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Adaptive Quaternion Attitude Control of Aerodynamic Flight Control Vehicles

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Conventional quaternion based methods have been extensively employed for spacecraft attitude control where the aerodynamic forces can be neglected. In the presence of aerodynamic forces, the flight attitude control is more complicated due to aerodynamic moments and inertia uncertainties. In this paper, a robust nero-adaptive quaternion controller based on back-stepping technique for vehicle with aerodynamic actuators is proposed. The presented control lawconsists of a neural network based adaptive part and an additional term which ensures the robustness of the system. Actually, the first term is designed to approximate and cancel out the matched uncertainties and the second term is used toensure the robustness of system against approximation error of the neural network.The Lyapunov direct method is applied to derive the learning laws for the neural network weights and adaptive gain. Also,theultimately boundedness of the error signals is guaranteed based on theLyapunov's stability criterion. The benefit of the presented method is evaluated through simulation of an aerodynamic control vehicle.

Keywords: Nero adaptive control, Quaternion attitude control, Aerodynamic control

Introduction1

Attitude control of flight vehicles has been a momentousissue such that a lot of publication shave been presented in this areaduring the past decades. The plenitude of attitude control outcomes can be divided almost into two categories: first, the methods which control the attitude by using the physical attitude parameters such as Euler angles which sufferfrom singularity that impedes large orientation maneuvers [1,2] and second, the methods which are based on quaternions that describe an Euler axis and the body-fixed frame rotation with respect to that axis. In these methods, singularities obstacle that were discovered in Euler angles approaches are completely removed [3-7].A large number of optimal and nonlinear control approaches are proposed to control the quaternion attitude based on minimization of some performance objectives like minimal time or control energy [8]. A nonlinear proportional -derivative control method is presented in [9], which globally stabilizes the dynamic in finite time. A robust nonlinear control based on H_∞ approach is proposed for spacecraft attitude maneuver in [10], which is designed such that L_2 gain from the exogenous disturbance to the performance measure in the closed loop system becomes less than a desired scalar.A quaternion feedback control based on back stepping approach is presented in [11] to stabilize a micro satellite attitude. This proposed controller stabilizes the equilibrium points in the closed-loop system uniformly asymptotically.

In practical applications, there are uncertainties which should be considered in system dynamic when the controller is designed, for instance, in fuel consumption, outgassing etc. One possible approach to handle the model uncertainty is adaptive control.

In this area, the adaptive output feedback attitude control of a spacecraft based on using

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the Chebyshev neural networks isinvestigated in presence of dynamic uncertaintiesin [12].

In [13], an adaptive sliding tracking control algorithm is extended to track a desired time-varying attitude of a satellite in the presence ofexternal disturbances and inertia uncertainties. In this work, reaction wheels are used as actuators and are modelled in addition to the spacecraft dynamics. The attitude control of a rigid satellite withexternal disturbance and actuator failure uncertainties is presented using adaptive control method in [14]. The parameters of external disturbance and failure uncertainties are estimated directly by adaptive rules. Also, the desired stability and output tracking properties of control scheme are analyzed.

In the previously mentioned works, there is a common point where the relevance between torque and deflection angles and atmospheric moment coefficients are neglected.For instance, when the attitude control for the tactical projectile operated in the low atmosphere is under study, these relationships become important and cannot be omitted.However, the researches in this area aresorely found [15-16].

In [15], a non-adaptive robust control scheme based on the quaternion feedback for attitude control of aprojectile which employs thrust vector controlis proposed. The control law consists of two parts: the nominal feedback part and an additional term to guarantee the robustness against the plantuncertainties. Quaternion attitude control of a projectile model, which is nonlinear inaerodynamics with atmospheric moment and inertia coefficients uncertainties together with bounded disturbances, is presented in [16]. In this work, a free chattering adaptive sliding mode controlleris designed based on back stepping technique to stabilize the state variables of the closed loop system to a small region of there ference states.

In this paper, a relatively simple neural network based adaptive attitude control of aerodynamic control projectile is considered. In contrary to [3-15], here the quaternion dynamic model which is nonlinear in aerodynamics with inertial and aerodynamic coefficients uncertainties is considered. Also, in contrary to [16] which assumed that the uncertainties satisfy a bound which is linear with respect to a portion of states norm, here the neuralnetwork which was demonstrated as a universal smooth function approximator, is applied to cancel out the unknown unstructured uncertainty adaptively [17].

In this paper, first the back-steppingapproach is applied to construct the baseline controller, and thenthe uncertainties are cancelled out using an additive nero-adaptive control term.In addition, an augmented term as a robustifying control part, is applied in the control law to eliminate the neural network approximation error.

The paperis organized as follows: section 2 describes the dynamic model of system and the problem of control associated with this system is also clarified. The procedure ofthe control designwhich is based on back-stepping technique, and approximation properties of the NN is addressed in section 3. Section 4 provides analytical results on stability proof of the closed-loop system. Simulation results are presented in section 5. Conclusions are also given in section 6.

