

## Scientific-Research Article

# Orbit and Attitude Control of a GEO Satellite in Station Keeping Flight Mode

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*In this study, the performance requirements influencing the orbital and attitude control system of a geostationary satellite in the station-keeping flight mode considering the coupling effect of both attitude and orbital motion is determined. Controlling and keeping the satellite in its orbital window have been done using a set of four thrusters located on one side of the satellite body, by considering the coupling effect of the attitude motion on orbital motion. The satellite's orbital motion could be disturbed by the attitude motion in the allowable orbital window. The main factors conducting this behavior are derived utilizing the satellite attitude and orbital dynamic equations of the motion. In the mathematical analysis of this study, the effects of environmental perturbations originating from the oblateness of the Earth, third mass gravity like the sun and the moon, and solar radiation pressure on the satellite dynamic behavior are also considered. Afterward, the condition of using four installed thrusters on one side of the satellite and the reaction wheels in order to control the satellite orbital and attitude motion is investigated. To reduce the satellite attitude errors, a proportional-derivative controller is employed to activate the reaction wheels properly. The satellite positions in north-south and east-west directions are controlled by a specific array of thrusters in order to maintain it in its predefined orbital window. The required amount of velocity variations for a duration of one year via some simulations may demonstrate the effectiveness of the proposed approach in enhancing the orbital maintenance procedure for the satellite.*

**Keywords**— Geostationary Orbit, Perturbation Forces, Attitude Control, Coupling of Attitude and Orbital Dynamics, Orbital Control, Station-Keeping

## Introduction

It is well established that environmental disturbing forces continually perturb the orbit and the attitude of geostationary satellites. This reason reveals that environmental disturbances should be considered in designing process of a station-keeping strategy for geostationary satellites. In 2005, the DEMETER1 satellite was launched from the platform Baikonur to the space in Kazakhstan. This satellite was the

second satellite from the project Myriade, which was created by CNES2 and launched to space. In the case of DEMETER satellite, the goal was to upgrade the technology and the performance in propulsion subsystems, and both the orbital and the attitude controls along with its scientific mission [1-3]. The main idea was to use four thrusters, all mounted on one side of the satellite. This idea, along with decreasing the weight of the propulsion subsystem relative to installing six or eight thrusters helps

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provide more space to use the other equipment that needs to have distance from the thrusters. The DEMETER satellite executes this property in the LEO orbit under the perturbation factors of this orbit. This paper attempts to extend the achievements of DEMETER satellite to a telecommunication geostationary satellite in station keeping and correction phase. To perform this adaptation, it is necessary to consider some factors affecting the attitude and orbital control subsystem's performance as follows:

- Related standards
- Satellite dynamics
- System requirements,
- Environmental perturbations acting on the satellite,
- Actuator limitations

The examination of geo belt and its related standards has been of interest to researchers from 20th century [4-6]. Gorove collected the laws of the geo orbit for the first time in his book [7]. Moreover, the environmental perturbations were examined generally by Shrivastava [8]. Kraige also identified and introduced the perturbation's forces and torques, particularly in the geostationary orbit [9]. The Bang-Bang controller strategy is one of the most straightforward on-off controllers which were studied by Dougherty [10]. The Dead-zone effect in the control process was then added to improve the controller performance conducting the on-off thrusters' for reducing the system fuel consumption [11]. The Schmidt trigger controller is similar to the bang-bang controller equipped with the dead-zone but performs the satisfactory operation. Agrawal et al. studied the Schmidt trigger controller considering different values rather than 0 and 1 for the controller design [12]. Reducing the fuel consumption of thrusters is one of the crucial challenges which researchers have been faced with for years. Kosari et al. optimized the arrangement of thrusters in order to minimize the fuel consumption and increase the control accuracy [13].

In order to deal with the station-keeping problem, C. Gazzino et al. [14] proposed a decomposition method. It was claimed that the decomposition of the overall station-keeping optimal control problem into several sub-problems has certain advantages with respect to the traditional direct methods in finding a reliable solution and also improving the required

solving time through reducing the number of optimization variables.

The presence of a high number of satellites in the geostationary orbit as well as the need for a high pointing accuracy for telecommunication satellites in this orbit, encourage the researchers to study the satellite's motion in geostationary orbit. Kosari and Beiglari simulated the transient process of the final phase of orbit correction using optimal control for satellites which should be operational in geostationary orbit [14]. Fakoor et al. provided a novel method to optimize the satellite's trajectory in its orbit [15]. Navabi et al. investigated the adaptive attitude control of space system [16]. They did the hardware-in-the-loop simulation and integrated adaptive back stepping to reach the attitude control of the spacecraft [17], [18].

Numerous studies have been performed on the satellite station-keeping dealing with different aspects of the problem. A possible approach to solve the station-keeping problem is to decompose the problem by breaking it up into a number of sub-problems. An example of decomposition-based approaches is the one proposed by D. Losa et al. [19] to solve the optimal station-keeping problem for a satellite equipped with electrical thrusters. Oleson studied the station-keeping of a Geo satellite on the north-south direction [20], and the east-west direction was also investigated by Borissov in 2015 [21].

