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Design and Comparison of MPC and LQR Control Methods for a Passenger Aircraft

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ABSTRACT

The paper compares the performance of two altitude controllers, a model predictive controller (MPC) and linear quadratic regulator (LQR), for aircraft in cruise flight and height change conditions. The design of the controllers is based on the linearized state space matrix of the aircraft's longitudinal motion around the trim conditions. The controllers' ability to track the desired altitude while satisfying input and state constraints is evaluated, and it is found that both controllers are effective in maintaining the desired height. However, the MPC controller performs less overshoot, settling time, and transient error than the LQR controller and achieves a more efficient control input by predicting the future behavior of the system. The proposed altitude controllers provide a promising solution for maintaining the desired aircraft altitude in cruise flight conditions, and the comparative analysis of the two control methods can assist in selecting the appropriate control strategy for a given aircraft system based on the desired performance requirements

Keywords: *MPC, LQR, Fixed-wing, Aircraft, Altitude control*

Introduction

With the development of the air transportation industry and the increasing use of flying devices to carry passengers, planes with less passenger capacity are expanding; jet planes are also a category of this group that usually has more agility and maneuverability [1]. The design of the control system determines whether there is sufficient knowledge of the dynamics and operation of the flying device. As we know, in real conditions, the relationships governing the dynamics of a flying vehicle are nonlinear and variable with time. On the other hand, we faced problems such as un-modeled dynamics, the coupling of the equations governing the flying vehicle, and the ambiguity of the effect of aerodynamic coefficients on parameters such as the angle of attack, the angle of side slip, etc. Suppose there is sufficient knowledge of the system, despite the mentioned problems. In this case, it is possible to use the basics of linear control or to use methods that are generalizations of linear control methods [2], [3], [4]. Flight control systems for passenger aircraft are still predominantly designed using classical control techniques. However, in recent years, modern methods have found more and more applications [5], [6]. The current flight control systems provide augmented stability and control. However, in the case of severe or unforeseen failures or changes in aircraft behavior (e.g., due to icing), the control system reverts to reversionary

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modes or even direct control; this implies that the control law functionality is partly reduced or abandoned. This behavior is undesirable, as the pilot's workload is not only increased due to failure but also due to aircraft control. Developments subsequently focus on maintaining functionality, even in the case of such failures. Research in this field includes reliable fault detection and diagnosis and control reconfiguration [7],[8]. One aspect is the adaptation of flight control laws to unforeseen circumstances and failures[9]. Although model predictive controllers provide many advantages over classical or modern control methods (such as PID or LQR), their practical applications have been limited to high-level path planning, guidance logic, and control of slow robots with less complex dynamics[10]. In the paper [11], a robust MPC-based autopilot is designed for a mini unmanned aerial vehicle, where the key features of the proposed technique are: (i) the control gain matrix is evaluated offline to guarantee the real-time feasibility of the MPC; (ii) the controller is robust to parametric model uncertainties (i.e., mass and inertia variations) and random bounded noise (i.e., gust). Lateral control during aircraft-on-ground deceleration is discussed in [12] based on linear, quadratic, and predictive control theories. One of the problems in passenger planes is the control of the plane in flight maneuvers.

Reference [13] introduces a new incremental predictive guidance method based on a generalized incremental predictive control approach, which utilizes both previous and present states for better robustness and tracking performance. The method is evaluated through computer simulations, showcasing significant improvements compared to standard generalized predictive control. [14] proposes a fault-tolerant control strategy based on MPC for a dual space tether system, aiming to continue missions in case of failure. A new and accurate model of a space tether is introduced, with the ability to modify the junction to the subsatellite, resulting in decreased fuel usage and successful control despite thruster and tether failures.

In this paper, the dynamic model of a passenger plane is derived using Euler's equations. The nonlinear model subsequently is linearized around the cruise flight, and the two controllers, model predictive control (MPC) and linear quadratic regulator (LQR) are compared for executing the desired maneuver. The selection of these controllers is based on the quadratic optimization in their control laws. The primary difference lies in applying online constraints and model prediction in MPC.

The outline of this paper is as follows: In section II, we will deal with the mathematical modeling of the Cessna Citation II (550) aircraft. In section III, the LQR and MPC controllers are described. In section IV, the numerical results will be investigated and compared and finally, the concluding remarks are stated in section V.