Model Description and Problem Statement

Consider the Euler dynamic equations of rotational motion of a vehicle in the body frame as [3]

$$\mathbf{J}\dot{\boldsymbol{\omega}} = \left[\boldsymbol{\omega}\times\right]\mathbf{J}\dot{\boldsymbol{\omega}} + \mathbf{M} \tag{1}$$

Where $\boldsymbol{\omega} = \left[\omega_x, \omega_y, \omega_z\right]^T$ is angular rate vector of roll, yaw and pitch channel, $\mathbf{M} = \left[M_x, M_y, M_z\right]^T$ is the input torque vector containing roll, yaw and pitch torques, respectively and $\mathbf{J}$ represent the inertia matrix.

Here, it is assumed that the aerodynamic forces cannot beneglected; so, the input moments includingaerodynamic control moments of aerodynamic surfaces are considered as follows:

$$\begin{cases} M_x = qSd\left(\dfrac{d}{2V}c_{lp}\omega_x + c_{l\delta}\delta_a\right) \\ M_y = qSd\left(c_{m\beta}\beta + \dfrac{d}{2V}c_{mr}\omega_y + c_{n\delta}\delta_r\right) \\ M_z = qSd\left(c_{m\alpha}\alpha + \dfrac{d}{2V}c_{mq}\omega_z + c_{m\delta}\delta_e\right) \end{cases} \tag{2}$$

Where $c_{m\alpha}, c_{m\beta}$ are pitch and yaw moment coefficients, respectively, c_{lp}, c_{mr}, c_{mq} are roll, yaw and pitch damping coefficients and

$c_{l\delta}, c_{n\delta}, c_{m\delta}$ are deflects moment coefficients of roll, yaw and pitch, respectively. Also, $q, S, d, V, \alpha, \beta$ represent dynamic pressure, reference area and diameter of vehicle,amplitude of speed and pitch and yaw attack angles, respectively and $\delta_a, \delta_r, \delta_e$ arerespectivelyaileron,rudder and elevator commands. Substituting (2) into (1) yields,

$$\mathbf{J}\dot{\boldsymbol{\omega}} = [\boldsymbol{\omega}\times]\mathbf{J}\boldsymbol{\omega} + \mathbf{A}_{\boldsymbol{\omega}}\boldsymbol{\omega} + \mathbf{A}_{\boldsymbol{\alpha}}\boldsymbol{\eta} + \mathbf{A}_{\boldsymbol{\delta}}\boldsymbol{\delta} + \mathbf{d}(t) \quad (3)$$

Where

$$\mathbf{A}_{\boldsymbol{\omega}} = \frac{qsd^2}{2V}\begin{bmatrix} c_{lp} & 0 & 0 \\ 0 & c_{mr} & 0 \\ 0 & 0 & c_{mq} \end{bmatrix}$$
$$\mathbf{A}_{\alpha} = qsd\begin{bmatrix} 0 & 0 & 0 \\ 0 & c_{m\beta} & 0 \\ 0 & 0 & c_{m\alpha} \end{bmatrix} \quad (4)$$
$$\mathbf{A}_{\boldsymbol{\delta}} = qsd\begin{bmatrix} c_{lp} & 0 & 0 \\ 0 & c_{n\delta} & 0 \\ 0 & 0 & c_{m\delta} \end{bmatrix}$$

and $\boldsymbol{\eta} = [\mu \quad \beta \quad \alpha]^T$, $\boldsymbol{\delta} = [\delta_a \quad \delta_r \quad \delta_e]^T$, and $\mathbf{d}(t)$ is bounded external disturbance vector. Let the parameters of system $\mathbf{J}, \mathbf{A}_{\boldsymbol{\omega}}, \mathbf{A}_{\boldsymbol{\alpha}}$ and $\mathbf{A}_{\boldsymbol{\delta}}$ have some additive uncertaintiesof $\Delta\mathbf{J}, \Delta\mathbf{A}_{\boldsymbol{\omega}}$, $\Delta\mathbf{A}_{\boldsymbol{\alpha}}$ and $\Delta\mathbf{A}_{\boldsymbol{\delta}}$, respectively. Considering these uncertainties, the dynamic equation (3) can be rewritten as:

$$\mathbf{J}\dot{\boldsymbol{\omega}} = [\boldsymbol{\omega}\times]\mathbf{J}\boldsymbol{\omega} + \mathbf{A}_{\boldsymbol{\omega}}\boldsymbol{\omega} + \mathbf{A}_{\boldsymbol{\alpha}}\boldsymbol{\eta} + \mathbf{A}_{\boldsymbol{\delta}}\boldsymbol{\delta} + \boldsymbol{\Delta}(\boldsymbol{\omega},\boldsymbol{\eta},\boldsymbol{\delta}) \quad (5)$$

