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Investigating cooperative and non-cooperative interference elimination between aircraft using game theory

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This article is aimed to investigate the interference elimination between multiple aircraft using game theory. A differential game is used to eliminate the interference if all the interfering aircraft cooperate to eliminate the interference or if each makes a rational decision based on their own interests. All interfering aircraft calculate the interference elimination route in cooperative mode by defining the flight priority. In the non-cooperative state, the problem of eliminating the interference is investigated using the Nash equilibrium, and then the new path is calculated. A point mass model has been used to implement this problem, which is converted into a linear model by changing the control variable. The above problem is solved using the quasi-spectral numerical solution method. In order to validate the presented method, the problem of eliminating the interference between several aircraft in two-dimensional space has been studied, and the results show the appropriate performance of the presented method.

***Keywords**: Interference Elimination, Differential Game*

Introduction

With the increase in the number of aircraft, the problem of route interference and its solution has gained lots of attention as several aircraft may be in the same area at the same time, so they are forced to modify the flight path to avoid a collision. Like free flight [1], where each aircraft corrects its route in the interference area, this concept can be used to eliminate the interference between aircraft in other areas. Therefore, each aircraft acts as a decision-maker agent, and the problem of eliminating interference can be expressed as a game problem. In this problem, each aircraft is a player who must choose a path to eliminate the interference based on its goal and the decisions of other interfering aircraft. Since the variables of the state vector are a differential function in this problem, a differential game is proposed. Also, each aircraft can receive and share flight information with the help of communication technology, such as the ADS-B receiver. Therefore, the differential game is a game with complete information.

The problem of eliminating interference and the optimal path has been the focus of many

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researchers in recent decades. The problem of eliminating interference has been studied as an optimization problem using the Monte Carlo method [2], non-linear programming [3], mixed-integer non-linear programming [4], and the principle of minimization [5]. Reference [3] used non-linear programming to solve the problem of eliminating the concentrated interference between several aircraft by considering the state and control constraints. The norm of velocity and the path and direction angles are considered as a function of the cost. This research is aimed to eliminate the interference between multiple aircraft based on game theory, and the cost function is the sum of the control vector norm, which is a more suitable criterion for the least fuel consumption or control effort. Moreover, The problem of the maximum number of aircraft in an interference elimination problem has been solved using the mixed-integer non-linear programming method [4]. This reference used velocity control to solve the interference, which cannot solve the interference in one line. Likewise, the aircraft's functional limitations are not considered in this reference. Reference [6] has investigated the problem of eliminating the interference between several airplanes so that the interference between the airplanes in two-dimensional and three-dimensional space has been solved using three-dimensional rotation maneuvers. The paths to eliminate the interference are quasi-optimal, and each aircraft returns to its original path after the interference elimination maneuver. Based on statistical information, reference 7 has tried to eliminate the interference between the planes in the modeling stages of the problem of eliminating interference as a system with several independent agents and based on the priority of the maneuver of each of the agents. The research has investigated different objective functions of flight. Then, based on this information, the routes without interference were calculated using the multivariable optimization method. Therefore, the objective function is modeled based on the interests of airlines and air navigation service providers. In reference [8], the problem of eliminating interference between multiple aircraft has been studied. In this article, the separation condition is considered cylindrical, which cannot be implemented due to the lack of continuity. Therefore, the oval condition is considered instead. Also, in solving the problem of eliminating the interference between several airplanes, the interference elimination was done in stages and two by two. In other words, the elimination was not implemented simultaneously among all aircraft. Also, fixed obstacles and no-fly zone are not considered in the environment of interference elimination.

Reference 9 has investigated the problem of the movement of wheeled robots on aircraft using game theory. Also, the coordinated flight game theory and pursuit and escape problems [12], [13] have been considered for the set of unmanned aerial vehicles [10], [11]. Reference [14] has investigated the movement of several agents in the aircraft with fixed obstacles as a differential game. This research considers velocity as a control vector and position as a system state vector. The condition of non-collision is considered an inequality constraint between two factors. The control vector and state have no limitations in this research. The difference between reference [9] and reference [14] is only in the dynamics of the investigated device. Also, reference [15] has validated the problem of eliminating the interference between several factors using the differential game theory in two-dimensional space without considering the constraints of the state vector and expression control and by simulating the movement of the interference between two aerial vehicles.

The pseudospectral method is a direct solution that converts ordinary and partial differential equations into non-linear equations and the integral actuator into a sum actuator [16]. Despite equality and inequality constraints, the Pseudospectral method writes the problem into a non-linear programming problem (NLP) with state vector estimation and control using Legendre polynomials [17]. This method has been used to solve non-linear optimal control problems related to optimizing the path of various devices [17]-[20].

The problem of eliminating interference is expressed as a differential game and has been investigated in two cooperative and non-cooperative modes. Therefore, the actual limitations of an air taxi have been considered to implement the interference elimination routes. In defining the interference elimination problem as a differential game, it is assumed that each aircraft knows the flight information of other interfering aircraft. The stated problem has been solved using the Pseudospectral numerical solution method.

In the following, part II presents the dynamic model of the aircraft's point mass and the collision condition between two aerial vehicles. Differential games and the expression of the interference problem as a cooperative differential game are presented in section III. After that, in part IV, the pseudospectral solution method is explained, and the differential game problem is transformed into non-linear programming. The simulation results are presented in part V based on the proposed hypotheses. Finally, conclusions and suggestions for future work are presented.

