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Investigation of Flow Separation Phenomenon for Supersonic Convergent–Divergent Nozzles

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The nozzle, an end-element of the propulsive process cycle, represents a critical part of any aerospace vehicle. The task of accelerating and efficiently exhausting combusted and reactive gases according to the delivered thrust represents the main objective of the propulsion system design. Flow separation in supersonic convergent–divergent nozzles has been the subject of several experimental and numerical studies in the past. Now, with the renewed interest in supersonic flights and space vehicles, the subject has become increasingly important, especially for aerospace applications (rockets, missiles, supersonic aircrafts, etc). Flow separation in supersonic nozzles is a basic fluid dynamics phenomenon that occurs at a certain pressure ratio of chamber to ambient pressure, resulting in shock formation and shock/turbulent-boundary layer interaction inside the nozzle. From purely gas-dynamics point of view, this problem involves basic structure of shock interactions with separation shock, which consists of incident shock, Mach reflections, reflected shock, triple point and slip lines. In this article A Review on Flow Separation Phenomenon for Supersonic Convergent–Divergent Nozzles has been investigated.

Keyword: *gas-dynamics, Flow separation, supersonic convergent divergent nozzles, shock interactions, Mach reflections.*

Introduction

The nozzle, an end-element of the propulsive process Cycle, represents a critical part of any aerospace vehicle. The task of accelerating and efficiently exhausting combusted and reactive gases according to the delivered thrust represents the main objective of the propulsion system design. As such, propulsive convergent divergent (C-D) axisymmetric rocket nozzles have evolved since the early period of use till nowadays. From technical and historical point of view, it is possible to sort axisymmetric nozzle types by the shape of their divergent profile as conical, ideal de- Laval, truncated ideal contour (TIC) and sorts of modern thrust optimized contour (TOC) nozzles. Several viscous phenomena, such as boundary layers with adverse pressure gradients, induced separation, and recirculation bubbles, shear layers may additionally occur and can strongly affect the flow-field inside the nozzle (Figs. 5, 6). Previous studies on supersonic nozzles [2, 3] have shown that shock-wave/boundary layer interaction (SWBLI) occurring in highly overexpanded nozzles may exhibit strong unsteadiness that cause symmetrical or unsymmetrical flow separation. In rocket design

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community, shock-induced separation is considered undesirable because an asymmetry in the flow can yield dangerous lateral forces, the so called side-loads, which may damage the nozzle [4]. This phenomenon has received significant attention in the past and it is still an active subject of research, whose primarily motivation is to improve nozzle performance under overexpanded flow conditions and to mitigate against nozzle side-loads produced by shock unsteadiness as well as asymmetric boundary-layer separation. This condition [5] occurs because the nozzle expansion ratio is too large for a given nozzle pressure ratio (NPR). Supersonic nozzle flow is driven to expand below exit backpressure, and a nonisentropic, discontinuous shock jump is required to adjust. Ideally, flow stays attached downstream of the shock, but in reality, it usually separates from the nozzle flap if the expansion ratio is high enough or the flap angle steep enough.

Flow separation on supersonic convergent-divergent nozzles: literature review

Several experimental studies, performed on either subscale [4,6-8] or full-scale [4] optimized nozzles, corroborated by different numerical simulations [9–13], demonstrated the existence of two distinct separation processes, namely the Free Shock Separation (FSS), in which the boundary layer separates from the nozzle wall and never reattaches (Fig. 5), and the restricted shock separation (RSS) characterized by a closed recirculation bubble, downstream of the separation point, with reattachment on the wall (Fig. 6). In fact, the earliest studies attributed the cause of the measured side loads to asymmetric FSS that yields a tilted separation surface as reported by [4, 14]. Subsequently, in the early 70s, during cold-flow subscale tests for the J-2S engine development, Nave and Coffey [4], in a study that can be considered the pioneer milestone for the field, observed that the highest value of side loads takes place during the transition from FSS structure to different kind of separated nozzle flow structures, which had not been noticed before. In particular, the pressure downstream of the separation point showed an unsteady behavior with strong oscillations, and finally jumped to values quite above the ambient pressure. They attributed this behavior to the reattachment of the separated flow to the nozzle wall, and because of the limited extension of this separated region, they called it restricted shock separation (RSS).

