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Assessment of the Existing Models to Estimate the Fatigue Life under Both Multiaxial in Phase and 90° Out of Phase Loading Conditions

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The objective of this paper is to evaluate the validity of commonly used multiaxial fatigue criteria for different multiaxial loading conditions. The best criterion is identified through comparative analysis. The assessment is based on the experimental research findings of the SAE notched shaft, which is used as a benchmark. The relative performance of the three existing multiaxial fatigue theories based on the strain criteria and the critical plane approaches are investigated. There is no fixed strain life criterion that can predict the fatigue life best for different loading cases. Among all the critical plane models considered, Fatemi and Socie model gives best fatigue life prediction for both multiaxial in phase and 90° out of phase loading cases.

NOMENCLATURE

B	Bending moment
b	Fatigue strength exponent
b_0	Pure torsional fatigue testing strength exponent
c	Fatigue ductility exponent
c_0	Pure torsional fatigue testing fatigue ductility exponent
E	Young's modulus
N	Cycle numbers
n'	Cyclic hardening exponent
T	Torque
ε	Local strain
ε'_f	Fatigue ductility coefficient

Subscripts

eq	Equivalent
e	Elastic
f	Fatigue
τ	Shear stress
τ'_f	Pure torsional testing fatigue strength
Δ	Range
γ'_f	Pure torsional fatigue testing ductility coefficient
σ	Normal stress or strength
σ_{ij}	Local elastic-plastic stress tensor
σ'_f	Fatigue strength coefficient
$\sigma_{n,\max}$	The maximum normal stress on the maximum shear strain plane (γ)
ν	Poisson's ratio
ν^*	Is the elastic-plastic Poisson's ratio
1, 2, 3	Principal directions
p	Plastic

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INTRODUCTION

There are different Engineering methods to estimate the fatigue life under multiaxial conditions. All of them have generally some restrictions depending on the assumptions made to simplify the model. The fatigue failure processes caused by cyclic stressing or straining may be classified as: (i) low-cycle fatigue (LCF) and (ii) high-cycle fatigue (HCF). The Stress based (S-N) approach is commonly used for HCF but an idea is recently presented that makes it possible to extend its application to the LCF class, too [1]. Local strain-life approach is normally used for LCF class that has gained acceptance as a useful method to evaluate the notched specimen fatigue life. The strain-life method uses material properties obtained from smooth specimen laboratory strain controlled fatigue tests.

Most real design situations, including rotating shafts, connecting links, drillstring, automotive and aircraft components, *etc.* involve the multiaxial state of cyclic stress, where two or more non-zero principal stresses fluctuate under the combination of two or more load components and/or the structures geometry. Components subjected to multiaxial loading are most often analyzed by the assumptions made to convert it to the "equivalent uniaxial fatigue" case. The simplifying assumptions are generally invalid for the specific load sequence or component being considered and give a poor life prediction compared to the experimental tests. The fact that fatigue is essentially a directional process (damage and cracking take place on a particular plane) has not been considered in their analysis. The extension of the mentioned approach to the nonproportional loading case is even more disputable. Therefore much greater research emphasis has been placed to understand the mechanisms of the fatigue damage accumulation under multiaxial loading. A somewhat different approach based on predicting the extent of damage in a specific direction and plane has been proposed. It is referred to as the critical plane approach.

Sonsino [2] shows that the hypothesis of the equivalent effective stress (EES), which is based on local coordinate stresses in weld toes considering the maximum stressed material volume governed by the stress gradient and the phase displacement between the local normal and shear stresses, is capable of transferring the local multiaxial stress states with constant and variable principal stress directions into an S-N curve determined under uniaxial load, irrespective of the stress concentration level given by the geometry and the machining of the welded joint. Varvani-Farahani [3] proposed a parameter that gives a good correlation between multiaxial fatigue life spans in various in-phase and out-of-phase straining conditions. Liu and Wang [4] reaffirm the effectiveness of the virtual strain-energy (VSE) method for predicting fatigue life and demonstrate its inherent ability to describe fatigue behavior such as crack orientations and crack growth characteristics. Some multiaxial high-cycle fatigue criteria based on the so-called critical plane approach was reviewed by Carpinteri and Spagnoli [5]. Bonnen, *et. al.*, [6] conducted a series of experiments using either axial or torsional overloads in order to determine their effects on the torsional fatigue of normalized SAE1045 steel tubes. They determined that, in the high cycle regime, both types of overload had the same impact on fatigue life. Kueppers and Sonsino [7] developed a suitable calculation procedure based on a combination of local normal and shear stresses in the critical plane for multiaxial loaded aluminium. Lee, *et. al.*[8] describe a simple damage model for fatigue life predictions of welded joints under nonproportional, constant, and variable amplitude loading histories.

Laboratory testing to measure fatigue life is really complicated if not impossible for complex loading cases. The general trend is to perform elasto-plastic analysis by finite element method and then use equivalent uniaxial stress or stain criterion to determine fatigue life. However the elasto-plastic finite element analysis with cyclic stress-strain histories is an inconvenient and expensive procedure. The equivalent uniaxial stress-strain criterion deals only with multiaxial or biaxial proportional loading conditions, and its application to nonproportional loading cases after simplified assumptions renders poor results. Therefore, it is proposed in this paper to use the stress-strain results of a linear static finite element analysis to predict the fatigue life by fatigue software. The finite element based "MSC-PATRAN" software is used to calculate stress-strain distribution curve and "MSC-Fatigue" software to predict the fatigue life [9]. The in phase and 90° out of phase loadings are solved as examples.

