Pages: 101-114 / Vol. 16/ Issue 1 / Winter-Spring 2023
Journal of Aerospace Science and Technology
www:jast.ias.ir

Scientific-Research Article

Investigating the relative motion of two satellites corresponding to the orbital collocation strategy

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ABSTRACT

Keywords: *GEO-orbital space, Relative motion Collocation, Collocation strategy, Geometric modeling of relative orbit*

The ever-increasing demand for placing satellites in the geostationary orbit has caused the revision and change of the conventional mechanism of allocating orbital slots. Therefore, collocation approaches and station-keeping of several satellites with a common position have been developed to improve the utilization of the capacity of the geostationary orbit. This, in turn, increases the complexity and sensitivity of the modeling, guidance, and control processes. However, new restrictions are added to the problem of maintaining a common location, such as maintaining the minimum separation distance between satellites to prevent possible interference. Employing a collocation strategy is essential, especially for effective control of high-demand orbital regions that will lead to space congestion.

Controlling the relative motion of satellites by maintaining a safe distance between them is the main rule in collocation. This article investigates the problem of the relative motion of satellites corresponding to collocation strategies. Then, the results are implemented and compared using a solution based on geometrical modeling of relative orbit and the concepts of spherical geometry. In this regard, the relative orbital elements of the two satellites are calculated using the presented relative motion modeling. Also, the relative position of the satellites is obtained. The case studies and evaluations confirmed that the inclination and eccentricity separation strategies are suitable options for meeting the fuel consumption requirements and providing more space for collocated satellites than other strategies.

Introduction

Today, there is an urgent need for collocation to effectively use the crowded area of space at GEO altitude. High-demand and high-congestion orbital regions, such as Asia and Europe, require several satellites from different countries and companies to increase their telecommunication channels. As a result, terrestrial users can access services such as Internet access and telephone capabilities to provide companionship. Therefore, several satellites need to share the same longitude to meet the growing volume of space communications. For example, Luxembourg's ASTRA and France's Eutelsat missions have successfully collocated 7 and 5 satellites within ±0.1 deg slots, respectively [1].

The collocation of several satellites in a GEO slot creates the risk of interference between satellites. The most destructive interference is the collision between two collocated satellites. For the arrangement of two satellites, the relative motion and the positioning algorithm are the key components that affect flight quality and mission efficiency.

The relative motion of one or more collocated satellites in the geostationary orbit must be limited

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DOI: 10.22034/jast.2023.376290.1142
Submit: 03.01.2023/ Accepted: 05.04.2023
Print ISSN:2345-3648 Online ISSN: 1735-2134

to a specific slot defined by latitude and longitude [2].In these missions, it is necessary to check the relative motion between the satellites and adopt a suitable control strategy to reduce the amount of fuel required by the satellite, increase its lifespan, and ensure goal achievement [3].

Over the last decade, satellite collocation has gained more interest because of the growing number of GEO satellites.

Juana et al. (2010) examined the third generation of Meteosat, the primary source of geostationary observations in Europe and Africa. According to the International Telecommunication Union regulations, 3 or 4 satellites must be collocated in a GEO slot with a longitude range of ±0.1. This collocation brings unprecedented challenges for European meteorological satellites, the most important of which is the collocation of satellites. At the same time, the fuel needed for Maneuvers is minimized to maximize mission length. This research has addressed the main challenge of the collocation of Meteosat satellites and presented an example of the collocation plan under the separation of eccentricity/inclination for three Meteosat satellites[4].

Rausch and Howell (2010) and Rausch (2012) presented the concept of relative orbit control for collocated geostationary spacecraft. The ground station controls the leader spacecraft, responsible for determining the orbit and controlling other spacecraft. Each orbit is explained relative to the leader regarding the orbital element difference. Moreover, linear mapping is used to quickly convert from the relative orbit measurement to the orbital element difference. It has been shown that this concept is suitable for spacecraft collocated using eccentricity/inclination separation vectors. The relative line of sight limitation between spacecraft in longitude is also considered, and this algorithm is developed to provide practical cases[6][5].

Lee et al. (2013) provide a mathematical model to establish an acceptable relative distance concerning the uncertainties of determining the orbit and the difference of orbital parameters for each pair of collocated satellites and the algorithm. It is proposed to build such relationships to face the challenge of placing satellites in the same position. In addition, algorithms are provided for assigning longitude, eccentricity, and inclination to each satellite so that the mathematical model can be used to design a geostationary satellite collocation strategy[7].

Emam and Abd Elghany (2015) evaluated the collocation between geostationary and geosynchronous satellites. Change in orbit between collocated satellites in the range of ±0.09º east-west and ±0.07º north-south is evaluated. Then, one of these satellites is placed in the GEO orbit. Next, the effect of this change is studied and evaluated on the security of collocated satellites inside the flight window. Several collocation scenarios have been investigated to adjust the location of two satellites within the flight window to maximize the distance between them and perform the mission correctly[8].

Bruijn et al. (2016) have developed a method to determine the station-keeping maneuvers of a set of collocated satellites in a geostationary slot. Hierarchical leader-follower control is used to control the follower satellite relative to the leader satellite. The station-keeping method minimizes the required fuel and limits the number of maneuvers while ensuring a safe distance between the satellites. This method is shown for one week for four satellites with different mass, surface, and propulsion systems. Then it has been shown that if the duration of the maneuver cycle is reduced to one day, this method allows 16 satellites to be placed in one slot without consuming additional fuel[9].

Hwang et al. (2016) consider that geostationary satellites' orbital slots and frequency bands are insufficient for their efficiency in Korea. The Communication, Ocean, Meteorological Satellite (COMS) and the two Geo Compsat 2 satellites (Kompsat 2) 2 A and B will be placed in the same window in 2018 and 2019, in orbit of 128.2 degrees East, respectively. Therefore, a collocation strategy is needed for the safe operation of satellites. The most common strategy is to control orbital inclination and eccentricity vectors for operation within a narrow control range. According to the operational scenario of three satellites, the longitude separation method and the new combined control strategy of inclination and eccentricity vector have been used. It has been validated by calculating the relative position difference between both satellites[10].

