Pages: 31-37 / Vol. 16/ Issue 2 / Summer- Fall 2023
Journal of Aerospace Science and Technology
www:jast.ias.ir

Scientific-Research Article

On the Effects of Stiffness Ratio in Nonlinear Aeroelastic Stability of a Folding Wing

Sayed Hossein Moravej Barzani [1] , **Mahdi Mortazavi**[2]*, **Hossein Shahverdi**[3]

1-2- Department of Mechanical Engineering, Faculty of Engineering, University of Isfahan, Iran
3- Department of Aerospace Engineering, Amirkabir University of Technology ,Tehran, Iran

ABSTRACT

Keywords: : *Folding wing- nonlinear aeroelasticity- fully intrinsic equations- stiffness effects- folding angle.*

In this paper, the effects caused by the combination of folding angles simultaneously with changing the stiffness ratio of different parts of a folding wing are investigated. The geometrically exact fully intrinsic equations are employed to simulate the wing's nonlinear dynamic behavior. The important advantages of these geometrically exact equations can be seen as complete modeling without simplifying assumptions in large deformations, low-order nonlinearities, and thus less complexity. In this research, folding angles have been used in the geometrically exact fully intrinsic beam equations and the combination of different folding angles is studied. The applied aerodynamic loads in an incompressible flow regime are determined employing Peter's unsteady aerodynamic model. In order to check the stability of the system, first the resulting non-linear partial differential equations are discretized and then linearized about the nonlinear steady-state condition. By obtaining the eigenvalues of the linearized system, the stability of the wing is evaluated. Furthermore, the investigation of the effects of stiffness on the flutter speed and frequency of the folding wing for various folding angles is another achievement of this study. It is observed that the combination of folding angles can significantly delay the flutter speed and improve the performance of the bird.

Introduction

The possibility of planform changes (extension, folding, sweep) in the airplane wing turns it into a morphing wing. One of the states of changing the wing's planform is folding, which can improve the bird's performance by choosing the appropriate parameters. Many researchers have studied the dynamic behavior of folding wings. One of the most prominent works in the aeroelastic study of folding wings is the research of Lee and Chen [1]. In their study, the nonlinear aeroelastic characteristics of a folding wing with free-play or piecewise nonlinearities at the inboard and outboard hinges are investigated. Ameri, et al. [2] presented a simulink multibody model for active winglets. The results of this research have shown a non-negligible dependency of the dynamic transient behavior on the shape variation. Tang and Dowell [3] used a model for the theoretical and experimental aeroelastic study of folding wings. This study was

1 PhD.
2 Associate Professor (Corresponding Author) Email: * ma.mortazavi@eng.ui.ac.ir
3 Associate Professor

10.22034/jast.2023.396484.1149
Submit: 09. May. 2023/ Accepted: 22. May 2023
Print ISSN:2345-3648 Online ISSN: 1735-2134

then extended by combining the von Karman strain theory with the three-dimensional vortex lattice aerodynamic model [4] to investigate the limit cycle oscillations of folding wings. Liska and Dowell [5] developed an analytical solution methodology to find the flutter speed of a two-segment uniform folding wing. In this research, different aerodynamic models were studied and compared, and the effect of each on flutter speed and folding angle was examined. Zhao and Hu [6] studied the flutter characteristics of a folding wing based on the substructure synthesis and aerodynamic doublet lattice method. This study demonstrated that the flutter characteristics of the folding wing are very sensitive to the folding angle. Ajaj, et al. [7] investigated the aeroelasticity of cantilever wings equipped with flared hinge folding wingtips. In this study, the wing structure is modelled using Euler–Bernoulli beam theory, and unsteady. Theodorsen's aerodynamic theory is employed for aerodynamic load predictions. Also, the influence of tip mass, hinge stiffness, folding angle, hinge-line angle, and nonlinear hinge behavior are investigated. Moravej Barzani and Shahverdi [8] used geometrically exact fully intrinsic beam equations for the first time to investigate the aeroelastic instability of a folding wing. They proved that the geometrically exact fully intrinsic beam equations can model the folding angles for the aeroelastic analysis more accurately.