MULTIAXIAL LIFE PREDICTIONS

Several multiaxial fatigue theories have been presented in the literature. A common characteristic of all of the approaches is that the required material properties can be determined from standard uniaxial fatigue test data. However, there still exists a lack of agreement on the answer to the question of which model is the most appropriate one?

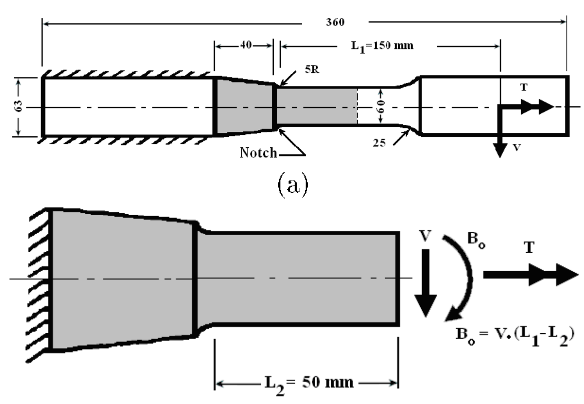


Figure 1. SAE notched shaft geometry and loading [13].

The SAE notched shaft is taken as an example in this paper to evaluate the validity of the existing fatigue theories applied to multiaxial loading.

Multiaxiality assessment of the problem is carried out to determine the criterion that is best to predict fatigue life. The equivalent strain fatigue life prediction approach can predict the fatigue life within some range of accepted factors for multiaxial proportional loading. Hoffmann and Segger [10] tried to improve the traditional effective strain approach by making some assumptions to consider the notch and the proportional loading effect. They extended Neuber's rule to multiaxial loading case by defining notch root equivalent stress and strain (based on von Mises yield criterion and Henckey's flow rule). For more information on elasto-plastic notch stress and strain behavior refer to Zeng and Fatemi's articles [11, 12]. Fatemi et al analyzed fatigue behavior and deformation of notched vanadium-based micro-alloyed forging steel specimens under constant amplitude axial and torsion loads [12].

Fash [13] performed an experimental investigation on the SAE notched shaft under an in phase loading condition. He also performed a finite element analysis to evaluate the three dimensional stress-strain fields. He then used the obtained strains and effective strain criteria to predict the fatigue life of the SAE notched shaft. Maximum principal and shear strain criteria were also considered. Use of one element at the notch area (due to the lack of computer resources) caused the larger scattering of Fash's results.

SAE notched shaft has been analyzed by existing models and criteria to investigate to find the best model to predict multiaxial loading fatigue life. The finite element based software MSC-Fatigue and critical plane approach is proposed to predict fatigue life.

GEOMETRY AND MATERIAL PROPERTIES FOR THE SAE NOTCHED SHAFT

The geometry details of the SAE notched shaft are shown in Figure 1. The shaft is made of hot rolled and normalized SAE 1045 steel with the 5 mm notch radius. Its monotonic and cyclic parameters are summarized in Table 1.

The monotonic and the cyclic stress-strain curves may be obtained from the material parameters. The curves show that the material is cyclically hardened.

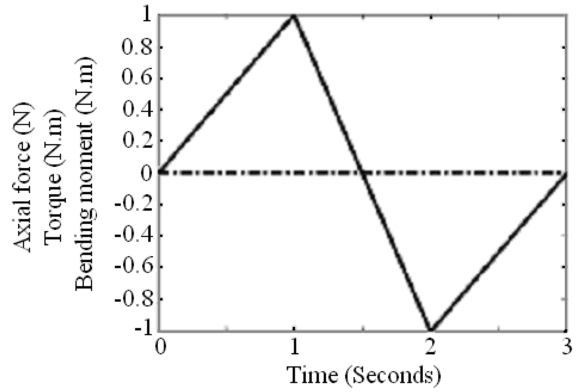


Figure 2. Normalized in-phase loading histories.

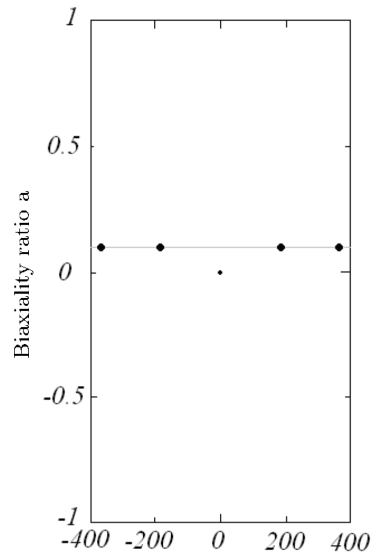
The geometry of the SAE notched shaft is modeled in MSC-PATRAN using 3D-solid elements. Loading and boundary conditions are shown in Figure 1. Figure 1 also shows the loads acting on a cross-section at 50 mm from the notch root (transverse force, bending and torsional moment).

Normalized in phase loading histories for axial, bending moment and torque are shown in Figure 2. The time histories here represent one full cycle. They are scaled during stress superposition within "MSC-Fatigue" according to the applied scale factor. Figure 3 shows biaxiality ratio and phase angle distribution over the time histories (in phase loading case, Figure 2). As it can be seen α and θ are constant over the time history. Therefore, the problem is multiaxial proportional loading, where the strain-based life prediction models are applicable.