Strategies take to in-orbit control of geostationary satellites typically consist of two types of corrective actions, namely attitude control and station-keeping maneuvers. In this study, a new arrangement for four thrusters is investigated in a geostationary orbit satellite, which in turn could help alleviate the coupling effect of attitude dynamics in the translational motion of the satellite in the station keeping mode. Consideration of the satellite attitude motion on the relative positioning in the allowable operational orbital window is justified when the distance of the satellite from the ideal position which is defined by the position's error, is low. While, in the longer distances, in addition to increasing the calculation value, it may cause the divergence of the calculations. Also, for convenience, the calculations of other terms except for the first term can be considered as the perturbation terms. The first section of this article entitles the equations of motion, modelling the environmental perturbation, and the effects of the disturbances on the satellite. Then, the control of the satellite is described in the

next section, and at the end, results and conclusions are explained.

### Equations of Motion

In this section, both the orbital and the attitude motion of the satellite are discussed. Then, the environmental perturbation forces affecting the satellite motion are explained. The interaction effect of satellite attitude dynamics on orbital motion which could be considered as a disturbing force is also investigated in orbital window. In this research, five different coordinates frame of Earth-Centered Inertial frame (ESI), Earth-Centered Earth Fixed frame (ECEF), Orbit Reference Frame (ORF), Gaussian Frame, and the body frame have been used for modeling derived from previous studies [4] [19]. It is worth mentioning that in the body coordinates  $i$ ,  $j$ , and  $k$  are perpendicular to each other in the direction of the three axes of the satellite which pass through the center of mass of the satellite.

### Satellite motion modelling

The simplified equation in the absence of perturbation which defines the orbital motion of a satellite around Earth can be represented as follows:[4]

$$\ddot{\vec{r}} = -\mu \frac{\vec{r}}{r^3} \quad (1)$$

In which  $\mu = 398601.2 \text{ km}^3 \text{ s}^{-2}$ . On the other hand, the differential equation discussed Keplerian orbit neglecting the perturbation parameters are provided in equation (1) with the initial variations of  $r(0)$  and  $v(0)$ .

The variations in Keplerian parameters discussing an orbit in space are considered as follows:

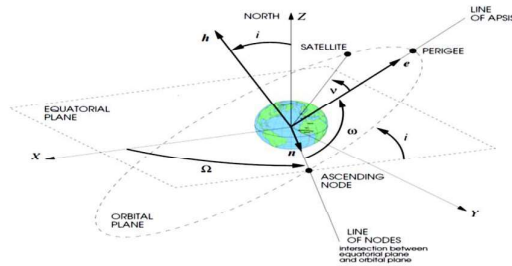


Fig. 1 Classic Orbital Elements [19]

$$\frac{da}{dt} = \frac{de}{dt} = \frac{d\Omega}{dt} = \frac{d\omega}{dt} = \frac{di}{dt} = 0; \quad \frac{dM}{dt} = n. \quad (2)$$

The angular velocity vector of the body frame relative to inertial could be defined as follows:

$$\omega_{BI} = \omega_{BR} + \omega_{RIB} \quad (3)$$

In which  $\omega_{BR}$ , is the angular velocity of the body frame relative to the reference orbital frame, and the  $\omega_{RIB}$  is the angular velocity of the reference orbital frame relative to the inertia frame which is defined in the body frame.

The angular momentum vector for a rigid body is also provided in equation (4):

$$h = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = [I]\omega \quad (4)$$

It is worth mentioning that,  $\omega_x$ ,  $\omega_y$  and  $\omega_z$  are the components of the body angular velocity relative to the inertial coordinate. The angular momentum “h” includes both the rigid body momentum  $h_B = [h_x, h_y, h_z]^T$  and the momentum of “momentum wheel” equipment such as reaction wheels  $h_w = [h_{wx}, h_{wy}, h_{wz}]^T$ . Also,  $[I]$  is the satellite moment of the inertia matrix.

By differentiation of Eq. (4) with respect to time and substituting h components, it could be concluded that:

$$\begin{aligned} T = T_c + T_d = & + [I_{xx}\dot{\omega}_x + h_{wx} \\ & + (I_{zz}\omega_y\omega_z - I_{yy}\omega_y\omega_z) \\ & + (\omega_y h_{wz} - \omega_z h_{wy})] i \\ & + [I_{yy}\dot{\omega}_y + h_{wy} \\ & + (I_{xx}\omega_x\omega_z - I_{zz}\omega_x\omega_z) \\ & + (\omega_z h_{wx} - \omega_x h_{wz})] j \\ & + [I_{zz}\dot{\omega}_z + h_{wz} \\ & + (I_{yy}\omega_y\omega_x - I_{xx}\omega_x\omega_y) \\ & + (\omega_x h_{wy} - \omega_y h_{wx})] k \end{aligned} \quad (5)$$

in which T represents the sum of the external torques acting on the satellite which includes the control torque represented by  $T_c$  and the perturbation torque showed by  $T_d$ .