Where

$$\begin{aligned}\boldsymbol{\Delta}(\boldsymbol{\omega},\boldsymbol{\eta},\boldsymbol{\delta}) = {} & \mathbf{J}\Delta\mathbf{J}\boldsymbol{\Omega}\mathbf{J}\boldsymbol{\omega} + \mathbf{J}\Delta\mathbf{J}\boldsymbol{\Omega}\Delta\mathbf{J}\boldsymbol{\omega} + \boldsymbol{\Omega}\Delta\mathbf{J}\boldsymbol{\omega} + \\ & (\Delta\mathbf{A}_{\boldsymbol{\omega}} + \mathbf{J}\Delta\mathbf{J}\mathbf{A}_{\boldsymbol{\omega}} + \mathbf{J}\Delta\mathbf{J}\Delta\mathbf{A}_{\boldsymbol{\omega}})\boldsymbol{\omega} \\ & + (\Delta\mathbf{A}_{\boldsymbol{\alpha}} + \mathbf{J}\Delta\mathbf{J}\mathbf{A}_{\boldsymbol{\alpha}} + \mathbf{J}\Delta\mathbf{J}\Delta\mathbf{A}_{\boldsymbol{\alpha}})\boldsymbol{\eta} + \\ & (\Delta\mathbf{A}_{\boldsymbol{\delta}} + \mathbf{J}\Delta\mathbf{J}\mathbf{A}_{\boldsymbol{\delta}} + \mathbf{J}\Delta\mathbf{J}\Delta\mathbf{A}_{\boldsymbol{\delta}})\boldsymbol{\delta} + \mathbf{d}(t)\end{aligned} \quad (6)$$

On the other hand, under some mild conditions, kinematic equations are defined in terms of the aerodynamicangles μ, β, α as follows [18]:

$$\dot{\boldsymbol{\eta}} = \mathbf{R}(\boldsymbol{\eta})\boldsymbol{\omega} \quad (7)$$

Where

$$\mathbf{R}(\boldsymbol{\eta}) = \begin{bmatrix} -\cos\alpha\cos\beta & -\sin\alpha\cos\beta & -\sin\beta \\ \sin\alpha & -\cos\alpha & 0 \\ -\cos\alpha\tan\beta & -\sin\alpha\tan\beta & 1 \end{bmatrix} \quad (8)$$

According to Euler's eigenaxis rotation theorem, the attitude of rigid-body can be determined by a rotation angle λ about an eigenaxis. Based on such angle λ and axis $\boldsymbol{e} = \{e_1, e_2, e_3\}^T$, the well-known Hamilton rotation quaternion is defined as [19]:

$$(\mathbf{q}, q_4) = \left(\sin\frac{\lambda}{2}\mathbf{e}, \cos\frac{\lambda}{2}\right) \quad (9)$$

Where q_4 and $[q_1, q_2, q_3]^T$ denote the scalar part and vector part of the quaternion, respectively, which satisfy $\mathbf{q}^T\mathbf{q} + q_4^2 = 1$.

The quaternion kinematic differential equation describing the error between a desired spacecraft body-fixed frame $B_d$and the spacecraft body-fixed frame B become [20]:

$$\begin{cases} \dot{\mathbf{q}}_{B_dB} = \frac{1}{2}[\boldsymbol{\omega}_{BI}\times]\mathbf{q}_{B_dB} + \frac{1}{2}q_{B_dB,4}\boldsymbol{\omega}_{BI} \\ \dot{q}_{B_dB,4} = -\frac{1}{2}\boldsymbol{\omega}_{BI}^T\mathbf{q}_{B_dB} \end{cases} \quad (10)$$

With $\boldsymbol{\omega}_{BI} = \boldsymbol{\omega}_{B_dB}$ considering a rate stabilization maneuverwhere $\boldsymbol{\omega}_{B_dI} = \mathbf{0}$. For simplicity, the error quaternion $\mathbf{q}_{B_dB}$ will be represented by $\mathbf{q} = [q_1, q_2, q_3]^T$ and $\boldsymbol{\omega}_{BI}$ will be representedby $\boldsymbol{\omega} = [\omega_x, \omega_y, \omega_z]^T$ for the remainder of the paper. So the error kinematic differential equation of the quaternion can be rewritten as:

$$\begin{cases} \dot{\mathbf{q}} = \frac{1}{2}[\boldsymbol{\omega}\times]\mathbf{q} + \frac{1}{2}q_4\boldsymbol{\omega} \\ \dot{q}_4 = -\frac{1}{2}\boldsymbol{\omega}^T\mathbf{q} \end{cases} \quad (11)$$

Now, consideringthe entire system including (5),(7) and (11) in the presence of matched uncertainties $\boldsymbol{\Delta}(\boldsymbol{\omega},\boldsymbol{\eta},\boldsymbol{\delta})$,the goal is to design a suitable combined control law which stabilizes the quaternions tracking error.