Dynamic model and interference problem

In order to accurately investigate the problem of eliminating interference, it is inevitable to use an accurate and realistic model of the dynamics of the aerial vehicle's body. In addition to the main forces on the aircraft, the used model should also include the functional limitations of the aircraft. Also, a minimum safe distance is considered based on the rules of low-altitude flight to avoid collision between two aircraft, which must be maintained during the interference elimination maneuver. Each of these topics will be examined below.

Aircraft dynamics model

This article uses the three degrees of freedom model, considering all the forces affecting the aircraft. This model is used in many path optimizations and interference elimination problems [1], [3], [20]. The limitations of the aircraft in the dynamic model are considered state and control vector constraints to enable them to track the obtained route. In this model, the engine thrust vector is in line with the vehicle's velocity vector, the constant mass of the vehicle, and the flat and fixed ground.

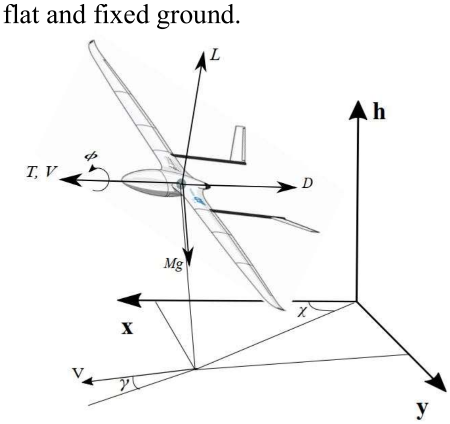

Figure 1. Aircraft and local coordinate device

Considering the above assumptions, the dynamic model of the aircraft is as follows.

$$\dot{x}_i = V_i \cos(\gamma_i)\cos(\chi_i) \tag{1}$$

$$\dot{y}_i = V_i \cos(\gamma_i)\sin(\chi_i) \tag{2}$$

$$\dot{h}_i = V_i \sin(\gamma_i) \tag{3}$$

$$\dot{V}_i = \frac{(T_i - D_i)}{m_i} - g\sin(\gamma_i) \tag{4}$$

$$\dot{\gamma}_i = \frac{L_i cos(\varphi_i) - m_i g cos(\gamma_i)}{m_i V_i} \tag{5}$$

$$\dot{\chi}_i = \frac{L_i sin\,(\varphi_i)}{m_i V_i cos(\gamma_i)} \tag{6}$$

In the above equations, $i = 1.2 \dots . Q$ is the number of interfering aircraft. $x_i . y_i$ is the position of each aircraft on the horizon plane and h_i is the height above the ground, m_i is the aircraft's mass, and is considered constant. γ_i is the direction angle, χ_i is the side angle and $L_i . D_i$ is the drag and lift force of the aircraft. The control inputs of the aircraft are: load coefficient $n_i = L_i/m_i g$ which is produced by the control surface of the elevator, the angle φ_i which is produced by the combination of the rudder and aileron control surfaces and T_i is the propulsion force. Since the aircraft has functional and structural limitations, the aircraft's control and state variables have limitations during the interference elimination. The non-linear motion dynamics (1-6) can be expressed in a linear form by changing the following variable [36] to facilitate the analysis of the interference elimination problem.

$$\ddot{x} = \bar{u}_1 . \quad \ddot{y} = \bar{u}_2 . \quad \ddot{h} = \bar{u}_3 \tag{7}$$

In the above equations, $u_1 . u_2 . u_3$ are new control variables. By deriving relation to time from equations (1-3) and substituting equations (4-6) and using equation (7), the non-linear dynamics becomes the dynamics of the following state space.

$$\begin{aligned} \dot{z}_i &= A_i z_i + B_i u_i \\ p_i &= C_p z_i \\ v_i &= C_v z_i \end{aligned} \tag{8}$$

$z_i = \left[p_i^T \ v_i^T\right]^T$ is the state vector that includes the position vector $p_i \in \mathbb{R}^3$ and the velocity vector $v_i \in \mathbb{R}^3$. The vector $u_i = \left[\bar{u}_{i_1} \ \bar{u}_{i_2} \ \bar{u}_{i_3}\right]^T$ is the new control input.

$$A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \otimes I_3 \ . B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes I_3 \quad (9)$$
$$C_p = [1 \quad 0] \otimes I_3 \ . C_v = [0 \quad 1] \otimes I_3$$

$I_3 \in \mathbb{R}^{3\times3}$ is the identity matrix and $\otimes$ represents the *Kronecker* coefficient. The primary control variables can be calculated from the new control variables using the following equations.

$$\varphi = \operatorname{atan}\left[\frac{\bar{u}_2 \cos\chi - \bar{u}_1 \sin\chi}{\cos\gamma(\bar{u}_3 + g) - \sin\gamma(\bar{u}_1 \cos\chi + \bar{u}_2 \sin\chi)}\right] \quad (10)$$

$$n = \frac{\cos\gamma(\bar{u}_3 + g) - \sin\gamma(\bar{u}_1 \cos\chi + \bar{u}_2 \sin\chi)}{g \cos\varphi} \quad (11)$$

$$T = [\sin\gamma(\bar{u}_3 + g) + \cos\gamma(\bar{u}_1 \cos\chi + \bar{u}_2 \sin\chi)]m + D \quad (12)$$

$$\tan\chi = \frac{\dot{y}}{\dot{x}} \quad (13)$$

$$\sin\gamma = \frac{\dot{h}}{V} \quad (14)$$

A collision between two aircraft

Collision is a situation when the distance between two aircraft is less than a safe limit. In the flight space outside the control of the air traffic center, the rules of visual flight prevail; therefore, the safe distance is not defined as a specific value [21]. Hence, a default value is considered for this safe distance.
The condition of non-collision between two aircraft i and j is equal to:

$$\left\|Z_i - Z_j\right\| \geq R_s \quad (15)$$

Where Z is the position of each aircraft in three-dimensional space, R_s is the safe distance between two aircraft and actuator $\|\dots\|$ is the second norm of the vector. As the defined field of view is 1500 meters in the plane and 600 feet in height, this article considers a spherical safe distance with a radius of 150 meters (this distance can change and be any other safe value). The initial distance for the beginning of the interference elimination maneuver is 1500 meters.

