}(\theta) \begin{bmatrix} \dot{\theta}_1 \dot{\theta}_2 \\ \dot{\theta}_1 \dot{\theta}_3 \\ \dot{\theta}_2 \dot{\theta}_3 \end{bmatrix} + N_{Cf}(\theta) \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \\ \dot{\theta}_3^2 \end{bmatrix}, \\ \tau_{IF} &= B_{\tau 0}(\theta) \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + N_{R\tau 0}(\theta) \begin{bmatrix} \dot{\theta}_1 \dot{\theta}_2 \\ \dot{\theta}_1 \dot{\theta}_3 \\ \dot{\theta}_2 \dot{\theta}_3 \end{bmatrix} + N_{C\tau 0}(\theta) \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \\ \dot{\theta}_3^2 \end{bmatrix}, \end{aligned} \quad (17)$$

Here Coriolis and centrifugal effects have been written separately, where N_{Rf} , N_{Cf} , $N_{R\tau 0}$, and $N_{C\tau 0}$ are rigid manipulator coriolis and centrifugal interaction matrices (forces, and torques), respectively.

Interaction Forces and Torques Acting Between the Rigid Manipulators and the Base

In this section the effects of rigid manipulators, or those terms that are only functions of q_m , are investigated. First, the inertia forces and torques are discussed,

or those generated by accelerating the links of each rigid manipulator. Next, the nonlinear centrifugal and coriolis forces and torques are discussed. The following performance measure provides a quick and accurate measure of the ability of the l^{th} manipulator in generating effective interaction forces and torques:

$$\left| B_f^{(l)T}(q_m^{(l)}) B_f^{(l)}(q_m^{(l)}) \right|, \left| B_{\tau 0}^{(l)T}(q_m^{(l)}) B_{\tau 0}^{(l)}(q_m^{(l)}) \right| \quad (18)$$

Alternately, by defining:

$$B^{(l)} = \begin{bmatrix} B_f^{(l)}(q_m^{(l)}) \\ B_{\tau 0}^{(l)}(q_m^{(l)}) \end{bmatrix} \quad (19a)$$

the combined ability of the robot to generate interaction forces and torques may be evaluated as:

$$\left| B^{(l)T}(q_m^{(l)}) B^{(l)}(q_m^{(l)}) \right| \quad (19b)$$

where B_f and $B_{\tau 0}$ are inertia-like matrices. These are important for two reasons. First, the rigid manipulators must have enough inertia to effectively apply interaction forces and torques to the base. Second, there are locations in the workspace where these matrices become singular, which in turn presents a problem since they are inverted in the control scheme. However, the more important consideration is that these “inertial singularities” represent physical limitations in that an inertial force or torque cannot be created in one or more degrees of freedom. As an example, consider a three degree of freedom manipulator, Figure 3. By considering the variation of the B_f matrix throughout the workspace, a few important features become apparent. These singularities consist of some of the kinematic singularities plus additional dynamically singular configurations. These are driven by the columns of B_f when the matrix contains:

1. Linearly dependent columns, which indicate that the forces created by two or more joints are parallel. This scenario occurs when the last two joints are aligned.

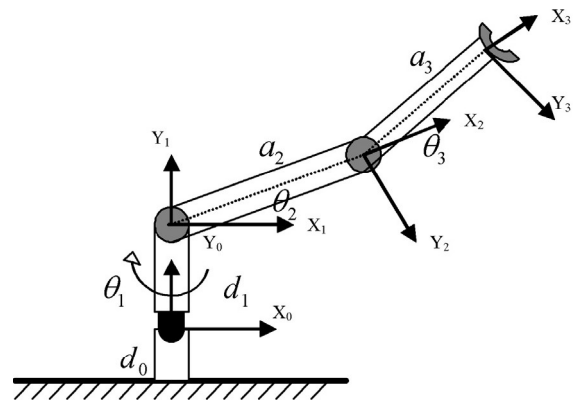


Figure 3. A three DOF manipulator.