Combined controller design

For the aforementioned system equations, back-stepping control technique will be applied in this section to design attitude controller. First, lets define new state variables as $\mathbf{x}_1 = [\boldsymbol{\eta}^T, 1-|q_4|, \mathbf{q}^T]^T$ and $\mathbf{x}_2 = \boldsymbol{\omega}$, then, the transformed equations of the system including Eqs. (5),(7) and (11) can be rewritten as:

$$\begin{cases} \dot{\mathbf{x}}_1 = [\dot{\boldsymbol{\eta}}^T, -\dot{q}_4 \operatorname{sgn} q_4, \dot{\mathbf{q}}^T]^T = \frac{1}{2}\left[\boldsymbol{\omega}^T \mathbf{R}(\boldsymbol{\eta})^T + \Delta\mathbf{f}, \boldsymbol{\omega}^T \mathbf{q} \operatorname{sgn} q_4, \boldsymbol{\omega}^T [\mathbf{q}\times]^T + \boldsymbol{\omega}^T q_4\right]^T \\ \qquad = \overbrace{\frac{1}{2}\begin{bmatrix} \mathbf{R}(\boldsymbol{\eta}) \\ \operatorname{sgn} q_4 \mathbf{q}^T \\ [\mathbf{q}\times] + q_4 \mathbf{I}_{3\times3} \end{bmatrix}}^{\mathbf{Q}()} \boldsymbol{\omega} = \mathbf{Q}(\mathbf{x}_1)\mathbf{x}_2 \\ \mathbf{J}\dot{\mathbf{x}}_2 = [\mathbf{x}_2 \times]\mathbf{J}\mathbf{x}_2 + \mathbf{A}_\omega \mathbf{x}_2 + \mathbf{A}_\alpha \boldsymbol{\eta} + \mathbf{A}_\delta \boldsymbol{\delta} + \Delta(\mathbf{x}_2, \mathbf{x}_1, \boldsymbol{\delta}) \\ \mathbf{y} = \mathbf{C}\mathbf{x}_1 \end{cases}$$

$$\begin{cases} \dot{\mathbf{x}}_1 = [\dot{\boldsymbol{\eta}}^T, -\dot{q}_4 \operatorname{sgn} q_4, \dot{\mathbf{q}}^T]^T = \frac{1}{2}\left[\boldsymbol{\omega}^T \mathbf{R}(\boldsymbol{\eta})^T + \Delta\mathbf{f}, \boldsymbol{\omega}^T \mathbf{q} \operatorname{sgn} q_4, \boldsymbol{\omega}^T [\mathbf{q}\times]^T + \boldsymbol{\omega}^T q_4\right]^T \qquad 12(a) \\ \qquad = \overbrace{\frac{1}{2}\begin{bmatrix} \mathbf{R}(\boldsymbol{\eta}) \\ \operatorname{sgn} q_4 \mathbf{q}^T \\ [\mathbf{q}\times] + q_4 \mathbf{I}_{3\times3} \end{bmatrix}}^{\mathbf{Q}()} \boldsymbol{\omega} = \mathbf{Q}(\mathbf{x}_1)\mathbf{x}_2 \\ \mathbf{J}\dot{\mathbf{x}}_2 = [\mathbf{x}_2 \times]\mathbf{J}\mathbf{x}_2 + \mathbf{A}_\omega \mathbf{x}_2 + \mathbf{A}_\alpha \boldsymbol{\eta} + \mathbf{A}_\delta \boldsymbol{\delta} + \Delta(\mathbf{x}_2, \mathbf{x}_1, \boldsymbol{\delta}) \qquad 12(b) \\ \mathbf{y} = \mathbf{C}\mathbf{x}_1 \end{cases}$$

Where $\mathbf{C} = \left[\mathbf{0}_{4\times3}, \mathbf{I}_{4\times4}\right]$. According to back stepping approach, first $\mathbf{x}_2$ is considered as an input for subsystem (12-a), and a virtual control signal is designed such thatit guarantees the input to state stability of this subsystem;then, the stability of overall system will be proven. So lets define the new state vector as:

$$\mathbf{z}_2 = \begin{bmatrix} z_2^1 & z_2^2 & z_2^3 \end{bmatrix}^T := \mathbf{x}_2 - \mathbf{v}(\mathbf{x}_1) \qquad (13)$$

where $\mathbf{v}(\mathbf{x}_1)$ is the virtual control signal vector, which is designedas:

$$\mathbf{v}(\mathbf{x}_1) = -k_1 \mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1 \qquad (14)$$