Differential game

In the problem of eliminating interference, each aircraft can be considered a player or decision-maker in the cooperative differential game. The considered differential game has been investigated in a centralized framework so that each aircraft knows the flight information of the others. Also, the number of aircraft in an interference elimination problem remains constant. In general, a differential game with N players is defined as follows.
If the goal of each player $i = 1 \dots Q$ is to choose a strategy (u_i) that minimizes the objective function J_i.

$$J_i = K_i\big(x(t_f), t_f\big) + \int_0^{t_f} L_i(x, u_1, \dots, u_Q, t)dt \ , \quad (16)$$

Considering the following differential constraint

$$\dot{x} = f\big(x, u_1 \ , \dots, u_Q \ , t\big) \ \ x(t_0) = x_0 \quad (17)$$

Where $x \epsilon R^n$ is the state vector, $u \epsilon U$ is the selected strategy from the set of strategies available to the player. Also, x_0 is the initial conditions of the game and t_f is the final time of the game. In the differential game problem, the control vectors $u(t)$ and state $x(t)$ may be bounded or unbounded based on the physical conditions of the problem.

$$x_l \leq x(t) \leq x_u \qquad u_l \leq u(t) \leq u_u \quad (18)$$

The subscripts l,u indicate the lower and upper limits for the state and control vectors. Also, there may be constraints on the state vector or control vector in the initial conditions, final conditions and along the way. These can be equality or inequality constraints.

$$g_{eq}\big(x(t), u(t)\big) = 0 \qquad g_{ineq}\big(x(t), u(t)\big) \leq 0 \quad (19)$$

In the above equation, g_{eq} and g_{ineq} are equality and inequality constraints of the problem, respectively.
There are different solution methods for the above differential game with different assumptions. Besides, there are three methods to solve a differential game: Nash equilibrium, Minimax, and Non-inferior method [22]. The Nash equilibrium method is used for non-cooperative or competitive games. The Minimax method is for games where there is no communication between players, and players only know that the others are playing Nash. Finally, the Non-inferior method is a cooperative and negotiated mode of the

differential game. This article uses the Non-inferior method and Nash equilibrium. For more information about the Minimax method, refer to the reference [23].

Non-inferior method

In game theory, Non-inferior methods are used when cooperation and exchanging opinions are intended to reach the best solution. The best solution can be found among the answers with the following condition.

Definition. The strategy (answer) is the cooperative differential game belongs to the set of $\mu = \{\mu_1 . \dots . \mu_Q\}$ so that for each answer set $\phi = \{\phi_1 . \dots . \phi_Q\}$

$$\{J_i(\mu) \leq J_i(\phi)\,, i = 1, \dots, Q\}\, only\ if\ \{J_i(\mu) = J_i(\phi)\,, i = 1, \dots, Q\} \tag{20}$$

Finding the solution set μ is equivalent to solving the optimal control problem with the vector optimality criterion. The answer to this problem is actually solving the set of N-1 parameters of an optimal control problem. Therefore, the answer to the game problem from the Non-inferior method is equivalent to the optimization of the following cost function.

$$J = \mu_1 J_1 + \cdots + \mu_Q J_Q \tag{21}$$

for $\mu = \{\mu_1 . \dots . \mu_Q\}$, so that

$$\sum_{i=1}^{Q} \mu_i^2 = 1\,, \mu_i \geq 0\,, i = 1, \dots, Q \tag{22}$$

In negotiation game problems, which is equivalent to finding a μ vector, other constraints should be applied to the problem [22]. In this article, the μ coefficients are based on the delay time of each aircraft relative to its designated route. Hence, an aircraft with a higher time delay has a higher effect coefficient, and all aircraft with the same delay have similar coefficients.

Definition. The effect coefficient of each aircraft is defined as the ratio of the delay time of each aircraft to the total delay time of all others.

$$\mu_i = \left(\frac{\Delta t_i + 1}{\sum_{i=1}^{Q} \Delta t_i + Q}\right)^{0,5} \tag{23}$$

In the above equation, Δt_i is the flight delay of the i-th aircraft.

The response of the above differential game can be calculated using numerical solution methods in the optimal control to solve the above problem.