As it is noted, the translational and rotational motion of a satellite are not independent in the reality, and this fact could affect the accuracy of simulation results. Therefore, in this research, the impact of the mentioned assumption will be investigated.

To see the interaction of translational motion and attitude motion on each other, the six-degree-of-freedom equation of motion of a rigid body can be executed. For this goal, we consider the satellite as a rigid body and derive the equation of motion as follows: [22]

$$\frac{d^2X}{dt^2} = \frac{d^{*2}X^*}{dt^2} + \omega \times (\omega \times X^*) + 2\omega \times \frac{d^*X^*}{dt} + \frac{d\omega}{dt} \times X^* \quad (6)$$

In equation (6)  $X$  represents the motion of the object in the inertial coordinates,  $X^*$  is the motion of the object in the orbital reference frame, and  $\omega$  is the relative velocity of the satellite in the reference coordinate.

### Environmental Perturbation modeling

In the expression of the Keplerian orbit, the existence of any object other than the two namely objects is neglected, and the two objects themselves are considered as the complete spheres with linear weight distribution, which is different from reality. These differences are generally added as perturbation dynamics to the main equations. In this research, the most influential perturbation factors include the gravity of the third object, the effect of Earth oblateness, and the solar radiation pressures.[4]

### Earth Oblateness

Since the applied force from the earth on an external object is a conservative force, it could be obtained as a gradient of a scalar potential function. If the earth was considered as integrated and concentrated or as a homogeneous sphere, we could do so, but unfortunately this assumption cannot be applied because the earth is considered as an oblate spherical object and its mass and distribution are inhomogeneous; subsequently, we add this force difference to the satellite according to the following relationships (7)-(10): [5]

$$a_g = a_{gx}X + a_{gy}Y + a_{gz}Z \quad (7)$$

$$a_{gx}X = GM_{\oplus} (R_{\oplus}^2 a_{gx20} + R_{\oplus}^2 a_{gx22} + R_{\oplus}^2 a_{gx30} + R_{\oplus}^2 a_{gx31} + R_{\oplus}^2 a_{gx32} + R_{\oplus}^2 a_{gx33}) \quad (8)$$

$$a_{gy}Y = GM_{\oplus} (R_{\oplus}^2 a_{gy20} + R_{\oplus}^2 a_{gy22} + R_{\oplus}^2 a_{gy30} + R_{\oplus}^2 a_{gy31} + R_{\oplus}^2 a_{gy32} + R_{\oplus}^2 a_{gy33}) \quad (9)$$

$$a_{gz}Z = GM_{\oplus} (R_{\oplus}^2 a_{gz20} + R_{\oplus}^2 a_{gz22} + R_{\oplus}^2 a_{gz30} + R_{\oplus}^2 a_{gz31} + R_{\oplus}^2 a_{gz32} + R_{\oplus}^2 a_{gz33}) \quad (10)$$

In above relations,  $\oplus$  denotes the earth properties and  $a_g$  represents the accelreactions in different directions which are declared in the [5].

### Acceleration of the third body

A third object, such as the sun or the moon, creates a perturbation force on the satellite motion that could cause changes in the Keplerian orbit parameters. In an n-body system, the sum of the applied forces on the  $i_{th}$  body could be defined as follows: [4]

$$F_i = G \sum_{j=1}^{j=n} \frac{m_i m_j}{r_{ij}^3} (r_j - r_i) \quad . \quad i \neq j \quad (11)$$

Which  $m$  shows the mass of each body, and  $r$  is the distance from the coordinate. As a result, the perturbation acceleration happen by the sun and moon gravity forces can be written as follows:

$$a_a = GM_{\odot} \left[ \frac{r_s - r}{(r_s - r)^3} - \frac{r_s}{(r_s)^3} \right] + GM_{\oplus} \left[ \frac{r_m - r}{(r_m - r)^3} - \frac{r_m}{(r_m)^3} \right] \quad (12)$$

In which  $\odot$  and  $\oplus$  note the sun and earth, respectively. Also, the “s” and “m” subscripts show the parameters related to the sun and moon, respectively. The distance from the satellite is represented by  $r$ . To calculate the acceleration due to the sun and the moon, we need to obtain their center distance from the Earth's center and the center of mass of the satellite at any time. [5].

### Solar radiation pressure

Solar radiation includes a wide range of waves with various wavelengths from X-rays to radio waves. Also radiating of this radiation, which consists of electrons and ionized nuclei, may apply physical pressure on any surface of the satellite. This pressure is proportional to the momentum flux of the radiation. The average solar energy flux of the radiation is proportional to the inverted square of the distance from the sun. The solar pressure acting on the satellite is obtained as follows:[5]

$$a_p = -C_R P_s \frac{S}{m} \frac{r_s - r}{|r_s - r|} \quad (13)$$

### Satellite motion affected by perturbation

Geosynchronous satellite has orbital period equal to the Earth's rotation period around its symmetry axis. The satellite motion in such an orbit is harmonic with the Earth rotational motion. In such orbits, there is no limit on the eccentricity and the inclination. Relative to the inertial frame, the Earth's rotation period is the same astronomical day equals

to 86164.1, which it can be concluded that the semimajor axis equals to 42164.157 kilometers.