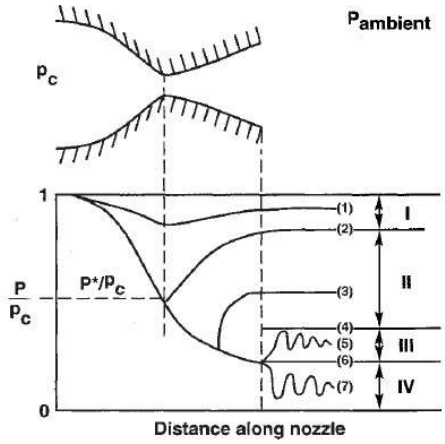

Figure 1. Pressure vs distance for converging-diverging nozzle with various pressure ratios [9].

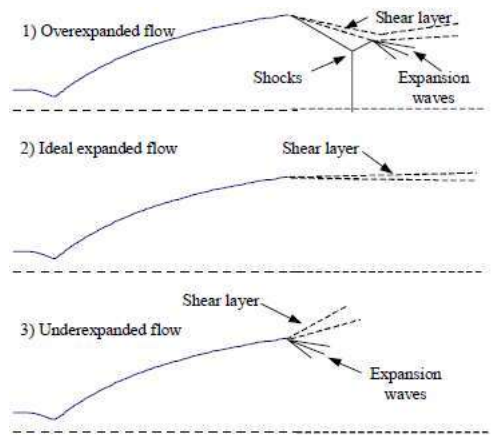

Figure 2. Schematic of a flow phenomenon of a nozzle [15].

A typical inviscid one-dimensional nozzle flow can be characterized into four types as shown in Fig. 1, depending on its corresponding ambient pressure. An overexpanded nozzle flow is shown as type III under pressure condition 5 in Fig. 1 where condition 4 indicates an internal shock standing right at the exit plane, and condition 6 is the design condition the exit pressure is equal to the ambient pressure. Ideally, the overexpanded nozzle flow is supersonic within the entire portion of the nozzle downstream of the throat, and the exit-plane pressure is uniformly lower than the ambient pressure. However, for an actual nozzle under ambient pressure condition 5, complicated shock wave and boundarylayer interactions can lead to the separation of the boundary layer on the nozzle wall and result in a highly nonuniform flow at the nozzle exit plane

[9]. Fig 2. Shows the Schematic of a flow phenomenon of a nozzle.

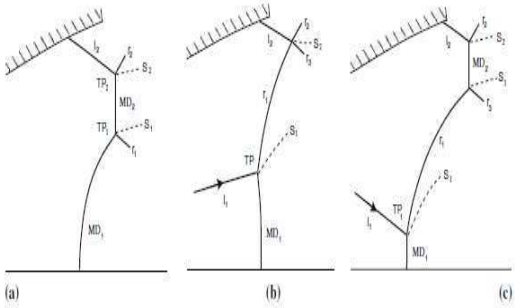

Figure 3. Schematic illustration of shock interactions and cap-shock pattern in over expanded supersonic nozzles [1].

According to Fig.3, It is worth noting that, in case (**a**), the curvature of the Mach disk is due to the upstream flow non uniformities, characterized by a strong vortical pressure gradient but no internal shock, whereas, in cases (**b**) and (**c**), the cap shock pattern is mainly caused by the impingement of the internal shock with the central Mach disk. The reflected shock resulting from this interaction meets later with the incident shock (arising from the boundary-layer region) and form either regular reflection (RR) as in case (**b**) or Mach Reflection (MR) as in case (**c**), depending on their respective slopes. In the latter case, the MR corresponds to an annular Mach disk. Note also that the internal shock is only observed in nozzles with thrust-optimized, parabolic or compressed contours, and it is induced shortly downstream of the nozzle throat at the inflection point where wall curvature suddenly changes from a convex to a concave contour shape. i_1 internal shock, i_2 incident shock, *r* reflected shock, *TP* triple point, *S* slip line, MD_1 central Mach disk, MD_2 annular Mach disk [1]. More recently, during the studies motivated by the development of the Ariane 5 Vulcain engines, experiments made on both subscale [16–18] and full-scale rocket nozzles [19,20] confirmed that the highest values of side loads take place during the transition from FSS to RSS, as indicated by Nave and Coffey. Nevertheless, there was no clear explanation of the cause of the flow reattachment to the wall. The evidence of the flow reattachment in the J-2S sub-scale nozzle was first confirmed by numerical simulations of Chen et al. [9]. In addition, those calculations revealed a trapped vortex behind the recompression shock. Later, Nasuti, Onofri and M. Sellam [12,21–23,61] stressed the role played by the centerline vortex on the side-loads generation, and suggested a possible explanation for its formation mainly based on the key role played by the flow gradients upstream of the Mach disk in the nozzle core. According to that explanation, an inviscid mechanism is the principal cause of vorticity generation. In particular, the driving role is played by the non-uniformity of the flow impinging on the Mach disk. Because of this upstream flow non-uniformity, and because of the downstream quite-uniform pressure, the shock strength cannot be constant along its surface and its shape takes a curved shock profile, rather than a flat one.