Nash equilibrium

Definition. If $\overline{\boldsymbol{U}}_i$ is the set of acceptable strategies for the i-th player, and if the following equation holds for all values of $i = 1. \dots . Q$ [17], the strategy $u_i^*(\dots)$ is a Nash equilibrium strategy.

$$J_i(u_1^* . \dots . u_Q^*) \leq J_i(u_1^* . \dots . u_{i-1}^* . u_i . u_{i+1}^* . \dots . u_Q^*) . \forall u_i \in \overline{\boldsymbol{U}}_i \tag{24}$$

Assumption 1. In the differential game (16) and (17), if the objective function and the dynamics of the players can be written as follows:

$$f(X . u_1 . \dots . u_Q . t) = f_0(t . X) + \sum_{i=1}^{Q} f_i(t . X) u_i \tag{25}$$

$$L_i(X . u_1 . \dots . u_Q . t) = \sum_{j=1}^{Q} L_{ij}(t . X . u_j)\, . i = 1 . \dots . Q \tag{26}$$

$$\lim_{|u_i| \to \infty} \frac{L_{ii}(t . X . u_1 . \dots . u_Q)}{u_i} = +\infty .\ i = 1 . \dots . Q . \tag{27}$$

Also, if $\overline{\boldsymbol{U}}_i$ is a closed and convex set in the space R^{m_i} and the condition (27) is satisfied, then the differential game will have a unique solution [37].

Nash equilibrium is a set of players' strategies, such that each player's strategy is the best choice against the strategy of other players. In other words, in the Nash equilibrium strategy, none of the players tend to deviate from their decision. Also, if assumption 1 holds true in the differential game, the game has a unique solution.

Therefore, the problem of eliminating interference between several aircraft can be modeled as a differential game. According to the dynamics of the aircraft movement in equation (8) and the standard format of the differential game in equation (17), if Q number of aircraft interfere in the interference space, the dynamics of the interference elimination game is described as follows:

$$\dot{X} = AX + BU \tag{28}$$

Where $\boldsymbol{X} \in \mathbb{R}^{6Q}$ is the state vector of the game, $\boldsymbol{U} \in \mathbb{R}^{3Q}$ is the control vector of all aircraft and $\boldsymbol{A}.\boldsymbol{B}$ are the square matrix of the coefficients of the state and the control vector, respectively, which are defined as follows:

$$X = \left[z_1^T. \dots . z_Q^T\right]^T \tag{29}$$

$$U = \left[u_1^T. \dots . u_Q^T\right]^T \tag{30}$$

$$A = \begin{bmatrix} A_1 & 0_{6\times6} & \cdots & 0_{6\times6} \\ 0_{6\times6} & A_2 & 0_{6\times6} & \vdots \\ \vdots & 0_{6\times6} & \ddots & 0_{6\times6} \\ 0_{6\times6} & \cdots & 0_{6\times6} & A_Q \end{bmatrix}. A_i = \begin{bmatrix} 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3} \end{bmatrix}. i = 1. \dots . Q \tag{31}$$

$$BU = \begin{bmatrix} bu_1 \\ \vdots \\ bu_Q \end{bmatrix}. bu_i = \begin{bmatrix} 0_{3\times1} \\ u_i \end{bmatrix}. i = 1. \dots . Q \tag{32}$$

Also, according to the equation (15), the condition of non-collision between two aircraft i and j is defined as follows.

$$\widetilde{D}_{ij} = \widetilde{D}_{ji} = \left\|p_i - p_j\right\| \geq R_s. i \neq j$$
$$p_i = X[6 \times (i-1) + 1 : 6 \times (i-1) + 3]. i = 1. \dots . Q$$
$$p_j = X[6 \times (j-1) + 1 : 6 \times (j-1) + 3]. j = 1. \dots . Q \tag{33}$$

Where the equation (33) is applied as an inequality constraint in the differential game.

The objective function can be the least control effort, the least amount of velocity changes, the least amount of deviation from the path, and others. In this research, the objective function is the least control effort. Therefore, the objective function of each aircraft is as follows:

$$J_i = \int_0^{t_f} u_i^2 dt \tag{34}$$

Finding the Nash equilibrium for problems (28)-(34) is equivalent to simultaneously solving the Q number of the optimization problem so that each problem has $\boldsymbol{Q-1}$ number of inequality constraints. Using the necessary condition of the first order, we have the optimality and the Hamiltonian function.

$$. \lambda = H_i = u_i^2 + \lambda(\boldsymbol{AX} + \boldsymbol{BU}) \quad [\lambda_1. \dots . \lambda_{6Q}] \tag{35}$$

Where λ_i is the pseudo-state vector for the i-th aircraft. If the control u_i^* for the i-th aircraft is a Nash equilibrium, all the Hamiltonian functions are minimized with respect to u_i . According to Pontryagin's principle of minimization, the following condition must be satisfied for all values of u_i [17].

$$H_i\left(x^*. u_1^*. \dots . u_Q^*. \lambda. t\right) \leq H_i\left(x^*. u_1^*. \dots . u_{i-1}^*. u_i. u_{i+1}^*. \dots . u_Q^*. \lambda. t\right) \quad \forall u_i \in \overline{\boldsymbol{U}}_i. t \in [0. t_f] \tag{36}$$

In order to apply inequality constraints by defining the Lagrangian function using inequality constraints (34), we have [17]:

$$\boldsymbol{L}_i = H_i + \bar{\bar{\gamma}}_i \times \widetilde{D}_{ij}. \bar{\bar{\gamma}}_i = \left[\bar{\bar{\gamma}}_1. \dots . \bar{\bar{\gamma}}_{Q-1}\right]. j = 1. \dots . Q . i = 1. \dots . Q . i \neq j \tag{37}$$