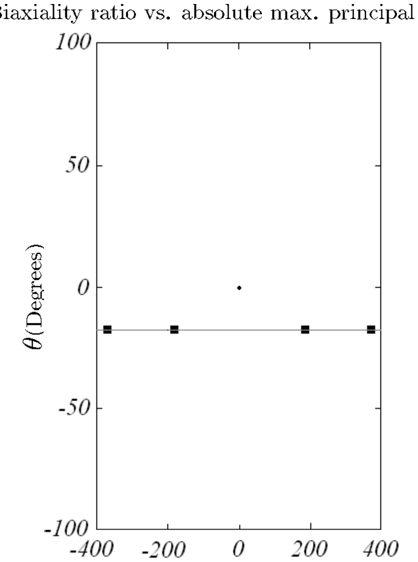
The created finite element model in MSC-PATRAN, and its stress-strain distribution are then exported into MSC-Fatigue to predict fatigue life. The actual bending and torque loading histories at any time are analyzed as static load cases to calculate stresses using the principle of superposition to produce stress histories for all or selected nodes of the model. Then "MSC-Fatigue" takes the output stress or strain of the "MSC-PATRAN" as input and estimates the fatigue damage for each supplied node or element of the model. The "MSC-Fatigue" provides tools to assess the multiaxiality by calculating the biaxiality ratio α (the ratio of the smallest to the largest absolute in-

Table 1. SAE 1045 HR material properties [nCode].

Monotonic		Strength					Poisson's ratio	
		Yield (0.2%)		Ultimate			Elastic υ_e	Plastic υ_p
		380 MPa		621 MPa			0.3	0.5
Cyclic	Axial	K'	n'	σ'_f	b	ε'_f	c	E
		1258 MPa	0.208	948 MPa	−0.092	0.26	−0.445	202000 MPa
	Torsional	K'_0	n'_0	τ'_f	b_0	γ'_f	c_0	G
		614 MPa	0.217	505 MPa	−0.907	0.413	−0.445	79000 MPa



(a) Biaxiality ratio vs. absolute max. principal stress.



(b) Phase angle vs. absolute max. principal stress.

Figure 3. Proportional loading time history distribution (0 to 3 Secs).

plane principal stress), and the phase angle θ (absolute maximum principal stress angle with the X-axis). The multiaxiality assessment is carried over the time histories of α and θ vs. absolute principal stress plots (Figure 3). The scattering spectrum of the dominant value in these plots indicates the stress state multiaxiality (uniaxial, proportional or nonproportional) cases as follows;

Case 1: Uniaxial stress state- there is only one dominant principal stress for the loading history with fixed phase angle. No special algorithm corrections are needed to convert elastic stresses and strains to elastic-plastic case and the Neuber correction technique is adequate [14].

Case 2: Proportional multiaxial stress state- both the two principal stress ratio and phase angle remains constant during the loading histories. Actions must be taken to consider the non-uniaxiality fact of the loading.

Case 3: Nonproportional multiaxial stress state- Either the biaxiality ratio or the phase angle changes are significant during the time history.

STRAIN BASED APPROACH FOR MULTIAXIAL IN PHASE LOADING

The most popular criteria proposed in the literature are the strain-based criteria [15]. The validities of the strain based criteria are investigated by predicting the fatigue lives of the SAE notched shaft under different proportional multiaxial loadings. Absolute maximum principal strain, maximum shear strain and von Mises strain criteria are considered in this paper. The predicted "MSC-Fatigue" and the experimental fatigue lives are compared to evaluate the above mentioned criteria. "MSC-Fatigue" uses Neuber correction to convert the elastic stresses and strains into the elasto-plastic case.

The following equations are used to predict fatigue life for each criterion:

- by absolute maximum principal strain:

$$\left|\frac{\Delta\varepsilon_1}{2}\right| = \frac{\sigma'_f}{E}(2N)^{\delta} + \varepsilon'_f(2N)^{c} \tag{1}$$

where, $\Delta\varepsilon_1$, ε'_f, N, and σ'_f are principal axial strain range, fatigue ductility coefficient, cycle numbers and fatigue strength coefficient, respectively.

- by maximum shear strain:

$$\frac{\Delta\gamma_{\max}}{2} = 1.3\,\frac{\sigma'_f}{E}(2N)^{b_0} + 1.5\varepsilon'_f(2N)^{c_0} \tag{2}$$

where $\Delta\gamma_{\max}$ is the maximum shearing strain range, b_0 and c_0 are fatigue strength and ductility exponents respectively, obtained from pure torsional fatigue testing.

- By von Mises or octahedral shear strain:

$$\varepsilon_{eq} = C\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{\frac{1}{2}} \tag{3}$$

where;

$$C = \frac{\sqrt{2}}{2}\left(\frac{1}{1+\nu^*}\right) \tag{4}$$

$$\nu^* = \frac{\varepsilon_e\nu_e + \nu_p\varepsilon_p}{\varepsilon_e+\varepsilon_p} \tag{5}$$

