Science Article

Intelligent Sensor Fusion in High Precision Satellite Attitude Estimation Utilizing an Adaptive Network-Based Fuzzy Inference System

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In this study, Adaptive Network-Based Fuzzy Inference System (ANFIS) is presented with a sensor data fusion approach to estimate the satellite attitude. The active sensors are the sun and earth sensors. Satellite attitude dynamics, including attitude quaternion and angular velocities are estimated simultaneously utilizing the measured values by the sensors. The Extended Kalman Filter (EKF) is employed to verify and evaluate the efficiency of the presented method. Additionally, the neural networks with Radial Basis Function (RBF) and Multi-Layer Perceptron (MLP) are also designed to prove the superiority of the proposed ANFIS network among the smart methods of sensor data fusion for satellite attitude estimation. Root Mean Square Error (RMSE) as a numerical criterion and graphical analysis of residues are utilized to evaluate the simulation results. The simulations confirm that the obtained estimations from ANFIS network have more accuracy in modeling of nonlinear complex systems compared to EKF, MLP, and RBF networks. In general, using intelligent data fusion, especially ANFIS, reduces attitude estimation error and time in comparison to the classical EKF method.

Keywords: Attitude estimation; Data fusion; ANFIS; Extended Kalman Filter; Neural network

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Introduction

One of the important subsystems of a satellite is Attitude Determination and Control Subsystem (ADCS), which is utilized to keep the satellite in a desirable attitude and position. ADCS employs several types of sensors to determine the satellite direction toward a particular reference, such as the sun or stars. Therefore, different processing methods and sensor data fusion approaches should be considered in ADCS design [1, 2].

All data raised from the sensor measurements are uncertain. The fusion process of information leads to establishment of a database, which is broader, stronger, and more accurate than any individual databases created by each independent sensor. Environmental complexities and uncertainties, errors and limitations in the capability of the sensors, prevent the creation of complete and accurate information by each sensor, independently.

The fusion of information is known as one of the best methods to achieve useful information with maximum reliability. Obtained information from

different sensors is combined in data fusion process to achieve more comprehensive results [3]. Features such as reduction of uncertainty and increasing estimation speed and reliability, are the other advantages of utilizing sensor data fusion.

For some reasons, such as noise sources, sensor data cannot be utilized as an input for control system directly. Therefore, it is necessary to remove the noise from measurement data by employing appropriate filters, before entering them into the control system. On the other hand, common filters are not suitable for employing in precise applications due to time delay.

Kalman filter which is known as a sensor data fusion method is a suitable option for utilizing in satellites due to its ability in removing noise and estimation of the system state variables simultaneously.

Neural networks are also a part of model-free intelligent systems that are able to model a highly nonlinear and complex system by utilizing a collection of neurons [4]. Because each neuron in the network is affected by activities of the other neurons, despite eliminating some of the grid cells or in the case of malfunction of some neurons, it will be possible to reach the correct answers. In addition, based on the interpolation mechanism, network can provide a suitable output when faced with an un-experienced situation. The existence of a parallel structure also increases the processing speed in the form of hardware implementation. In fact, these networks, by processing experimental data, transfer the rules and knowledge which are hidden behind the information to the network structure. Therefore, they are able to learn the general rules based on fulfillment of calculations upon the primary data and examples. These networks have shown very high performance for estimation and approximation [5].

As mentioned before, due to important role of ADCS in keeping the satellite in suitable position and situation, multiple researches have been done in this area. However, it is noteworthy to know that despite the strategic importance of satellite control in Geosynchronous Earth Orbit (GEO) and economic nature of the issue, generally, in many of the references, only the general topic and exerted methods have been mentioned and the precise executive details have been refused to be presented.

