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Modification in Calibration of Strapdown Inertial Navigation System Due to Compensation of Inherent Error in Fixture Manufacturing

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It is always easier to measure the deviation of a product from its design commitments, in comparison to making it identical to plan obligations. In this paper, this simple fact is utilized to compensate the inherent error of fixture fabrication by a mathematical modeling of possible sources of error, in calibration of strapdown inertial navigation system (INS). Since an INS should be mounted on the fixture, all deviations are completely transferred to INS. Compensating the inherent errors of a fixture surely guarantees a more accurate calibration, but its effectiveness depends on specific factors. Proposed method can upgrade a fixture with any manufacturing quality to accuracy level of measurement tool. This technique is explained by two examples. Data of industrial-grade strapdown INS is used to proof of claims.

Keywords: *Fixture - strapdown inertial navigation - INS calibration - error compensation*

Introduction

An INS estimates a vehicle's position, attitude, and velocity as a function of time in a specific navigation frame. There are two types of inertial systems, stable platform and strapdown. In a stable platform system, the inertial sensors are isolated from the rotations of the moving object. A strapdown INS has the inertial sensor assembly (ISA) fixed relative to vehicle body [1]. During all-inertial navigation, inertial sensors are the only means for calculations of linear and angular motion. Unlike aided navigation, there is no correction by external sources and any error in navigation will increase exponentially over time. Therefore in all-inertial navigation, accurate position and velocity measurements can only be achieved by means of an accurate INS. The accuracy is mostly related to sensors grade and their calibration.

Calibration consists of comparing the output of the instrument or sensor under test against the output of

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an instrument of known accuracy when the same input is applied to both instruments. This procedure is carried out for a range of inputs covering the whole measurement range of the instrument or sensor [2]. Calibration removes repeatable errors that are the result of manufacturing imperfections.

Calibration of an INS has been studied seriously in recent decades [3-5]. Calibration of the INS done by means of an accurate two or three axis turn table. This table places the INS in different positions and rotates it at different rates in order to motivate the accelerometers and gyroscopes of the INS by force of the earth's gravity and rotation and moreover, the rotation of turn table itself. The accuracy of calibration cannot cross the accuracy of turn table.

The accuracy of an instrument is a measure of how close the output reading of the instrument is to the correct value [2]. Accuracy has an important role in successful calibration. There are two stages in procedure of calibration. First stage is testing and data acquisition, and second one is data analysis and estimation. In offline calibration, these stages are fully independent, and after completing data acquisition, the estimation phase begins. But in online calibration, there is a live communication between data acquisition and estimation stages. For an accurate calibration, specific commitments in both stages are required. For data acquisition, cases such as turn table, fixture, INS installation and sampling time affect on calibration quality. On the other hand, during analysis and estimation, factors like accurate geographical constants including azimuth and longitude, estimation method and quantization error impress calibration.

This study focuses on the fixture which is important in reliable data acquisition. The fixture is an important component because any deviation from its designing commitments identically transfer to INS. Therefore, there should be a way to compensate these probable inherent errors, because an ideal fixture manufacturing is impossible. Inherent error in fixture manufacturing means dissimilarity between designing commitments and produced fixture. By assumption of measurable deviations, a mathematical compensation is possible. As measurement tools are always more accurate than manufacturing devices, a measuring-based compensation is a step forward.

Inherent errors in fixture manufacturing import defects in process of calibration. There are two strategies to retrieve them. First approach suggests a modification in initial and final values of each test, in case of new tilt and index angles according to measured deviations. This method needs correction for all tests, and any further change in tests or their order demonetize all modifications. A more practical methodical technique will be discussed hereafter, which is easier to implement, and also independent of tests arrangement.

Implementation

In an INS calibration and about calculation of the analytical (actual) values, some assumptions must be considered. One of the most important ones is specific relations between coordinates. It will be shown that it is feasible to compensate any deviation of fixture fabrication in the specific part of analytical values calculation, which is relevant to relation between the coordinates.

In calculation of analytical values, a parametric equation must be solved. This equation depends on the selected coordinates, as well as installation status of fixture on turn table and INS on fixture.