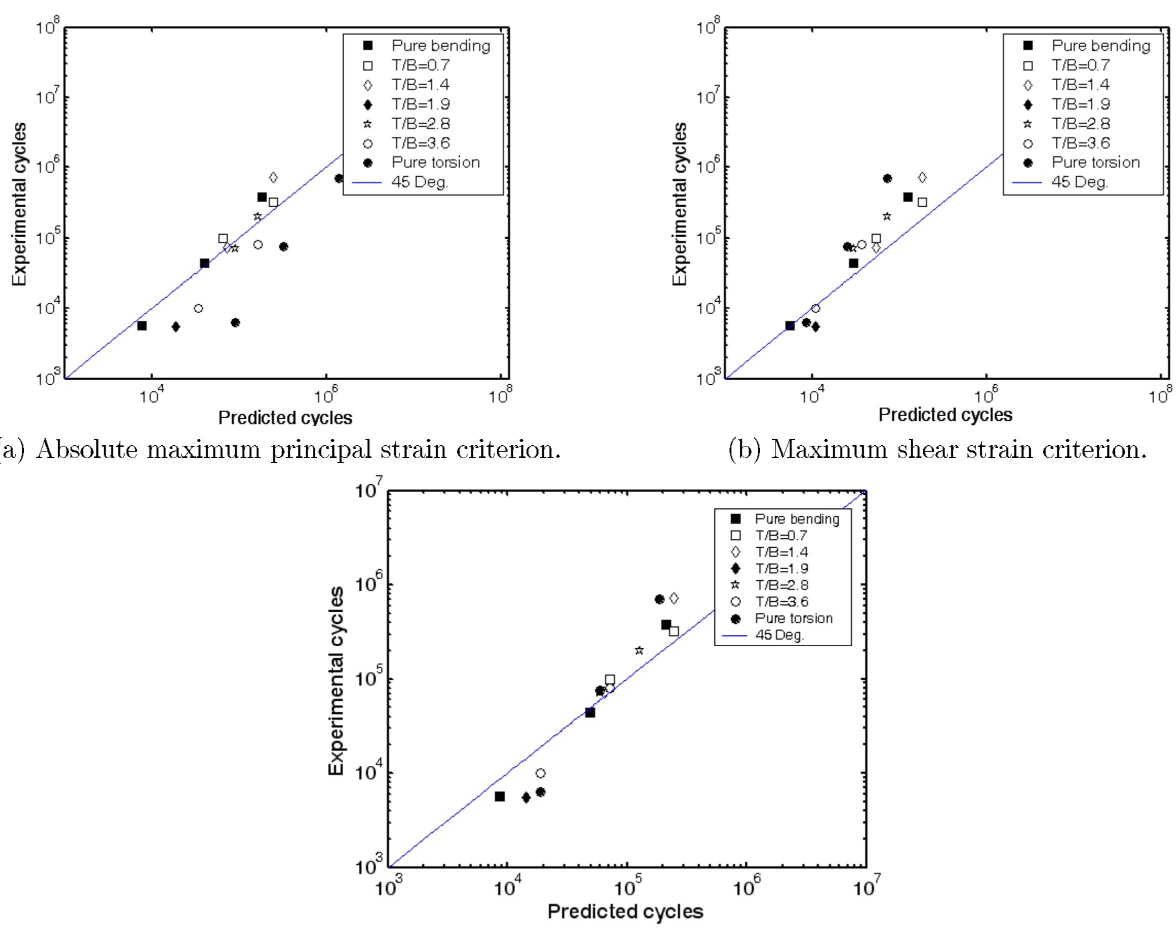
ε_{eq} is the equivalent normal strain, $\sigma_1, \sigma_2, \sigma_3$ are principal stresses in the principal directions 1, 2 and 3, ε_e , ε_p , ν_e and ν_p are elastic and plastic strains and Poisson's ratio, respectively. The calculated values of the equivalent strain are used directly in the uniaxial strain life equation to get fatigue life (N) as follows:

$$\varepsilon_{eq} = \frac{\sigma'_f}{E}(2N)^b + \varepsilon'_f(2N)^c \qquad (6)$$

The above calculated and experimentally measured fatigue life cycles are compared for different bending, torsion and combined bending-torsion loading conditions. The results are plotted as is shown in Figures 4a to c. These figures show that there is no fixed strain life criterion that predicts the fatigue life for loading conditions. Selecting a criterion to predict the fatigue crack initiation depends on the fatigue domain (LCF or HCF regions).

1. Figure 4a shows that absolute maximum principal strain criterion does not predict properly when torsion dominates over bending moment and so for pure torsion case this criterion gives very poor estimates. This criterion cannot be used solely to predict proportional multiaxial fatigue life. The method works fine when bending moment dominates over torsion.
2. Figure 4b shows that maximum shear strain criterion gives conservative results in HCF regions. Although, this criterion shows good prediction in LCF regions, it cannot be considered as a reliable approach to predict in HCF regions.
3. Von Mises criterion takes the effects of all strain components present in the analysis and so delivers results with better accuracy than the other two strain life criteria (Figure 4c).

Hoffmann and Seeger [10] suggested a method to extend the Neuber correction to multiaxial proportional loading case. This method is an extension of strain life criteria with some simplified assumptions to deal with notch effect in the structural component. The predicted and experimentally measured fatigue life



(a) Absolute maximum principal strain criterion.

(b) Maximum shear strain criterion.

(c) Von Mises criterion.

Figure 4. Experimental vs. predicted fatigue life cycles, B=Bending moment, T=Torsion.

cycles are shown in Figure 5. While maximum shear strain method is conservative in predicting the proportional loading in HCF regions, and while absolute maximum principal strain criterion cannot predict fatigue life well when shear stress dominates, Hoffmann-Seeger version of strain life criterion is the best choice for fatigue life prediction for proportional loading in the strain based approaches.

For the nonproportional loading case most of the Hoffmann and Seeger assumptions are violated, therefore the critical plane models are used.

CRITICAL PLANE METHODS

Critical plane approach must be used for the mobile stress tensor case, *i.e.* rotating principal stress direction and also varying biaxiality ratio over the load time histories. Critical plane should not be confused with the crack plane. Cracks typically start in shear mode and after a transition period grow in opening mode (mode I). During the transition period, the crack initiation life may include growth in a number of planes. The plane with the most accumulated damage is called the critical plane. This method is used to rotate the coordinate system of the stresses and strains by means of a tensor rotation such that the x–y plane of the new coordinate system lies in the critical plane.