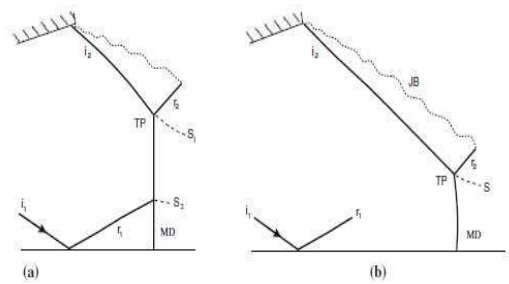

Figure 4. Schematic illustration of shock interactions near the nozzle lip for two different pressure ratios at full (**a**) and over (**b**) flowing nozzle regimes [1].

According to Fig.4, It is worth noting that the cap-shock pattern is unlikely to appear in the over expanded jet plume, since the Mach disk is out of reach of the internal shock. This observation is confirmed by various experimental and numerical studies showing that, for such high chamber pressure operations, only classical Mach reflection configuration is possible. Further increase of the nozzle chamber pressure will result in a reduction of the height of the Mach disk, which moves further downstream of the nozzle exit until a smooth transition from MR to an apparent regular reflection (aRR), characterized by a very small (and not easily visible) Mach disk. The aRR configuration appears due to the fact that in axisymmetric flows the RR solution is theoretically impossible [24]. JB is jet boundary (see Fig. 3 for other notations) [1].

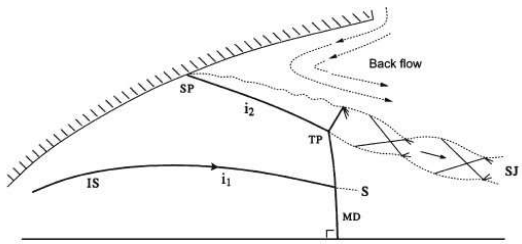

Figure 5. Free shock separation. IS internal shock, SJ supersonic jet, SP separation point (see Fig. 3 for other notations) [1].

As a consequence, a rotational flow occurs behind the Mach disk with velocity and entropy gradients that become larger for increasing flow non-uniformity upstream, and thus can generate vortical structures. The centerline vortex, whose size and growth rate are mainly controlled by viscous effects, acts as an obstruction for the main exhausting jet that therefore deviates towards the wall. As a consequence, a radial flow component is generated behind the shock that tends to reattach the separated region to the wall, thus switching the flow structure of the separated region from FSS to RSS. Behind the reattachment point, due to the flow impingement on the wall, a sudden increase of pressure occurs. Summarizing, the flow non-uniformity may generate a curved-shock profile with a downstream trapped vortex (also referred to as "cap-shock", Figs. 3, 6), which acts as a driver for the transition from FSS to RSS, and in turns generates the highest wall pressure peaks. This conclusion is largely proven by experiments [16–20, 25, 26,57,58,59]. Concerning the causes of the generation of the recirculating region, a different interpretation was given by Hagemann and Frey, who indicated it as a consequence of the direct or inverse Mach reflection of the internal shock that typically characterizes the flow field in parabolic nozzles. Following this point of view, they suggested that truncated ideal contour (TIC) nozzles would be a better choice to avoid intense side loads [18,27,28,55,56]. However, recent experiments [29, 30] showed that even in TIC nozzles significantly high-amplitude side-loads may occur in particular at low pressure regimes, confirming earlier findings [2] of symmetrical/unsymmetrical boundary-layer separations and subsequent side-loads generation in conical nozzles. Although the physical mechanism that drives the unsymmetrical boundary-layer separation in axisymmetric nozzles is not yet well understood, the phenomenon is mainly governed by one or both of the two mentioned flow separation structures: FSS and RSS. Fig. 7 shows an over expanded converging-diverging nozzle at the moment of separation.