An attitude estimator has been proposed based on Kalman filter which considers attitude quaternion, scale factor, gyro bias ratio, and star tracker bias as its state variables [6]. Of course, it is in a situation that angular velocity of the satellite has not been taken into account as a system state variable. Hajiyev et al. proposed a combination of EKF and Singular Value Decomposition (SVD) to estimate the attitude angles, angular velocities, and gyro biases of a small satellite. Robust Kalman filtering by using both modified methods of EKF and Unscented Kalman Filter (UKF) for attitude estimation in Pico satellites considering evaluated faults is proposed in [8 and 9]. Myung et al. proposed a UKF that can estimate the state variables of the satellite system, which includes quaternion and angular velocities of the satellite, and is able to estimate gyro sensor setting variables. In this reference, it is supposed that measurement includes quaternion vector, so the satellite attitude measurement vector is assumed linear which improves the estimation error and makes the filter design easy [10]. Ejiang and Chang employed a Kalman filter for data fusion of sensors, which used the weighted combination of error covariance matrix of each sensor for error covariance matrix of the system [11]. Star sensor, GPS, and gyro were intended to determine the satellite attitude. A gain-scheduled EKF was also proposed to decrease the computational load in nano-satellite attitude determination process [12]. The Kalman gain was determined analytically with the aid of sensor parameters, instead of computing the online Kalman gain. Souza et al. investigated the antenna pointing system for satellite tracking. Kalman filter based conical scan technique was employed for estimation [13]. Choi proposed a new method utilizing GPS observations for real-time navigation based on unscented filtering [14]. Effects of low frequency errors of star sensor with a novel multiple-model of Kalman filter is introduced in [15]. Kalman filter has also been used in [16-18] to estimate the attitude parameters. A Kalman filter has been presented in Ref. [19] to utilize a seven-component angular-momentum state vector including the parameters of angular momentum in an inertial and body frame, and a rotation angle to estimate the attitude of a spinning spacecraft. The constraint that was applied to the filter was the same magnitude of the angular momentum vector in body and inertial frames, which greatly facilitate the measurement of sensitivity matrices.

A Locally Linear Neuro-Fuzzy model (LLNF) with a locally linear model tree learning algorithm was suggested to estimate the Euler angles of a

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Low Earth Orbit (LEO) satellite based on observations of the sun and magnetic sensors [20]. Yue et al. discussed about the application of neural networks in navigation issue and the integration of Global Positioning System (GPS) with Inertial Navigation System (INS) to present a new integration scheme for spacecraft attitude determination [21].

As it can be found from the above literature, although the classical approaches of multi sensor data fusion have been examined in several attitude estimation researches, but application of smart methods in designing satellite attitude determination system is not still so prevalent.

In this article, to achieve high accuracy in attitude estimation of a GEO satellite, equations of motion and model of sensors will be expressed for the assumed satellite. Simulation of satellite attitude estimation is carried out with different views on sensor data fusion as follows:

• EKF as one of the classical methods
• MLP, RBF artificial neural networks and ANFIS network as intelligent methods

The efficiency of sensor data fusion in satellite attitude estimation in comparison with single sensor methods is proved. Finally, simulation of algorithms is implemented in MATLAB and the estimated results obtained from these strategies will be compared with each other to show the ANFIS capabilities.

Satellite equations of motion

The equations of motion for a satellite can be defined as follows [22]:

$$\dot{\omega} = -J^{-1}\omega \times J\omega + J^{-1}u \tag{1}$$

In which, $\omega \in R^3$ is defined as angular velocity vector of the body, J is the moment of inertia matrix and $u \in R^3$ is input control torque. Now, attitude quaternion is employed to express the attitude kinematics [22]:

$$\dot{q} = \frac{1}{2}\Omega(\omega)q = \frac{1}{2}\Xi(q)\omega \tag{2}$$

In which

$$q = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}^T = \begin{bmatrix} q_{13}^T & q_4 \end{bmatrix}^T \tag{3}$$

and,

$$\Omega(\omega) = \begin{bmatrix} -[\omega\times] & \omega \\ -\omega^T & 0 \end{bmatrix} \tag{4}$$

$$\Xi(q) = \begin{bmatrix} q_4 I_3 + [q_{13}\times] \\ -q_{13}^T \end{bmatrix} \tag{5}$$

$$[\omega\times] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{6}$$

Model of sensors

Sun sensor measures vector of the sun direction in satellite body frame. Regarding the satellite orbital position and the position of Earth in its orbit around the sun, direction of the sun vector is determined in the orbital frame. So, the measured vector by the sun sensor is obtained from the following equation:

$$V_s^b = V_o^b V_s^o + w \tag{7}$$

In which w is a random variable with zero mean and the normal Gaussian distribution with standard deviation of σ_s. In equation (7), the rotation matrix, which relates the body frame to orbital frame, is a direction cosine matrix that is defined in [1].

The basis for modeling of the earth sensor is also similar to the sun sensor. Whenever the earth direction is specified in satellite orbital frame based on orbital data, the earth vector in satellite body frame that has been measured by the earth sensor will be obtained through the following equation (8).

$$V_E^b = V_o^b V_E^o + w \tag{8}$$

In which measurement without bias and Gaussian distribution of noise with standard deviation of σ_s is assumed.