The present study is based on the geometrically exact fully intrinsic beam theory which was initially developed by Hegemier and Nair [9] and then revised by Hodges [10]. The important advantages of these geometrically exact equations can be seen as complete modeling without simplifying assumptions in large deformations, low-order nonlinearities, and thus less complexity. This beam theory has been used by several researchers to investigate the aeroelastic instability of the wing. For example, Sotoudeh and Hodges [11] studied the sweep angle and position of joint effects on the static deformation and dynamic stability of a wing subjected to a follower force. They showed that the static and dynamic stability of the joint-wing is different from conventional wings. Moravej Barzani, et al. [12] investigated the aeroelastic stability of swept wings employing geometrically exact fully intrinsic beam equations. The results showed that by utilizing the fully intrinsic beam equations, the swept wing aeroelastic instability can be determined more accurately. Amoozgar, et al. [13] studied the bend-twist elastic coupling and taper ratio effects in combination with the pre-twist angle to investigate the aeroelastic instability of the wing. Moravej Barzani, et al. [14] investigated the effect of structural nonlinearity on the aeroelasticity of span morphing wings using Peter's unsteady aerodynamic model and the exact fully intrinsic beam equations. They showed that the overlapping mass of the fixed and moving segments and morphing length have significant effects on the aeroelastic stability of telescopic wings. In this study, folding angles have been implemented in the geometrically exact fully intrinsic beam equations[10] and the combination of different folding angles is studied. Investigation of the effects of some important parameters such as stiffness ratio on the speed and frequency of the folding wing flutter in combination with the folding angles, is one of the other achievements of this article which has not been given much attention in previous research.

Governing equations

A schematic of a folding wing is shown in Fig. 1. The wing comprises three parts that have changed surface with folding angles θ_1 and θ_2.

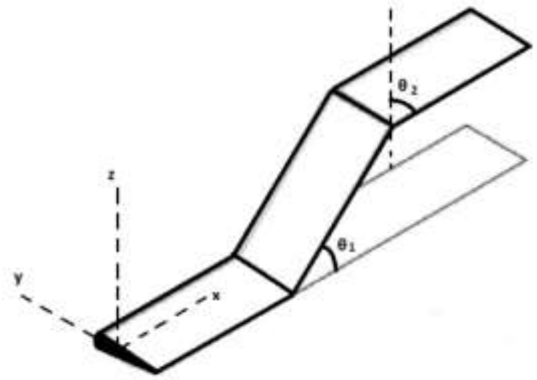

Fig. 1 A schematic of a folding wing.

This study uses geometrically exact fully intrinsic equations to simulate the wing's nonlinear dynamic behaviour.

The geometrically exact fully intrinsic equations can be expressed as [10]

$$\begin{aligned} &F'_B + \widetilde{K}_B F_B + f_B = \dot{P}_B + \widetilde{\Omega}_B P_B \\ &M'_B + \widetilde{K}_B M_B + (\widetilde{e}_1 + \widetilde{\gamma})\Omega_B + m_B \\ &\qquad = \dot{H}_B + \widetilde{\Omega}_B H_B + \widetilde{V}_B P_B \\ &V'_B + \widetilde{K}_B V_B + (\widetilde{e}_1 + \widetilde{\gamma})\Omega_B = \dot{\gamma} \\ &\Omega'_B + \widetilde{K}_B \Omega_B = \dot{k} \end{aligned} \qquad (1)$$

where $\dot{(\,)}$ denotes the absolute time derivative, $(\,)'$ denotes the derivative with respect to the beam reference line and, F_B and M_B are the internal force and moment measures, P_B and H_B are the linear and sectional angular momenta, and κ and γ denote the moment and force strain measures, Ω_B and V_B are the angular and linear velocity measures. Also, f_B and m_B are the external forces and moments. Furthermore, $K_B = k_b + \kappa$ is the beam curvature vector in which k_b is the initial twist and curvature. By using the cross-section stiffness matrix, the generalized strains (γ, κ) and the generalized forces (F_B, M_B) are related together as follows

$$\begin{Bmatrix} \gamma \\ k \end{Bmatrix} = \begin{bmatrix} R & S \\ S^T & T \end{bmatrix} \begin{Bmatrix} F_B \\ M_B \end{Bmatrix} \tag{2}$$

where, S, R and T are the cross-sectional flexibility matrices.