The critical plane is unknown a priori. Hence, the method must compute damage parameter on a number of candidate planes to identify the one with the maximum amount of damage. The basic steps for a critical plane analysis are:

- Transforming the state of stress and strain to a coordinate system on the nominee plane,
- Counting cycles on that plane,
- Evaluating the damage due to each cycle using the selected damage model,
- Summing up the damages for all cycles counted on a single plane,
- Repeating the above steps for all selected planes in the search space.

Then the plane with the maximum amount of damage will be the critical plane. In the critical plane with normal strain criterion, the normal strain amplitude, which is half the strain range, *i.e.* $\Delta\varepsilon_n/2$, is also calculated. Fatigue life (N) is then predicted by solving the Eq. (4) for N after replacing the left hand side by normal strain amplitude as follows [16];

$$\frac{\Delta\varepsilon_n}{2} = \frac{\sigma'_f}{E}(2N)^b + \varepsilon'_f(2N)^c \tag{7}$$

The calculated (predicted) and the experimentally measured fatigue life cycles for normal strain criterion are plotted in Figure 6. Figure 6 shows that for pure bending or when the normal strain (stress) is a major part of the critical strain (stress) developed in the notched component due to multiaxial loading ($B > T$), normal strain criterion works well.

The critical plane approach with normal strain criterion deals with the normal strain amplitude in the critical plane, while fatigue cracks normally start

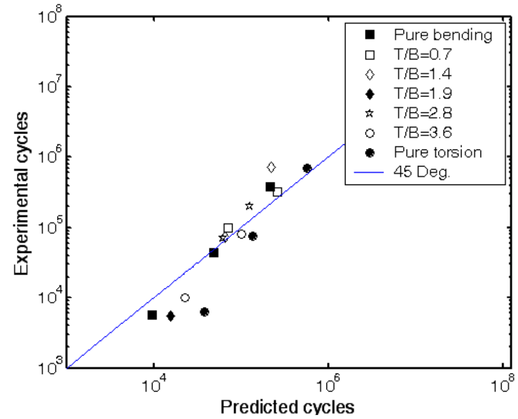


Figure 5. Experimental vs. predicted fatigue life cycles using Hoffmann and Seeger method.

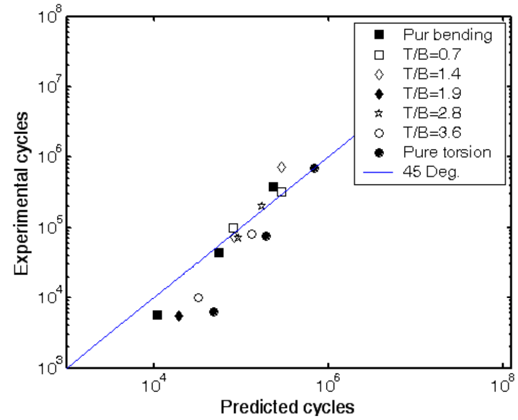


Figure 6. Experimental vs. predicted fatigue life cycles using critical plane with normal strain criterion.

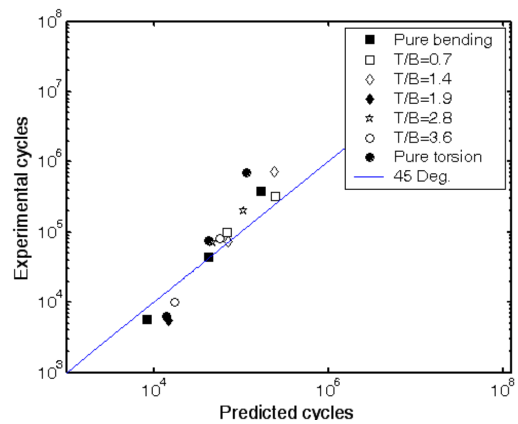


Figure 7. Experimental vs. predicted fatigue life cycles using critical plane with shear strain criterion.

in a shear mode. Therefore, life prediction with this criterion cannot deliver good results when applied to torsion test cases. In the critical plane with shear strain criterion, the shear strain amplitude, $\Delta\gamma/2$, is also calculated. Fatigue life is then predicted with the following equation [16],

$$\frac{\Delta\gamma}{2}=\frac{(1+\upsilon_e)\,\sigma'_f}{E}\,(2N)^{b_0}+(1+\upsilon_p)\,\varepsilon'_f\,(2N)^{c_0} \tag{8}$$

Predicted and experimentally measured fatigue lives for shear strain criterion are shown in Figure 7.

Like maximum shear strain criterion in strain-life method, critical plane approach based on shear strain criterion has good performance in LCF regions approximately. Shear strain criterion is conservative in predicting the fatigue lives on HCF region too. Fatemi and Socie [17] proposed a model, which is;

$$\frac{\Delta\gamma}{2}\left(1+k\frac{\sigma_{n,\max}}{\sigma_y}\right)=\frac{\tau'_f}{G}\,(2N)^{b_0}+\gamma'_f\,(2N)^{c_0} \tag{9}$$

where $\frac{\Delta\gamma}{2}$ is the largest amplitude of shear strain in the critical plane, $\sigma_{n,\max}$ is the maximum normal stress in the maximum shear strain amplitude plane, k is a material constantly fitted to merge uniaxial and torsional fatigue test data, τ'_f, γ'_f, b_0 and c_0 are the fatigue strength coefficients, fatigue ductility coefficients, fatigue strength and ductility exponents, respectively obtained from pure torsional fatigue testing.