Model of Extended Kalman Filter

Extended Kalman filter method is a well-known approach for satellite attitude estimation in noisy environments. EKF is widely employed for fusion of sensor data with regard to the simplicity and the robustness. However, since it must solve many kinematic and dynamic equations of the satellite at

any moment, a long delay will be imposed on system; and therefore, subsystems will lose their synchronicity.

To utilize this approach in noisy environments, we must assume that disturbances and noise of observations exerted on the model is Gaussian. Otherwise, the filter will lose the ability to provide the correct estimation. On the other hand, the first requirement in applying this method and most of non-intelligent data fusion methods is to have an appropriate model for the control system along with parameters of the model. Additionally, Kalman filter employs first-order approximation for linearization based on Taylor expansion that is not relatively a good approximation for complex nonlinear systems. It is obvious that, lack of the aforementioned information, or even rough approximation of them leads to a false estimation in navigation and situation parameters. Improper selection of initial values for state variables and error covariance matrix of Kalman filter will lead to divergence of the filter and to distance from the actual values.

Here, the Extended Kalman Filter (EKF) is utilized to verify and evaluate the efficiency of the presented method.

Suppose that system and measurement equations are defined as follows [23]:

$$x_k = f(x_{k-1}, u_{k-1}, w_{k-1})$$

$$z_k = h(x_k, v_k)$$

$$\tag{9}$$

In which the random variables w_k and v_k are process and measurement noise respectively, and have Gaussian distribution [23] i.e.

$$p(w) \sim N(0, Q)$$

$$p(v) \sim N(0, R)$$

$$\tag{10}$$

The Jacobian matrixes of partial derivatives of f and h with respect to x which is shown by A and H respectively, can be defined as:

$$A_{[i,j]} = \frac{\partial f_{[i]}}{\partial x_{[j]}}(\tilde{x}_{k-1}, u_k, 0)$$

$$H_{[i,j]} = \frac{\partial h_{[i]}}{\partial x_{[j]}}(\tilde{x}_k, 0)$$

$$\tag{11}$$

Time updating equations of EKF between two sampling intervals t_{k-1}^+ and t_k^-, denoted as t, and the related measurement update algorithm at $t = t_k^+$ could be extracted as follows [22].

$$\dot{\overline{x}}_t = f(\overline{x}, t), \overline{x}(t_{k-1}) = \hat{x}_{k-1}$$

$$\overline{A}_t = A\big|_{x=\overline{x}_t}$$

$$\dot{\overline{P}} = \overline{A}_t\overline{P}_t + \overline{P}_t\overline{A}_t^T + Q_t, \overline{P}(t_{k-1}) = \hat{P}_{k-1}$$

$$\tag{12}$$

$$\hat{x}_k = \overline{x}_k + K_k(z_k - h(\overline{x}_k, t_k))$$

$$H_k = H\big|_{x=\overline{x}_t}$$

$$K_k = \overline{P}_k H_k^T (H_k \overline{P}_k H_k^T + R_k)^{-1}$$

$$\hat{P}_k = (I - K_k H_k)\overline{P}_k$$

$$\tag{13}$$

Where for example the symbols $\overline{\theta}$ and $\hat{\theta}$ for the variable θ represent an estimate before and after the measurement update phase.

Initial Conditions and Filter Parameters

In order to solve the recursive equations given in equations (12) and (13), some initial conditions must be considered to estimate the state variables and covariance of early estimation. The following values are chosen as the initial condition:

$$\hat{x}_q(0) = \begin{bmatrix} 0.7067 & -6 \times 10^{-5} & 0.0251 & 0.7071 \end{bmatrix}$$

$$\hat{x}_w(0) = \begin{bmatrix} 0.019 & 0.020 & 0.021 \end{bmatrix}$$

$$P = \begin{bmatrix} 10^{-2}I_{4\times4} & 0_{3\times3} \\ 0_{4\times4} & \dfrac{\pi}{180}I_{3\times3} \end{bmatrix}$$

For the correct operation of the filter, the availability of the sun and earth vectors in the orbital frame is necessary. Thus, the problem of attitude estimation is dependent on the orbit estimation. To check the attitude estimator's performance, it is assumed that the sun and earth vectors in orbital frame are defined as in equations (14) and (15).

$$S^o = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T \tag{14}$$

$$E^o = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T \tag{15}$$

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It should be noted that accurate estimation of attitude does not depend on the values of the vectors, but correct estimation of the vectors by the orbit propagator is important.