Furthermore, the generalized velocities (V, Ω) and the generalized moments (P, H) can be converted to each other using the cross-sectional inertia matrix as

$$\begin{Bmatrix} P_B \\ H_B \end{Bmatrix} = \begin{bmatrix} \mu\Delta & -\mu\tilde{\xi} \\ \mu\tilde{\xi} & I \end{bmatrix} \begin{Bmatrix} V_B \\ \Omega_B \end{Bmatrix} \tag{3}$$

Where ξ is the cross-sectional mass centroid offset from the beam reference axis, μ is the mass per unit length, I is the inertia matrix per unit length, and Δ is the identity matrix.

By employing Peters' unsteady aerodynamic model, the aerodynamic loads can be presented as[15]

$$f_a^n = \rho b^n \begin{Bmatrix} f_{a1}^n \\ f_{a2}^n \\ f_{a3}^n \end{Bmatrix}$$

$$m_a^n = 2\rho b^{n^2} \begin{Bmatrix} m_{a1}^n \\ m_{a2}^n \\ m_{a3}^n \end{Bmatrix}$$

$$f_{a1}^n = 0$$

$$f_{a2}^n = -\left(C_{l_0}^n + C_{l_\beta}^n \beta^n\right) V_T^n V_{a_3}^n + C_{l_a}^n \left(V_{a_3}^n + \lambda_0^n\right)^2 - C_{d_0}^n V_T^n V_{a_2}^n$$

$$f_{a3}^n = \left(C_{l_0}^n + C_{l_\beta}^n \beta^n\right) V_T^n V_{a_3}^n - C_{l_a}^n V_{a_3}^n b/2 - -C_{l_a}^n V_{a_2}^n \left(V_{a_3}^n + \lambda_0^n - \Omega_{a_1}^n b^n/2\right) - C_{d_0}^{n\mathcal{E}} V_T^n V_{a_3}^n$$

$$m_{a1}^n = (C_{m_0}^n + C_{m_\beta}^n \beta^n) V_T^{n^2} - C_{m_\alpha}^n V_T^n V_{a_3}^n - b^n C_{l_\alpha}^n / 8 V_{a_2}^n \Omega_{a_1}^n - -b^{n^2} C_{l_\alpha}^n \dot{\Omega}_{a_1}^n / 32 + b^n C_{l_\alpha}^n \dot{V}_{a_3}^n$$

$$m_{a2}^n = 0$$

$$m_{a3}^n = 0$$

$$\lambda_0^n = \frac{1}{2} \{b_{\inf low}\}^T \{\lambda^n\}$$

$$[A_{\inf low}]\{\dot{\lambda}^n\} + \left(\frac{V_T^n}{b^n}\right)\{\lambda^n\} = \left(-\dot{V}_{a_3}^n + \frac{b^n}{2}\dot{\Omega}_{a_1}^n\right)\{c_{\inf low}\} \tag{4}$$

where b is the semichord, ρ is the air density, and a is the aerodynamic reference axis. Also, λ is a column matrix which includes states of inflow, and $\{b_{\text{inflow}}\}$, $\{c_{\text{inflow}}\}$, $[A_{\text{inflow}}]$ are constant matrices derived in Peters, et al. [16] work. Also,

$$\overline{V}_a^n = C_a^{n\,T} \overline{V}^n - \tilde{\overline{y}}_{mc}^n C_a^{n\,T} \overline{\Omega}^n$$

$$\overline{\Omega}_a^n = C_a^{n\,T} \overline{\Omega}^n$$

$$V_T = \sqrt{V_{a_2}^2 + V_{a_3}^2}$$

$$\sin\alpha = \frac{-V_{a_3}}{V_T} \tag{5}$$

C is the rotation matrix with the superscripts indicating the two reference frames in between which it transforms. [17]