Mathematical modeling

The control laws developed in this research are evaluated in the Cessna Citation II PH-LAB aircraft, as shown in Fig 1, with the corresponding aerodynamic model coefficient which is derived from the flight test data [15]. The aerodynamic derivatives are driven from flight tests [16] while the vehicle dimension, and mass, and inertia properties are listed in Table 1.

Fig 1. Cessna Citation II PH-LAB [9].

Table 1. Cessna Citation II PH-LAB dimension and mass properties [15].

Dimension	
b	15.9m
$\bar{c}$	2.09m
S	$30m^2$
Mass and inertia	
m	4,157kg
U_0	$225\frac{m}{s}$
I_{xx}	$12392kg.m^2$
I_{yy}	$31501kg.m^2$
I_{ZZ}	$41908kg.m^2$
I_{xz}	$2252.2kg.m^2$

The dynamic equations of the aircraft are extracted as:

$$\dot{u} = X_u u + X_w w - g\cos\theta_0 \theta + X_{\delta e}\delta e \quad (1)$$
$$\dot{w} = Z_u u + Z_w w + U_0 q - g\sin\theta_0 \theta + Z_{\delta e}\delta e \quad (2)$$
$$\dot{q} = M_u u + M_w w + M_q q + M_{\delta e}\delta e \quad (3)$$
$$\dot{\theta} = q \quad (4)$$
$$\dot{h} = -w + U_0\theta \quad (5)$$

where g is the earth's gravity, U_0 is the initial speed of the plane, u is the horizontal speed, w is the vertical speed, q is the angular speed around the longitudinal axis, and the pitch angle θ, X_u is stability derivative in the direction of the horizontal body axis relative to horizontal speed, X_w is stability derivative in the direction of the horizontal body axis relative to vertical speed, X_{δ_e} is control derivative in the direction of the horizontal body axis relative to the elevator deflection, Z_u is stability derivative in the direction of the vertical body axis relative to horizontal speed, Z_w is stability derivative in the direction of the vertical body axis relative to the vertical speed, Z_{δ_e} is control derivative in the direction of the vertical body axis relative to the elevator deflection, M_u represents the change in pitching moment caused by a change in forward speed, M_w is the change in pitching moment caused by a change in vertical speed, M_q is the change in pitching moment caused by a change in the rate of pitch angle, M_{δ_e} is the change in pitching moment caused by a change in elevator deflection. The matrices A and B in the state space model of the system are

$$A = \begin{bmatrix} X_u & X_\omega & 0 & -g\cos\theta_0 & 0 \\ Z_u & Z_\omega & U_0 & -g\sin\theta_0 & 0 \\ M_u & M_\omega & M_q & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & U_0 & 0 \end{bmatrix}, B = \begin{bmatrix} X_{\delta e} \\ Z_{\delta e} \\ M_{\delta e} \\ 0 \\ 0 \end{bmatrix} \quad (6)$$

Table 2. The value of stability and control coefficient [15]

Stability & control coefficients			
X_u	-0.0072	Z_{δ_e}	0
X_w	0.1532	M_u	0
X_{δ_e}	15.8181	M_w	-0.0593
Z_u	-0.6753	M_q	-1.3017
Z_w	-1.7716	M_{δ_e}	-24.0336

Flight control design

LQR Controller

The goal of this controller is to find a control input that minimizes a cost function in the following form

$$J = \frac{1}{2}\int_{t_0}^{t_f} [(x(t) - x_d(t))^T (t) Q (x(t) - x_d(t)) + u_c^T(t) R u_c(t)] dt \quad (7)$$
$$\dot{x}(t) = Ax(t) + Bu_c(t) \quad (8)$$

where, x(t) is the state vector and is equal to $[u \quad w \quad q \quad \theta \quad h]^T$, $u_c(t)$ is the control input equal to δe, A is the system state matrix, B is the control matrix, and R and Q are positive and non-negative definite weight matrices.
As a result, the control law will written as follows.

$$u_c(t) = -K[(x(t) - x_d(t))]^T \quad (9)$$

Model Predictive Control

The utilization of MPC necessitates using a model to anticipate the system's behavior in the future. The process model enables the system to estimate the future actions of the process. Fig 2 illustrates the block diagram of the control system, which is predicated on model-based prediction.