Where k_1 is positive constant. Lets select a Lyapunov candidateas $V_1 = \frac{1}{2}\mathbf{y}^T\mathbf{y}$, by differentiating V_1 and substituting (11) - (13) yields:

$$\begin{aligned} \dot{V}_1 = \mathbf{y}^T\dot{\mathbf{y}} = \mathbf{x}_1^T\mathbf{C}^T\mathbf{C}\mathbf{Q}(\mathbf{x}_1)\mathbf{x}_2 &= -k_1\mathbf{x}_1^T\mathbf{C}^T\mathbf{C}\mathbf{Q}(\mathbf{x}_1)\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1 + \mathbf{x}_1^T\mathbf{C}^T\mathbf{C}\mathbf{Q}(\mathbf{x}_1)\mathbf{z}_2 \\ &\le -k_1\left\|\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1\right\|^2 + \left\|\mathbf{x}_1^T\mathbf{C}^T\mathbf{C}\mathbf{Q}(\mathbf{x}_1)\right\|\left\|\mathbf{z}_2\right\| \end{aligned} \qquad (15)$$

So if $\mathbf{z}_2 = 0$, then $\dot{V}_1 \le -k_1\left\|\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1\right\|^2$, by integrating both side of later inequality,

$$\int_0^\infty \left\|\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1\right\| \le \frac{V_1(0) - V_1(\infty)}{k_1} \qquad (16)$$

Since the right hand side of (16), is bounded, then, according to Barbalet's lemma:

$$\lim_{t\to\infty}\left\|\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1\right\| = 0 \qquad (17)$$

On the other hand, from definition of $\mathbf{Q}$ we have:

$$\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1 = \frac{1}{2}\begin{bmatrix} \mathbf{R}(\boldsymbol{\eta}) \\ \operatorname{sgn} q_4 \mathbf{q}^T \\ [\mathbf{q}\times] + q_4\mathbf{I}_{3\times3} \end{bmatrix}^T \begin{bmatrix} \mathbf{0}_{4\times3} \\ \mathbf{I}_{4\times4} \end{bmatrix} \left[\mathbf{0}_{4\times3}, \mathbf{I}_{4\times4}\right] \begin{bmatrix} \boldsymbol{\eta} \\ 1 - |q_4| \\ \mathbf{q} \end{bmatrix} = \frac{1}{2}\operatorname{sgn} q_4 \mathbf{q} \qquad (18)$$

and consequently, from (17) and (18) $\lim_{t\to\infty}\mathbf{q} = 0$.

Now,the following control signal is proposed which stabilizestheoverallsystem, including $\mathbf{x}_1$ and $\mathbf{x}_2$

$$\boldsymbol{\delta} = \mathbf{A}_\delta^{-1}\begin{pmatrix} -[\mathbf{x}_2\times]\mathbf{J}\mathbf{x}_2 - \mathbf{A}_\omega\mathbf{x}_2 - \mathbf{A}_\eta\boldsymbol{\eta} - \mathbf{k}_2\mathbf{z}_2 \\ -\mathbf{Q}^T\mathbf{C}^T\mathbf{C}\mathbf{x}_1 + \mathbf{J}\dot{\mathbf{v}} - \mathbf{u}_{ad} - \mathbf{u}_R \end{pmatrix} \qquad (19)$$

With $\mathbf{k}_2 = diag(k_{21}, k_{22}, k_{23})$ where k_{2i} $i = 1,2,3$ are positive constants, also, $\mathbf{u}_{ad}$, $\mathbf{u}_R$ are NN adaptive and robustifyingcontrolvectorterms, respectively, which are designed to cancel out the vector of matched uncertainties $\Delta(\mathbf{x}_2, \mathbf{x}_1, \boldsymbol{\delta})$ and are illustrated in the next section.

Neuro adaptive control design

The vector function $\Delta(\mathbf{x}_2,\mathbf{x}_1,\boldsymbol{\delta}) = \left[\Delta_1(\mathbf{x}_2,\mathbf{x}_1,\boldsymbol{\delta}), \Delta_2(\mathbf{x}_2,\mathbf{x}_1,\boldsymbol{\delta}), \Delta_3(\mathbf{x}_2,\mathbf{x}_1,\boldsymbol{\delta})\right]^T$ is unknown and should be estimated and eliminated to design a suitable stabilizing control law. In fact, Δ_i $i = 1,2,3$ are estimated by applying appropriate multi layer perceptron (MLP) to produce the appropriate adaptive part u_{adi} $i = 1,2,3$ of the control law in (19) for eliminating the influence of the unknown signals Δ_i $i = 1,2,3$ on the system performance. It is shown that multilayer perceptrons are universal approximators which can be applied to estimate any sufficiently smooth function on an appropriate compact set with any desired degree of accuracy. So,a set of ideal weights $\mathbf{w}_i^*$ and $\mathbf{V}_i^*$ on the compact set Ω_ζ can be foundsuch that[21].