and y_{mc} is a position vector (row matrix) from the beam reference axis to the midchord and can be written as

$$y_{mc}^n = [0 \quad \overline{y}_{ac}^n - \frac{b^n}{2} \quad 0] \tag{6}$$

Solution methodology

For solving the governing equations, a finite difference discretizing scheme is used (as seen in the work of Chang [15]). To determine the stability of the wing, first the nonlinear steady-state of the system is obtained by dropping all time-dependent variables and solving the resulting nonlinear equation by employing the Newton–Raphson method. Then, aeroelastic stability of the wing is sought by investigating the eigenvalues of this linearized system. The compact form of resulting discretized equations can be written as

$$[\mathbf{A}]\{\dot{\mathbf{X}}\} + [\mathbf{B}(\mathbf{X})]\{\mathbf{X}\} = 0 \qquad (7)$$

where $\{\mathbf{X}\}$ is a vector contains the structural and aerodynamic states.

Results and discussion

To check the validity of the developed aeroelastic model, first, the flutter speed and frequency of the Goland wing is obtained and compared with those reported in the literature. The properties of this wing are presented in Table 1. According to Table 2, It is clear that the obtained results are in very good agreement with those reported by Patil [18] with a maximum difference of 0.3%.

Table 1: The properties of the Goland wing.

Parameter	Value
l(m)	6.1
c(m)	1.83
m(kg/m)	35.7
$EI_{\text{spanwise}}(\text{Nm}^2)$	9.765 x 10^6
$GJ(\text{Nm}^2)$	9.89 x 10^5
Spanwise elastic axis	33% chord
Center of gravity	43% chord
I(50% chord)(kg. m)	8.64

Table 2: The comparison of the flutter speed and frequency.

	Patil (1999)	Present	Difference (%)
u_f(m/s)	135.64	136.06	0.3
ω_f(rad /s)	70.2	70.09	0.2

It should be noted that $v_f = \frac{u_f}{b\omega_\alpha}$ denotes reduced flutter speed, where ω_α is the first uncoupled torsional frequency. Also, the following nondimensional parameter is used for each segment

$$\psi = \frac{EI}{GJ} \qquad (8)$$

In the first step, the effects of changing the angle of θ_1 on the aeroelastic stability of the Goland wing are investigated based on the work of Moravej Barzani and Shahverdi [8]. Fig. 2 shows that at the same time as the first folding angle (θ_1) increases, the reduced flutter speed has decreased to the angle range of 30-40 degrees. Then it suddenly increases and reaches an optimal value in the range of 40-50 degrees. In the following, it will have a downward trend up to about 80 degrees and the highest value at an angle of 90 degrees. Also, the trend of flutter frequency is similar to flutter speed changes.

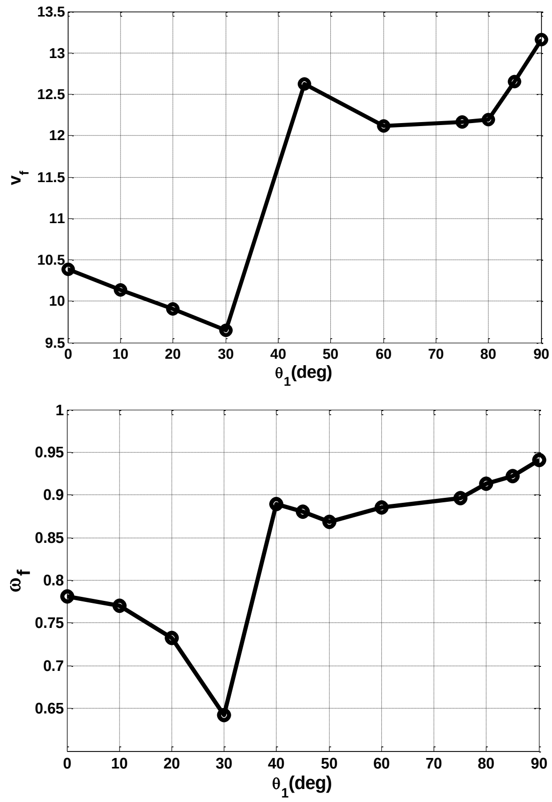

Fig. 2 Reduced flutter speed and flutter frequency vs the first folding angle (θ_1) changes [8].

