

## Science Article

# Neural Network Backstepping control of a reentry vehicle in the presence of uncertainties

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*In this paper, a neural network backstepping controller is designed for the control of a reentry vehicle. The backstepping control system is applied to the nonlinear six degree of freedom dynamics of the reentry vehicle for tracking the desired input. The neural network is used for estimation of nonlinear parts of backstepping controller during entry to atmosphere and to estimate the nonlinear terms as well as the external disturbances. Numerical simulations have been performed to verify the performance of the proposed control method.*

**Keywords:** Reentry vehicle, Backstepping control, Neural network estimation, Feedforward neural network.

## Introduction

The dynamic model of a reentry vehicle is highly nonlinear, multivariable, considerable coupling and contains uncertain parameters due to the wide flight envelope and the high Mach number. Furthermore, it is difficult to measure or estimate the atmospheric properties and aerodynamic characteristics at the hypersonic flight altitude. As a result of this, the linear control methods cannot be used to design a control system for reentry vehicles. It is essential to improve the speed and the robustness of the control system.

Therefore, nonlinear control methods, such as dynamic inversion, backstepping control, have been the main techniques used for hypersonic flight control.

Backstepping design method is an effective design approach for nonlinear control systems. That offers unique advantages in dealing with nonlinear system problems. Taking cascade linear or nonlinear system by selecting the suitable Lyapunov function and constructing virtual

control laws step by step makes a process in which a stable control system can be achieved [1]. This method can guarantee the global stability of the closed loop system in the case that regulating and tracking have asymptotic behavior. The traditional adaptive backstepping design methods acquire the system to be parameterized [2]. In this paper, based on backstepping control algorithm, a nonlinear neural network backstepping controller is derived for plants with arbitrary relative degree. The backstepping controller is designed in two steps and for each step a nonlinear function is estimated by Bayesian regularization Neural.

## Equations of motion

The motion of 6-DOF unpowered rigid flight vehicle can be separated in to the transitional motion of the center of mass using a flight path coordinate system and the rotational motion of a body fixed coordinate system about the center of mass (rotational or attitude motion). The center of the motion of mass is caused by the forces that act

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on the vehicle. It is used for generating trajectory and designing a guidance law. Most applications assume steady coordinated turns such that the sideslip angle is zero. The equations of translational motion are given as follows [3,4]:

$$\dot{h} = v \sin \gamma \quad (2)$$

$$\dot{\phi} = \frac{v \cos \gamma \sin \chi}{(Re+h) \cos \theta} \quad (3)$$

$$\dot{\theta} = \frac{v \cos \gamma \cos \chi}{(Re+h)} \quad (4)$$

$$\dot{v} = -\frac{D}{m} - g \sin \gamma \quad (5)$$

$$\dot{\gamma} = \frac{L \cos \sigma}{mv} - \left( \frac{g}{v} - \frac{v}{Re+h} \right) \cos \gamma \quad (6)$$

$$\dot{\chi} = \frac{L \sin \sigma}{mv \cos \gamma} + \frac{v \cos \gamma \sin \chi \tan \theta}{Re+h} \quad (7)$$

where  $h$ ,  $\phi$ ,  $\theta$ ,  $v$ ,  $\gamma$ ,  $\chi$  are altitude, latitude, longitude, velocity, flight path angle, and heading angle respectively.  $L$  and  $D$  are lift and drag, respectively.  $g = \frac{\mu}{(Re+h)^2}$  is the acceleration due to gravity with  $\mu$  being the earth's gravity constant and  $Re$  is radius of the earth. The rotational equations governing the attitude dynamic of the rigid vehicle during the reentry flight are given as follows:

$$\dot{\alpha} = -p \cos \alpha \tan \beta + q - r \sin \alpha \tan \beta - \frac{\cos \sigma}{\cos \beta} (\dot{\gamma} - \dot{\phi} \cos \chi - \dot{\theta} \cos \phi \sin \chi) + \quad (7)$$

$$\frac{\sin \sigma}{\cos \beta} (\dot{\chi} \cos \gamma - \dot{\phi} \sin \chi \sin \gamma) +$$