Geographical frame (NED) is preferred reference coordinates because in this coordinate the earth gravity and rotation have simpler presentation. Earth gravity and rotation in this coordinates should be transformed into body coordinate of the INS, by using specific direction cosine matrix (DCM). DCM is 3×3 matrix C_n^m like , while elements in *i*th row and the *j*th column represents the cosine of the angle between the *i* axis of the "m" frame and the *j* axis of "n" frame [6]. Because always there is a turn table in calibration of INS, it is more feasible to use table coordinates as a mediator in computation of C_N^B. This is done according to Eq. (1). Using a mediator makes the computation of DCM simpler and more comprehensible.

$$C_N^B = C_{table}^B \times C_N^{table} \quad (1)$$

In calculation C_N^{table} of there is no trace of body coordinates, so this matrix is independent of any inherent error in fixture build. Compensation of inherent error in fixture manufacturing is only possible by modification in C_{table}^B. Since C_{table}^B is dependent on the fixture shape, it is impossible to suggest a general procedure for all fixtures. But there is a specific correction matrix for each fixture's configuration. Correction matrix is same promised technique that helps to compensate inherent errors of fixture in mathematical method. The efficiency of compensation is only limited by the measurement restrictions.

As a result, this method suggests to spot all measurable sources of error in computation of C_{table}^B

. Because compensation procedure is highly related to fixture design and the INS status, two examples are presented to explain the proposed technique.

Example 1

For calibration, the azimuth will be measured relative to table situation, by means of sensor-based methods or accurate optical tools. Since calibration's turn table is anchored, the measured azimuth is valid, unless the position of table is changed. A flat fixture is required for vertical installation of the INS. This prevalent fixture duplicates the azimuth of table to mounted INS.
One possible source of error is difference between the azimuth of table and the azimuth of fixture, which is considered to be same as INS azimuth. This difference happens when tilt axis of table has deviation from symmetry line of fixture, as shown is Fig. 1. A careless installation of fixture on table or INS on fixture produces this deviation. A permanent and constant deviation, which surely caused by the fixture manufacturing, can be corrected. Fig. 2 shows the coordinates of table (t) and body frames (B) relative to geographical frame (NED). Difference between geographical north (N) and x_t, which coincides with tilt axis of table, is measured azimuth. According to Fig. 2 C_N^{table}, is equal to,

$$C_N^{table} = \begin{bmatrix} cos(A) & sin(A) & 0 \\ -sin(A)\,cos(tilt) & cos(A)\,cos(tilt) & sin(tilt) \\ sin(tilt)\,sin(A) & -sin(tilt)\,cos(A) & cos(tilt) \end{bmatrix} \quad (2)$$

For an accurate fixture, azimuth deviation, , is zero, and DCM of table to body frame become,

$$C_{table}^{B} = \begin{bmatrix} 0 & 0 & -1 \\ -sin(index) & cos(index) & 0 \\ cos(index) & sin(index) & 0 \end{bmatrix} \quad (3)$$

As g_N and ω_{ie}^{N} are known, Eqs. (2) and (3) are needed to calculate g_B and ω_{ie}^{B} , according to Eq. (4).

$$\begin{cases} g_B = C_{table}^{B}(C_N^{table} \times g_N) \\ \omega_{ie}^{B} = C_{table}^{B}(C_N^{table} \times \omega_{ie}^{N}) \end{cases} \quad (4)$$

Any difference between x_t and $z_B$is azimuth deviation, and after an accurate measurement could be compensated in C_{table}^{B} . This is ineffective on C_N^{table}because geographical and table coordinates are fully independent of azimuth deviation. Any malfunction in fixture built-up can only influences the first term of Eq. (1). Recalculation of this DCM highly related to fixture design as well as limitations of turn table. Therefore, it is enough to recalculate C_{table}^{B} , in presence of nonzero azimuth deviation (α). This leads to $C_{table}^{\acute{B}}$, which is presented in Eq. (5).

$$C_{table}^{B'} = \begin{bmatrix} 0 & 0 & -1 \\ -sin(index+\alpha) & cos(index+\alpha) & 0 \\ cos(index+\alpha) & sin(index+\alpha) & 0 \end{bmatrix} \quad (5)$$

By using $C_{table}^{\acute{B}}$, instead of C_{table}^{B} , in calculation of analytical values in all calibration steps, there is a chance to compensate this source of error, only if an accurate measurement of α be available. In addition, $C_{table}^{\acute{B}}$ could be used to investigate the effect of inaccurate azimuth measurement on calibration procedure. Plotting calibration error for specific range of α , reveals the sensitivity of calibration quality to azimuth angle.
In a flat fixture there is another source of error, which is leveling of fixture deck. Modeling of this error is more complicated in comparison to the azimuth deviation. Although, leveling problem is so farfetched.