Assuming that the earth and sun vectors are perpendicular to each other in the orbit frame, it gives much information from these two sensors. In order to have a criterion from actual data and the data infected with noise, it is necessary to solve satellite equations per initial conditions and specified control torque to obtain the real data for the system. After that, data will be contaminated by noise, and estimation of the desired states takes place. Therefore, with regard to equations (7) and (8) we will have:

$$z_{1(sun)} = q_1^2 - q_2^2 - q_3^2 + q_4^2 + v_1$$
$$z_{2(sun)} = 2(q_1 q_2 + q_3 q_4) + v_2 \qquad (16)$$
$$z_{3(sun)} = 2(q_1 q_3 - q_2 q_4) + v_3$$

$$z_{1(earth)} = 2(q_1 q_2 - q_3 q_4) + v_4$$
$$z_{2(earth)} = -q_1^2 + q_2^2 - q_3^2 + q_4^2 + v_5 \qquad (17)$$
$$z_{3(earth)} = 2(q_2 q_3 - q_1 q_4) + v_6$$

The noise mean value for all sensors are also assumed to be equal to zero and standard deviation of the noise of the sun and earth sensors is considered equal to 0.001. The parameters used in dynamic part of the model and filter are related to geosynchronous satellite. We will have:

$$J = \begin{bmatrix} 1218.6 & -5.3 & -1.8 \\ -5.3 & 442.8 & -8.4 \\ -1.8 & -8.4 & 1429.4 \end{bmatrix} kg.m^2$$

$$\omega_{o/b}^o = 7.27 * 10^{-5} rad / s$$

The measurement matrix and linearized system matrix in the Kalman filter equations which denoted as H and A, respectively, are defined as follows:

$$H = \begin{bmatrix} \dfrac{\partial z_{sun}}{\partial q} & 0_{3\times 3} \\ \dfrac{\partial z_{earth}}{\partial q} & 0_{3\times 3} \end{bmatrix} \qquad (18)$$

$$A_{7\times 7} \cong \frac{\partial F}{\partial x} = \begin{bmatrix} \dfrac{\partial \dot{q}}{\partial q} & \dfrac{\partial \dot{q}}{\partial \omega} \\ 0_{3\times 4} & \dfrac{\partial \dot{\omega}}{\partial \omega} \end{bmatrix} \qquad (19)$$

Multi-Layer Perceptron (MLP) neural network

Neural Networks include a series of processing elements called nodes or neurons which are internally connected to each other by synapses, or axons.

Each neuron sends stimulations or input data to the outputs after processing, and each output can itself be as an input for the next neurons. The connections between neurons have weighting coefficients, and organization of neurons and their connection manner are usually fixed, but weighting coefficients and biases can change. Recursive algorithms express neural network behavior as a learner system. The learning capability of a neural network means the ability to configure and update network parameters and synaptic weights based on experience of conditions and getting new information [24].

In this study, a Multi-Layer Perceptron network with an intermediate layer has been used to estimate the attitude. This structure contains static arrangement, but can be used to control the dynamical systems. For this purpose, it is necessary to bring forward all the information at any moment which includes dynamics of the system as network input.

If we show any input data with x_i that i = 1,2, ..., n, then the output of mth neuron from the first layer will be calculated from the following equation.

$$z_m = g\left(\sum_{i=0}^{n} x_i w_{im}\right) \qquad (20)$$

The matrix w with elements w_{im}, is a $(n+1)\times M$ dimensional matrix where n is the size of every input data and M is the number of neurons in the hidden layer. z_m is the input of output layer neurons. Therefore, the output of jth neuron can be calculated as:

$$y_j = g\left(\sum_{m=0}^{M} z_m u_{mj}\right) \qquad (21)$$

The matrix U with elements u_{mj}, is a $(m+1) \times J$ dimensional matrix where J is the number of neurons in the output layer.

The function g in the hidden layer is a non-linear function and works as a nonlinear mapping, but in the output layer, it is a linear function. In this research, g for the hidden layer is the intended function $tanh(.)$. The intended cost function for neural network is in accordance with the following equation.

$$SSE = \sum_{q=1}^{Q} \left\| y^q - \hat{y}^q \right\|^2 \qquad (22)$$

In this equation, y^q is the amount of a desirable output that the neural network must have per data. And $\hat{y}^q$ is the amount of actual output of the network per the same data. To update the weights, Steepest Descent method has been used according to equation (23).

$$w_{im}{}^+ = w_{im}{}^- - \rho \frac{\partial SSE}{\partial w_{im}}\bigg|_-$$
$$u_{mj}{}^+ = u_{mj}{}^- - \rho \frac{\partial SSE}{\partial u_{mj}}\bigg|_- \qquad (23)$$

In which, ρ is the learning rate that is adjusted according to the desired function. In general, for training the neural network, the following algorithm can be considered:
Starting from one initial condition for U, W
Calculation of SSE with U, W available for data
Update of U, W using steepest descent method
Go back to step 2 with new U, W until terminal condition is met.