Luo and Sun (2017) presented a safe design scenario method for orbital rendezvous missions that considered the safety constraints of collocated satellites. A quantitative index considering trajectory uncertainty is introduced to analyze scenario parameters' safety performance, including V-bar keeping positions and y-by trajectory radius. In addition, a comprehensive analysis has been done to find the risk regions for maintaining the position and the appropriate semi-major axis of the elliptical flight trajectory, considering the safety requirements of the target satellite and collocated satellites. In addition, a geometric method has been developed to design an optimal and feasible safe orbital rendezvous scenario. This method has been tested by designing four orbital rendezvous scenarios in situations with and without a collocated satellite. The safety performance and velocity increase of these scenarios have been compared. It was found that the collocated satellite significantly affects the scenario's impact [11].

Qi et al. (2019) have proposed a Coulomb tether double-pyramid satellite formation whose center of mass moves along the geostationary orbit. Then, the motion equations of this system are obtained from Lagrange's equations. Moreover, the derivative of the Hamiltonian is expressed as a function of the damping coefficients of the boundaries. The exact solution of the satellite loads for the spinning and non-spinning cases can be obtained for the three-satellite configuration. If the number of satellites in the flight arrangement exceeds three, an approximate solution is obtained using symmetry. Finally, numerical simulations show that the proposed formation can be used for a set of collocated geostationary satellites [12].

Satpute and Emami (2019) provide details on developing a planning algorithm for several GEO-collocated satellites to perform simultaneous station-keeping and momentum-unloading maneuvers. This research aims to minimize the required fuel while ensuring the minimum safe separation distance between satellites in a specific GEO slot. This algorithm has used the leader-follower structure to define the relative orbital elements of satellites equipped with four gimbals and electric thrusters. It has solved the convex optimization problem with unequal constraints, including the requirements of momentum unloading to determine optimal maneuvers. Furthermore, the proposed algorithm has been investigated based on fuel consumption, the satisfaction of constraints, and satellite performance, using numerical simulations that consider the dominant perturbations in the GEO-orbital environment [13].

Schwarz and Knopp (2019) apply the multi-input/multi-output concept to collocation satellites and discuss the potential for further increasing the capacity of a GEO-orbital slot. They show that the achievable capacity in fixed satellite service applications highly depends on the collocation method. The geometry between satellites must be maintained to obtain a permanent capacity increase, which requires implementing a coordinated station-keeping strategy. The effect of the satellite positioning error has been investigated, which is caused by the inaccuracy of determining the orbit or the propulsion system on the capacity. Monte Carlo simulations, including two satellites, show that even with a conservative assumption of satellite positioning accuracy, a capacity increase of at least 1.7 can be achieved for more than 90% of all observations compared to a single input/single output satellite system [14].

Sun et al. (2020) have used the double quaternion to model the electromagnetic force and the relative motion of collocated satellites according to torque and electromagnetic force features. Therefore, the model of the electromagnetic force is presented based on dual quaternion using the matching between torque and magnetic force and describing the capability of the dual quaternion. And then, the simultaneous position and state motion equations for diffraction imaging of the system have been extracted by GEO collocation. Finally, the numerical simulation of two orbital states has been done to verify the new dynamic model [15].

The geostationary satellites must also fly along a particular orbital corridor and occupy a very limited area defined by the longitude of the point center of the slot. This approach has increased modeling processes, guidance, and control complexity and sensitivity. New constraints are added to the problem of maintaining a common station, such as maintaining the minimum separation distance between the satellites. In addition to spatial-temporal interference of satellites, interferences may be due to interference with the sensor's field of view or satellite antenna frequency interference. Therefore, applying the collocation strategy is very important, especially for effectively controlling high-demand orbital slots that will lead to space congestion.

This article investigates the problem of the relative motion of satellites in longitude separation strategy, coordinated station-keeping strategy, halo separation strategy, and the e-i separation strategy. It also analyzes them using geometrical modeling of the relative orbit based on spherical coordinates, considering the effect of environmental perturbations. In this regard, the

relative orbital elements of two satellites are also calculated using the geometrical modeling of the relative orbit. Finally, the relative position of the satellites is obtained using them.
In part 2, satellite collocation requirements to prevent interference are presented. In part 3, the geometrical modeling of the relative orbit for two satellites is presented, and the relative position vector of the two satellites is obtained. Part 4 examines the types of collocation strategies, and the relative motion of satellites in collocation strategies is extracted using the proposed model. Then, the optimal strategy is selected.

Collocation requirements

The collocation requirements are considered to avoid satellite interference. Satellite collocation requirements are mainly concerned with customer service considerations, including the avoidance of controllable satellite collisions and the reliability of the orbital control strategy. These requirements can simply be presented as a set of constraints for collocated satellites[16].
Four types of geometric constraints have been identified for the station-keeping of GEO satellites collocation:

- absolute station-keeping
- Collision avoidance (relative station-keeping)
- Establishing an ISL and avoiding frequency interference
- Avoiding interferences of sensors

Investigating the relative motion is necessary to analyze the limitations mentioned for the collocation of GEO satellites.

Absolute station-keeping

Keeping a GEO satellite in the correct orbital slot is very important. GEO satellite station-keeping helps maintain the satellite's position in its assigned GEO slot. It is essential that the satellite remains stationary in relation to the earth and occupy a well-defined location at the equator, shown in Figure 1.

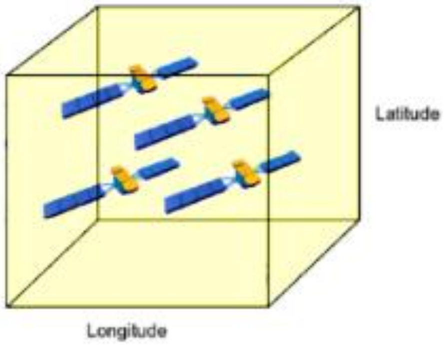

Figure 1: GEO orbital slot [16].

A satellite's location is defined as latitude and longitude. Removing a satellite from the station-keeping window can result in frequency interference between adjacent satellites or a physical interaction between them. Therefore, satellite locations should remain within both latitude and longitude boundaries. Equation (1) shows the relation for this constraint.

$$latitude_{satellite} = latitude_{center} \pm 0.1 \quad (1)$$
$$longitude_{satellite} = longitude_{center} \pm 0.1$$

Collision avoidance

The main rule in collocation is to keep satellites at a safe distance from one another in order to control their relative motion. As shown in figure 3, in collocation, each satellite should maintain its position relative to the other satellites to avoid interference in addition to the absolute station-keeping. This type of station-keeping is known as relative station-keeping.