Predicted and experimentally measured fatigue life cycles for Fatemi and Socie criterion are shown in Figure 8. It can be seen that Fatemi and Socie model gives good fatigue life prediction for different loading cases.

The Smith, Watson and Topper (SWT) model [18] for multiaxial loading is based on the strain amplitude normal to the critical plane, $\Delta\varepsilon_n/2$ and maximum normal stress in the critical plane, $\sigma_{n,\max}$,

$$\sigma_{n,\max}\frac{\Delta\varepsilon_n}{2}=\frac{{\sigma'_f}^2}{E}\,(2N)^{2b}+\sigma'_f\varepsilon'_f\,(2N_f)^{b+c} \tag{10}$$

The calculated (predicted) and experimentally measured fatigue life cycles for critical plane with SWT model are shown in Figure 9.

The SWT model predicts good fatigue life when the normal stress (strain) is greater than the shear stress (strain) in the critical region of the structural component ($B > T$). The reason is that this method is developed taking into consideration the tensile mode of fatigue failure. This approach overestimates the fatigue lives in LCF severely.

All the critical plane approaches are compared in Figure 11 with the experimentally measured life cycles for different loading conditions. It shows that among all the critical plane approaches investigated, Fatemi and Socie model gives approximately satisfactory fatigue life prediction for different loading cases. Because the model takes the effect of both shear and tension modes of the fatigue failure.

NONPROPORTIONAL MULTIAXIAL LOADING CRITICAL PLANE METHODS

Some engineering components and structures such as aircraft, automobile and rotary drilling rig are subjected to multiaxial nonproportional loading conditions. In plane biaxiality ratio varies over the time history of the cyclic loading and also does the angle of maximum principal stress. The design of these components requires the life prediction under nonproportional multiaxial loading.

The strain based methods and also Hoffmann and Seeger method with simplified approximations are not valid for these nonproportional multiaxial loading. These approaches have some limitations when applied to nonproportional loading paths. A main drawback of the equivalent strain (or stress) approaches is that they are generally nonconservative under nonproportional loading [15]. From a physical observation perspective,

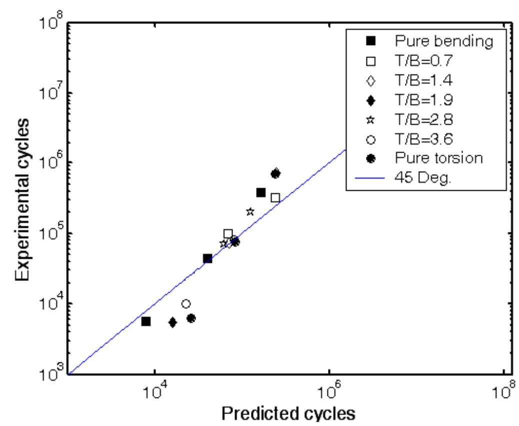


Figure 8. Experimental vs. predicted fatigue life cycles using Fatemi and Socie critical plane model.

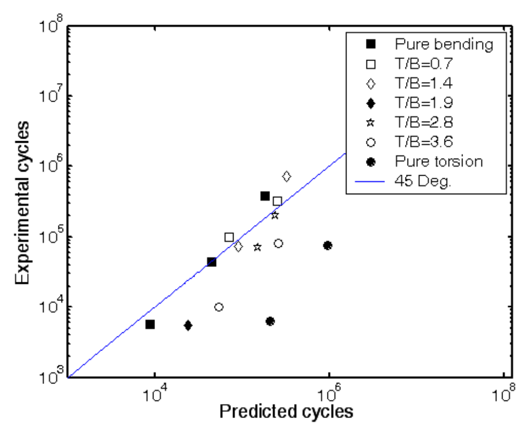
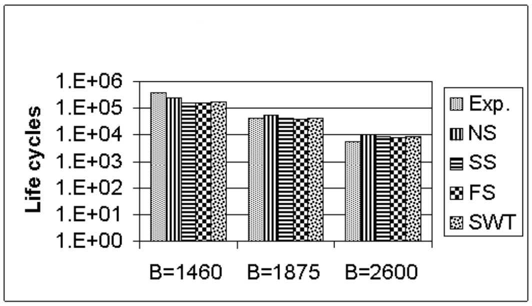


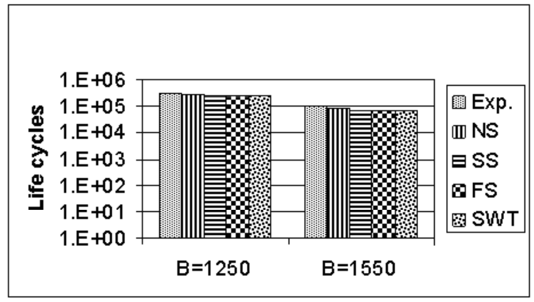
Figure 9. Experimental vs. predicted fatigue life cycles using SWT critical plane model.

it is seen that fatigue crack initiation is a directional process; therefore, critical plane approaches are there which should predict fatigue life better for nonproportional multiaxial loadings over all other existing strain life methods. The four critical plane approaches mentioned, normal strain, shear strain, Fatemi and Socie, and SWT critical plane approaches are selected here and so evaluated under nonproportional loading. The equations of these critical plane approaches are given in the previous section.