$\bar{\bar{\gamma}}_i$ is the coefficient of the Lagrange function and $\times$ represents the inner multiplication of the two vectors. The necessary condition is those Lagrange functions (37) should be minimized with respect to all $\boldsymbol{u_i}$ values:

$$\frac{\partial L_i}{\partial u_i}\Big|_{u_i = u_i^*} = 0 \tag{38}$$

We also have state and pseudo state vectors:

$$\dot{X} = -\frac{\partial L_i}{\partial \lambda}. X(t_0) = X_0 \tag{39}$$

$$\dot{\lambda_i} = -\frac{\partial L_i}{\partial X} = -\left(\frac{\partial H_i}{\partial X} + \frac{\partial \widetilde{D}_i}{\partial X}\right). \lambda_i(t_f) = 0 \tag{40}$$

$$\bar{\bar{\gamma}} \geq 0. \ \bar{\bar{\gamma}} D\left(x^*(t_f). t_f\right) = 0 \tag{41}$$

$$\bar{\bar{\gamma}}_i(t) \geq 0. \ \bar{\bar{\gamma}}_i \widetilde{D}_{ij} = 0. \dot{\bar{\bar{\gamma}}}_i(t) \leq 0 \tag{42}$$

Equation (41) is used to apply equality constraints (final conditions), and equation (42) as complementary slackness conditions for indirect application of inequality constraints. The set of equations (28)-(42) shows the problem of eliminating interference using game theory, where each of the Hamiltonian functions is minimized relative to the control vectors. This problem is actually equivalent to the simultaneous solution Q of the optimal control problem, such that the state vector is $\boldsymbol{X} \in \mathbb{R}^{\boldsymbol{6Q}}$, the pseudo-state vector is $\boldsymbol{\lambda_i} \in \mathbb{R}^{\boldsymbol{6Q}}. \boldsymbol{i} = \boldsymbol{1} \dots \boldsymbol{Q}$ and the control vector is $\boldsymbol{U} \in \mathbb{R}^{\boldsymbol{3Q}}$. Also, in this problem, there is an $\sum_{\boldsymbol{i}=\boldsymbol{2}}^{\boldsymbol{Q}}(\boldsymbol{i}-\boldsymbol{1})$ number of inequality constraints to prevent collision. For the above problem, if the Hamiltonian function is defined as (43), the optimal control problem becomes a differential game problem and it reduces the number of quasi-state differential equations from $Q \times \boldsymbol{6Q}$ to $6Q$, which reduces the calculation volume. The new Hamiltonian function is defined as follows:

$$\bar{H} = \sum_{i=1}^{Q} u_i^2 + \bar{\lambda}(\boldsymbol{AX} + \boldsymbol{BU}) \\ .\bar{\lambda} \in \mathbb{R}^{6Q} \tag{43}$$

And therefore, the Lagrangian function will be as follows:

$$\bar{L} = \bar{H} + \bar{\bar{\gamma}} \times \widetilde{D}_{ij}. \quad \bar{\mu} = [\bar{\bar{\gamma}}_1. \dots . \bar{\bar{\gamma}}_{Q-1}]. j = 1. \dots . Q \,. i = 1. \dots . Q \,. i \neq j \tag{44}$$

In this case, the optimal condition of the Lagrange function with respect to the control vector will be as follows:

$$\frac{\partial L_i}{\partial u_i}|_{u_i=u_i^*} = \frac{\partial \bar{\boldsymbol{L}}}{\partial u_i}|_{u_i=u_i^*} = 0 \tag{45}$$

For state and pseudo state vectors, we will also have:

$$\dot{X} = -\frac{\partial L_i}{\partial \lambda_i} = -\frac{\partial \bar{L}}{\partial \bar{\bar{\lambda}}} = AX + BU. \ X(t_0) = X_0 \tag{46}$$

$$\dot{\lambda}_i = -\frac{\partial L_i}{\partial X_i} = -\frac{\partial \bar{L}}{\partial X_i} = -\left(\frac{\partial \bar{H}}{\partial X_i} + \frac{\partial \widetilde{D}_i}{\partial X_i}\right). i = 1. \dots .6Q. \lambda_i(t_f) = 0 \tag{47}$$

$$\bar{\bar{\gamma}} \geq 0. \ \bar{\bar{\gamma}}\widetilde{D}\big(x^*(t_f). t_f\big) = 0 \tag{48}$$

$$\bar{\bar{\gamma}}_i(t) \geq 0. \ \bar{\bar{\gamma}}_i\widetilde{D}_{ij} = 0. \dot{\bar{\bar{\gamma}}}_i(t) \leq 0 \tag{49}$$

By comparing the set of equations (21) and (34), it can be easily proved that the elimination of cooperative interference with the same priority coefficient is the Nash equilibrium point of the problem. In other words, in the case of any non-cooperative interference elimination, the decision of each of the players will be the same as non-cooperative interference elimination with equal priority. It should be noted that if the objective function of the aircraft is different or if they have an objective function that depends on the state or time variable, the response of the Nash equilibrium will not be the same as the cooperative state.

For the differential equations (43) and (49) with the conditions (48) and (49) with the initial conditions of the state vector and the final condition of the pseudo-state vector, it is challenging to obtain a closed solution for this set of constrained differential equations. Therefore, numerical methods are used to solve it. This article investigates this problem using the pseudo-spectral direct solution method. In addition to accurately estimating the state vector and optimal control, the pseudo-spectral method also estimates the pseudo-state vector with high accuracy [31]. Unlike other numerical solution methods, such as projectile and multi-projectile, this method is not sensitive to the initial guess of the pseudo-state vector. Considering the non-linear nature of the problem, the initial guess for the pseudo-state vector is very difficult [29], [38]. Therefore, the pseudo-spectral method has been used to solve the problem of eliminating interference as a differential game. In the following, the pseudo-spectral method is described.