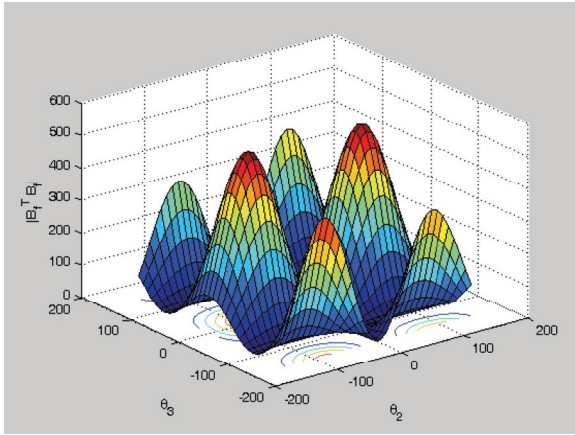


Figure 4. Force Reaction of a 3 DOF Manipulator on a Base

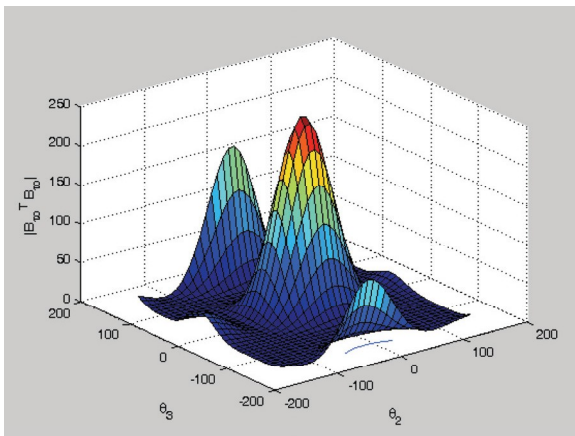


Figure 5. Torque Reaction of 3 DOF Manipulator.

2. A column of zeros, which indicates a location in the workspace where the motion of a joint cannot create any interaction forces. This occurs when the system center of mass is aligned along an axis of rotation. These inertial singularities depend on the location of the center of mass of the system.

In other words, variation of the interaction forces and torques is affected by the joint space configuration of the manipulator.

The nonlinear rigid manipulator effects (N_f , $N_{\tau 0}$) may become significant in certain workspace regions. However, with proper choice of vibration control feedback gains, the amplitude of the commanded joint motion can be limited to ensure the inertia effects remain dominant. Furthermore, under these conditions, the nonlinear effects can be linearized near an operating point.

SIMULATION RESULTS

Simulations were implemented in Matlab (V.7) for a three DOF manipulator mounted on a flying base. The configuration is shown in Figure 3, with dimensions and properties as given in Table 1. The resulting equations

of motion take the form of Eq. (17). It is evident that for single arm manipulator, l equals one, and could be omitted from equations. Using the Recursive Newton-Euler method, the inertia force matrix B_f is square and the variation in force performance, as quantified by the performance measure in Eq. (17), is shown in Figure 4.

The inertia torque reaction of the three DOF manipulator can be seen in Figure 5 by evaluating the torque performance measure defined in Eq. (18). However, the inertia torques created by accelerating joints 2 and 3 are always parallel so this evaluation was made for joints 1 and 2 only. The combined force and torque reaction of the manipulators can be evaluated by using Eq. (19). It is important to note that even if the interaction forces or torques are desirable independently, having one will always bring the other. Thus, it becomes important to evaluate the combined force and torque performance of the manipulator, as shown in Figure 6.

The interaction forces and torques produced by the rigid manipulator are given by Eq. (17). In these equations, the last two terms are nonlinear parts. The nonlinear rigid motion effects also vary throughout the workspace. The nonlinear Coriolis and centripetal effects can be seen in Figures (7-10). There are several important points to be made. First, the magnitude of the nonlinear forces is relatively small. In addition,

Table 1. Dimensions and Properties of Rigid Manipulator.

Parameters	Links			
	0	1	2	3
$a_i(\text{m})$	-	-	1.50	0.80
$d_i(\text{m})$	0.15	0.15	-	-
$m_i(\text{kg})$	1.00	1.00	5.00	3.00
$r_{ci}(\text{m})$	-	-	0.75	0.40
$I_{xxi}(\text{kg-m}^2)$	0.015	0.015	0.011	0.008
$I_{yyi}(\text{kg-m}^2)$	0.015	0.015	0.011	0.008
$I_{zzi}(\text{kg-m}^2)$	0.007	0.007	0.002	0.001

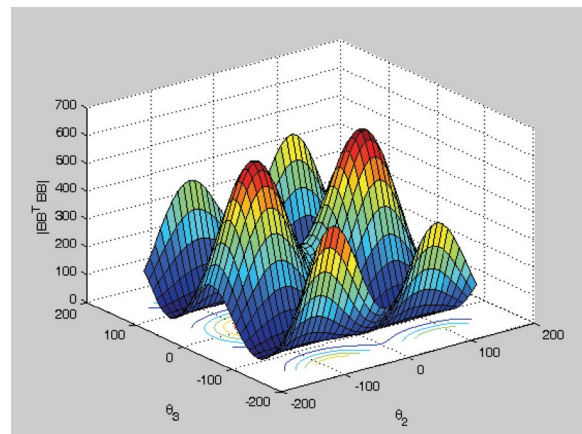


Figure 6. Combined Torque and Force Reaction of 3 DOF Manipulator.