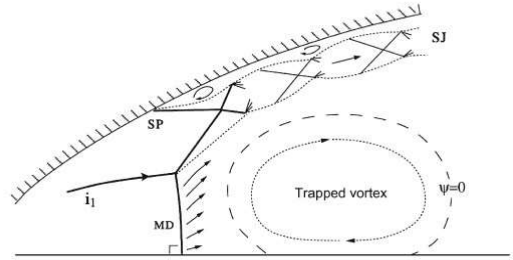

Figure 6. Restricted shock separation (RSS) [1].

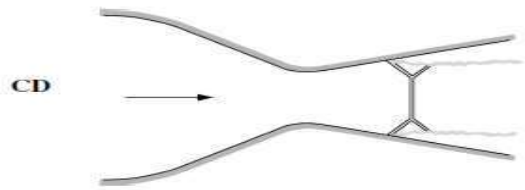

Figure 7. An over expanded converging-diverging nozzle at the moment of separation [5].

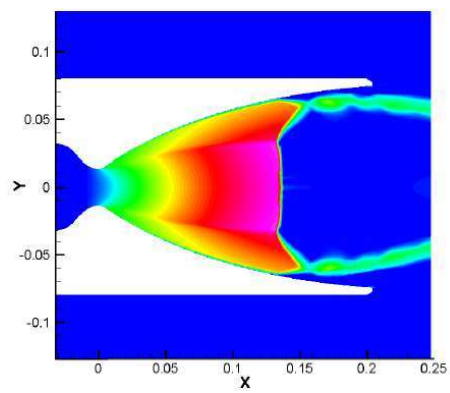

Figure 8. Mach plot at normal plane depicting Free Shock Separation (FSS) at NPR=40 [64].

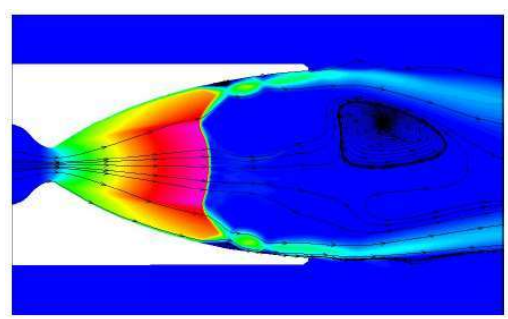

Figure 9. Mach plot at normal plane depicting Restricted Shock Separation (RSS) at NPR=30 [64].

In Fig. 8 FSS is captured with a higher side tilted towards the wall. With further *NPR* decrease qRSS goes
to RSS. RSS depicted Fig. 9 imposes recirculation zones between separation and reattachment points at the wall. In the main flow trapped vortex or counter rotating vortex pair is formed being trapped inside the sequence
region between cap shock and the end. Vortex motion inside allows "breathing" of the trapped flow which is by number of authors related [38,40] to FSS/RSS transition and oscillations. The recirculation formed near the wall effectively controls the pressure imposed on the wall. For the FSS, flow separates due to adverse pressure gradient and the back flow enters the nozzle rapidly increasing the pressure as the back flow turns and forms long loop the slope of the pressure profile starts to decrease going to its plateau ambient value [64]. This two type of flow separations occur during transient startup or shutdown even if the nozzle operates with full-flow at steady-state chamber pressure. In a typical rocket engine, the chamber pressure rises from the ambient pressure to the steady-state operating value [9,12,31]. Flow separation momentarily occurs when the chamber pressure is relatively low, such as to yield wall pressure much lower than the ambient one in some location of the divergent section. During this transient startup period, the separated flow is first governed by the FSS structure. Then the FSS is replaced by the RSS pattern when the chamber pressure exceeds a certain critical value. Hysteresis of the FSS↔RSS transition is also clearly identified and several peaks of side-loads are measured by different groups in Europe [32,33], USA [4] and Japan [34]. In spite of many studies on the subject, the mechanisms of shock-wave propagation and related side-loads generation are quite complex, and fundamental knowledge of supersonic flow physics in presence of shock reflection at wall, shock/shock and shock/boundary layer interactions is still needed. Khan and Shembharker [35] presented a viscous flow analysis of a convergent divergent nozzle. Adamson and Nicholls [36], analyzed nozzle jets experimentally and presented an analytical method for calculating the position of shock inside the nozzle, whereas Lewis and Carlson [37] experimentally determined the distance of the first Mach disc in under expanded supersonic nozzles issuing gas from the nozzle exit plane. Fig.10 shows the FSS - dimensionless wall pressure profile