Before training the neural network, it is worth to note that the input data should be normalized so that the neurons could work in their own linear area and do not enter into the saturated area. To stop learning, we utilize a technique to divide data into training, testing, and validation data. A part of data is used to build the network through training and testing. The other part of data that none of the estimators have seen, is used to validate and compare the methods. From the existing data, 70% are used for training, 20% for testing, and 10% have been considered for validation. The network learning is done according to both training and testing data sets so that by using the training data, the network is trained, and then, one-step readout

is carried out. In the other words, assuming that learning is finished, observations of the test data are given to the network, and the actual output of the network is compared with the desired output. With every epoch the error of training data output is reduced; because, the network operates on the basis of reduction in training data output error. Additionally, the error of test data output also decreases by increasing the number of epoch. Usually, the output error, according to training data is less than the error with regard to test data; because, network learning has taken place using training data. Reduction in the error of test data output will continue until the network reaches to a balance state between generalization and specialization. Then, due to excessive accommodation of network parameters with training data, slope of reduction in errors of test data output decreases and in some cases, the output errors increase. Figure (1) shows the error curve on each epoch for training and test data.
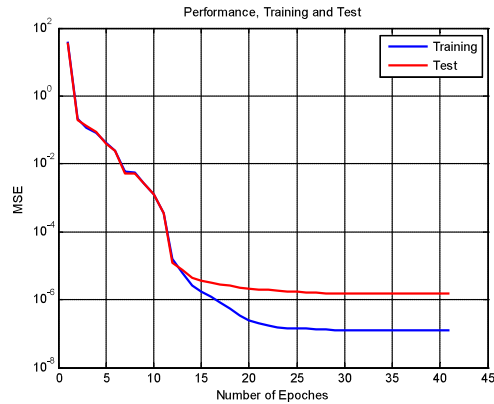

Fig. 1 Mean square error for training and test data vs. number of epoch

The optimal number of epochs is assumed where the distance between two curves increases and test error reduction rate decreases. For instance, the value of 20 in Figure (1) is virtually the optimal number of epochs. After finding the best number of epochs, the number of neurons in the hidden layer must be specified. The number of neurons in the hidden layer cannot be obtained with conventional optimization methods; because, it is a discrete parameter, while the parameters of the optimization methods are usually real numbers and could be differentiated. For this purpose, considering the optimal number of epochs, the number of neurons is increased from low to high and a model is made in any case.

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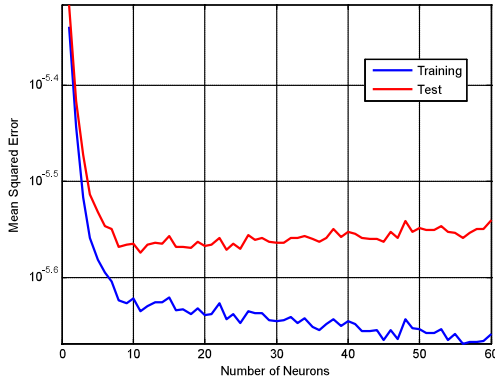The error curve relative to the number of neurons is plotted in Figure (2) for training and test data.

Fig. 2 Mean square error for training and test data vs. number of neurons in hidden layer

As it could be found from Fig. 2, by increasing the number of neurons, each model will be a reference for the previous model, and consequently, the training data error will be reduced. Also, test data error decreases, and then, starts to increase, or if the data was rich, it remains constant. The optimal number of neurons is considered where the distance between two curves increases and the rate of reduction in test error decreases. The value of 10, in Figure (2), is approximately the optimal number of neurons in the hidden layer.