Example 2

In special purpose fixtures, when there is specific angles between its planes, more errors are conceivable. For example, as shown in Fig. 3, the tilt axis of turn table has limited range of zero to 90 degree only in counter clockwise direction. By using a flat fixture, negative direction of "X" axis has no opportunity of motivation. This is possible to recompense this drawback by changing the shape of fixture to split this drawback over two directions of other axles.
A configuration like Fig. 4 leads to best motivation of all axles in both directions, while there is no discontinuity in calibration for fixture exchanging. This configuration guarantees the optimal calibration for mentioned turn table. This fixture arrangement motivates the INS optimally and is the simplest formation in sense of manufacturing. As there is no difference between the coordinates of turn table and geographical frames in this fixture, from what proposed in Fig. 2, C_N^{table}will be identical to Eq. (2).
Using a fixture like Fig. 4, C_{table}^{B} changes the entirely. For extremely accurate fixture, C_N^{table} is,

$$C_{table}^{B} = \begin{bmatrix} cos(I) & cos(90-I) & cos(90) \\ cos(45)\,cos(90+I) & cos(45)\,cos(I) & cos(135) \\ cos(45)\,cos(90+I) & cos(45)\,cos(I) & cos(45) \end{bmatrix} \quad (6)$$

It is obvious that C_{table}^{B} is independent of tilt angle, because changing in tilt is equal to rotation of all axles of both INS and table altogether, at the same time. So, tilt variation cannot change the angles between coordinates of INS body and table. First step in compensation of possible manufacturing errors of fixture is recognition of error sources. For this configuration, there are three main sources of error (Fig. 5). Two of these faults originate from two important angles in fixture structure. The angle between its two plates, which should be 90°, and INS setting up angle, which should be 45°. The other error is related to junction of plates and its uniformity α_1. supposed to be zero, While α_2 and $\alpha_3$has been measured precisely, and will be modeling in following.

Modeling of these deviations is more difficult in contrast to azimuth deviation, mostly because of more complicated C_{table}^{B} . Importing these two angles straightly in C_{table}^{B} is difficult. Using more intermediate coordinates comforts this problem. Four intermediate coordinates applied to participate α_2 and α_3 in computation of C_{table}^{B}. First coordinat e recompe nses 45° orientati on of INS. Second one envisages α_2, while third coordinat es returns INS

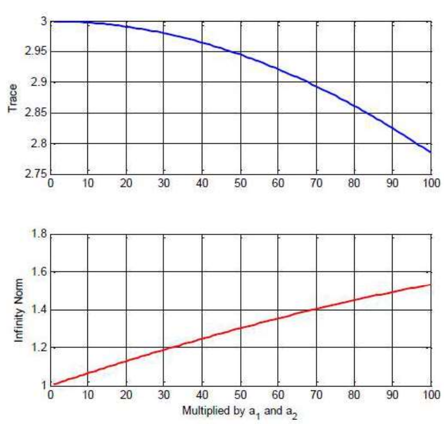

Fig. 6 Change of trace and ∞-norm of correction matrix (7), for specific range of variation in α_2 and α_3

position to 45° situation. Last coordinates visualizes α_3. This process is not unique and may vary. In each C_{Bn}^{Bm}coordinate the calculated, and according to Eq. (7), C_{B4}^{B} attained.

$$C_{B4}^{B} = C_{B1}^{B} \times C_{B2}^{B1} \times C_{B3}^{B2} \times C_{B4}^{B3} \qquad (7)$$

For proposed fixture C_{B4}^{B}, stated in Eq. (8), when $B_4$is body coordinates by consideration of nonzero angles of α_2 and α_3 .