In what follows, the first folding angle (θ_1) is assumed to be fixed at an optimal value (45^0) that causes a suitable flutter speed. Then the effects caused by the combination of the first folding angle (θ_1) and the second folding angle (θ_2) on the flutter speed and frequency for different values of the second folding angle are investigated. Also, the effects of changing the stiffness of each segment are studied.

Fig. 3 shows that applying the reasonable second folding angle simultaneously with the presence of the first folding angle has been able to have a positive effect on the flutter speed by about 40%. Also, the flutter frequency has changed by about 20% in the investigated range. Furthermore, The effects of changing the stiffness ratio of the first part simultaneously with changing the socond folding angle, on the reduced flutter speed and frequency are shown in Fig. 3. The results show that a 30% reduction (increase) in stiffness ratio of the first part caused an 8-10% increase (decrease) in the flutter speed. Also, the decreasing trend of the flutter frequency is clear.

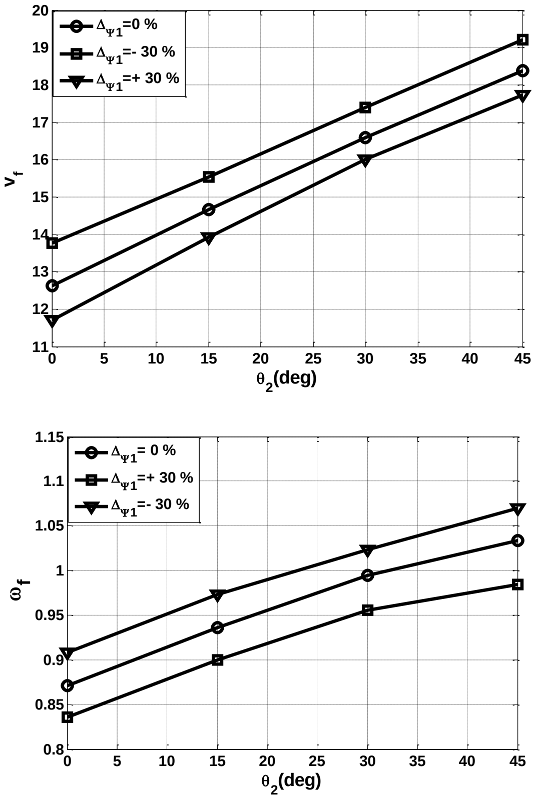

Fig. 3 Reduced Flutter speed and flutter frequency vs the second folding angle (θ_2) for different values of the stiffness ratio of the first part ($\theta_1 = 45^0$).

The effects of changing the stiffness ratio of the second part and changing the second folding angle, on the flutter speed and frequency, are shown in Fig. 4. The results show that a 30% reduction in the stiffness ratio of the second part caused a 2-3% increase in the flutter speed. Also, it can be seen that the trend of the results is consistent with the case in which the first part of the wing was investigated. On the other hand, it can be observed that the change in the stiffness ratio of the second part has had less impact than the first part.