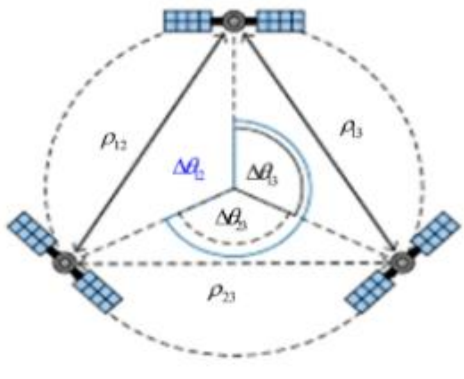

Figure 2: Relative station-keeping of satellites [17].

Multi-satellite collocation in a GEO orbital slot created a collision between satellites. The collision between the two collocated satellites is the most destructive interference. These hazards are reduced by keeping the minimum distance between satellites in an orbital slot. Equation (2) shows the relation for this constraint.

$$\vec{r} \geq d_{min} \tag{2}$$

Establishing an ISL and avoiding frequency interference

Inter-satellite links are defined as communication links between satellites. In other words, radio or optical links provide satellite communication without requiring intermediate ground stations. In collocation, the inter-satellite link coordinates satellites and transfers information between them. Also, increasing the level of automaticity of satellites leads to reducing the minimum distance between satellites, which allows more satellites to be placed in the same slot.
Based on the results of using the inter-satellite link, there may be interference between two satellite links and tracking signal detection with other satellites. Therefore, measuring the angle of view of two satellites is necessary to avoid interferences. The angle of view for both satellites is computed using equation (3) as follows.

$$\beta = \frac{\lambda}{2D} \tag{3}$$

The angle of view for both satellites is shown in figure 3.

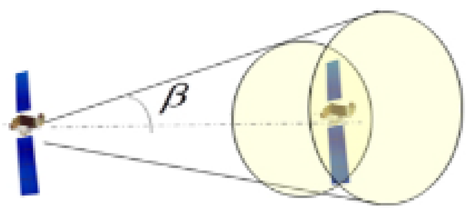

Figure 3: Angle of view of two satellites [6].

According to the angle of view required to establish an inter-satellite link, there is a constraint to the difference in inclination vector between the two satellites, shown in (4).

$$|\delta i| \approx \Delta\Omega tan\beta \tag{4}$$

Avoiding interference of sensors

The payload of GEO satellites is typically used for communication, earth observation, and navigation. Observation or transmission of the signal is impaired when an object or satellite passes through the sensor's field of view. Satellites may also produce shadows on each other's solar panels and block the complete absorption of energy. Due to their frequency, the analyses show that these problems are significant for collocated satellites in an orbital slot.
As a result of the field of view of collocated satellites, sensor cone constraint occurs. For example, figure 4 shows the star sensor cone.

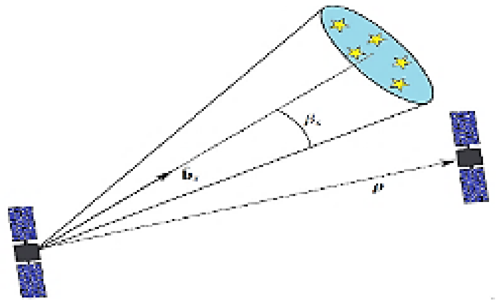

Figure 4: The geometrical embodiment of sensor cone constraint[۱۸]

For a sensor with bore-sight unit vector $\vec{b}_s$ and half-cone angle β_s and relative distance $\vec{r}$, The sensor cone avoidance constraint is then defined as equation (5).

$$\hat{r}^T \vec{b}_s - \|\hat{r}\|_2 \cos\beta_s < 0 \tag{1}$$

Table 1 shows the Common values the half-cone angle and the bore-sight unit vector.

Table 1: Common specifications of earth and star sensors [18]

Sensor	bore-sight unit vector	half-cone angle
earth	$[-1,0,0]^T$	9°
star	$[0, cos30^\circ, sin30^\circ]^T$	26°

Geometrical modeling of the relative orbit

In this section, the velocity and position vectors of the target satellite are obtained geometrically relative to the base satellite. The subscript B indicates the base satellite and the subscript T indicates the target satellite. As seen in Figure 5, the Kepler orbit of two satellites is depicted on the sphere for geometric interpretation.
The poles P_B and P_T orbital poles of the satellites are shown, too. The image of the intersection point of two orbital planes on the surface of the sphere is shown by I_P and the relative position of the target satellite with respect to the base satellite is expressed by the azimuth angle α and the elevation angle δ. The angle α at the point H is perpendicular to the angle δ.

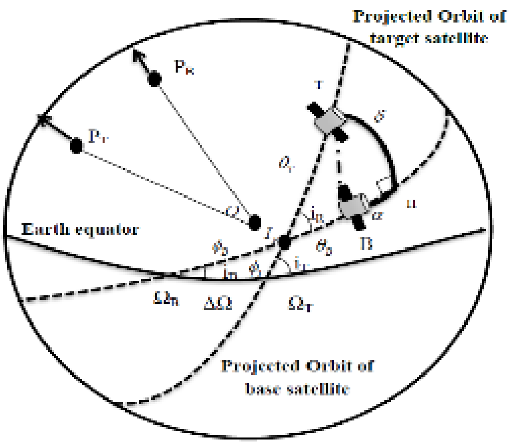

Figure 5: Geometry for modeling relative motion on the surface of a sphere [19].