$$C_{B4}^{B} = \begin{bmatrix} cos(\alpha_2) & \frac{\sqrt{2}}{2}sin(\alpha_2)\,[cos(\alpha_3) - sin(\alpha_3)] & -\frac{\sqrt{2}}{2}sin(\alpha_2)\,[sin(\alpha_3) + cos(\alpha_3)] \\ -\frac{\sqrt{2}}{2}sin(\alpha_2) & \frac{1}{2}[cos(\alpha_3)\,\varphi - sin(\alpha_3)\,\omega] & \frac{1}{2}[-sin(\alpha_3)\,\varphi + cos(\alpha_3)\,\omega] \\ \frac{\sqrt{2}}{2}sin(\alpha_2) & \frac{1}{2}[cos(\alpha_3)\,\omega + sin(\alpha_3)\,\varphi] & \frac{1}{2}[-sin(\alpha_3)\,\omega + cos(\alpha_3)\,\varphi] \end{bmatrix} \qquad (8)$$

when φ and ω are,

$$\begin{cases} \varphi = 1 + cos(\alpha_2) \\ \omega = 1 - cos(\alpha_2) \end{cases} \qquad (9)$$

By using Eqs. (2), (6) and transpose of Eq. (8) there is an opportunity to recalculate the analytical values in viewpoint of error compensation. For example, earth gravity and rotation in new body coordinates (B_4) become,

$$\begin{cases} g_{B4} = C_{B}^{B4}\left(C_{table}^{B}(C_{N}^{table} \times g_N)\right) \\ \omega_{ie}^{B4} = C_{B}^{B4}\left(C_{table}^{B}(C_{N}^{table} \times \omega_{ie}^{N})\right) \end{cases} \qquad (10)$$

What proposed in Eq. (8) is exclusive to the fixture presented in Fig. 4. But similar technique could be applied for any fixture with desired arrangement, as well as any possible source of error. The outcome matrix has two main specifications. First, like all DCMs, its determinant should be equal to one. Indeed, it should be valid for all matrixes of Eq. (7). Second one is limited to this matrix. When $\alpha_i = 0$, $i = 1,2,3,\ldots$ this matrix become a unitary matrix. This is clear because $\alpha_i = 0$ pulls out all sources of error, and under this situation, this matrix should be ineffective matrix. Setting α_2 and α_3 to zero, make Eq. (8) a unitary matrix. It is valid for first example, because $\alpha_i = 0$ turns Eq. (5) equal to Eq. (3).

Size of deviation, in case of α_2 and α_3, impresses on specific features of correction matrix (C_{B4}^{B}). Two of them are trace and ∞-norm. For an ideal fixture, the trace of correction matrix should be 3. Any deviation from ideality decreases the trace of matrix. Since this matrix, for any value α_2 of α_3 and , has a determinant equal to one, the trace number demonstrates deviation from being an unitary matrix. This matrix is attainable independently by investigation and measurement of the fixture itself and indicates the quality of fixture manufacturing. ∞-norm of this matrix is another criterion for evaluation of fixture fabrication. ∞-norm has reverse behavior in comparison to trace value. For an accurate fixture, ∞-norm will be one, and for any inherent error, this number grows. Fig. 6 shows behavior of these two criterions for correction matrix of Eq. (8). At a specific interval, variation of

∞-norm is greater than the trace value. As deviations are minor in practice, ∞-norm is more helpful for conclusion.
It should be noted that the correction matrix is calculated for deviation in positive or negative direction. For instance and in first example, Eq. (5) is valid only when $Z_B$is after X_t in clockwise turn. If fixture malfunction put Z_B before X_t,α should be considered negative in Eq. (5). In example 2, Eq. (8) calculated for deviations according to Fig. 5. For deviation in opposite directions, α_2 and α_3 should be negate in Eq. (8).