Radial Basis Function (RBF) neural network

RBF network is a three-layer neural network and is defined as a general function approximator; unlike the MLP network, it can be easily analyzed. RBF network basis is Stone theorem that has been represented in equation (24).

$$\forall \varepsilon > 0; \exists M, C_i, w_i \rightarrow$$
$$\hat{f}(U, [w_i]_{i=1}^M, [C_i]_{i=1}^M) = \sum_{i=1}^M w_i h(\|U - C_i\|) \qquad (24)$$

Where $h(x) = e^{-\frac{1}{2}x}$. According to Stone theorem, there are three degrees of freedom in RBF network called network parameters as follows:

Gaussian functions location (C_i)
Gaussian functions weight (w_i)
The number of Gaussian functions (M)
The hidden layer in RBF network is Gaussian function instead of sigmoid, and here, comparison criterion is norm.
In this method, like MLP network for training, measurement of the earth and sun sensors are given to the network as an input in addition to quaternion and angular velocities. It should be noted that quaternion and angular velocities are obtained from solving the satellite's kinematics and dynamics equations using rank 4 Runge-Kutta method as target. The same valid data set, collected for the MLP network has been utilized. The first, second, and third categories have been used for training, testing, and validation, respectively.

Adaptive Network-Based Fuzzy Inference System (ANFIS) network

Utilizing Neural networks and ANFIS Network for attitude estimation can greatly enhance the speed and accuracy of estimation; because, the learning phase which is very time consuming, takes place before the satellite launch phase. Therefore, in space operation, only the matrix multiplication and addition operations are executed in a matter of seconds.
ANFIS network provided with increased learning ability can be used in real time for compensation and proofing estimation. Moreover, since in practical applications of engineering, the observation noise of measurement sensors is not Gaussian, using the proposed method in comparison with EKF, which is a conventional and classical method to estimate satellite attitude, will have fewer errors.
ANFIS network's feedforward formulas with two inputs and two labels for each input are obtained through equations (25) and (26) [26].

$$w_i = \mu_{A_i}(x) \times \mu_{B_i}(y), \quad i = 1, 2$$

$$\overline{w}_i = \frac{w_i}{w_1 + w_2}, \quad i = 1, 2 \qquad (25)$$

$$f_1 = p_1 x + q_1 y + r_1$$

$$f_2 = p_2 x + q_2 y + r_2 \qquad (26)$$

$$f = \frac{w_1 f_1 + w_2 f_2}{w_1 + w_2} = \overline{w}_1 f_1 + \overline{w}_2 f_2$$

In this study, hybrid learning rule "Gradient Descent" and "Least Square Error (LSE)", has been utilized to update the parameters of ANFIS structure. Therefore, Takagi and Sugeno's fuzzy

if-then rule has been employed for this purpose. This method has been introduced by Jang in [26] for updating the parameters of the ANFIS structure.

Here, like MLP and RBF networks, measurement of the earth and sun sensors is given to the network as the input data in addition to quaternion and angular velocities as target. Like before, the same collected valid data set is utilized for learning, testing, and validation of network as mentioned in the previous methods.

Results of Simulations

In this section, the results of simulations extracted from extended Kalman filter, MLP network, RBF network, and ANFIS network have been presented. Initially, state variables are estimated using EKF, and then, relying on the results of the estimation, these strategies are compared with each other. To validate and compare these methods, 10% of validation data set, seen by none of the estimators, has been used.

The RMSE numerical criterion and graphical analyses of curve of residuals have been used for validation of the estimations. The curve of residuals has higher degree of certainty compared to numerical criterion, and contains widespread information on different aspects of an estimate. The residuals of a fitted model are defined as the difference between the responses observed in each step and the corresponding prediction of calculated response by the model. Mathematically, the definition of residual for i^{th} observation in the data set is as follows:

$$e_i = y_i - f(x_i, \theta) \qquad (27)$$

Where y_i is the ith response existing in the data set, x_i is the inputs corresponding with that output, and $f(x_i, \theta)$ is the output obtained from the model.

The various analyses have been carried out on the curve of residuals. The most important analysis includes investigating the zero mean of residuals, residuals independency, testing whiteness, and uniformity of distribution of residues with minimum number of outlier data. If the model is fitted properly on the data, the distribution of residuals will be randomized and the relationship between the input and response variables will have a statistical relationship. Therefore, if residuals are

randomly distributed around zero, it indicates that probably the model has been correctly fitted on the data. In other words, if a non-random structure is evident in the residuals, it becomes clear that the model is poorly fitted on the data. Residuals must be independent of each other and have zero mean. Additionally, few residuals are allowed to exist out of triple the standard deviation of residuals from the mean ($\pm 3 \sigma$). The outlier areas with high number of residuals must be investigated in more detail to determine the cause of the poor fitness of the model.

The residuals whiteness test includes investigation of whether the correlation coefficients between residuals are small enough until the desired lag or not.