The subject of latitude is introduced to transfer between orbital elements and angular positions on the sphere. Latitude is equal to the arc length from the ascending node to the current angular position of the satellite. Latitude can be expressed as follows.

$$u_j = \phi_j + \theta_j = \omega_j + v_j \qquad j = B, T \tag{6}$$

The arcs ϕ_j and θ_j are the distance of the ascending node Ω_j, from the point I_P, and from I_P to the current angular position of the satellite, respectively. To obtain the relative motion of the satellite, the key parameter is the relative inclination angle i_R, which is the angle between the two orbital planes in point I_P. Spherical triangle $\Delta\Omega_B\Omega_T I_P$ is used for calculation i_R. Because i_R is not equal to the difference between the inclination angle of two orbits $i_R \neq i_T - i_B$, the law of cosines should be used for trigonometric angles.
In which the relative ascending node is defined as follows.

$$\Delta\Omega = \Omega_T - \Omega_B \tag{2}$$

First, the angles of the target satellite relative to the base satellite are extracted in terms of the orbital elements related to the angle (i.e., Ω, i, ω, v). Figure 2 shows the spherical triangle $\Delta\Omega_B\Omega_T I_P$ which is used to obtain ϕ_B and ϕ_T. Also, Figure 6 shows the detailed specifications of the spherical triangle.

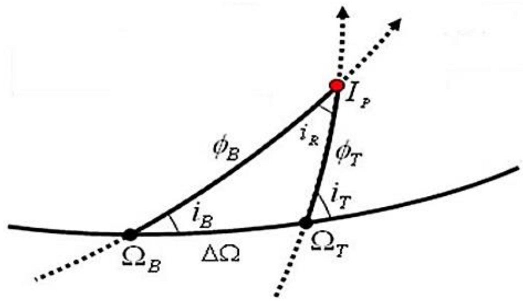

Figure 6: Spherical triangle to calculate ϕ_B and ϕ_T [19]

The law of sines is applied to the spherical triangle to calculate $sin\phi_B$.

$$\sin\phi_B = \frac{\sin\Delta\Omega \sin i_T}{\sin i_R} \tag{3}$$

Based on the law of cosines for the angles in the spherical triangle $\Delta\Omega_B\Omega_T I_P$, another geometrical relationship is found for calculation $\cos\phi_B$.

$$\cos\phi_B = \frac{\cos(180^\circ - i_T) + \cos i_B \cos i_R}{\sin i_B \sin i_R} \tag{4}$$

After dividing equation (9) by equation (10), we will have.

$$\phi_B = \arctan\left[\frac{\sin\Delta\Omega \sin i_B \sin i_T}{-\cos i_T + \cos i_B \cos i_R}\right] \tag{5}$$

To calculate $\sin\phi_T$, the law of sines is also used for the spherical triangle $\Delta\Omega_B\Omega_T I_P$ shown in Figure 6.

$$\sin\phi_T = \frac{\sin\Delta\Omega \sin i_B}{\sin i_R} \tag{6}$$

Also, based on the laws of cosines, it can be obtained that:

$$\cos\phi_T = \frac{\cos i_B + \cos(180^\circ - i_T)\cos i_R}{\sin(180^\circ - i_T)\sin i_R} \tag{7}$$

We divide equation (12) by equation (13) and the result is as follows.

$$\phi_T = \arctan\left[\frac{\sin\Delta\Omega \sin i_B \sin i_T}{\cos i_B - \cos i_T \cos i_R}\right] \tag{8}$$

In this case, a celestial sphere is made, whose satellite poles are the basis of its geographic poles. This is shown in Figure 7. In the celestial sphere, two spherical triangles $\Delta P_B P_T I_P$ and ΔTHI_P are also shown. The arc $P_B T$ can be found by subtracting δ from 90 degrees.

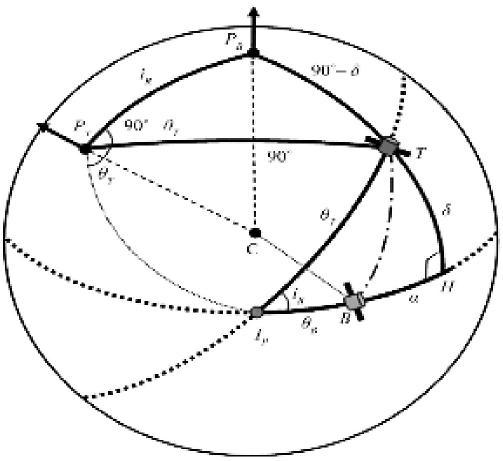

Figure 7: Calculation geometry α and δ [19].

The angle δ is obtained from the spherical triangle $\Delta P_B P_T I_P$. By applying the law of cosines for the sides of the spherical triangle, we reach the following equation.

$$\cos(90-\delta)=\sin i_R \sin 90^\circ \cos(90^\circ-\theta_T)+\cos i_R \cos 90^\circ \qquad (9)$$
$$\sin\delta=\sin i_R \sin\theta_T$$

Therefore, the angle δ is obtained from the following equation.

$$\delta=\arcsin\left[\sin i_R \sin\theta_T\right] \qquad (10)$$

Based on the law of cosines for the sides of the two sides of the spherical triangle ΔTHI_P, the angle α results in the following equations.

$$\cos\theta_T=\sin\delta\sin(\theta_B+\alpha)\cos 90^\circ+\cos\delta\cos(\theta_B+\alpha) \qquad (11)$$

And

$$\cos\delta=\sin\theta_T \sin(\theta_B+\alpha)\cos i_R+\cos\theta_T \cos(\theta_B+\alpha) \qquad (12)$$

Substituting $\cos\delta$ from equation (18) in equation (17) we will have.

$$\tan(\theta_B+\alpha)=\frac{\sin\theta_T \cos i_R}{\cos\theta_T} \qquad (13)$$

Therefore, the angle α is obtained from the following equation.

$$\alpha=-\theta_B+\arctan\left[\frac{\sin\theta_T \cos i_R}{\cos\theta_T}\right] \qquad (14)$$

Using the definition of the latitude argument in equation (6), the angles α and δ are obtained as follows.

$$\alpha=\arctan\left[\cos i_R \tan(\omega_T+\nu_T-\phi_T)\right]+(\phi_B-\omega_B-\nu_B),\ 0^\circ\le\alpha<360^\circ \qquad (15)$$

$$\delta=\arcsin\left[\sin i_R \sin(\omega_T+\nu_T-\phi_T)\right],\ -90^\circ\le\alpha<90^\circ \qquad (16)$$

For a simple analysis of the relative motion of the satellite, the angles α and δ can be directly used to determine the angular position of the target satellite relative to the base satellite[20].