Test and Evaluation

In past section, two examples proposed. At this point, data of an industrial level strapdown INS will be used to examine the calibration quality in proposed fixture configurations, with and without inherent error compensation.
In matter of first example, since azimuth variation has no effect on estimation of accelerometer's parameters, only gyroscopes result examined. Result for 2° deviation in azimuth angle, with and without compensation, illustrated in Fig. 7. Mean absolute error (MAE) of calibration tests, with final estimated coefficients, with and without azimuth compensation expressed in Table (1). It should be noted that azimuth compensation does not always reduce the MAE, but actualizes it. Compensation of azimuth angle mostly changes the acceleration terms in gyroscope's error model. 2° deviation is not normal and only selected for exaggerated results.

Table 1. MAE of calibration tests, with and without azimuth compensation

	Gyro. X	Gyro. Y	Gyro. Z
Original	0.396	0.492	0.304
Compensated	0.389	0.436	0.289

Since azimuth has effective role in alignment stage, 18 minutes deviation in azimuth considered to illustrate the effect of compensation in consequences of alignment. Results specified in Table (2). It shows importance of azimuth in alignment which happens in a stationary situation.

Table 2. Effect of 18 minutes deviation of azimuth in alignment, with and without compensation

	Align. No.	Gyro. X	Gyro. Y	Gyro. Z
Original	1	-1.6737	-1.6221	-2.2543
Compensated	1	-1.6748	-1.6232	-2.3182
Original	2	-1.6550	-1.6232	-2.2436
Compensated	2	-1.6555	-1.6242	-2.3075
Original	3	-1.6660	-1.6499	-2.2415
Compensated	3	-1.6665	-1.6509	-2.3055

For second example, a fixture just like structure of Fig. 4 used to calibrate the INS. Utilized fixture, inspected by accurate tools, and its deviations in case α_2 of α_3 and measured. The angle which should be 90°, was 90.268°, and the one should be 45°, actually was 45.012°. So, it is necessary to negate both and in Eq. (8). As α_2 and α_3 are known, the correction matrix is attainable, and new analytical values should be calculated. Before checking the effect of correctness on calibration, it is useful to check variation of the analytical values. This is illustrated in Fig. 8, for accelerometer of axis "X". In calibration procedure, first twelve tests have changing only around tilt shaft of table. According to Fig. 8, in tests 6 and 12 compensated values are zero, because in these two test INS experience 90° of tilt angle. By consideration of Fig. 4 and for 90° of tilt angle, "X" axis of INS should be zero for any deviation. The random vibration test is selected to challenge the efficiency of implemented modification, because this test excites all terms of applied error model, while the INS does not experience any trajectory-based movement. 300 seconds offline navigation performed by data that recorded during INS test. Original and compensated calibration coefficients used and improvement in elements of position efficient and its effectiveness improves over time. Although the random vibration is naturally a static test, but the value of compensation is undeniable.

Conclusion

This study claims that it is practically possible to compensate the inherent error in structure of fixtures with modifications in specific DCM.
Consequently and by precise measurement of fixture, it is feasible to calculate C_X^B the , while B is shared body frame between INS and fixture, and X is same shared body frame but with all possible errors consideration. Assuming that all deviations has been measured, and utilized in C_X^B . Then

C_X^B will be a numerical matrix. Fixture has a precise fabrication only if trace of C_X^B be sufficiently close to 3, or ∞- norm of C_X^B be adequately close to 1. For an ideal fixture, C_X^B should be unitary matrix, and these proposed criterions shows how far the C_X^B is from a unitary matrix. The ∞-norm is C_X^B a better criterion because has greater growth in deviation increment.

Correction matrix also indicates how effective the errors are in calibration quality. For example, it can be used to determine how important the initial azimuth measurement is in alignment and calibration procedures; or impression of 1% error about leveling problem on growth of error.

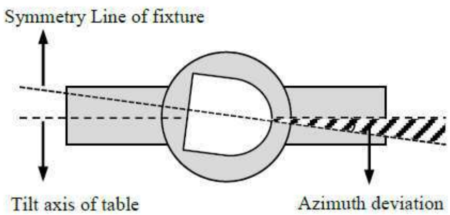

Fig. 1: Azimuth deviation because of difference between symmetry line of fixture and tilt axis