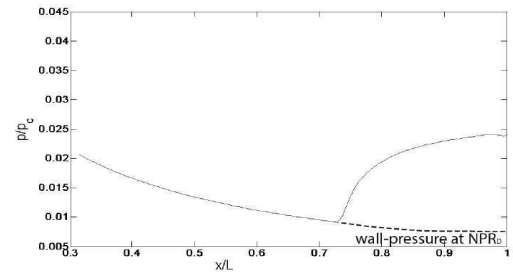

Figure 10. FSS - dimensionless wall pressure profile [64].

Special case on supersonic Nozzle Flow Separation (NFS)

The special case[1] on NFS focuses on the integration of theory, modeling, and experiments for the study of shock-wave interactions in supersonic nozzles, and helps to provide a basis for future work in this area.

This issue includes some original and/or review papers from specialists in the various aspects of supersonic nozzles (theory, advanced measurements, and numerical simulations). some of the contributions are experimental investigations, and some of the other is dedicated to numerical simulations [20,38,39]. In the framework of this study, several experimental techniques have been used with different nozzle shapes (planar or bell ideal and optimized contours) to elucidate the phenomenon of boundary layer separation and shock interactions. For example, Papamoschou et al. [40] presented an experimental study of the dynamics of the shock-wave/ boundary-layer interaction in a planar nozzle. Optical resolutions as well as pressure information have been used to highlight the important characteristics of this unsteady flow with in particular the low-frequency movements of the separated shock, oscillating in a "piston-like" manner. Stark and Wagner [30] summarized recent findings achieved on TIC nozzles, with emphasis on separation criteria and understanding of side-loads generation. Some interesting data are included in their paper with regard to the Mach disk shape and location. Although these data are directly linked to the specific nozzle shape and driving gas conditions, a simple, and useful separation criterion has emerged from their study. The authors proposed an explanation of a flow phenomenon observed for very low-pressure ratios. Boundary layer relaminarization and subsequent transition from laminar to turbulent flow separation is shown to create the potential for a tilted Mach disk that directs

flow towards the wall and causes large side-loads. On the other hand, Verma [41] dedicated his study to flow separation and shock unsteadiness in thrust optimized parabolic (TOP) nozzles conducted in the DLR P6.2 cold-gas subscale test facility. Several data, obtained from time-resolved wall pressure measurements, high-speed schlieren and strain-gauges, highlighted the unsteady character of the shock motion in the separation region. The physical mechanisms responsible for the origin of flow unsteadiness, for various separation modes and their contribution towards generation of side-loads, are discussed. Also, Tomita et al. [42] presented an overview of an experimental study dedicated to a small-scale TOP nozzle with both cold and hot gases. Different aspects of the flow behavior, during the transient process, have been described through the use of various and complementary experimental methods. Surface pressure and unsteady forces measurements, surface flow qualification by liquid crystal responding to shear stress, shadowgraph pictures, etc. Detailed experimental results, confirmed by numerical simulations, have been discussed and a new mechanism, so-called "separation jump", has been identified as a reason for the measured unsteady nozzle side-loads amplification. To conclude the experimental part, Nurnberger- Genin and Stark [43] investigated the flow transition between two operating modes of a dual-bell nozzle. Particular attention has been paid to the hysteresis effect during the transition from the first mode (flow separated at the inflection) to the second one (fully attached flow) and back. In order to characterize the side-load behavior, the transition duration as well as the separation front velocity have been measured and analyzed. From numerical point of view, it is worth noticing that the modeling challenge is to predict the boundary layer in nozzles at a very high-Reynolds number to adequately simulate the interaction of shock-waves with large and small scale turbulence and associated phenomena. One of the major stumbling blocks for computing nozzle flows is the near-wall turbulence. Fig. 11 shows Distribution of pressure and phenomena occurring in radius of nozzle for FSS (top) and RSS (bottom) and Fig. 12 shows Schematic of the effect of the interlayer boundary layer impact waveform in the following conditions: a) Ramp flow. And b) Shock reflection. And c) Step induced separation.