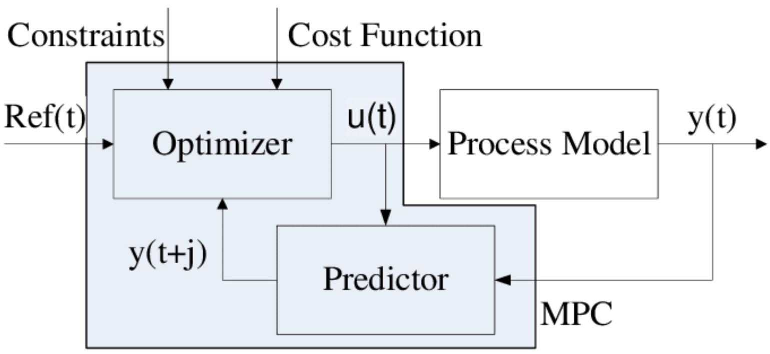

Fig 2. Genral MPC block diagram.

Control oriented model is a time-invariant linear system, therefore discrete-time equations of the system will be as follows:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu_c(k) \\ y(k) &= Cx(k) + Du_c(k) \end{aligned} \tag{10}$$

According to equation (10), as well as the relationship between the state and control variables $x(k), u_c(k)$, the prediction of the state $x(k+1)$ can be obtained as equation (11).

$$x_{n\times 1}(k+1|k) = A_{n\times n}x_{n\times 1}(k|k) + B_{n\times m}u_{c_{m\times 1}}(k|k)$$

$$x(k+2|k) = A^2x(k|k) + ABu_c(k|k) + Bu_c(k+1|k) \tag{11}$$

$$\vdots$$

$$x\left(k+N_p\middle|k\right) = A^{N_p}x(k|k) + A^{N_p-1}.B.u_c(k|k) + \cdots + A^{N_p-N_c}.B.u_c(k+N_c-1|k)$$

In the above relationship, N_p is the prediction horizon and N_c is the control horizon. The prediction equation for a system model can be written as equation (12)

$$X_{nN_p\times 1} = F_{nN_p\times n}x_{n\times 1}(k|k) + \Phi_{nN_p\times mN_c}U_{mN_c\times 1} \tag{12}$$

Where $X \in R^{nN_p}$, $F \in R^{nN_p\times n}$, $\Phi \in R^{nN_p\times mN_c}$ and $U \in R^{mN_c}$ and F and Φ are shown in equations (13), (14)

$$F \triangleq \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^{N_p-1} \\ A^{N_p} \end{bmatrix}_{nN_p\times n} \tag{13}$$

$$\Phi \triangleq \begin{bmatrix} B & \bar{0} & \dots & \bar{0} \\ AB & B & \dots & \vdots \\ \vdots & \vdots & \ddots & \bar{0} \\ A^{N_p-1}.B & A^{N_p-2}.B & \dots & A^{N_p-N_c}.B \end{bmatrix}_{nN_p\times mN_c} \tag{14}$$

Implementation and Results

To compare the efficacy of the designed controllers, we employed identical optimization conditions and conducted a comparative analysis of the results. Specifically, we designed the controller by considering the weight matrices Q and R under similar conditions and defined them as follows:

$Q = \text{diag}([10,10,10,10,10])$

$R = 0.1$

The MPC controller is designed with a sampling time of one second, a prediction horizon of 30, and a control horizon of 8. However, due to constraints inherent in the aircraft and actuator during cruise flight, we defined the control constraint for the elevator angle as follows:

$$-0.5\ rad \leq \delta e \leq 0.5\ rad$$

To compare the performance of two controllers in a specific flight maneuver, Fig 3 illustrates that both controllers exhibited similar path-following characteristics.

Moreover, Fig 4 and Fig 5 depict the aircraft's angle of attack under LQR and MPC controllers, respectively, when the aircraft seeks to adjust its height by 200 meters. Notably, the LQR control's angle of attack reaches the stall angle, causing the aircraft to stall. Conversely, the maximum angle of attack observed under MPC control is lower than the stall angle.

Finally, Fig 6 and Fig 7 display the perturbed velocity of the aircraft, which indicates that both controllers exhibit almost identical behavior.