According to the time variable of the orbital elements, the derivatives α and δ are obtained as follows.

$$\dot{\alpha}=-\dot{\theta}_B+\frac{-\dot{i}_R \sin i_R \tan\theta_T}{1+(\cos i_R \tan\theta_T)^2}+\frac{\dot{\theta}_T \cos i_R(1+\tan^2\theta_T)}{1+(\cos i_R \tan\theta_T)^2} \qquad (17)$$

$$\dot{\delta}=\frac{\dot{i}_R \cos i_R \sin\theta_T+\dot{\theta}_T \sin i_R \cos\theta_T}{\sqrt{1+(\sin i_R \sin\theta_T)^2}} \qquad (18)$$

Derivatives i_R, θ_j, ϕ_B and ϕ_I of satellites are as follows.

$$\dot{i}_R=\frac{\dot{i}_B\left(\sin i_B \cos i_T-\sin i_T \cos i_B \cos\Delta\Omega\right)}{\sin i_R}+\frac{\dot{i}_T(\sin i_T \cos i_B-\sin i_B \cos i_T \cos\Delta\Omega)}{\sin i_R}+\frac{\Delta\dot{\Omega}\sin i_B \sin i_T \sin\Delta\Omega}{\sin i_R} \qquad (19)$$

$$\dot{\theta}_j=\dot{\omega}_j+\dot{\nu}_j-\dot{\phi}_j,\quad j=B,T \qquad (20)$$

$$\dot{\phi}_B=\frac{(\sin\Delta\Omega\sin i_B \sin i_T\dot{)}(-\cos i_T+\cos i_B \cos i_R)}{\left(-\cos i_T+\cos i_B \cos i_R\right)^2+\left(\sin\Delta\Omega\sin i_B \sin i_T\right)^2}-\frac{(-\cos i_T+\cos i_B \cos i_R\dot{)}(\sin\Delta\Omega\sin i_B \sin i_T)}{\left(-\cos i_T+\cos i_B \cos i_R\right)^2+\left(\sin\Delta\Omega\sin i_B \sin i_T\right)^2} \qquad (21)$$

$$\dot{\phi}_T = \frac{(\sin\Delta\Omega \sin i_B \sin i_T)^{\cdot}(\cos i_B - \cos i_T \cos i_R)}{(\cos i_B - \cos i_T \cos i_R)^2 + (\sin\Delta\Omega \sin i_B \sin i_T)^2} - \frac{(\cos i_B - \cos i_T \cos i_R)^{\cdot}(\sin\Delta\Omega \sin i_B \sin i_T)}{(\cos i_B - \cos i_T \cos i_R)^2 + (\sin\Delta\Omega \sin i_B \sin i_T)^2} \quad (22)$$

The position vectors of the base and target satellites should be written as the following vectors.

$$\vec{r}_B = \begin{pmatrix} r_B & 0 & 0 \end{pmatrix}^T \quad (23)$$

$$\vec{r}_T = \begin{pmatrix} r_T \cos\delta\cos\alpha & r_T \cos\delta\sin\alpha & r_T \sin\delta \end{pmatrix}^T \quad (24)$$

The relative position vector $\hat{r}$ is obtained from the difference of two vectors $\hat{r}_T$ and $\hat{r}_B$.

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r_T \cos\delta\cos\alpha - r_B \\ r_T \cos\delta\sin\alpha \\ r_T \sin\delta \end{pmatrix} \quad (25)$$

Figure 8 displays the relative position error resulting from the comparison of equations and simulation results for two collocated satellites.

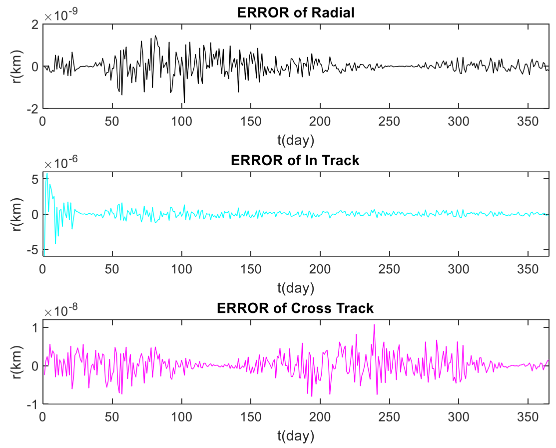

Figure 8: Relative position error diagram

As can be seen, there is a great agreement between the simulation obtained from STK software and the model results. The finding suggests that relative position errors have been reduced due to the consideration of other perturbations on the satellite compared to the reference[20].

collocation strategies

Two steps should be considered in GEO-satellite collocation strategies. In the first step, several critical issues should be considered, including orbit design with constraint conditions for collocated satellites based on collocation position accuracy, characteristics of orbital perturbations, and fuel needed for the satellite. On the other hand, more issues should be considered in the second stage, including designing the algorithm of east/west and south/north maneuvers is essential to keep the collocated satellites under the conditions of the orbital limitations of the first stage throughout the lifetime of the satellites. There are several methods for collocating satellites in existing geostationary slots. These different methods are basically divided into four categories.

Longitude separation strategy

In fact, it is not a collocation strategy. A complete longitude separation strategy is a simple method to divide the longitude range into smaller bands. Each spacecraft performs station-keeping maneuvers independently in the small band assigned to it. This strategy is limited since it assumes that 1) the initial range is very large (at least 0.2 degrees) and 2) the number of satellites collocated with this method is small (two or three satellites). This strategy is shown in Figure 9.

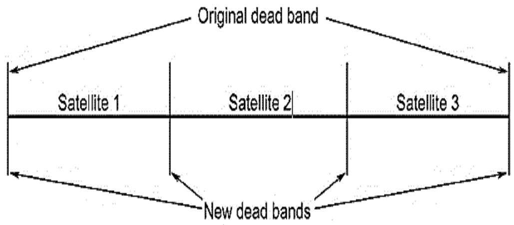

Figure 9: longitude separation strategy

The main advantage of this method is that coordination between control centers for different satellites is not required, and each satellite is controlled entirely independently from the others. The main problem of this method is excessive fuel consumption due to performing many east/west maneuvers[1].

Coordinated station-keeping strategy

Coordinated station-keeping requires minimal coordination and modification of mission operations. This strategy ensures a safe distance along the radial paths and the cross path between the position-keeping maneuvers under the planning of the maneuvers. The radial and cross separation distance at the intersection of the longitude path makes it possible to avoid calculating the radial separation distance, which is always difficult to estimate with high accuracy after a few days. A simplified analytical presentation can be derived that helps to explain the relative motion between satellites and leads to the design and development of a software tool to determine the optimal maneuvering steps and scope for Coordinated station-keeping.