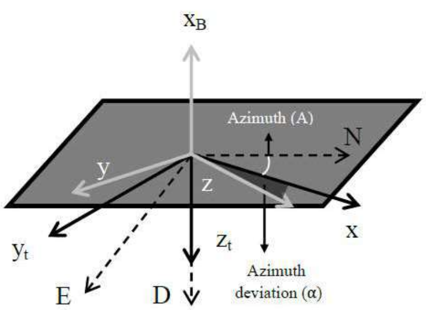

Fig. 2: Coordinates diagram of a flat fixture presented in Fig. 9. It is obvious that compensation is

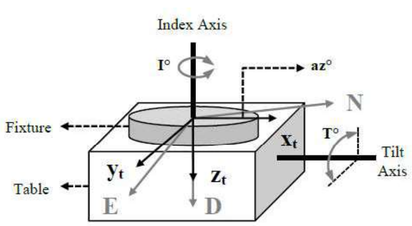

Fig. 3: Coordinates of turn-table and geographical frame (NED)

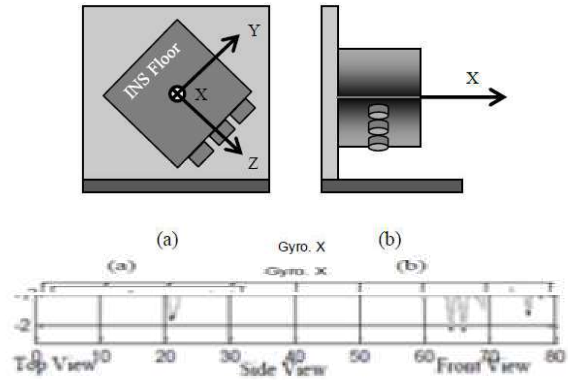

Fig. 4: Special purpose fixture (a) : back view, (b) : side view

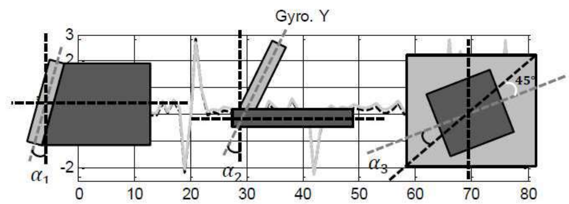

Fig. 5: Possible errors in structure of the fixture shown in Fig. 4

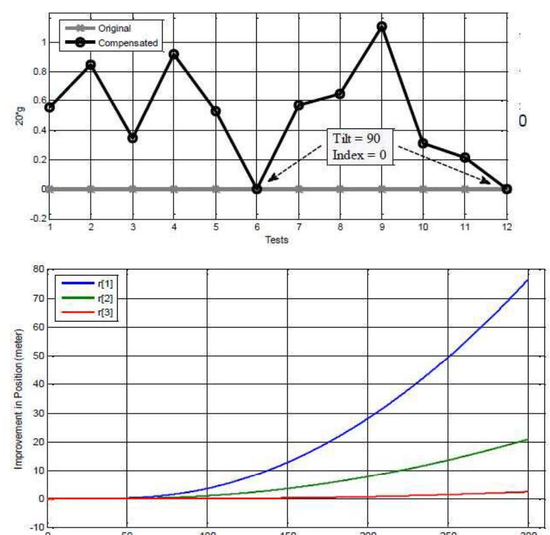

Fig. 9: Improvement in position during random vibration test, because of using correction matrix in calibration

References

1- Curey K. R., M. E. Ash, L. O. Thielman, C. H. Baker, Proposed IEEE Inertial Systems Terminology Standards and Other Inertial Sensor Standards, 2004.

2- Moris A. S., *Measurement & Instrumentation Principles*, Third Edition, Butterworth-Heinemann, 2001.

3- Britting, K. R., *Inertial Navigation Systems Analysis*, John Wily and Sons, New York, 1971.
4- Widnall, W. S., Grundy, P., *Inertial Navigation System Error Models*. TR-03-73, Intermetrics, Inc., Cambridge, Mass., 1973.
5- Arshal, G., Error Equations of Inertial Navigation. *Journal of Guidance, Control, and Dynamics*, Vol. AC-22, No. 6, 1987, pp. 899 – 905
6- Titterton, D. H. and J. L. Weston, *Strapdown Inertial Navigation Technology*, Third Edition, Peter Peregrinus Ltd, Chapter 3, Page 42