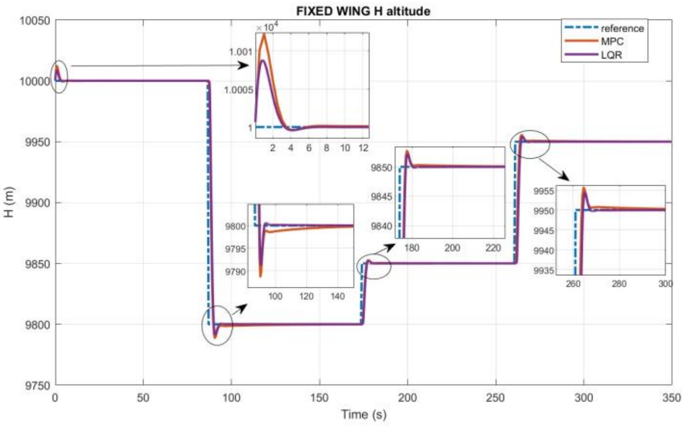

Fig 3. Aircraft altitude response to the given scenario using the LQR controller and MPC

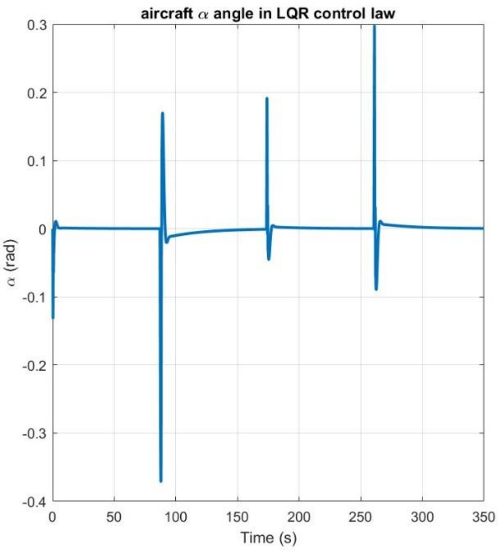

Fig 4. Aircraft angle of attack in LQR control

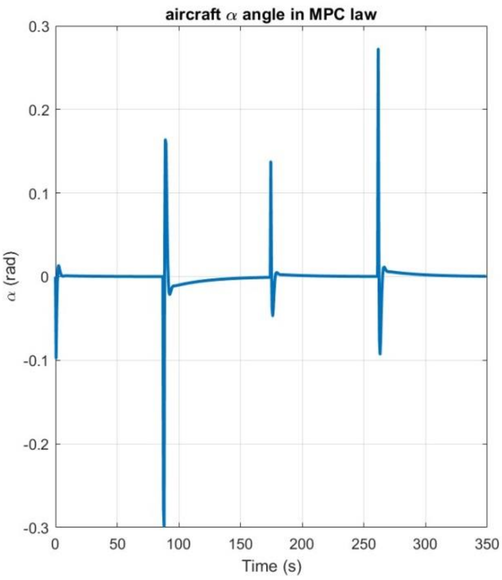

Fig 5. Aircraft angle of attack in MPC control.