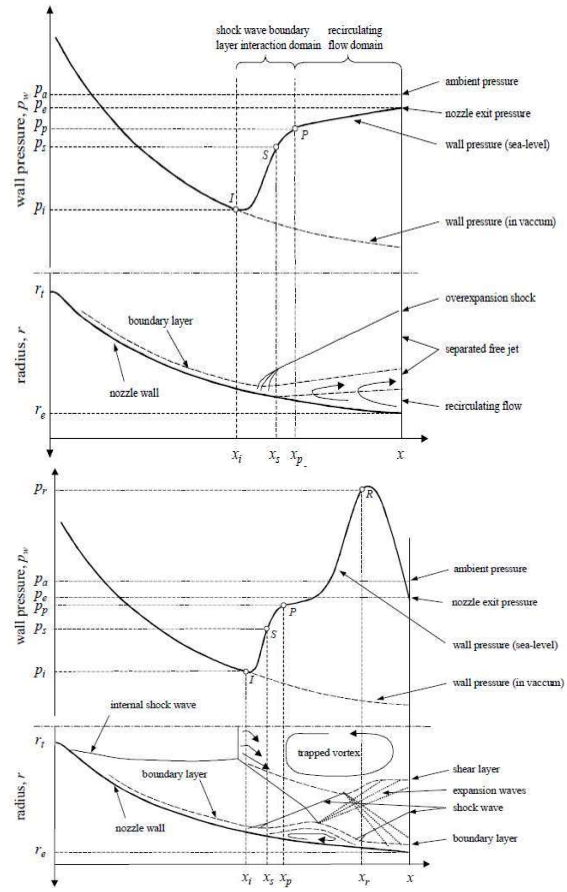

Figure 11. Distribution of pressure and phenomena occurring in radius of nozzle for FSS (top) and RSS (bottom) [15].

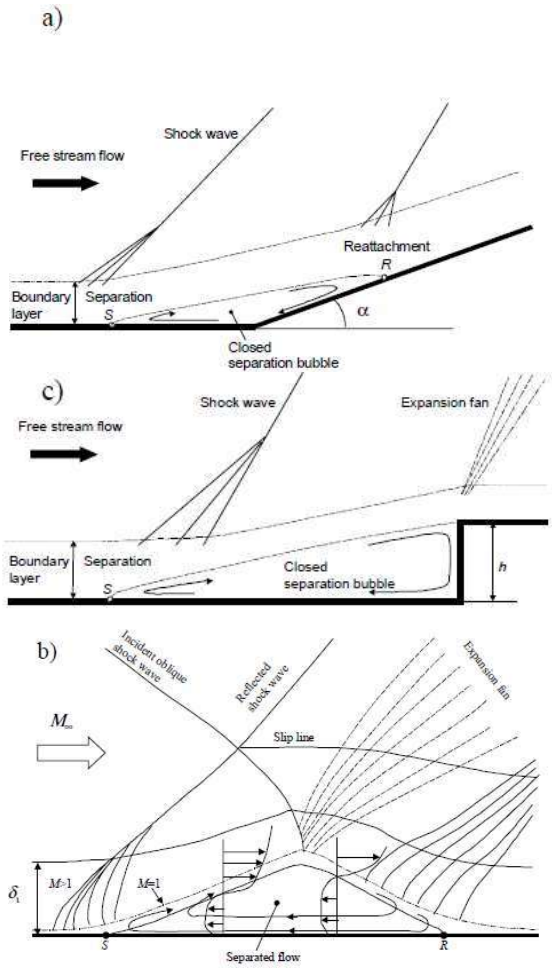

Figure 12. Schematic of the effect of the interlayer boundary layer impact waveform in the following conditions: a) Ramp flow. And b) Shock reflection. And c) Step induced separation [15].