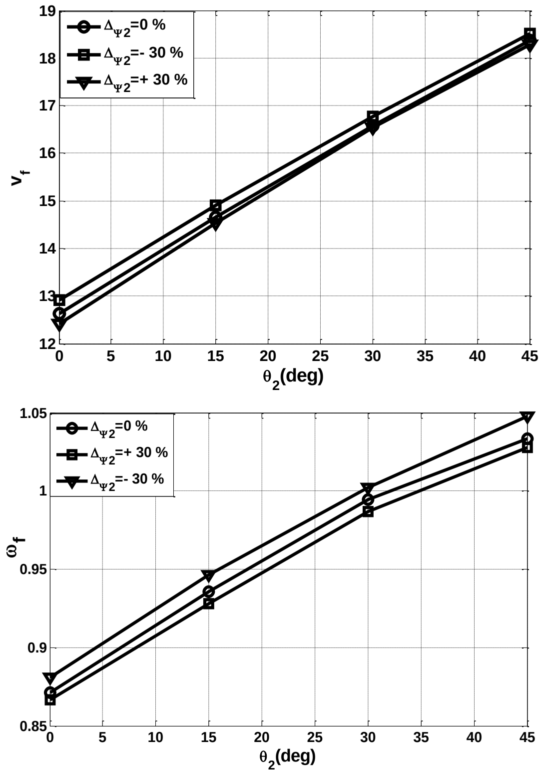

Fig. 4 Reduced Flutter speed and flutter frequency vs the second folding angle (θ_2) for different values of the stiffness ratio of the second part ($\theta_1 = 45^0$).

The effects of changing the stiffness ratio of the third part and changing the second folding angle, on the flutter speed and frequency are shown in Fig. 5. The results show that a 30% reduction in the stiffness ratio of the third part caused an increase in about 1% in the flutter speed and frequency. Also, it can be observed that the change in the stiffness ratio of the third part has had less impact than the second part. Thus, by comparing the results obtained for other parts, it can be concluded that the stiffness ratio of the first part has a more important impact on the flutter speed and frequency than other parts [8]. It should be noted that the positive effect of increasing the second folding angle simultaneously with the presence of the first folding angle exists in all situations and plays an important role.

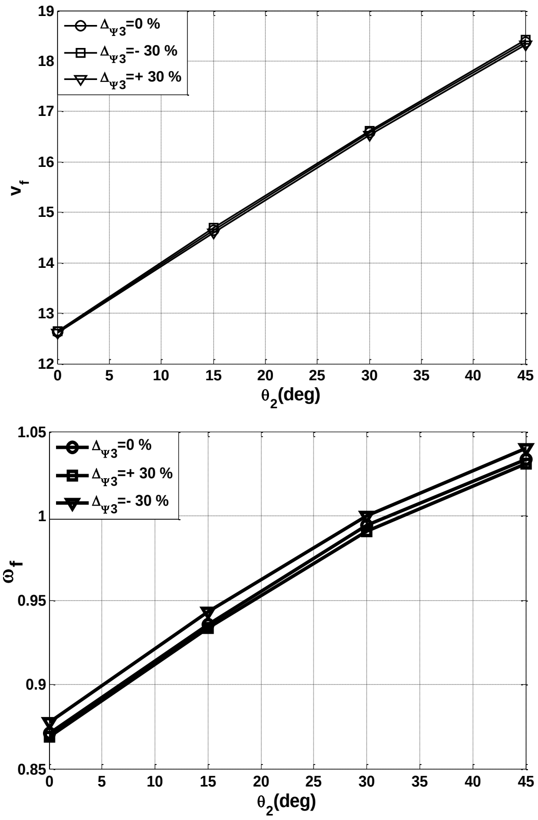

Fig. 5 Reduced Flutter speed and flutter frequency vs the second folding angle (θ_2) for different values of the stiffness ratio of the third part ($\theta_1 = 45^0$).

Concluding Remarks

A schematic of a folding wing is shown in Fig. In this study, the effects caused by the combination of folding angles simultaneously with changing the stiffness ratio of different parts of a folding wing were investigated. Peters' unsteady aerodynamic model and the geometrically exact fully intrinsic beam equations were used for aerodynamic and structural modelling, respectively. By obtaining the eigenvalues of the linearized system, the stability of the wing is evaluated.

The obtained results were validated with those available in the literature, and a good agreement was obtained. Finally, the effects of changes in folding angles and stiffness ratio on the flutter speed and frequency were studied in different cases. The results of this study are summarized as follows:

1) The combination of folding angles can significantly delay the flutter speed and improve the performance of the bird.
2) Like the first folding angle, the second folding angle is also effective in improving the flight envelope.
3) The effect of folding angles on the flutter speed and frequency is greater than the stiffness ratio and this is seen more in the second and third parts.
4) Choosing the optimal angles for each wing is different, but the importance of choosing the right one is very high.