The goal of the coordinated station-keeping strategy is to minimize or avoid the risk of collision between two or three GEO satellites with special coordination in maneuver planning in the operational centers of the mission. A plan of coordinated station-keeping between two collocated satellites in the control range with the same longitude can be described after finding the longitude changes between the station-keeping maneuvers. With the help of this strategy, it is possible to collocate more satellites than with ground station-based methods. Figure 10 shows the plan of the coordinated station-keeping strategy of two satellites. This strategy has not been used operationally for collocation [1].

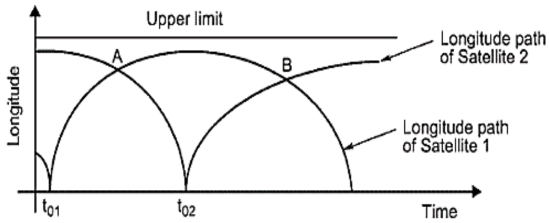

Figure 10: Plan of the coordinated station-keeping strategy of two satellites [21].

The specifications of three GEO satellites collocated with coordinated station-keeping strategy are listed in Table 2.

Table 2: Orbital parameters of satellites

	Satellite1	Satellite2	Satellite3
a(km)	42164.3	42164.3	42164.3
e	0.00032155	0.00041068	0.00039867
i(deg)	0.03327	0.04603	0.04782
Ω(deg)	337.795	34.9	136.5
ω(deg)	292.303	245.991	267.787
M(deg)	105.996	95.173	331.777

Figures 11 show the simulation of three satellites in coordinated station-keeping strategy mentioned in table 2 with STK software.

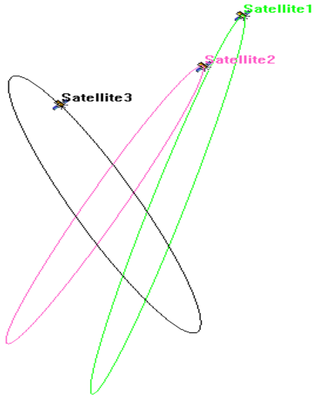

Figure 11: Simulation of three satellites in coordinated station-keeping strategy with STK software

The relative motion of three GEO satellites in a coordinated station-keeping strategy with the characteristics listed in Table 2 is obtained from relative motion modeling. Their relative position is shown in Figure 12.

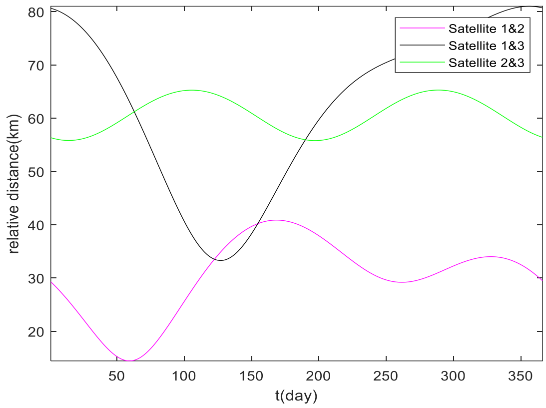

Figure 12: Relative position of three satellites in coordinated station-keeping strategy

As shown in Figure 12, in the coordinated station-keeping strategy, the relative distance between satellites with the characteristics listed in Table 2 obtained from relative motion modeling varies between 15 and 80 Km.

Eccentricity/inclination separation strategy

This strategy assumes two or more satellites are located in the same latitude/longitude control box. Instead of the separation of longitude (east-west direction), the separation is achieved by ensuring the separation between two eccentricity vectors (radial direction) and also two inclination vectors (north/south direction or tangent to the path) of the orbit of collocated satellites. The eccentricity/inclination separation strategy works well for satellites from the same company or mission of roughly similar designs with relatively large solar panels. In such a scenario, all satellites operate with the same mission control center with a perigee sun-pointing control strategy to minimize eccentricity. The advantage of this method is that each satellite uses the entire longitude control window while maintaining its safe distance.

This separation strategy has been successfully implemented for five satellites collocated in the range of 13ºE±0.1 east for Eutelsat of France and seven satellites collocated in the range of 19º degrees east for Luxemburg Astra. The optimal eccentricity/inclination separation strategy has the following rules:

- Maximizing the minimum relative distance among collocated satellites
- Minimizing the number of maneuvers
- Keeping the satellites within the control range (in latitude and longitude)

The concept of eccentricity vector and inclination vector is shown in Figure 13.

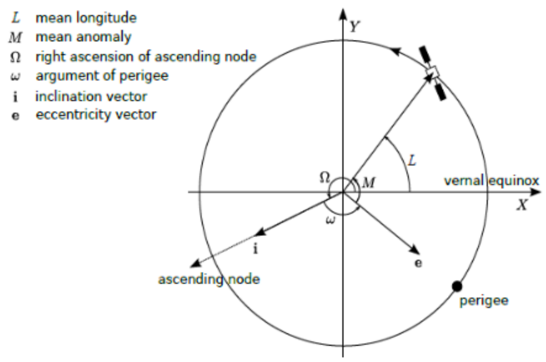

Figure 13: inclination and eccentricity vectors in geostationary orbit [18].