$$\Delta_i(\mathbf{x}_2,\mathbf{x}_1,\boldsymbol{\delta}) = \mathbf{w}_i^{*T}\boldsymbol{\sigma}(\mathbf{V}_i^{*T}\boldsymbol{\zeta}) + \varepsilon_i \qquad \forall\boldsymbol{\zeta}\in\Omega_\zeta \qquad (20)$$

where $\boldsymbol{\zeta} = \left[\mathbf{x}_2, \mathbf{x}_1, \boldsymbol{\delta}\right]^{\mathrm{T}}$, $\mathbf{V}_i^* \in R^{N\times m}$ and $\mathbf{w}_i^* \in R^m$ are synaptic weights connecting the input layer to the hidden layer and the hidden layer to the output layer, respectively; $\boldsymbol{\sigma} = \left[\sigma_1 \cdots \sigma_m\right]^T$ is a vector of nonlinear activation functions of neurons in the hidden layer, and $|\varepsilon_i| \le \varepsilon_{Mi}$ where ε_{Mi} depends on the network architecture. The ideal constant weights $\mathbf{w}_i^*$ and $\mathbf{V}_i^*$ are defined as:

$$\left(\mathbf{w}_i^*, \mathbf{V}_i^*\right) \triangleq \arg \min_{(\mathbf{w}_i, \mathbf{V}_i) \in \Omega_{\mathbf{w}}} \left\{ \sup_{\zeta \in \Omega_\zeta} \left| \mathbf{w}_i^T \boldsymbol{\sigma}\left(\mathbf{V}_i^T \boldsymbol{\zeta}\right) - \Delta_i \right| \right\} \quad (21)$$

where $\Omega_{\mathbf{w}} = \left\{ (\mathbf{w}_i, \mathbf{V}_i) \middle| \|\mathbf{w}_i\| \le M_{\mathbf{w}i}, \|\mathbf{V}_i\|_{\mathrm{F}} \le M_{\mathbf{V}i} \right\}$, in which $M_{\mathbf{w}i}$ and $M_{\mathbf{V}i}$ are positive numbers, and $\|\cdot\|_{\mathrm{F}}$ denotes the Frobenius norm. Since Δ_i can be modeled by using NN, so a MLP NN is employed to construct the adaptive control part as:

$$u_{adi} = \mathbf{w}_i^T \boldsymbol{\sigma}\left(\mathbf{V}_i^T \boldsymbol{\zeta}\right) \quad (22)$$

Nevertheless, in an adaptive closed loop operation, the neural network weights may be different from ideal weights mentioned in (21). Therefore, an approximation error should be formulated to derive appropriate training lawssothat they reduce this error and achieve acceptable closed loop performance. Defining $\boldsymbol{\sigma}_i := \boldsymbol{\sigma}\left(\mathbf{V}_i^T \boldsymbol{\zeta}\right)$, it is shown in [21, 22] that:

$$\Delta_i - u_{adi} = \tilde{\mathbf{w}}_i^T \left(\boldsymbol{\sigma}_i - \dot{\boldsymbol{\sigma}}_i \mathbf{V}_i^T \boldsymbol{\zeta}\right) + \mathrm{tr}\left(\tilde{\mathbf{V}}_i^T \boldsymbol{\zeta} \mathbf{w}_i^T \dot{\boldsymbol{\sigma}}_i\right) + \delta_i(t) \quad (23)$$

Where $\tilde{\mathbf{w}}_i = \mathbf{w}_i^* - \mathbf{w}_i$, $\tilde{\mathbf{V}}_i = \mathbf{V}_i^* - \mathbf{V}_i$ and $\dot{\boldsymbol{\sigma}}_i = diag\left(\dot{\sigma}_1(\nu_{i1}), \ldots, \dot{\sigma}_m(\nu_{im})\right)$ denotes the derivative of vector $\boldsymbol{\sigma}$ with respect to the input signals ν_{ij}, $j = 1, \ldots, m$, where $[\nu_{i1}, \ldots, \nu_{im}]^T = \mathbf{V}_i^T \boldsymbol{\zeta}$ and $m$is the number of neurons in the hidden layer also:

$$\begin{aligned} |\delta_i(t)| &\le \varepsilon_{Mi} + 2\sqrt{m} M_{\mathbf{w}i} + \alpha M_{\mathbf{w}i} \|\tilde{\mathbf{V}}_i\|_{\mathrm{F}} \|\boldsymbol{\zeta}\| \\ &+ \alpha M_{\mathbf{V}i} \|\tilde{\mathbf{w}}_i\| \|\boldsymbol{\zeta}\| \end{aligned} \quad (24)$$

where α is upper bound of $\|\dot{\boldsymbol{\sigma}}\|$.

The adaptation rules for the weights of the neuro-adaptive control part u_{adi}, defined in (22), is proposed as:

$$\begin{cases} \dot{\mathbf{w}}_i = \gamma_{\mathbf{w}} z_2^i \left(\boldsymbol{\sigma}_i - \dot{\boldsymbol{\sigma}}_i \mathbf{V}_i^T \boldsymbol{\zeta}\right) \\ \dot{\mathbf{V}}_i = \gamma_{\mathbf{V}} z_2^i \boldsymbol{\zeta} \mathbf{w}_i^T \dot{\boldsymbol{\sigma}}_i \end{cases} \quad (25)$$

where $\gamma_{\mathbf{V}}$ and $\gamma_{\mathbf{w}}$ are learning coefficients.