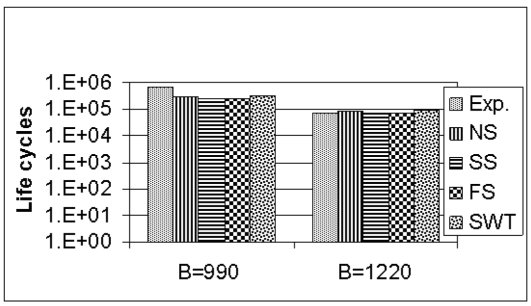
MSC-Fatigue supports multiaxial critical plane methods both for the proportional and nonproportional loading. Multiaxial assessment for 90° out-of-phase loading problem is done in MSC-Fatigue.



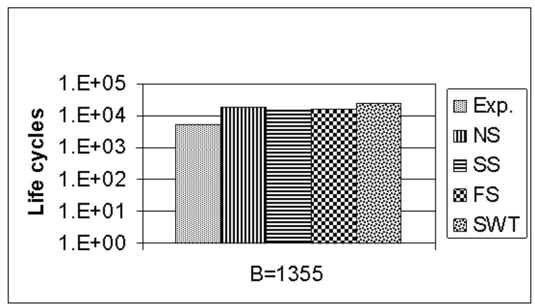
(a) Pure bending.



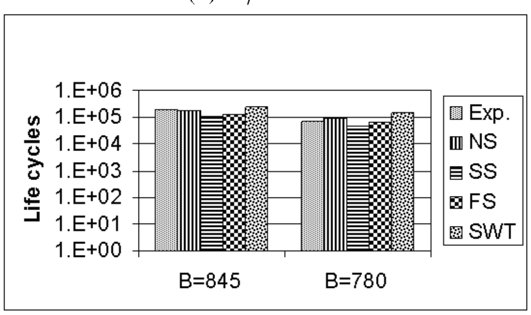
(b) $T/B = 0.7$



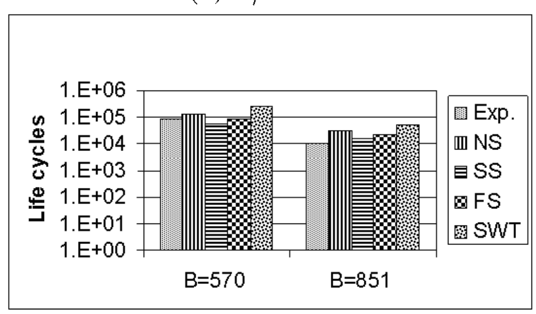
(c) $T/B = 1.4$



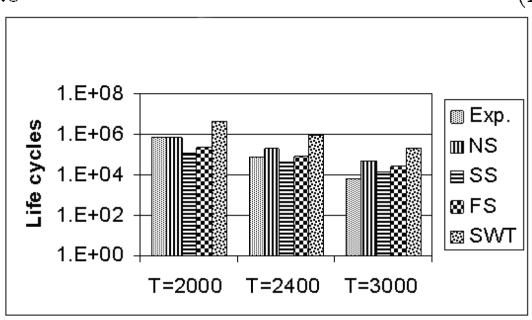
(d) $T/B = 1.9$



(e) $T/B = 2.8$



(f) $T/B = 3.8$



(g) Pure torsion.

Figure 10. Comparison among critical plane and experimental fatigue lives for proportional loadings, B and T in N.m; Exp.=Experimental, NS=Normal Strain, SS=Shear Strain, FS=Fatemi and Socie, SWT=Smith, Watson and Topper.

The stress results from the static load cases are divided by the values of static loads applied to the "MSC-PATRAN" finite element model. Normalized 90° out of phase loading histories are shown in Figure 12. They are scaled during stress superposition within "MSC-Fatigue". Figure 13 shows biaxiality ratio and phase angle of maximum principal stress distribution absolute maximum principal stresses over 90° out of phase loading time histories. The analysis is done with bending moment and torque applied in the model.

Figure 13 shows that biaxiality ratio and phase angle distribution are mobile over the time history, so the problem is a multiaxial nonproportional loading case.

Fatigue life predictions for different multiaxial nonproportional loading conditions are carried out for the critical plane approaches. For different bending, torsion and combined bending-torsion loading conditions, the critical plane approaches reported experimentally measured life cycles [16] are compared in Figure 13. Figure 13 shows that among all the critical plane approaches described, Fatemi and Socie model gives best fatigue life prediction for different loading conditions approximately.

This is because this model takes the effect of both shear and tension modes of fatigue failure. One of the advantages of this critical plane approach is that the effect of strain hardening on fatigue, which often becomes serious under nonproportional multiaxial loadings, can be accounted for with the maximum stress term $\sigma_{n,\max}$, in the appropriate critical plane parameter. Although the Fatemi and Socie model could be an effective tool for multiaxial fatigue life prediction under complex loading conditions, the approach still needs to be tested more for LCF and HCF regions and in the presence of mean stress effects.

CONCLUSIONS

There is no general strain fatigue life criterion suitable to predict the fatigue life of different combinations of torsion and bending loadings. The choice of selecting a criterion to predict the fatigue crack initiation depends on the fatigue domain (LCF or HCF). Strain-based approaches are suitable fatigue life predictors for proportional loadings (Figure 5a to c).

Equivalent strain approaches measure average strain in a small volume of the material; therefore, they may not be used to predict fatigue life of Torsion/Bending load cases. Moreover, these methods are unsuitable for the nonproportional loading cases. Among the strain based approaches, Hoffmann and Seeger method seems to be more reliable (Figure 5), but it does not work for nonproportional loading conditions.

The present study shows that critical plane models perform better than strain based approaches for both proportional and nonproportional loading (Figures 6-10 and 13). The plane where damage takes its maximum value is called critical plane.