The correlation between residuals means that the number of correlations in the data has not been described by the made model.

In Table (1), RMSE of satellite attitude determination parameters per different estimation methods has been gathered. In addition, the residual mean for each of 7 considered outputs has been presented in Table 2.

Table 1. RMSE error in estimation quaternion and angular velocities for satellite attitude estimation by several well-known methods ($\times 10^{-3}$)

RMSE	EKF	MLP	RBF	ANFIS
q1	9.75	11.8	8.52	**4.04**
q2	7.96	6.9	5.42	**4.04**
q3	3.49	10.5	7.28	**4.14**
q4	3.90	12.5	8.38	**4.03**
w1	72.53	61.2	53.33	**52.03**
w2	62.86	45.2	42.37	**38.04**
w3	9.75	32.1	25.07	**28.70**

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Table 2. Residuals mean in estimation quaternion and angular velocities for satellite

attitude estimation by several well-known methods ($\times 10^{-3}$)

RES-MEAN	EKF	MLP	RBF	ANFIS
q1	0.39	0.760	0.16	**0.04**
q2	0.08	0.0416	0.02	**0.14**
q3	0.16	0.1	0.10	**0.03**
q4	0.08	0.3940	0.23	**0.09**
w1	0.55	9.041	1.59	**0.51**
w2	2.70	4.790	1.12	**0.14**
w3	4.35	1.462	0.57	**0.49**

It can be found that according to two criteria namely lower RMSE and closeness of residual mean to zero, in total, ANFIS method is the best estimator compared to other methods.

Now, we investigate various aspects of its correct fit on the data to check the validity of the ANFIS model. Figures (3) and (4) show the distribution of residuals of estimated quaternion and angular velocities. It could be observed that residuals have been distributed uniformly and there is no specific pattern between them. Additionally, the available outliers are negligible against the total number of residuals. Moreover, the variance of residues does not take an increasing or decreasing trend, and has a horizontal-band pattern which shows constant variance of residues.
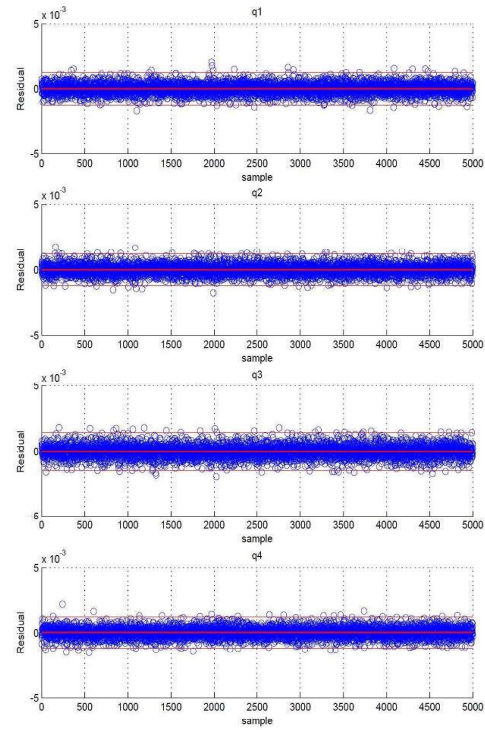

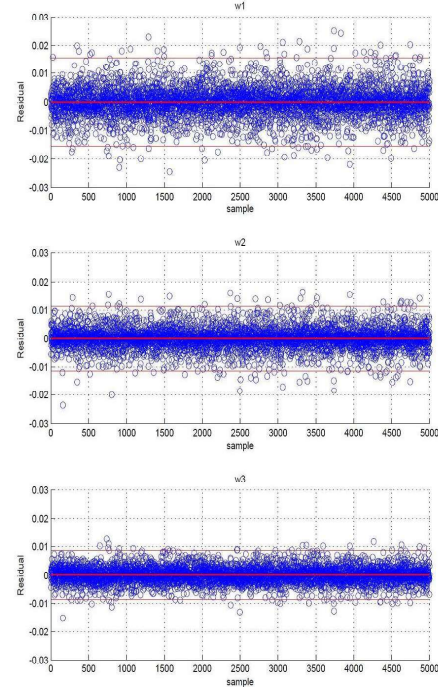**Fig. 3** The distribution of quaternion residuals – ANFIS model

Fig. 4 The distribution of angular velocities residuals - ANFIS model

The autocorrelation function has been utilized to evaluate the independency of the residuals. Figures