In previous works [10,26,44,45,60], a major step has been made in turbulence modeling, where state-of-the-art steady RANS and unsteady URANS methods have been used to simulate flow characteristics in supersonic nozzles and to conduct parametrical studies for both steady and transient flow regimes. Also, this approach has been effective for the analysis of different shock-waves structures and separation type (FSS, RSS). In this context, Nasuti and Onofri [46] discussed the somewhat physical mechanism that drives the Mach-stem curvature in typical over expanded nozzles using RANS method. The phenomenological explanation as well as the simplified description of the flow features helps in the understanding of the "Inviscid Separation" and "Restricted-Shock Separation" phenomena in separated over expanded nozzles. Also, Martelli et al. [38] presented a numerical study mainly focused on the transition between the two shock-separation patterns in a parabolic nozzle with an evidence of a hysteresis loop, depending on the initial conditions. For transient flow simulations, Wang [47] presented computational methodology to capture the side-loads physics in a representative rocket engine, using an engine system simulation to obtain a sequence for reproducing the inlet history as close as possible to the fire test. Additionally, Perrot and Hadjadj [39] examined numerically the transient flow in a supersonic ideal nozzle. Their computations provide engineers with detailed insights into the complex time evolution of the starting process, clearly showing the development and the effect of shock-wave propagation and early stages of boundary-layer separation from the nozzle wall. Deck [48] reported results of an advanced CFD investigation using a detached-eddy simulation (DES) approach on the unsteady nozzle flow under "end-effect" regime and also on side-load characteristics. DES stands as a promising solution for computing side-loads generation, since it combines the efficiency of a Reynolds-averaged turbulence Model near the wall with the fidelity of large-eddy simulation (LES) in separated regions. Fig. 13 shows Schematic the phenomenon of waves created at the moment of separation.

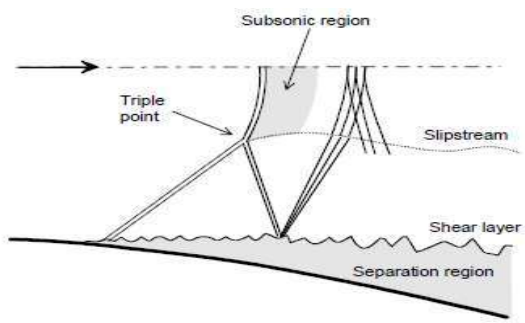

Figure 13. Schematic of the phenomenon of waves created at the moment of separation [15].

Guillaume Daviller et al [63] by Using Large-eddy Simulation Predicted Flow Separation and Side-loads in Rocket Nozzle. In his research he explained that this flow is characterised by a FSS regime and the main shock pattern position oscillates around the flow separation point. As a consequence a steady mesh refinement strategy is adapted to correctly discretise both the flow field and the shock. In order to have a better understanding of the flow characteristics, LES flow field obtained with the hybrid mesh is first analysed. Finally he explained that The methodology used to study the FSS allows to correctly predict the flow physics with a decrease

of the CPU cost (20% of gain). In particular, only 2% error on the separation point position is observed on the adapted mesh with a better representation of the flow separation leading to a gradually pressure rise downstream the main shock pattern. Indeed, it is shown that the mesh resolution in the recirculation region have a significant influence on the pressure rise downstream the SWBLI. His study represents a first step to understand the nozzle side loads which occur during rocket engine ignition and shutdown.

Conclusions and recommendations for future research

It should be evident from the very brief summary of different investigations above that flow separation in nozzles is an extremely difficult task and despite truly remarkable progress in computational and measurement capabilities, there are still many unresolved problems. According to several research that conducted on this subject, some suggestions are made as to where future efforts on nozzle flow investigations might be focused.