The orbit of the satellites is geostationary with unique specifications. For instance, their inclination is zero or close to zero and has a very small eccentricity of  $10^{-4}$ . For such orbits, it is difficult to define the next angle and it is, therefore, necessary to redefine the six orbital classical parameters. In this redefinition, the three parameters  $\omega$ ,  $\Omega$ , and  $\theta$  are all integrated into a single argument  $\alpha = \omega + \theta + \Omega$  that it is called the satellite's astronomical angle.

Instead of the average anomaly, we should use the average longitude, which is defined as follows. In the fixed earth orbits, the eccentricity is also very small, so that the location of the apocentre and the pericentre is questionable. The eccentricity is also redefined as the radial vector from the center of mass of the gravity toward the pericentre so that it depends on  $\omega$  and  $\Omega$ . In the following, the relations linked to the rewriting the Keplerian elements are mentioned. The motion equation of an object under the gravitational force is rewritten as follows in equations 14 to 19:[19]

$$x = [a \ P_1 \ P_2 \ Q_1 \ Q_2 \ l_\theta] \quad (14)$$

$$P_1 = e \sin(\omega + \Omega) \quad (15)$$

$$P_2 = e \cos(\omega + \Omega) \quad (16)$$

$$Q_1 = \tan\left(\frac{i}{2}\right) \sin \Omega \quad (17)$$

$$Q_2 = \tan\left(\frac{i}{2}\right) \cos \Omega \quad (18)$$

$$l_\theta = \Omega + \omega + M - \theta_t \quad (19)$$

The equation of motion of an object under gravitational force is rewritten as follows.

$$\frac{d^2 r}{dt^2} = -\mu \frac{r}{r^3} = \gamma_k \quad (20)$$

In above equation,  $\gamma_k$  is for the perturbation's terms. It can be said about the general state of any kind of perturbation that:

$$\frac{d^2 r}{dt^2} = \gamma_k + \gamma_p \quad (21)$$

In equation (21)  $\gamma_p$  and  $\gamma_k$  represent the perturbation and Keplerian accelerations generated by the Keplerian and perturbation forces, respectively.

### Satellite motion control

The satellite operational window is determined by the satellite orbit specifications. This window could be defined by a vector with three elements as follows:

$$p = [r \ \lambda \ \varphi] \quad (22)$$

Where  $r$  represents the distance of the center of mass of the satellite from the center of the Earth,  $\lambda$  denotes the location of the satellite in the east-west direction and  $\varphi$  denotes the location of the satellite in the north-south direction.

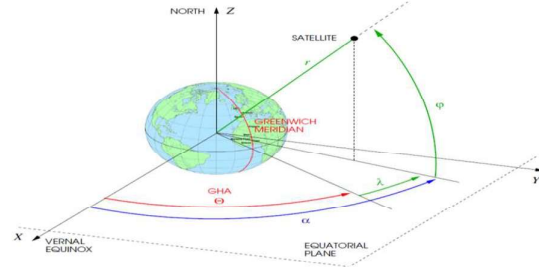


Fig.2 Station Keeping Elements [19]

The next instance for defining an orbital operational box is using two intervals for allowed variations in the window, for example,  $\lambda \Delta = \pm 0.5^\circ$ , which specifies that the satellite is allowed to move in the east-west direction by a five tenth of a degree.

The first element of the orbital location is  $r$ , which could be defined using the classical parameters as follows:

$$\cos v = \frac{\cos E - e}{1 - e \cos E} \quad (23)$$

$$r \cos(180 - v) = a e - a \cos E \quad (24)$$

$$r = a(1 - e \cos E) \quad (25)$$

$$r = a [1 - P_1 \sin(l_\theta + \theta) - P_2 \sin(l_\theta + \theta)] \quad (26)$$

The second element of the orbital window represents the position of the orbit in the east-west direction. As shown in Figure 2, the value of  $\lambda$  is obtained from the difference between  $\alpha$  and  $\theta$  (here  $\alpha = L$ ). This element is defined by the following relations using the Keplerian parameters and its redefinition:

$$\lambda = L - \theta = \omega + \Omega + v - \theta \quad (27)$$

$$\lambda = \omega + \Omega + M + 2e \sin M - \theta \quad (28)$$

$$\lambda = l_\theta + 2e \sin[l_\theta + \theta - (\omega + \Omega)] \quad (29)$$

$$\lambda = l_\theta + 2P_1 \cos(l_\theta + \theta) + 2P_2 \sin(l_\theta + \theta) \quad (30)$$

The third and the last element of the orbital window is the north-south element. This element is defined by the following relations using the Keplerian parameters and its redefinition:

$$\sin \varphi = \frac{z}{r} = \sin(\omega + v) \sin i \quad (31)$$

$$\varphi = 2 \tan \frac{i}{2} \sin(v + \omega) = 2 \tan \frac{i}{2} \sin(L - \Omega) \quad (32)$$

$$\varphi = -2Q_1 \cos L + 2Q_2 \sin L \quad (33)$$

$$\varphi = -2Q_1 \cos(l_\theta + \theta) + 2Q_2 \sin(l_\theta + \theta) \quad (34)$$

### Attitude motion control

In this research the attitude control of the satellite is performed using three reaction wheels placed on each axis of the satellite.