Adaptive robustifying term

The neural network based adaptive control law u_{adi} with adaptation rules (25) cannot remove the uncertainties Δ_i $i = 1,2,3$ completely and the approximation errors $\delta_i(t)$ $i = 1,2,3$ defined in (23), exist yet. So, to eliminate this error an additional control term u_{Ri} is proposed to augment thecontrol law. Using (24), it is shown that the upper bound of this error can be derived asfollows[21]:

$$\begin{aligned} |\delta_i| &\le \left(\varepsilon_{Mi} + 2\sqrt{m} M_{\mathbf{w}i}\right) + \alpha M_{\mathbf{V}i} \|\mathbf{w}_i^* - \mathbf{w}_i\| \|\boldsymbol{\zeta}\| + \alpha M_{\mathbf{w}i} \|\mathbf{V}_i^* - \mathbf{V}_i\|_F \|\boldsymbol{\zeta}\| \\ &\le \left(\varepsilon_{Mi} + 2\sqrt{m} M_{\mathbf{w}i}\right) + \alpha M_{\mathbf{V}i} \|\mathbf{w}_i^*\| \|\boldsymbol{\zeta}\| + \alpha M_{\mathbf{V}i} \|\mathbf{w}_i\| \|\boldsymbol{\zeta}\| + \alpha M_{\mathbf{w}i} \|\mathbf{V}_i^*\|_{\mathrm{F}} \|\boldsymbol{\zeta}\| + \alpha M_{\mathbf{w}i} \|\mathbf{V}_i\|_{\mathrm{F}} \|\boldsymbol{\zeta}\| \\ &\le \left(\varepsilon_{Mi} + 2\sqrt{m} M_{\mathbf{w}i}\right) + \alpha M_{\mathbf{V}i} M_{\mathbf{w}i} \|\boldsymbol{\zeta}\| + \alpha M_{\mathbf{V}i} \|\mathbf{w}_i\| \|\boldsymbol{\zeta}\| + \alpha M_{\mathbf{w}i} M_{\mathbf{V}i} \|\boldsymbol{\zeta}\| + \alpha M_{\mathbf{w}i} \|\mathbf{V}_i\|_{\mathrm{F}} \|\boldsymbol{\zeta}\| \\ &\le \varphi_i^* \left(1 + \|\boldsymbol{\zeta}\| \left(1 + \|\mathbf{V}_i\|_{\mathrm{F}} + \|\mathbf{w}_i\|\right)\right) = \varphi_i^* \chi_i \end{aligned} \quad (26)$$

where

$$\varphi_i^* = \max\left\{ \varepsilon_{Mi} + 2\sqrt{m} M_{\mathbf{w}i}, \alpha M_{\mathbf{w}i}, \alpha M_{\mathbf{V}i}, 2\alpha M_{\mathbf{V}i} M_{\mathbf{w}i} \right\}$$

and $\chi_i \triangleq 1 + \|\boldsymbol{\zeta}\| \left(1 + \|\mathbf{V}_i\|_{\mathrm{F}} + \|\mathbf{w}_i\|\right)$. Hence, $\delta_i(t)$ is bounded with the multiplication of unknown gain φ_i^* and the known function χ_i. To cancel out the approximation error $\delta_i(t)$, the augmented control term is introduced as:

$$u_{Ri} = \chi_i \, \varphi_i \tanh\left(z_2^i \big/ \mu_R \right) \quad (27)$$

with the following adaptation rule:

$$\dot{\varphi}_i = \gamma_\varphi \chi_i \left| z_2^i \right| \quad (28)$$

Where γ_φ is the learning coefficient and φ_i is estimation of the unknown parameter φ_i^*.

Here, to remove the chattering from control signal, the continuous function $\tanh(\cdot)$ with smoothing parameter μ_R is appliedinstead of the conventional discontinuous function $\mathrm{sign}(\cdot)$. As mentioned before, the universal approximation property of NNs ensures the boundedness of the approximation error. Therefore, it is always possible to find positive constants U_{Mi} $i = 1,2,3$ such that [21]:

$$|u_{Ri}| \le U_{Mi} \quad (29)$$

Stability analysis

This section presents closed loop stability analysis of the proposed control law. Using an extension of the Lyapunov theory, it is shown in the following theorem thatthe state trajectories of system (12)are ultimately bounded.