a) Much effort remains to be done on basic research of shockwaves phenomena with different type of shock interaction in nozzles. Of particular interest, the curvature of the Mach disk (convex or concave shapes) and the mechanism of formation of vortices on the contact surface as well as on the downstream subsonic flow merit a special attention. Knowing the key role played by the recirculation bubble on the dynamic of the large-scales fluid motion, specifically at the nozzle exit (end-effect regime), the question of inviscid/viscous interaction and nozzle flow stability reminds extremely important for future research and can be useful for understanding this phenomena.
b) In another side, accurate estimates of side-loads in nozzles require, in part, a detailed study of the flow behavior during start-up or shut-down processes. At transient regimes, this involves careful analysis of the shock interactions and the nature of the boundary layer. One of the not yet well understood phenomenon in transient nozzle flows concerns the early stage of the startup, when the recompression shock start to interact with the emerging boundary layer, exhibiting hence various complex and transitional flow structures, ranging from purely inviscid to laminar and then fully turbulent flow separations. This very short transient period is crucial for the nozzle life constraints since the flow is very sensitive to small perturbations which may rapidly evolve through the interaction process, leading to a strong side-loads generation. Also, during the startup or shutdown processes, the question of the influence of the time scale, fast (impulsive) versus slow (steady-state pressure increase) on different shock separation patterns is still open. We would expect, in this case, different side-loads amplification depending on how the flow acceleration occur for fast and slow transient regimes.
c) The side-load phenomena in different regimes can be investigated. Side-loads can be generated by: (Random pressure fluctuations, Transition of separation pattern, Aeroelastic coupling).
d) Shock-induced flow separation in different regimes can be investigated because boundary-layer separation in transonic and supersonic flows is of great importance in many practical applications, such as rocket propulsion systems, transonic airfoils, and refrigeration ejectors. These unsteady phenomena are associated with undesirable effects such as shock oscillations, wall-pressure fluctuations, boundary-layer separation, and vortex shedding, which are major causes of vibration, noise, and side-loads generation.
e) Another task which was not directly addressed is this special issue is that related to low-frequency oscillations of shock-induced turbulent separation. Although this phenomenon appears to not strongly depend on the nozzle geometry [49,45], since it has been also revealed in many other configurations such as ducts [51,52], wind tunnels [53] or ramps [54], its relevance in SWBLI applications and in particular the fluctuating pressure loads generated by translating shockwaves, pulsating separated flows, and expansions/contractions of the global flow field which can cause severe nozzle structural damage, cannot be ignored by designers of rocket nozzles. Indeed, much effort should be spent towards the identification of the origin of low-frequency shock movement as well as the physical mechanism that drive this phenomenon. Probably, the most efficient and profitable approach that can used to handle this problem would be clearly one in which computation and experiment are closely coupled.
f) Continued development of engineering side load model.
g) Studies of the physical nature of different types of separation [62] and the origin of the side loads.
h) Demonstration of further side load reduction experimentally.

i) In CFD, the potential exists not only for computing unsteady interaction properties but also for using DES and LES to explore the effects of different nozzle configurations and flow variations and to investigate the underlying physics. To validate unsteady approaches and improve numerical simulations of complex nozzle flows, additional information is needed from experiments. Experimental studies typically do not report the nozzle geometry effects or characterize the flow field inside the nozzle. Shock waves, internal nozzle boundary-layer data, and turbulence measurements in the shear layer at the separation are important for developing accurate numerical simulations.

j) The flow asymmetry which occurs in either planar or axisymmetric nozzle geometries is still an open question, and is clarified neither by experiment nor by CFD. In particular, for low nozzle pressure ratio (NPR) regimes, the nozzle throat and the boundary layer may play an important role. Knowing the importance of the phenomenon at transonic speeds and how the nature of the interaction depends critically on the state of the incoming boundary layer, the upstream conditions merit to be carefully investigated, in particular the nature of the boundary layer (laminar, transitional or fully turbulent) as well as the influence of small perturbations at the wall (like roughness) or/and the shape of throat (with or without internal shock). From hydrodynamic stability point of view, the mixing layer emanating from the separation point at transonic regime ($0.8< M <1.9$) may evolve differently than at high Mach number. Therefore, the influence of the nozzle Mach number merits to be addressed. Another still open question is: for low NPR regimes, is there any influence of downstream conditions, especially the confinement effect of the separated jet by the nozzle walls?

k) Finally, it should be recalled that the experimental analysis of separated nozzle flows in both transient and stabilized regimes in full-scale rocket nozzles is very difficult and expensive, because it would need flow visualizations and measurements inside the divergent section in the few seconds of the engine run (or milliseconds for the crucial part of the transient). Since the main finding is revealing the unsteady nature of the flow separation, which is not easily accessible by experiments in real configurations, the quantitative data of the CFD, if previously well validated through appropriate benchmark calculations, should help to understand and explain such flow behavior.

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