Solving the problem of interference elimination

As it does not have an analytical solution, the answer to the differential game problem stated in the previous part is obtained using direct solution methods. In addition to accurate estimation of the state vector and optimal control, the Pseudospectral method also estimates the pseudo-state vector with high accuracy [16]. This method has been used to solve the Nash equilibrium of game theory problems [24], [25]. In the following, solving this problem is explained using the Pseudospectral method.

Pseudospectral method

The pseudospectral method estimates the problem's solution using Legendre polynomials of order K in a set of specific points. These points are defined in the interval [-1.1] so that if the points are the roots of the Legendre polynomial, it is called Legendre-Gauss (LG). If the roots are obtained from the linear combination of the Legendre polynomial, it is called Legendre-Gauss-Radau (LGR). Also, if polynomial derivative roots are used, it is called Legendre-Gauss-Lobatto (LGL). LG, LGR, and LGL points are defined in the interval (-1,1), [-1,1) or (-1,1], and [-1,1], respectively. In this article, the LGR method has been used for solving the differential game problem. For more information about this method, refer to [16].

First, the problem should be defined in the interval [-1,1]. Therefore, by using the variable change, the time interval is changed from $t\epsilon\left[t_0.t_f\right]$ to the interval $\tau\epsilon[-1.1]$.

$$t=\frac{1+\tau}{1-\tau} \tag{50}$$

Then the state vector $x(\tau)$ is estimated using the Legendre basis interpolation functions $L_l(\tau)$ of order K+1 as below,

$$x(\tau)\approx\sum_{l=0}^{K}X_l(\tau)\,L_l(\tau),\qquad L_l(\tau)=\prod_{m=0,m\neq l}^{K}\frac{\tau-\tau_m}{\tau_l-\tau_m},\quad (l=0\ldots K) \tag{51}$$

In addition, the control vector for each aircraft is estimated using the Legendre polynomial of order K as below.

$$u_i(\tau)\approx\sum_{l=1}^{K-1}u_i(\tau)\,L_l^*(\tau)\,,\quad L_l^*(\tau)=\prod_{m=1,m\neq l}^{K}\frac{\tau-\tau_m}{\tau_l-\tau_m}\quad i=1\ldots Q,\ l=1\ldots K \tag{52}$$

The derivative of each of the polynomials at LG points can be expressed as a differential estimation matrix $D\in\mathbb{R}^{K\times K+1}$. If derived from equation (51) with respect to time:

$$\dot{X}(\tau)\approx\sum_{l=0}^{K}X_l\,\dot{L}_l(\tau)=\sum_{l=0}^{K}D_lX_l \tag{53}$$

After using the differential estimation matrix, the dynamic constraint of the system (17) turns into an algebraic constraint.

$$\sum_{l=1}^{K}D_{ml}X_l-\frac{t_f-t_0}{2}f\left(X_m,U_{1m}\ldots U_{Qm},\tau_m;t_0,t_f\right)=0,\ (m=1\ldots K) \tag{54}$$

In the above equation, we have $X_m\equiv X(\tau_m),U_m\equiv U(\tau_m)$ and $\tau_m\epsilon[-1.1)$. Other variables are defined as below.

$$X_0\equiv X(-1)\qquad X_f\equiv X_0+\frac{t_f-t_0}{2}\sum_{l=1}^{K}\omega_l f(X_l,U_{1l}\ldots U_{Ql},\tau_l;t_0,t_f) \tag{55}$$

ω_l are Gauss coefficients. In this article, these coefficients are considered the same for all points.

The continuous cost function (16) can be written in the following discrete form:

$$J_i=K_i\left(X_f,t_f\right)+\frac{t_f-t_0}{2}\sum_{l=1}^{K}\omega_lL_i(X_l,U_{1l}\ldots U_{Ql},\tau_l;t_0,t_f),\ i=1\ldots Q \tag{56}$$

$$\boldsymbol{J}=\mu_1J_1+\cdots+\mu_QJ_Q$$

Border constraints and along-the-path constraints (19) are described as follows:

$$\begin{aligned}&g_{eq}\left(X_l,U_{1l}\ldots U_{Ql},\tau_l;t_0,t_f\right)=0\\&g_{ineq}\left(X_l,U_{1l}\ldots U_{Ql},\tau_l;t_0,t_f\right)\leq 0\end{aligned} \tag{57}$$

The cost function (56) and the algebraic constraints (54), (55), and (57) constitute an NLP problem in such a way that its solution is the solution of the Non-inferior differential game problem. The NLP problem obtained has been solved using SNOPT software in MATLAB environment.

Simulation results

Information from an air taxi produced by Lilium Jet Mobility has been used for simulation. With a weight of 400 kilograms and a set of electric motors with a power of 320 kilowatts, this aircraft can fly at a speed of 85 meters per second [26]. The considered hypotheses are as follows.