$$\frac{\sin \sigma}{\cos \beta} (\dot{\phi} + \dot{\theta} (\cos \phi \cos \chi \sin \gamma - \sin \phi \cos \gamma))$$

$$\dot{\beta} = p \sin \alpha - r \cos \alpha$$

$$+ \sin \sigma [\dot{\gamma} - \dot{\phi} \cos \chi + \dot{\theta} \cos \phi \sin \chi] \quad (8)$$

$$+ \cos \sigma [\dot{\chi} \cos \gamma - \dot{\phi} \sin \chi \sin \gamma]$$

$$- \dot{\theta} \cos \sigma$$

$$(\cos(\phi) \cos(\chi) \sin(\gamma) - \sin(\phi) \cos(\gamma))$$

$$\dot{\sigma} = -p \cos \alpha \cos \beta - q \sin \beta - r \sin \alpha \cos \beta \quad (9)$$

$$+ \dot{\alpha} \sin \beta - \dot{\chi} \sin \gamma - \dot{\phi} \sin \chi \cos \gamma$$

$$+ \dot{\theta} [\cos \phi \cos \chi \cos \gamma + \sin \phi \sin \gamma]$$

$$\dot{p} = \frac{I_{zz} M_x}{I_{xx} I_{zz} - I_{xz}^2} + \frac{I_{xz} M_z}{I_{xx} I_{zz} - I_{xz}^2} + \frac{(I_{yy} - I_{zz}) I_{xz}}{I_{xx} I_{zz}} q r \quad (10)$$

$$\dot{q} = \frac{M_y}{I_{yy}} \quad (11)$$

$$\dot{r} = \frac{I_{xx} M_z}{I_{xx} I_{zz} - I_{xz}^2} + \frac{(I_{xx} - I_{yy}) I_{xz}}{I_{xx} I_{zz}} p q \quad (12)$$

where,  $\alpha$ ,  $\beta$  and  $\sigma$  denote, angle of attack (AOA), sideslip angle, and bank angle, respectively.  $p$ ,  $q$  and  $r$  are roll rate, pitch rate and yaw rate, respectively.  $M_x$ ,  $M_y$  and  $M_z$  are rolling moment, pitching moment, and yawing moment. The expressions of the lift ( $L$ ) and drag ( $D$ ) are as follows:

$$L = q S C_L(\alpha) \quad (13)$$

$$D = q S C_D(\alpha) \quad (14)$$

where  $C_L(\alpha)$  and  $C_D(\alpha)$  are the lift coefficient and drag coefficient, respectively, and they are the function of AOA. Aerodynamic area of vehicle is given by  $S$  and dynamic pressure is given with  $q = 0.5 \rho v^2$ .

### Backstepping Controller Design

The state equations in Eqs (7-9) can be rearranged as:

$$\dot{x}_1 = f_1 + g_1 x_2 + h_1 u \quad (15)$$

$$\dot{x}_2 = f_2 + g_2 u \quad (16)$$

where  $f_1 = [\hat{f}_1, \hat{f}_2, \hat{f}_3]$  and  $f_2 = [\hat{f}_{21}, \hat{f}_{22}, \hat{f}_{23}]$  estimated by NN.

$$g_1 = \begin{bmatrix} -\cos \alpha \tan \beta & 1 & -\sin \alpha \tan \beta \\ \sin \alpha & 0 & -\cos \alpha \\ -\cos \alpha \cos \beta & -\sin \beta & -\sin \alpha \end{bmatrix} \quad (17)$$

$$g_2 = \begin{bmatrix} I_{11} & 1 & I_{13} \\ 0 & I_{22} & 0 \\ I_{31} & 0 & I_{33} \end{bmatrix} \quad (18)$$