The required angular velocity to compensate for the satellite attitude deviation could be determined utilizing a proportional-derivative (PD) controller. The coefficients related to this controller were calculated utilizing optimization software.

#### Orbit motion control

In order to control the orbital motion according to the four thrusters mounted on one side of the satellite and to consider the coupling of the attitude in the orbital motion, which is one of the main innovations of this research, a control strategy is performed in order to achieve the least control effort and also the maximum time interval between two control attempts. For this purpose, firstly, the satellite control in the two directions of north-south and east-west is separately considered, and then, the equations are developed in such a way that a correction orbit is found for the satellite that has the longest time to correct again.

### North-South station keeping

Environmental disturbing forces applied on a geostationary satellite gradually perturb its orbit and attitude. As the satellite approaches the borders of its station-keeping window, the control system is activated, maintaining the satellite inside its permitted latitude and longitude limits.

The position of the satellite in the north-south direction can also be seen as a modification of the satellite's inclination. There are various strategies for satellite position control. In this research, the maximum circulation strategy is utilized in order to maximize the roaming time of the satellite in the operational box in the orbit. The velocity difference could be calculated as the needed speed to return the satellite to the operational box as follows:

$$i = \tan\left(\frac{i}{2}\right) \quad (35)$$

$$|\Delta i|^2 = \Delta Q_{1t}^2 + \Delta Q_{1t}^2 = \frac{\Delta v_{tN}^2}{4v_{sk}^2} \quad (36)$$

$$= 4 \tan\left(\frac{i_{max}}{2}\right)$$

$$\Delta v_{tN}^2 = 4 \tan\left(\frac{i_{max}}{2}\right) \cdot v_{sk}^2 \quad (37)$$

To reach the purpose of the chosen strategy, the equations need to be solved in such a way that the inclination at that time varies twice before the maneuver. In these equations  $\Delta v_{tN}$  is the variation

amount of the needed velocity,  $v_{sk}$  is the satellite velocity at the moment of starting the maneuver.

### East-West station keeping

Satellite control in the east-west direction can be considered in the orbit modification in the x-axis and y-axis in the earth-centered inertial frame. Controlling the orbit in this direction requires less fuel than the north-south direction, but the number of control firings may be more and the complexity of these efforts is more as well. A suitable strategy for this mode is the maximum circulation strategy. By matching the translational motion of the satellite in the east-west direction, we have a quadratic equation as follows:

$$\lambda = \alpha + \beta t + \frac{\gamma}{2} t^2 \quad (38)$$

This value could be considered for the moments before and after the thruster firing, and can be written as equal to the moment of effort:

$$\lambda = \alpha + \beta t + \frac{\gamma}{2} t^2 = \tilde{\alpha} + \tilde{\beta} t + \frac{\gamma}{2} t^2 \quad t = t_{start} \quad (39)$$

The following results can be obtained for an east-west motion:

$$\alpha - \tilde{\alpha} = \Delta\alpha \quad (40)$$

$$\beta - \tilde{\beta} = \Delta\beta \quad (41)$$

$$\lambda(t=0) = \alpha \quad (42)$$

$$\dot{\lambda}(t=0) = \beta \quad (43)$$

$$\Delta\beta = \dot{\lambda}(t_{start}^+) - \dot{\lambda}(t_{start}^-) \quad (44)$$

If  $\lambda = l_\theta$  as described in the section related to the rewriting of the Keplerian elements, we can conclude that:

$$\Delta\beta = \Delta l_{\theta(start)} = -3 n_k \frac{\Delta v_{tstart}}{v_{sk}} \quad (45)$$

$$\Delta v_{tstart} = -\frac{a_k \Delta\beta}{3} \quad (46)$$

$$\Delta\beta = \beta + \gamma t_{start} + 2\sqrt{\gamma \lambda_{max}} \quad (47)$$

Given the above equations, we can find the difference in velocity needed to bring the satellite back into the operational window.

### Case studies and numerical results

Previous sections have outlined the station keeping problem, as well as the necessity of solving it, the approaches employed to solve this problem, and also the proposed strategy.

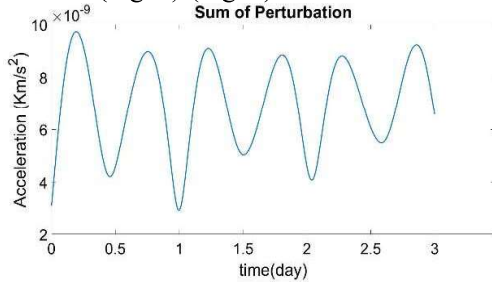
This section deals with the simulation results in presence of perturbation effects on the satellite. The coupling effect of translational and attitude motion



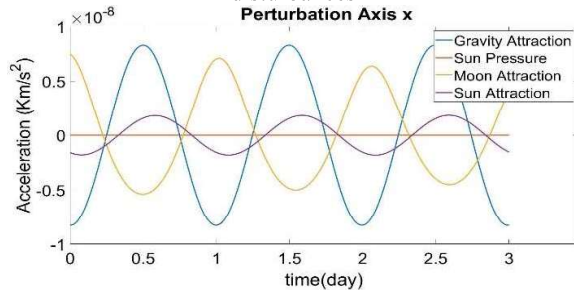
is examined, and the results of attitude control and position control are discussed. All simulations performed in this part have been implemented in MATLAB software.