Eccentricity and inclination vectors are defined according to equations (32) and (33).

$$\vec{e} = e\begin{bmatrix}\cos(\Omega+\omega)\\ \sin(\Omega+\omega)\end{bmatrix} \quad (26)$$

$$\vec{i} = \tan(\frac{i}{2})\begin{bmatrix}\cos\Omega\\ \sin\Omega\end{bmatrix} \quad (27)$$

The relative distance between two satellites in this strategy is calculated using equation (34)[7].

$$\begin{aligned}&\|\vec{r}\| = \sqrt{\delta R^2+\delta T^2+\delta N^2}\\ &\delta R = \delta a - a_{geo}\left(\delta e_x \cos(l)+\delta e_y \sin(l)\right)\\ &\delta T = a_{geo}\delta l + 2a_{geo}\left(\delta e_x \sin(l)+\delta e_y \cos(l)\right)\\ &\delta N = a_{geo}\left(\delta i_x \sin(l)-\delta i_y \cos(l)\right)\end{aligned} \quad (28)$$

As shown in equation (35), a necessary and sufficient condition must be met in order to guarantee that the minimum distance between satellites is not zero.

$$\delta e_x \delta i_y - \delta e_y \delta i_x \neq 0 \quad (29)$$

In this regard, $\delta\vec{e}$ is the difference of the eccentricity vectors of two satellites and $\delta\vec{i}$ is the difference of the inclination vectors of the two satellites [٩]. The acceptable range for δe and δi based on the minimum allowed relative distance of two satellites is obtained from equations (36) and (37).

$$\delta e \geq \frac{d_{\min}+|\delta a|}{a_{geo}} \quad (30)$$

$$\delta i \geq \frac{d_{\min}}{a_{geo}} \quad (31)$$

The maximum difference between the two satellites is about 2 to 3 km. If the limits mentioned in the strategy are respected, whether the satellites are controlled in a coordinated or non-coordinated manner, the minimum distance between the satellites is always maintained[7]. The specifications of two GEO satellites collocated with the e-i separation strategy are listed in Table 3.

Table 3: Orbital parameters of satellites

	Satellite1	Satellite2
a(km)	42164.3	42164.3
e	0.000512	0.000512
i(deg)	0.05	0.08
Ω(deg)	20	45
ω(deg)	33	67
M(deg)	90	31

Figures 14 show the simulation of two satellites in the e-i separation strategy mentioned in table 3 with STK software.

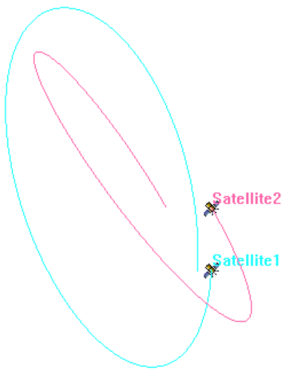

Figure 14: Simulation of two satellites in the e-i separation strategy with STK software

The relative motion of two GEO satellites in the e-i separation strategy with the characteristics listed in Table 3 is obtained from relative motion modeling, Clohessy -Wiltshire equations, and STK software. Their relative position is shown in Figure 15.

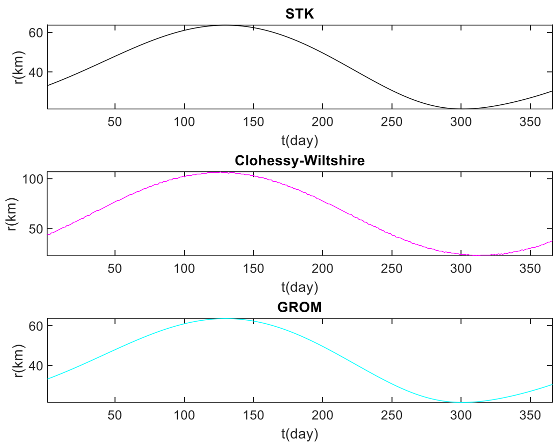

Figure 15: Relative position of two satellites in the e-i separation strategy

As shown in Figure 15, there is a significant difference between the results of STK software and Clohessy-Wiltshire equations due to the linearity of these equations. Furthermore, in the e-i separation strategy, the relative distance between satellites with the characteristics listed in Table 3 obtained from relative motion modeling varies between 34 and 60 Km and matches the STK software results.

Halo separation strategy

The concept of group orbit was first studied by Visher in 1979, Wadsworth in 1980, and later by Walker for GEO satellites in 1982. By correctly changing the orbital elements of the GEO orbit, it is possible to change the usual shape of the ground track into a circle or ellipse. Circular or elliptical sub-orbits are called halo sub-orbits relative to the terrestrial observer. It has been said that several satellites flying close to each other in the same or different sub-orbits form a flight formation. When these satellites fly on subcircular or near-circular orbits, they are in a halo formation. The Halo separation strategy is a collocation strategy that keeps several GEO satellites in a common halo sub-orbit.

This strategy is obtained by solving Hill's equations based on specific initial conditions, which guarantees a constant distance between satellites. However, as the number of satellites increases, the minimum distance is reduced, so a limited number of satellites can be collocated with this method [21].

$$r=\left(x^2+y^2+z^2\right)^{\frac{1}{2}}=z_0\left[4\xi^2+\left(1-3\xi^2\right)\cos^2\vec{\omega}t\right]^{\frac{1}{2}} \quad (32)$$

Based on the value of ξ, this equation can have infinite solutions. However, two values of ξ are considered here, the value $\sqrt{1/3}$ for the circular sub-orbit and the value $1/2$ for the elliptical sub orbit[224]. Based on this, the characteristics of satellites in the halo formation are calculated in the form of equation (38). Satellites in a halo formation have similar semi-major axis, eccentricity, inclination, perigee, and right ascension, while mean anomaly varies[1].

$$\begin{gathered}\omega=90^\circ \text{ or } 270^\circ, e=\frac{D'}{4a}, i=2e\sin\left(60^\circ \text{ or } 63.4^\circ\right)\\ a=a_{geo}, \Omega_i=\Omega_1+\frac{2\pi}{N}(i-1), M_i=l-\omega-\Omega_i\end{gathered} \quad (33)$$

D is the sub-orbit diameter, and N is the number of satellites. The radius of these sub-orbits is 30 km. The value of the minimum distance of the satellites is calculated using equation (40).

$$d_{\min}=D'\frac{\pi}{N} \quad (34)$$

Figure 16 shows an example of the formation of satellites based on the Halo strategy. The plane of these sub-orbits always faces the earth and makes an angle of 60 degrees with the plane of the equator[23].

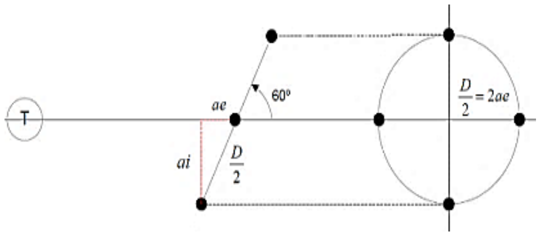

Figure 16: Halo formation [23]

The specifications of six GEO satellites collocated with halo separation strategy are listed in Table 4.