$$I_{11} = \frac{I_{zz}}{I_{xx}I_{zz} - I_{xz}^2} \quad (19)$$

$$I_{13} = \frac{I_{xz}}{I_{xx}I_{zz} - I_{xz}^2} \quad (20)$$

$$I_{22} = \frac{1}{I_{yy}} \quad (21)$$

$$I_{31} = \frac{I_{xz}}{I_{xx}I_{zz} - I_{xz}^2} \quad (22)$$

$$I_{33} = \frac{I_{xx}}{I_{xx}I_{zz} - I_{xz}^2} \quad (23)$$

In the design of the neural network backstepping controller as shown in Fig. 1,  $u$  is designed in such a way  $x_1$  tracks  $x_1^d$ . Equations in the real features include the aerodynamic uncertainties and the disturbance. As a result of this, the nonlinear functions  $f_1$  and  $f_2$  must be estimated  $\hat{f}_1$ ,  $\hat{f}_2$  are the estimation of  $f_1$  and  $f_2$ . The difference between  $x_1$  and  $x_1^d$  defined as:

$$z = x_1 - x_1^d \quad (24)$$

The following Lyapunov function candidate can be proposed:

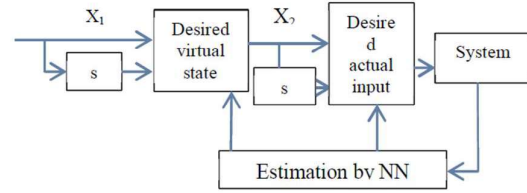
$$V_1 = \frac{1}{2} z_1^T z_1 + \frac{1}{2c_1} \hat{f}_1^T \hat{f}_1 \quad (25)$$

where,  $c_1$  is the positive constant, the time derivative of Eq (25) is semi-negative definite, if  $x_2$  and the corresponding adaptation law are chosen as:

$$x_2 = g_1^{-1}[-\hat{f}_1^T - h_1 u - \dot{x}_1^d - K_1 z_1] \quad (26)$$

where  $K_1$  is a positive definite gain matrix. It should be noted that the right side of Eq (26) includes the actual control input  $u$ , and during the maneuvers,  $h_1$  is close to zero. Therefore,  $h_1 u$  in Eq (26) can approximately be ignored in the design of the backstepping controller. Taking the time derivative of Eq (25) yields:

$$\begin{aligned} \dot{V}_1 &= z_1^T \dot{z}_1 + \frac{1}{c_1} \hat{f}_1^T \dot{\hat{f}}_1 = z_1^T [\dot{x}_1 - \dot{x}_1^d] + \frac{1}{c_1} \hat{f}_1^T \dot{\hat{f}}_1 = \\ & z_1^T [f_1 + g_1 x_2 + \hat{f}_1 - \dot{x}_1^d] + \frac{1}{c_1} \hat{f}_1^T \dot{\hat{f}}_1 = \\ & z_1^T [\hat{f}_1 + g_1 x_2 - \dot{x}_1^d] - z_1^T \hat{f}_1 + \frac{1}{c_1} \hat{f}_1^T \dot{\hat{f}}_1 \end{aligned} \quad (27)$$



**Fig. 1.** Neural Network Backstepping Control Scheme

Applying Eq.(26) to Eq(27) yields

$$\dot{V}_1 = -z_1^T K_1 z_1 \leq 0 \quad (28)$$

Therefore,  $x_1$  converges to  $x_1^d$ . The difference between  $x_2$  and  $x_2^d$  is defined as follows:

$$z_2 = x_2 - x_2^d \quad (29)$$

consider the following Lyapunov function candidate

$$V_2 = \frac{1}{2} z_1^T z_1 + \frac{1}{2} z_2^T z_2 + \frac{1}{2c_1} \hat{f}_1^T \hat{f}_1 + \frac{1}{2c_2} \hat{f}_2^T \hat{f}_2 \quad (30)$$

where,  $c_2$  is a positive constant. The time derivative of Eq (30) is semi-negative definite, if  $u$  is chosen as

$$u = g_1^{-1}[-\hat{f}_2 + \dot{x}_2^d - g_1^T z_1 - K_2 z_2] \quad (31)$$

where,  $K_2$  is a positive definite gain matrix. By substituting Eqs(24-26) and (29) into the time derivative of Eq (30), one can note that:

$$\begin{aligned} \dot{V}_2 &= z_1^T \dot{z}_1 + z_2^T \dot{z}_2 + \frac{1}{c_1} \hat{f}_1^T \dot{\hat{f}}_1 + \\ & \frac{1}{c_2} \hat{f}_2^T \dot{\hat{f}}_2 = z_1^T [f_1 + g_1 x_2 + h_1 u - \dot{x}_1^d] + z_2^T [f_2 + g_2 u - \dot{x}_2^d] + \\ & \frac{1}{c_1} \hat{f}_1^T \dot{\hat{f}}_1 + \frac{1}{c_2} \hat{f}_2^T \dot{\hat{f}}_2 \\ &= z_1^T [g_1 z_2 - K_1 z_1] + z_2^T [\hat{f}_2 + g_2 u - \dot{x}_2^d] - z_1^T \hat{f}_1 - z_2^T \hat{f}_2 + \\ & \frac{1}{c_1} \hat{f}_1^T \dot{\hat{f}}_1 + \frac{1}{c_2} \hat{f}_2^T \dot{\hat{f}}_2 \end{aligned} \quad (32)$$

Applying Eqs(26) and (31) and (32) yields:

$$\dot{V}_2 = -z_1^T K_1 z_1 - z_2^T K_2 z_2 \leq 0 \quad (33)$$

## The Estimation Procedure

The feedforward Neural Network is used for estimation of  $f_1$  and  $f_2$  in Eq (26-31). The network has 10 layers and based on multi-layer perception. It solves a data fitting problem by a two layer feedforward network trained with Bayseian regularization. Bayseian regularization is a network training function that updates the weight and bias values according to Levenberg-Marquardt optimization [5].

Regularization modifies the cost function by adding a term proportional to the square of the

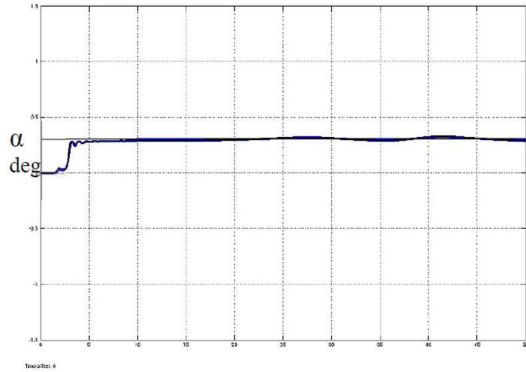
norm of the parameter vector  $\theta$ , hence, that the parameters  $\theta$  are obtained by minimizing:

$$\hat{V}_N(\theta) = \sum_{t=1}^N \varepsilon^2(t, \theta) + \lambda k \theta k^2 \quad (34)$$

where  $\lambda$  is a positive constant that has the effect of trading variance error in  $V_N(\theta)$  for bias error the larger the value of  $\lambda$ , the higher the bias and lower the variance of  $\theta$ .

### Simulation Results and Analysis

In order to verify the performance of the proposed control system, six degrees of freedom nonlinear simulations are carried out. The proposed neural network backstepping controller improves the performance of backstepping control and combines advantages of neural network with a nonlinear method for controlling a nonlinear system where all parameters change rapidly. The dynamic of reentry in section 1 is nonlinear and coupled. The backstepping controller designed in this study was added to the neural network algorithm.



**Fig. 2.** Angle of attack and its desired input.

The desired states of reentry during simulation are considered as follows:

$$x_1^d = [\alpha_d, \beta_d, \sigma_d]^T \quad (37)$$

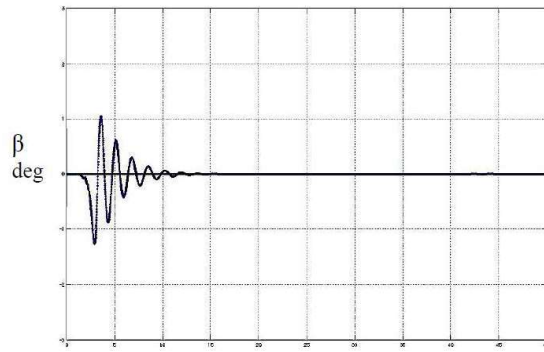
where,  $\alpha_d$  is a desired angle of attack, and  $\beta_d$  is the desired sideslip angle, and  $\sigma_d$  is the desired bank angle.