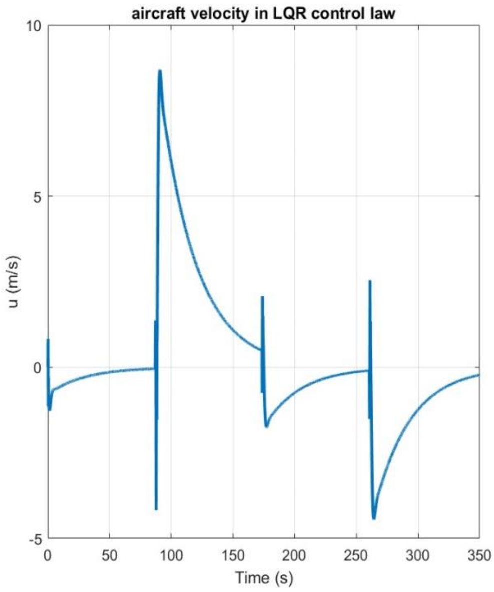

Fig 6. Perturbed longitudinal speed in LQR control

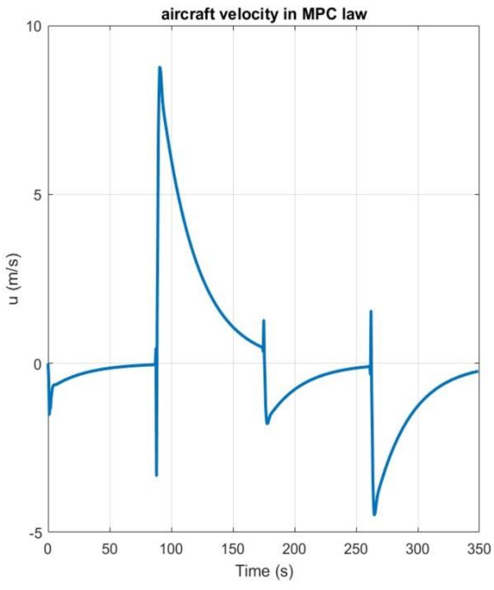

Fig 7 Perturbed longitudinal speed in MPC control

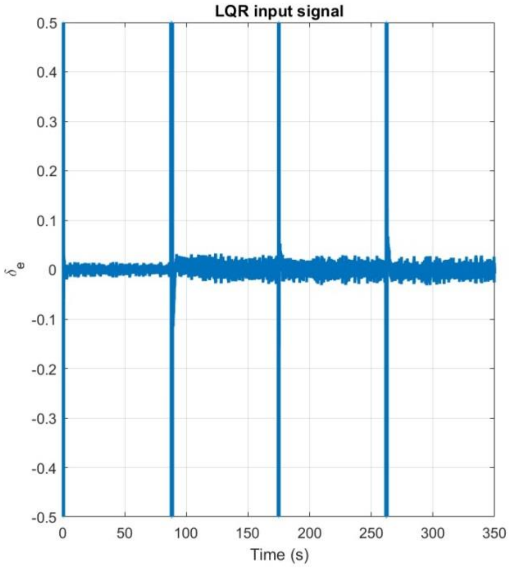

Fig 8. LQR control signal input (Elevator).

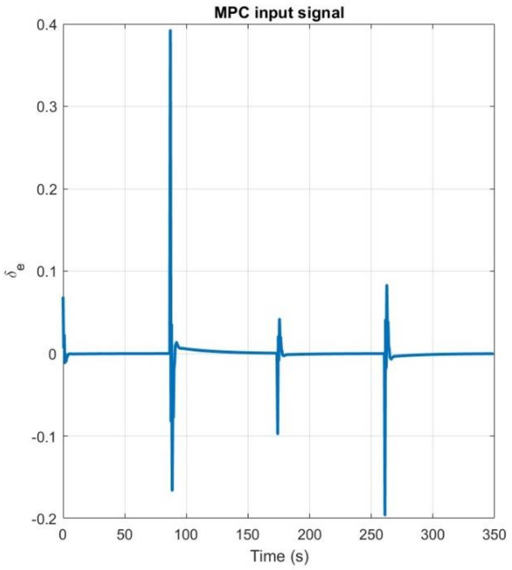

Fig 9. MPC control signal input (Elevator).

Based on the results in Fig 8, the LQR controller applies the control input (i.e., elevator) over its maximum range. Conversely, the control signal in the MPC, as illustrated in Fig 9, employs a reduced range to monitor the path. Notably, the control signal in the LQR approach exhibits a noise range of 0.02. This noise could be attributed to actuator fatigue or the potential for the actuator to disobey the control signal due to the actuator's dynamics. However, in an ideal setting, this noise does not exist in the MPC technique.

Considering the points mentioned above and the fact that the control signal of the LQR method goes out of the linear region when the saturation limit is reached, and the simulation results become invalid, we try the LQR controller again with different weight matrices by trying and error. The control signal and height error criteria are followed to get the desired value and a compromise between the values of the system state variables. Considering that our system is an airplane, it cannot tolerate many negative angles of attack. For this purpose, we tried to assign the largest coefficient to this variable. The selected weight matrices are as follows:

$$Q = \text{diag}\,[1 \quad 50 \quad 10 \quad 1 \quad 1]$$

$$R = 200000$$

In Fig 10, following the maneuver given by the two redesigned LQR controllers and MPC, we can see that the redesigned LQR controller, faced with the initial conditions of the climb angle of 0.1 radians, has two times overshoot and three times the settling time compared to the previous MPC controller. Also, in the following height changes, it reaches the desired value 10 to 15 times later than the MPC. Fig 11 shows the aircraft's angle of attack during the desired maneuver, and Fig 12 shows that the changes in the aircraft's speed have not changed significantly compared to the MPC method and are the same. In Fig 13, we can see that if the LQR method is to be a control signal in the linear range of the operator, its efficiency will be significantly reduced, and the MPC method will work better.