Table 4: Orbital parameters of satellites

	Satellite1	Satellite2	Satellite3	Satellite4	Satellite5	Satellite6
a(km)	42164.3	42164.3	42164.3	42164.3	42164.3	42164.3
e	0.000237	0.000237	0.000237	0.000237	0.000237	0.000237
i(deg)	0.023536	0.023536	0.023536	0.023536	0.023536	0.023536
Ω(deg)	0	60	120	180	240	300
ω(deg)	270	270	270	270	270	270
M(deg)	351.5	291.5	231.5	171.5	111.5	51.5

Figures 17 shows the simulation of six satellites in the halo separation strategy mentioned in table 4 with STK software.

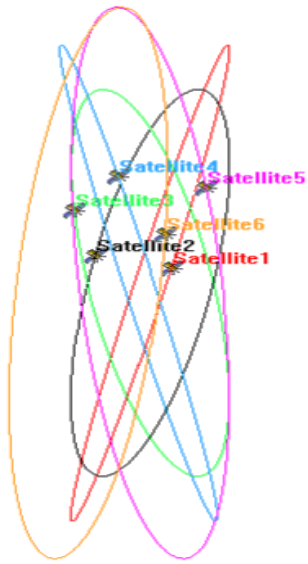

Figure 17: Simulation of six satellites in the halo separation strategy with STK software

The relative motion of two GEO satellites in the halo separation strategy with the characteristics listed in Table 3 is obtained from relative motion modeling, Hill equations, and STK software. Their relative position is shown in Figure 18.

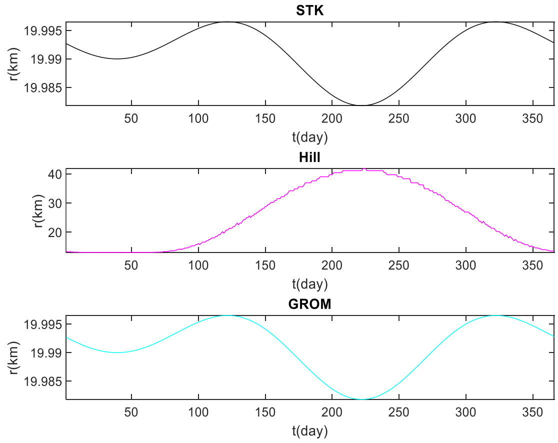

Figure 18: Relative position of two satellites in the halo separation strategy

As seen in Figure 18, there is a significant difference between the results of STK software and Hill equations due to the linearity of these equations. Furthermore, in the halo separation strategy, the relative distance between satellites with the characteristics listed in Table 3 obtained from relative motion modeling varies between 19.985 and 19.995 Km, which matches the STK software results.

The relative motion of six GEO satellites in the halo separation strategy with the characteristics listed in Table 4 is obtained from relative motion modeling. Their relative position is shown in Figure 19.

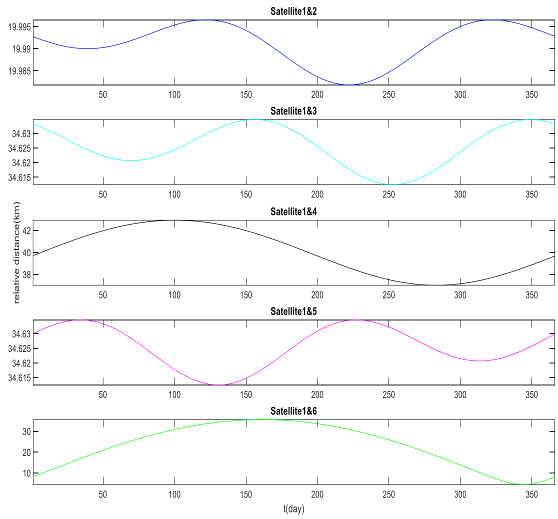

Figure 19: Relative position of six satellites in the halo separation strategy

As can be seen in Figure 19, in the halo separation strategy, the relative distance between satellites with the characteristics listed in Table 4 obtained from relative motion modeling varies between 8 and 45 Km.

Conclusion

Unlike other models used in the collocation strategies, this geometric method used in this article has less complexity. The resulting equations provide a complete form of relative motion and are also more accurate than other methods used for the relative motion of the satellite.

In collocation, satellite operation becomes complicated due to avoiding frequency interference and physical collision between satellites. Therefore, a proper strategy for station-keeping maneuvers of collocated satellites is fundamental. In the longitude separation strategy, the separation is done in one axis, which causes the non-optimal use of the orbital slot and the satellites' low reliability. Two satellites may interfere in at least two points in the coordinated station-keeping strategy. In the halo separation strategy, a fixed distance between satellites is guaranteed. However, as the number of satellites increases, the minimum distance decreases. Therefore, a limited number of satellites can be collocated with this method. In the eccentricity/inclination strategy, two satellites do not interfere at the collision points of the two orbits. The distance between the two satellites changes between 34 and 60 km. The advantage of this strategy is that each satellite can use the entire orbital window.

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Nomenclature

symbol	definition
$\vec{r}$	Relative position vector
$d_{\min}$	Minimum distance
β	Angle of view
λ	Wavelength
D	Antenna diameter
δi	Difference of inclination vectors
$\Delta\Omega$	Relative ascending node
$\vec{b}_s$	Bore-sight unit vector
β_s	Half-cone angle
P_B	Orbit poles of the Base satellite
P_T	Orbit poles of the Target satellite
I_P	Intersection point of two orbital planes
u	Spheral latitude
$\phi_{B,T}$	Distance from Ω to I_P
$\theta_{B,T}$	Distance from I_P to satellite's Current angular position of the satellites
Ω	Right ascension of ascending node
ω	Argument of perigee
ν	True anomaly
i	Inclination
i_R	Relative inclination
α	Relative Azimuth angle
δ	Relative Elevation angle
$\vec{r}_B$	Position vector of base satellite
$\vec{r}_T$	Position vector of base satellite
$\vec{e}$	Eccentricity vector
$\vec{\imath}$	Inclination vector
e	Eccentricity
a	Semi major axis
δR	Radial distance
δT	Tangential distance
δN	Normal distance
δe	Difference of eccentricity vectors
D'	Diameter of the halo sub-orbit
N	Number of satellites