The critical plane approach based on shear strain criterion has approximately good performance in LCF regions and is conservative in predicting the fatigue lives in HCF regions (Figure 7). Life prediction by critical plane approach based on normal strain criterion is not suitable for torsion cases, but works well for pure bending (Figure 6). The SWT model has good life predictions when the normal stress (strain) is greater than the shear stress (strain) in the critical region of the structural component (Figure 9). It overestimates the fatigue lives.

Among all the critical plane approaches considered, Fatemi and Socie model gives satisfactory fatigue life prediction for both multiaxial in phase and 90° out of phase loading cases (Figures 8, 11 and 14).

The main objectives of this paper are to:

1. Get discernment into usually used fatigue criteria for multiaxial proportional and non- proportional loadings.
2. Investigate existing fatigue models using finite element based "MSC-Fatigue" and compare those that predict the fatigue crack initiation for multiaxial loading conditions.

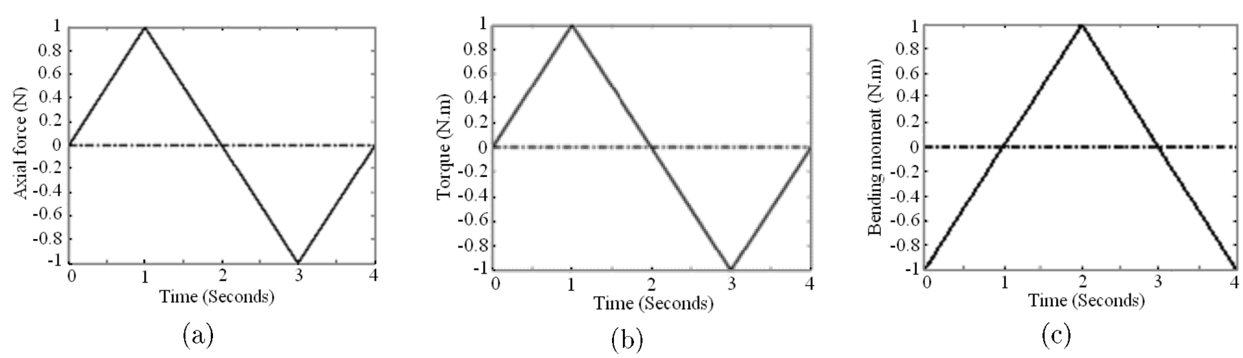
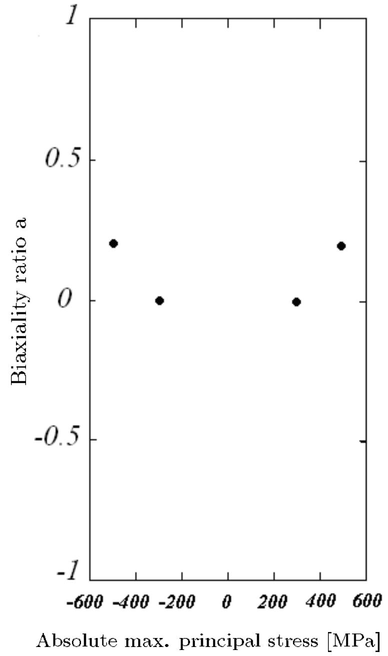
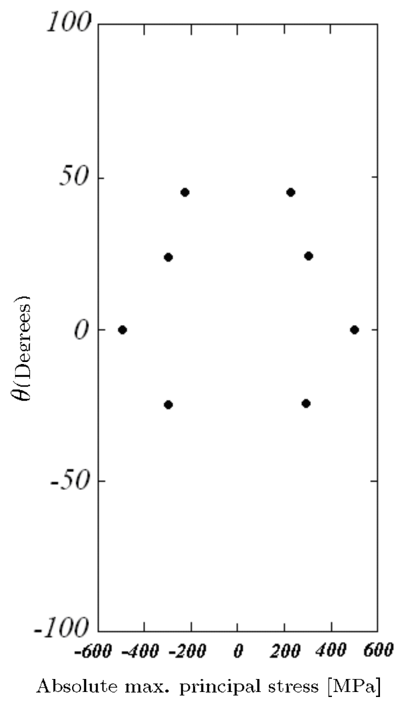


Figure 11. Normalized 90° out of phase loading history.



(a) Biaxiality ratio vs. absolute max. Principal Stress.



(b) Phase Angle vs. absolute max. Principal Stress.

Figure 12. Multiaxiality assessments for nonproportional loading distribution over the time history (0 to 4 Secs.)

3. Analyze the SAE test shaft by Finite Element Method, followed by a multiaxiality assessment to know the loading type and the suitable model to be used thereby.
4. Use the results of the finite element analysis with "MSC-Fatigue" to predict life for proportional and nonproportional multiaxial loading conditions.

Predicted fatigue lives are then compared to experimentally measured fatigue lives published in the literature [13 and 16].

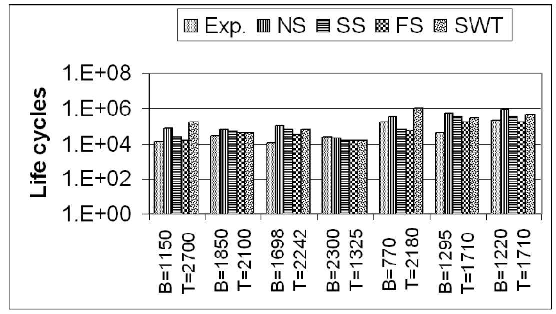


Figure 13. Critical plane and experimental fatigue lives comparison for 90° out of phase loadings, B and T in $N.m$.

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