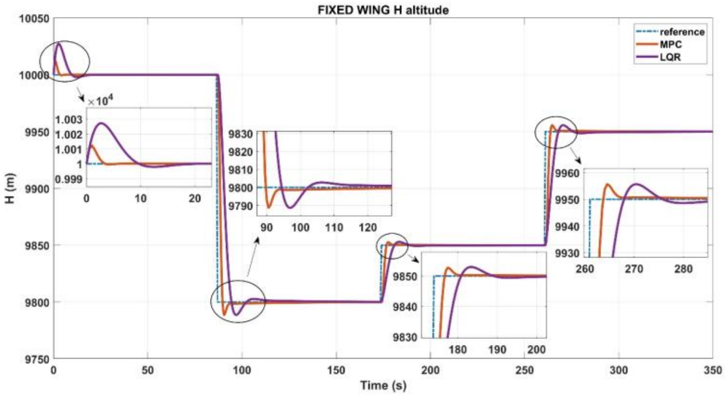

Fig 10. Aircraft altitude response to the given scenario using the MPC and redesigned LQR controller

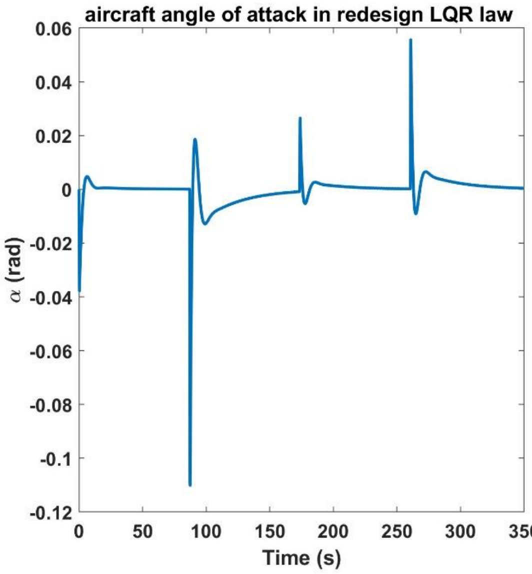

Fig 11. Aircraft angle of attack in redesigned LQR control.

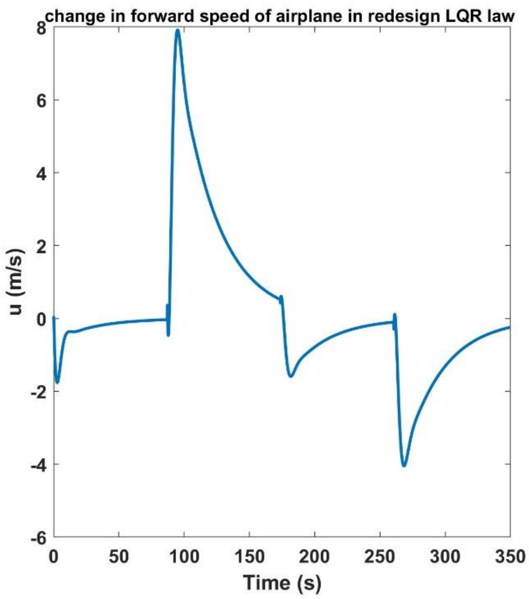

Fig 12. Perturbed longitudinal speed in redesigned LQR control

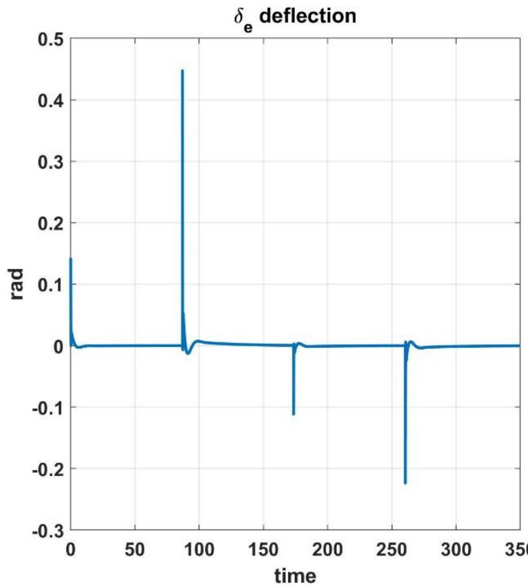

Fig 13. Redesigned LQR control signal input (Elevator).

Conclusion

This paper investigates the altitude adjustment of a passenger aircraft using two controllers: model predictive control (MPC) and linear quadratic regulator (LQR), accounting for the actuator limitations. After designing the aircraft's linear dynamics controller, the MPC controller exhibits better altitude adjustment behavior. Moreover, MPC control does not result in actuator saturation, whereas LQR control experiences saturation multiple times. Because of this problem, we chose another weighting matrix and observed that without saturation, the MPC controller performs better tracking with less error and faster response to commands compared to the LQR law. The control input or the elevator signal is also more practical in the MPC control system.

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