The desired values for all of angles are in terms of certain functions of time. it is also noted that because of its low rates of change in sideslip angle and zero initial value, this variable is a zero function of time.

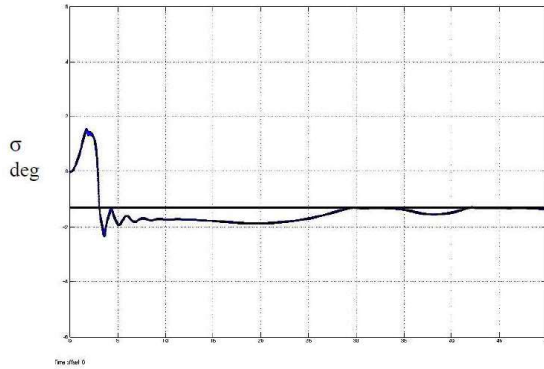
The initial conditions are assumed to be:

$$[\alpha_0 \beta_0 \sigma_0] = [0.030 - 73] \quad (35)$$

The initial values for height and velocity are 90 [km] and 5222 [m/s] respectively. The standard model considered in ref [6] is used as the model of the atmosphere. Furthermore, the earth is assumed to be spherical and aerodynamic coefficients are computed using Missile Datcom.



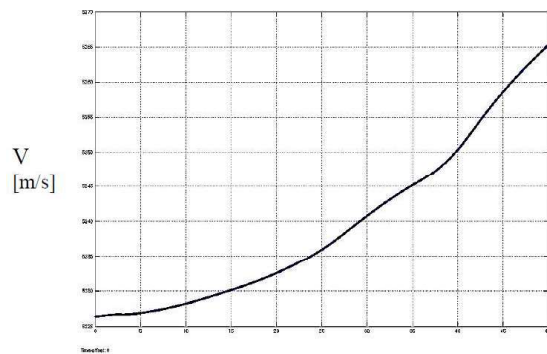
**Fig. 3.** Sideslip angle and its desired input



**Fig. 4.** Bank angle track desired input



**Fig. 5.** Height of vehicle during the flight



**Fig. 6.** Velocity of the vehicle during the flight.

## Conclusion

In this paper, the attitude controller based on neural network backstepping control is designed for reentry vehicle in spite of model uncertainty, and external disturbance. The developed Neural network backstepping controller (NNBC) scheme assures the stable tracking of the attitude angle with model uncertainty and external disturbance. The nonlinear functions terms are estimated by the feedforward neural network. The stability of the system is proven with Lyapunov theory, and the convergence of the tracking is ensured.

## References

- [1] Jingmei Zhang, Changyin Sun, Ruimin Zhang, Chengshen Qian, and Lei Xue, "Adaptive Sliding Mode Control for Reentry Attitude of Near Space Hypersonic Vehicle Based on Backstepping Design", IEEE/CAA Journal of Automatic Sinica, vol.2, no.1, 2015.
- [2] Jingmei Zhang, Changyian Sun, Rumin Zhang, Chengshan Qian, and Lei Xue, "Adaptive Backstepping Controller Design for Reentry Attitude of Near Hypersonic Vehicle", Springer, ISCI 2013, Lncs 8261, pp642-648, 2013.
- [3] Fang Wang, Qun Zong, and Bailing Tian, "Adaptive Backstepping Finite Time Attitude Control of Reentry RLV with Input Constraint", Hindawi Publishing Corporation Mathematical Problems in Engineering Volume, 2014.
- [4] F. Regan and S. Anandkrishnan, "Dynamics of Atmospheric Re-Entry", AIAA Education Series, Published by: American Institute of Aeronautics and Astronautics, Inc, 2012.
- [5] Seungho Yoon and Youdan Kim, "Constrained Adaptive Backstepping Controller Design for Aircraft Landing in Wind Disturbance and Actuator Stuck", Journal of Aeronautical and Space Sciences, 2012.

[6] Simon Haykin, "Neural Networks A Comprehensive Foundation", Prentice Hall International, Inc, Second Edition, 1999.

[7] Ashish Tewari, "Atmospheric and Space Flight Dynamics", Birkhauser, 2007.