As stated in part 3, the problem of eliminating cooperative and non-cooperative interference will have the same answer with certain assumptions. In this part, the problem of eliminating the interference between four interfering aircraft has been solved for cooperative mode with the same priority coefficient and inequality priority coefficient. In the case of eliminating interference with the same coefficient, the answer to the problem is equal to the elimination of non-cooperative interference (Nash equilibrium) (the priority coefficient is equal to $\mu_i = 0.5. i = 1 \ldots 4$). Also, it is assumed that all aircraft have entered the interference zone with a certain initial velocity. In the second case, it is assumed that one aircraft has a priority coefficient six times that of the others (in this case, the priority coefficient is equal to $\mu_i = 0.333\ .\ i = 1.2.3$ and $\mu_4 = 0.8165$). The initial conditions considered for this example are as follows:

$$v_0 = v_f = 10\frac{m}{s} \qquad 10 \leq v(t) \leq 85\, m/s$$

$$initial\ position = \begin{cases} AC_1 = [750;\ 0] \\ AC_1 = [750;\ 1500] \\ AC_1 = [0;\ 750] \\ AC_1 = [1500;\ 750] \end{cases}$$

$$final\ position = \begin{cases} AC_1 = [750;\ 1500] \\ AC_1 = [750;\ 0] \\ AC_1 = [1500;\ 750] \\ AC_1 = [0;\ 750] \end{cases}$$

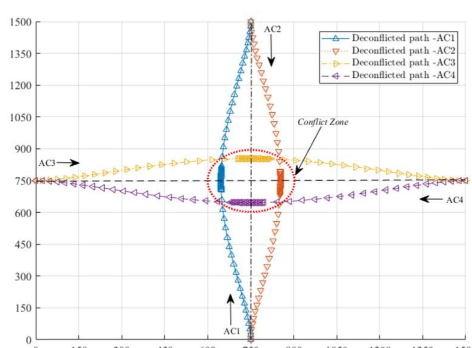

Figure 1. Cooperative interference elimination with the same priority coefficient (non-cooperative-Nash equilibrium)

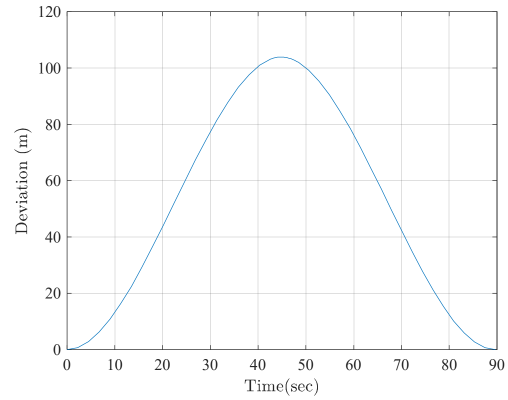

Figure 2. The amount of deviation of each aircraft during the interference elimination maneuver from the main route

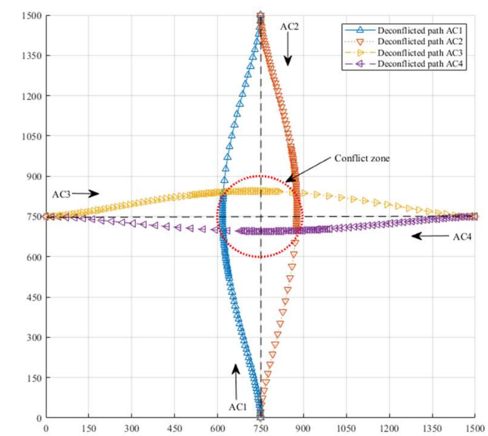

Figure 3. Eliminating the interference of four aircraft with different priority coefficients

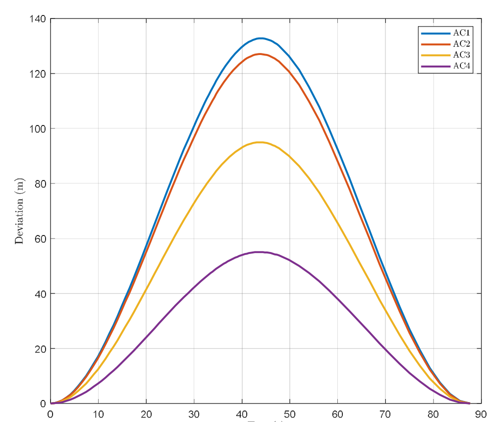

Figure 4. The amount of deviation of four aircraft per inequality priority coefficient

The performance characteristics of all four interfering aircraft are considered the same.

Therefore, if the flight priority is the same, the amount of deviation from the route for all the aircraft will be the same. as a result, the routes to eliminate interference will be the same. Figures 1 and 2 show this finding. These conditions are also the answer to the problem of non-cooperation. If one aircraft has a higher priority, the amount of aircraft deviation with a higher priority is less than the others. Since the path of one aircraft is affected by the others, there will be an unequal amount of deviation, shown in Figures 3 and 4.

Conclusion

This article uses the theory of differential games, the non-inferior method in cooperative and non-cooperative ways, and Nash equilibrium to solve the problem of eliminating interference based on flight priority. In the non-inferior game, the agreed criterion to resolve the interference can be the flight time delay of the number of passengers or other priorities. For example, the priority of an aircraft with a longer time delay is considered higher. Similarly, the definition of the Nash equilibrium in the non-cooperative game problem shows that the answer to the problem of eliminating interference with the objective function of the least control effort is the same as the answer to the problem of the cooperative game with the same priority. In other words, the Nash equilibrium is the same as the answer to the non-inferior problem. The above issue can also be implemented by the airlines for route allocation. Therefore, in the areas where there is a possibility of collision between several aircraft, their priority is defined according to the airline's agreement. To this end, the model of three degrees of freedom of aircraft movement has been used, and the functional limitations of the aircraft have also been applied. For further research, it is suggested that future papers focus on the answer to the problem of eliminating interference in cases where some interfering aircraft have different target angles.

References

[1] P. K. Menon, G. D. Sweriduk, and B. Sridhar, "Optimal Strategies for Free-Flight Air Traffic Conflict Resolution," *J. Guid. Control. Dyn.*, vol. 22, pp. 202–211, 1999.