(5) and (6) show the autocorrelation of residuals for different outputs. It can be observed that there is an insignificant dependency among the residuals and they have a high independency.
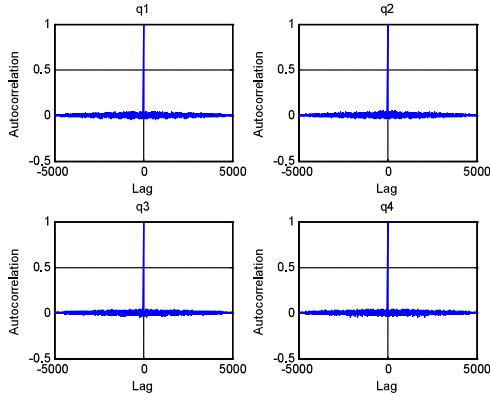

Fig. 5 The autocorrelation of residuals for quaternion - ANFIS model
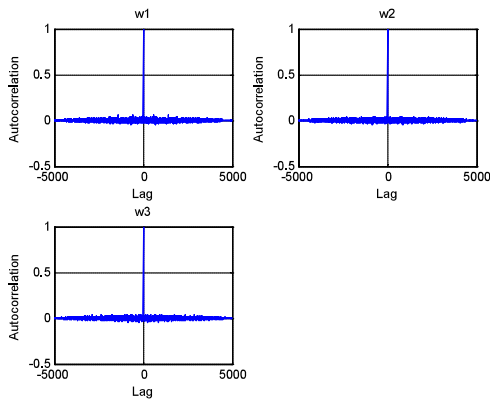

Fig. 6 The autocorrelation of residuals for angular velocities - ANFIS model

Figures (7) and (8) show the autocorrelation coefficients of residuals until the lag of 20, based on 95% confidence. Accordingly, the autocorrelation plot indicates that for the first 20 lags, almost all sample autocorrelations fall inside the 95% confidence bounds. This issue indicates that the residuals appear to be white and random with a probability of 95 percent.
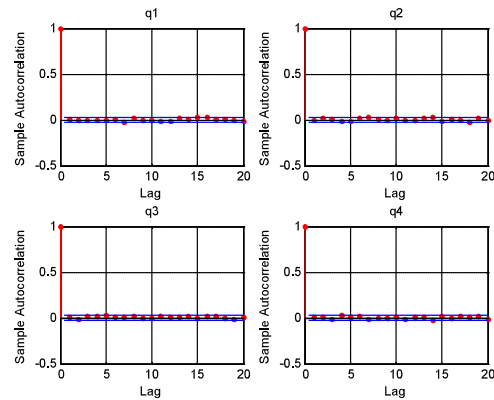

Fig. 7 The whiteness test of residuals for quaternion, 95% Confidence Band - ANFIS model
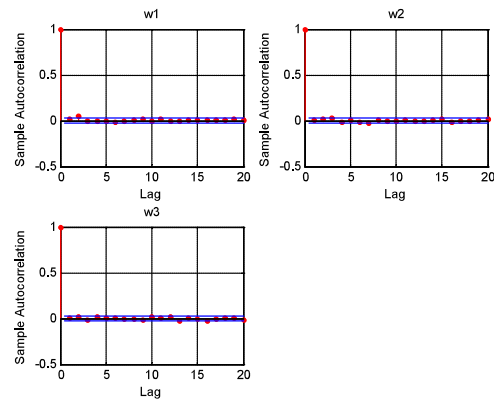

Fig. 8 Whiteness test of residuals for angular velocities, 95% Confidence Band - ANFIS model

Conclusion

In this study, simulation of satellite attitude estimation was carried out with different views on sensor data fusion i.e. EKF, MLP, RBF, and ANFIS networks. In addition, geosynchronous orbit was considered to solve the satellites attitude estimation problem. The efficiency of sensor data fusion in satellite attitude estimation in comparison with single sensor methods was proved.

The focus in the first part of the article was on the satellite modeling; including kinematics and dynamics of satellite and sensor's equations. The Gaussian assumption was made to provide the suitable conditions for the possibility of comparing different methods and presentation of an appropriate model for the control system.

Obtained results from different simulations showed that, estimation obtained from ANFIS estimator has the benefit of a higher accuracy compared to EKF, MLP, and RBF. The mean

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residuals of ANFIS were closer than to zero and its root mean square error was lower than other methods assuming their best performance. Additionally, we measured the credibility of the designed ANFIS network, utilizing different graphical analysis of the residual curves. The residuals were uniformly distributed and there was no specific pattern between them. Furthermore, with probability of 95%, residuals were white and had a high independency.

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