References

[1] D. H. Lee and P. Chen, "Nonlinear aeroelastic studies on a folding wing configuration with free-play hinge nonlinearity," in 47th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference 14th AIAA/ASME/AHS Adaptive Structures Conference 7th, 2006, p. 1734.

[2] N. Ameri, M. Lowenberg, and M. Friswell, "Modelling the dynamic response of a morphing wing with active winglets," in AIAA Atmospheric Flight Mechanics Conference and Exhibit, 2007, p. 6500.

[3] D. Tang and E. H. Dowell" ,Theoretical and experimental aeroelastic study for folding wing structures," Journal of Aircraft, vol. 45, pp. 1136-1147, 2008.

[4] P. J. Attar, D. Tang, and E. H. Dowell, "Nonlinear aeroelastic study for folding wing structures," AIAA journal, vol. 48 ,pp. 2187-2195, 2010.

[5] S. Liska and E. H. Dowell, "Continuum aeroelastic model for a folding-wing configuration," AIAA Journal, vol. 47, pp. 2350-2358, 2009.

[6] Y. Zhao and H. Hu, "Parameterized aeroelastic modeling and flutter analysis for a folding wing," Journal of Sound and Vibration, vol. 331, pp. 308-324, 2012.

[7] R. M. Ajaj, E. I. Saavedra Flores, M. Amoozgar, and J. E. Cooper, "A Parametric Study on the Aeroelasticity of Flared Hinge Folding Wingtips," Aerospace, vol. 8, p. 221, 2021.

[8] S. H .Moravej Barzani and H. Shahverdi, "Nonlinear aeroelastic stability analysis of a folding wing by using geometrically exact fully intrinsic beam equations," Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering ,p. 09544100231167728, 2023.

[9] G. Hegemier and S. Nair, "A nonlinear dynamical theory for heterogeneous, anisotropic, elasticrods," AIAA Journal, vol. 15, pp. 8-15, 1977.

[10] D. H. Hodges, "Geometrically exact, intrinsic theory for dynamics of curved and twisted anisotropic beams," AIAA journal, vol. 41, pp. 1131-1137, 2003.

[11] Z. Sotoudeh and D. Hodges, "Parametric study of joined-wing aircraft geometry," in 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference 18th AIAA/ASME/AHS Adaptive Structures Conference 12th, 2010, p. 2718.

[12] H. Moravej Barzani, M. Amoozgar, and H. Shahverdi, "Flutter Instability of Aircraft Swept Wings by Using Fully

Intrinsic Equations," Persian, Amirkabir Journal of Mechanical Engineering ,vol. 49, pp. 275-278, 2018.

[13] M. Amoozgar, S. A. Fazelzadeh, M. I. Friswell, and D. H. Hodges, "Aeroelastic stability analysis of tailored pretwisted wings," AIAA Journal, vol. 57, pp. 4458-4466, 2019.

[14] S. H. Moravej Barzani, H. Shahverdi, and M .Amoozgar, "Nonlinear aeroelastic stability analysis of a two-stage axially moving telescopic wing by using fully intrinsic equations," Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, p. 09544100221080117 ,2022.

[15] C.-S. Chang, "Vibration and aeroelastic analysis of highly flexible HALE aircraft," Georgia Institute of Technology, 2006.

[16] D. A. Peters, S. Karunamoorthy, and W.-M. Cao, "Finite state induced flow models. I-Two-dimensional thin airfoil ", Journal of aircraft, vol. 32, pp. 313-322, 1995.

[17] N. Nguyen and I. Tuzcu, "Flight dynamics of flexible aircraft with aeroelastic and inertial force interactions," in AIAA atmospheric flight mechanics conference, 2009, p. 6045.

[18] M. J. Patil, "Nonlinear aeroelastic analysis, flight dynamics, and control of a complete aircraft," Georgia Institute of Technology, 1999.