In this study the initial position of the satellite in Geo orbit. is  $r_0 = [42158 \ 0 \ 0]$  and its initial speed applied is  $v_0 = [0 \ 3.07483 \ 0]$ .

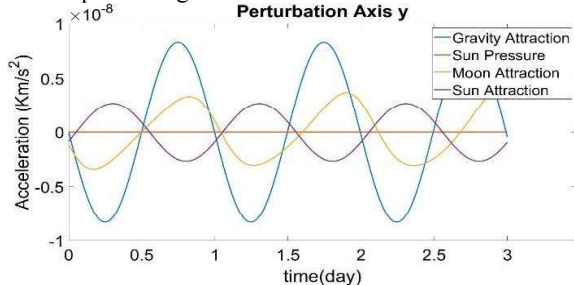
The total resultant perturbation acceleration exerted on the satellite in each direction is expressed and examined. (Fig. 1)-(Fig. 5).



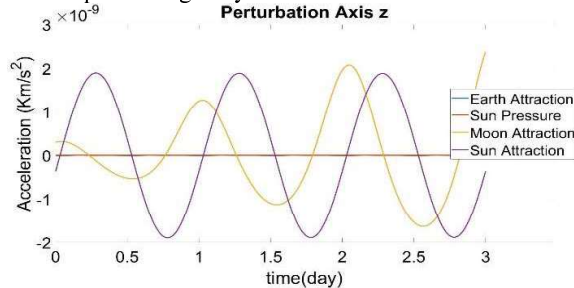
**Fig.3** Time history of the total environmental disturbances



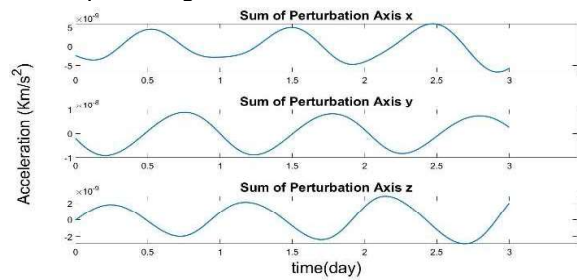
**Fig. 4** Time history of environmental disturbances decomposed along the X axis of the ECI reference frame



**Fig. 5** Time history of environmental disturbances decomposed along the y axis of the ECI reference frame



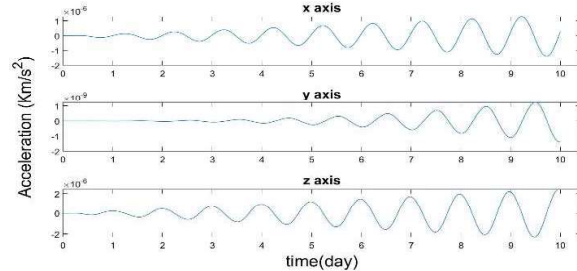
**Fig. 6** Time history of environmental disturbances decomposed along the z axis of the ECI reference frame



**Fig. 7** Time history of total environmental disturbances decomposed along the three axes of the ECI reference frame

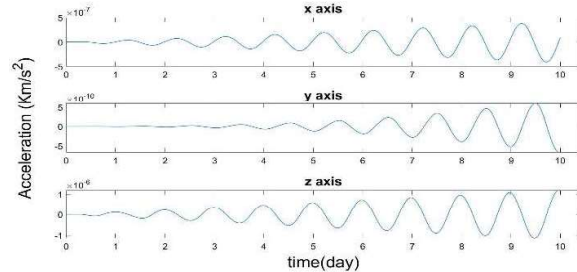
It is necessary to implement the motion simulation using the governing equation of the rigid body in the space to observe the mutual effect between the position and attitude motion.

Simulation results reveal that the angular velocity of the satellite may exert a force on the satellite position. In turn, this perturbation could move away the satellite from its position. If no control is applied to the satellite position, then, the satellite departs from the desired position and begins to approach the borders of its orbital window (Fig. 6).



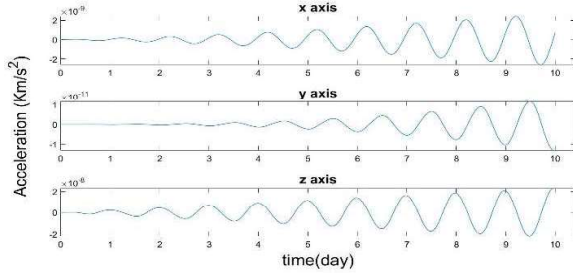
**Fig. 8** Attitude motion coupling effect on translational motion of the satellite

In the second case, we ignore 50% of the attitude perturbation and implement the simulations only in the presence of 50% of the perturbation, which the perturbation amplitude is still very high as can be seen in (Fig. 7).