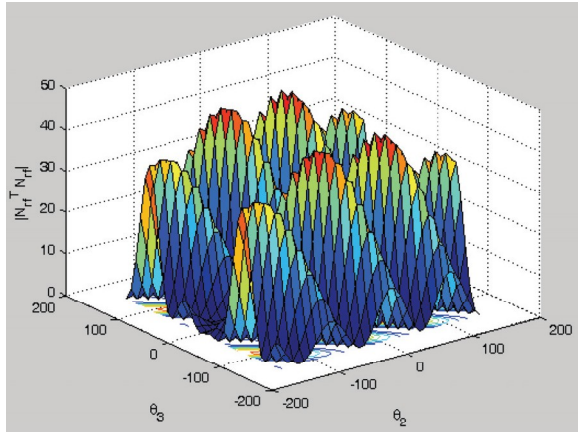


Figure 7. Three DOF Manipulator Reaction: Coriolis Forces.

the coriolis forces are the largest only in regions of poor inertia performance; so operation in these regions can be avoided by using the performance index. However, the centripetal forces become largest around kinematic singularity regions, while operation around these regions may be necessary for other aspects.

CONCLUSIONS

Explicit dynamics of a space free-flying robot with flexible appendages were investigated in this paper. The system was divided into two parts, the manipulator(s), and the main base body (spacecraft), which consists of flexible appendages. The recursive Lagrangian approach was used to describe dynamics model of the flexible base system. a Recursive Newton-Euler approach was employed for modeling the multi-manipulator system. The interacting forces and torques acting between the manipulators and the main body were also modeled and used for simulation studies. It was shown by simulation that the magnitude of the nonlinear forces is relatively small. Also, the magnitude of Coriolis forces becomes the largest near regions of small inertial forces, while the centrifugal forces may become considerably large in regions that inertia forces become large. Finally, the centrifugal torques are largest near to regions of small inertial torques.

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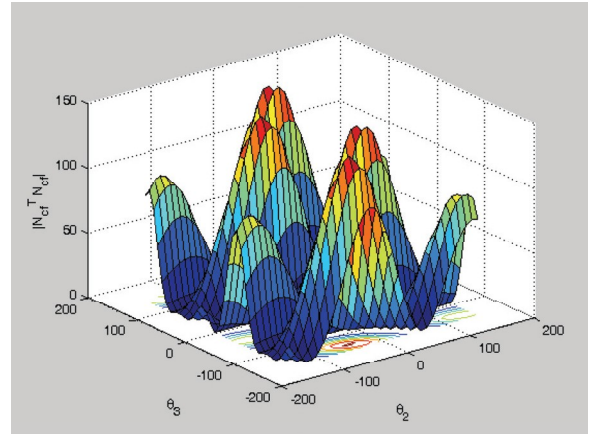


Figure 8. Three DOF Manipulator Reaction: Centrifugal Forces.

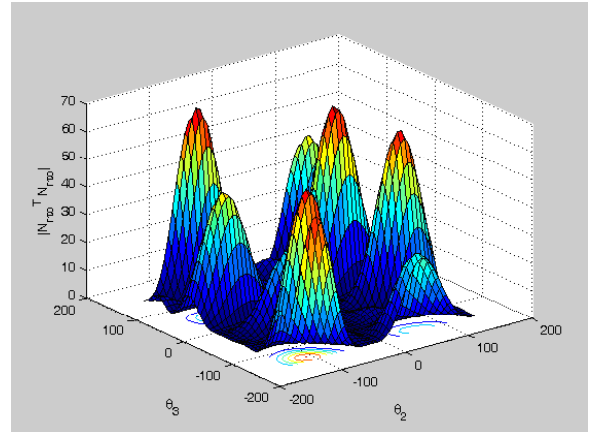


Figure 9. Three DOF Manipulator Reaction: Coriolis Torques.

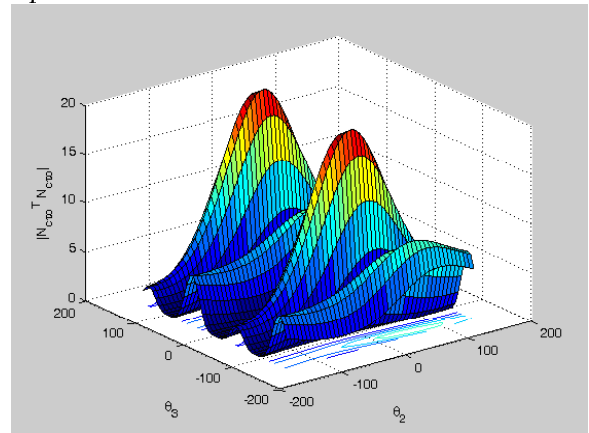


Figure 10. Three DOF Manipulator Reaction: Centrifugal Torques.

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