[2] A. L. Visintini, W. Glover, J. Lygeros, and J. Maciejowski, "Monte {Carlo} {Optimization} for {Conflict} {Resolution} in {Air} {Traffic} {Control}," *IEEE Trans. Intell. Transp. Syst.*, vol. 7, no. 4, pp. 470–482, 2006.

[3] A. U. Raghunathan, V. Gopal, D. Subramanian, L. T. Biegler, and T. Samad, "Dynamic Optimization Strategies for Three-Dimensional Conflict Resolution of Multiple Aircraft," *J. Guid. Control. Dyn.*, vol. 27, no. 4, pp. 586–594, 2008.

[4] S. Cafieri and D. Rey, "Maximizing the number of conflict-free aircraft using mixed-integer nonlinear programming," *Comput. Oper. Res.*, vol. 80, pp. 147–158, 2017.

[5] Y. Lu, B. Zhang, and X. Zhang, "Air conflict resolution algorithm based on optimal control," *Proc. 33rd Chinese Control Conf. CCC 2014*, no. c, pp. 8919–8923, 2014.

[6] Malaek, Seyed & Golchoubian, Mahsa. (2020). Enhanced Conflict Resolution Maneuvers for Dense Airspaces. IEEE Transactions on Aerospace and Electronic Systems. PP. 1-1. 10.1109/TAES.2020.2972422..

[7] E. Calvo-Fernández, L. Perez-Sanz, J. M. Cordero-García, and R. M. Arnaldo-Valdés, "Conflict-Free Trajectory Planning Based on a Data-Driven Conflict-Resolution Model," *J. Guid. Control. Dyn.*, vol. 40, no. 3, pp. 615–627, 2016.

[8] W. Chen, J. Chen, Z. Shao, and L. T. Biegler, "Three-Dimensional Aircraft Conflict Resolution Based on Smoothing Methods," *J. Guid. Control. Dyn.*, vol. 39, no. 7, pp. 1481–1490, 2016.

[9] T. Mylvaganam and M. Sassano, "Autonomous collision avoidance for wheeled mobile robots using a differential game approach," *Eur. J. Control*, vol. 40, pp. 53–61, 2018.

[10] W. Lin, "Distributed UAV formation control using differential game approach," *Aerosp. Sci. Technol.*, vol. 35, no. 1, pp. 54–62, 2014.

[11] D. Gu, "A differential game approach to formation control," *IEEE Trans. Control Syst. Technol.*, vol. 16, no. 1, pp. 85–93, 2008.

[12] P. K. A. Menon, "Optimal helicopter trajectory planning for terrain following flight," *J. Heat Transfer*, vol. 125, no. October, pp. 788–794, 1990.

[13] W. Lin, "Differential Games for Multi-agent Systems under Distributed Information," 2013.

[14] T. Mylvaganam, M. Sassano, and A. Astolfi, "A Differential Game Approach to Multi-agent Collision Avoidance," *IEEE Trans. Automat. Contr.*, vol. 62, no. 8, pp. 4229–4235, 2017.

[15] T. Mylvaganam, M. Sassano, and A. Astolfi, "A Differential Game Approach to Multi-agent Collision Avoidance," *IEEE Trans. Automat. Contr.*, vol. 62, no. 8, pp. 4229–4235, 2017.

[16] M. A. Patterson *et al.*, "an Overview of Three Pseudospectral Methods for the Numerical Solution of Optimal Control," *Aas 09*, pp. 1–17, 2009.

[17] T. Guo, J. Li, H. Baoyin, and F. Jiang, "Pseudospectral methods for trajectory optimization with interior point constraints: Verification and applications," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 49, no. 3, pp. 2005–2017, 2013.

[18] R. Dai, "Three-dimensional aircraft path planning based on nonconvex quadratic optimization," *Proc. Am. Control Conf.*, pp. 4561–4566, 2014.

[19] N. E. Smith, R. Cobb, S. J. Pierce, and V. Raska, "Optimal Collision Avoidance Trajectories via Direct Orthogonal Collocation for Unmanned/Remotely Piloted Aircraft Sense and Avoid Operations," no. January, 2014.

[20] P. Bonami, A. Olivares, M. Soler, and E. Staffetti, "Multiphase Mixed-Integer Optimal Control Approach to

Aircraft Trajectory Optimization," *J. Guid. Control. Dyn.*, vol. 36, no. 5, pp. 1267–1277, 2013.

[21] C. Aviation, "The Rules of the Air Regulations," no. 734, 1996.

[22] A. W. Starr and Y. C. Ho, "Nonzero-Sum Differential Games 1," *J. Optim. Theory Appl.*, vol. 3, no. 3, pp. 184–206, 1969.

[23] T. Başar, A. Haurie, and G. Zaccour, "Nonzero-sum differential games," *Handb. Dyn. Game Theory*, vol. 3, no. 3, pp. 61–110, 2018

[24] Z. Nikooeinejad, A. Delavarkhalafi, and M. Heydari, "A numerical solution of open-loop Nash equilibrium in nonlinear differential games based on Chebyshev pseudospectral method," *J. Comput. Appl. Math.*, vol. 300, pp. 369–384, 2016.

[25] P. Method, "Solving Nash Differential Game Based on Minimum Principle and Pseudo-spectral Method," no. 1, pp. 173–177, 2016.

[26] J. Holden, N. Goel, and UBER, "Fast-Forwarding to a Future of On-Demand Urban Air Transportation," *VertiFlite*, pp. 1–98, 2016.

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