**Fig. 9** Decomposed acceleration causing by the coupling perturbation with ignore 50% of the attitude perturbation

In the next case, 99% of the satellite attitude error is controlled, and the perturbation rate, in this case, is proportional to the other perturbations and is controllable as can be seen in (Fig. 8).



**Fig. 10** Decomposed acceleration caused by the coupling perturbation ignoring 99% of the attitude perturbation

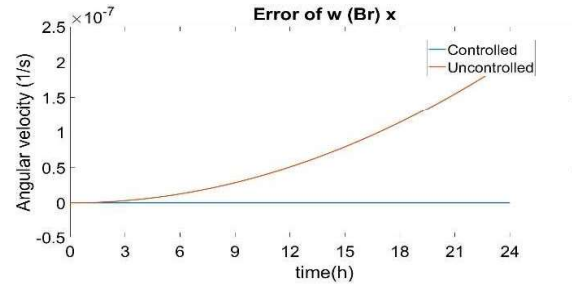
Simulation results show that to alleviate the effect of this type of perturbation on the satellite translational motion, it is necessary to control the satellite attitude.

As mentioned before, this perturbing force depends on both angular velocity and position error. Thus the reduction of this perturbation could be obtained by reducing each of these two states. It should be noted that reducing the position error leading to fuel consumption is much more important than reducing the angular velocity, just as in geosynchronous satellites. Consequently, it can be concluded that to reduce this type of perturbation force effect, no special process is required and if the position and the attitude of the satellite are controlled properly, the effect of this perturbation could reach to an acceptable value.

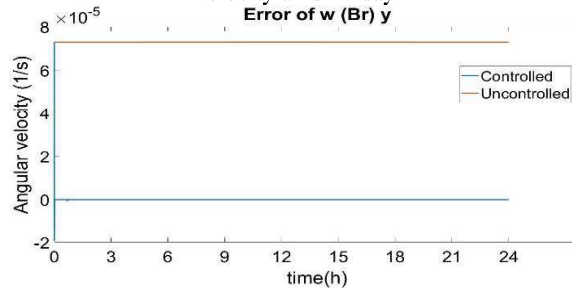
## Results of orbital and attitude control

### Attitude control

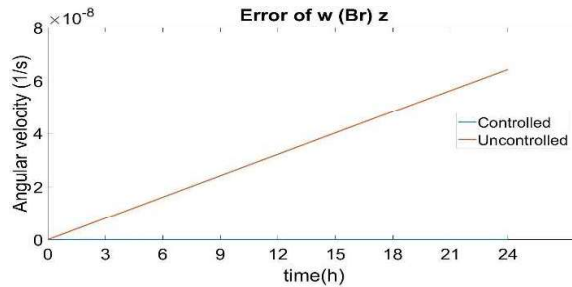
The attitude control in this research, as discussed in the previous chapter, is performed to minimize the intrinsic torque of the satellite and to reduce the perturbation acceleration due to the mutual effect between position and attitude on each other. Figs. 9-12 show the simulation results of the satellite attitude correction which has been performed by a PD compensator actuating three reaction wheels mounted on each controlling direction.



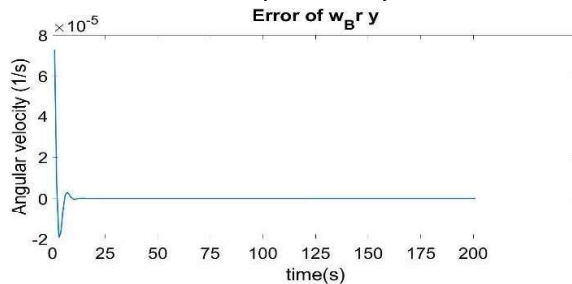
**Fig. 11** Compare controlled and uncontrolled angular velocity axis x 1day



**Fig. 12** Compare controlled and uncontrolled angular velocity axis y 1day



**Fig. 13** Compare controlled and uncontrolled angular velocity axis z 1day

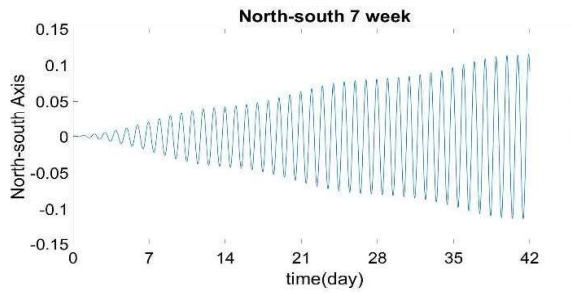


**Fig. 14** Angular velocity error in axis y in 300 seconds

### Orbital control (north-south station keeping)

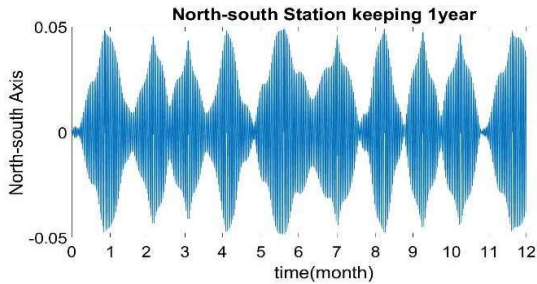
In the north-south direction, the satellite control was conducted by a thruster which applied force to the satellite in the z-direction of the body frame. Fig. 13 shows the satellite position in the north-south direction in the absence of a position controller.





**Fig. 15** Satellite behavior in North-south direction without control

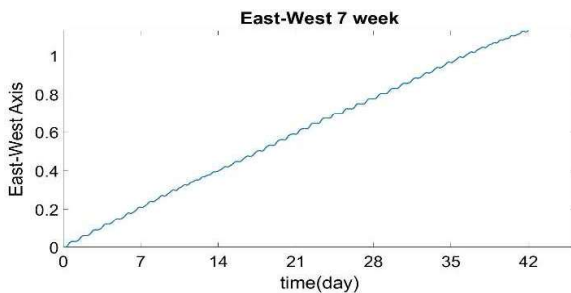
Fig. 14 indicated that during one year of operation, about 11 control maneuvers should be applied to preserve the satellite in its window in the north-south direction.



**Fig. 15** Satellite behavior in North-south direction with control

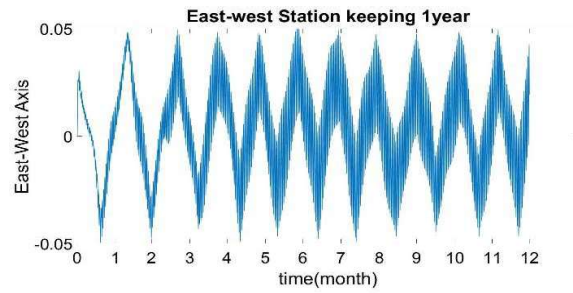
### East-west

The satellite is also controlled by a thruster in the east-west direction, with the difference that the force is applied to the satellite in the y-direction of the body frame. Fig. 15 shows the satellite position in the east-west direction in the absence of a controller.



**Fig. 17** Satellite behavior in East-West direction without control

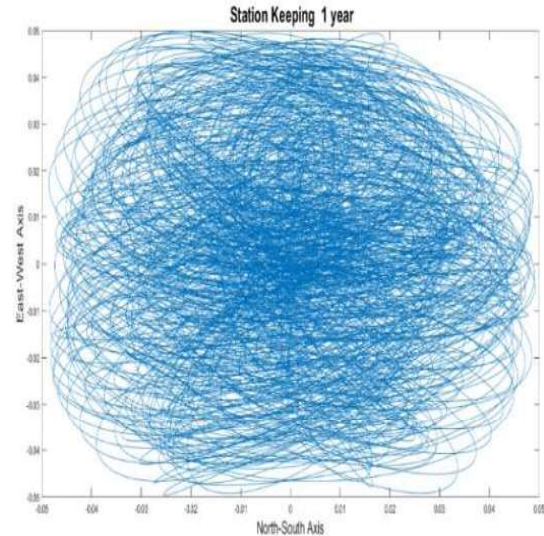
As it can be seen, after the first control maneuver, it needs another maneuver approximately every 17 days, and as specified in Fig. 25, during one year of operation, about 22 control maneuvers should be applied to preserve the satellite in its orbital window in the east-west direction (Fig. 16).



**Fig. 18** Satellite behavior in East-West direction with control

The amount of speed variation required for satellite control in the north-south direction for one year equals to 0.0509859 km/s, and the velocity value required for the west-west direction equals to 0.000495.

Satellite motion in one year in the predefined orbit window is represented in Fig. 17.



**Fig. 19** Satellite behavior in Geo orbit window for one year of operation

### conclusion

The effect of coupling the attitude and the orbital motion was missed in literature reviews, thus in this research, the impact of the coupled equations in addition to the environmental disturbances are included in the orbital equations of the satellite. In this study, an efficient strategy to solve the station-keeping problem of a GEO satellite in the presence of disturbances has been developed. According to this control strategy, the required velocity increment can be calculated based on the assessments performed on the position and the velocity deviations from the nominal reference trajectory. The deviations in orbital elements, environmental

disturbances, the satellite's instantaneous velocity, its mass and surface area, the magnitude of the force provided by the propulsion system, the maximum and minimum allowable values of action duration of thrusters are the main factors which have influence on the system performance. After describing the proposed approach, some case studies were presented and the satellite's translational motion in the presence of environmental disturbances was evaluated.

Based on the results obtained from simulations, it is observed that in the absence of the attitude control, the perturbation coupling effect of the attitude and orbit on the orbital motion increases and causes divergence and uncontrollability of the satellite. The satellite will be controllable when the perturbation effect of the coupling reduces considerably due to the complete control of the attitude motion executing the reaction wheels. Then, in the presence of the mentioned perturbation factors and also the residual amount of the coupling effect, a method is implemented to accurately maintain the satellite in its orbital window utilizing the introduced propulsion system. It is observed that the satellite will be controlled for one year by the velocity variation mentioned in the previous section.

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