Theorem 1: *Regarding to the combined adaptive controller (19), the neuro-adaptive control part* $\mathbf{u}_{ad}$ *in (22) with the adaptation rules (25) and the robustifying augmented control term* $\mathbf{u}_R$ *in (27), then, the error signals in the closed-loop system are ultimately bounded.*

Proof: Assume that $\mathbf{J}$ is positive definite matrix, the Lyapunovcandidateis defined as follows:

$$V = V_1 + \frac{1}{2}\mathbf{z}_2^T\mathbf{J}\mathbf{z}_2 + \frac{1}{2\gamma_{\mathbf{w}}}\sum_{i=1}^{3}\|\tilde{\mathbf{w}}_i\|^2 + \frac{1}{2\gamma_V}\sum_{i=1}^{3}\|\tilde{\mathbf{V}}_i\|_F^2 + \frac{1}{2\gamma_\varphi}\sum_{i=1}^{3}|\tilde{\varphi}_i|^2 \quad (30)$$

Notingthat $\mathbf{w}^*$ and $\mathbf{V}^*$ are the ideal constant weights of neural network, defined in (21), then, from (23), $\dot{\mathbf{w}} = -\dot{\tilde{\mathbf{w}}}$ and $\dot{\mathbf{V}} = -\dot{\tilde{\mathbf{V}}}$. Differentiating (30) and substituting from (13) yields:

$$\dot{V} = \dot{V}_1 + \mathbf{z}_2^T\left(\mathbf{J}\dot{\mathbf{x}}_2 - \mathbf{J}\dot{\mathbf{v}}\right) - \frac{1}{\gamma_w}\sum_{i=1}^{3}\tilde{\mathbf{w}}_i^T\dot{\mathbf{w}}_i - \frac{1}{\gamma_V}\sum_{i=1}^{3}\mathrm{tr}\left(\tilde{\mathbf{V}}_i^T\dot{\mathbf{V}}_i\right) - \frac{1}{\gamma_\varphi}\sum_{i=1}^{3}\tilde{\varphi}_i\dot{\varphi}_i \quad (31)$$

Using (12) gives:

$$\dot{V} = \dot{V}_1 + \mathbf{z}_2^T\left([\mathbf{x}_2\times]\mathbf{J}\mathbf{x}_2 + \mathbf{A}_\omega\mathbf{x}_2 + \mathbf{A}_\alpha\boldsymbol{\eta} + \mathbf{A}_\delta\boldsymbol{\delta} + \boldsymbol{\Delta}(\mathbf{x}_2,\mathbf{x}_1,\boldsymbol{\delta}) - \mathbf{J}\dot{\mathbf{v}}\right) - \frac{1}{\gamma_w}\sum_{i=1}^{3}\tilde{\mathbf{w}}_i^T\dot{\mathbf{w}}_i - \frac{1}{\gamma_V}\sum_{i=1}^{3}\mathrm{tr}\left(\tilde{\mathbf{V}}_i^T\dot{\mathbf{V}}_i\right) - \frac{1}{\gamma_\varphi}\sum_{i=1}^{3}\tilde{\varphi}_i\dot{\varphi}_i$$

applying the combined control law (19)

results:

$$\dot{V} = \dot{V}_1 - \mathbf{k}_2\mathbf{z}_2^T\mathbf{z}_2 - \mathbf{z}_2^T\mathbf{Q}^T\mathbf{C}^T\mathbf{C}\mathbf{x}_1 + \mathbf{z}_2^T\left(\boldsymbol{\Delta}(\mathbf{x}_2,\mathbf{x}_1,\boldsymbol{\delta}) - \mathbf{u}_R - \mathbf{u}_{NN}\right) - \frac{1}{\gamma_w}\sum_{i=1}^{3}\tilde{\mathbf{w}}_i^T\dot{\mathbf{w}}_i - \frac{1}{\gamma_V}\sum_{i=1}^{3}\mathrm{tr}\left(\tilde{\mathbf{V}}_i^T\dot{\mathbf{V}}_i\right) - \frac{1}{\gamma_\varphi}\sum_{i=1}^{3}\tilde{\varphi}_i\dot{\varphi}_i$$

Substituting $\dot{V}_1$ from (15) gives:

$$\dot{V} \le -k_1\mathbf{x}_1^T\mathbf{C}^T\mathbf{C}\mathbf{Q}(\mathbf{x}_1)\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1 - \mathbf{k}_2\mathbf{z}_2^T\mathbf{z}_2 + \mathbf{z}_2^T\left(\boldsymbol{\Delta}(\mathbf{x}_2,\mathbf{x}_1,\boldsymbol{\delta}) - \mathbf{u}_R - \mathbf{u}_{ad}\right) - \frac{1}{\gamma_w}\sum_{i=1}^{3}\tilde{\mathbf{w}}_i^T\dot{\mathbf{w}}_i - \frac{1}{\gamma_V}\sum_{i=1}^{3}\mathrm{tr}\left(\tilde{\mathbf{V}}_i^T\dot{\mathbf{V}}_i\right) - \frac{1}{\gamma_\varphi}\sum_{i=1}^{3}\tilde{\varphi}_i\dot{\varphi}_i \quad (32)$$

Which can be rewritten as:

$$\dot{V} \le -k_1\left\|\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1\right\|^2 - \lambda_{\min}(\mathbf{k}_2)\|\mathbf{z}_2\|^2 + \sum_{i=1}^{3}z_2^i\left(\Delta_i - u_{adi}\right) - \sum_{i=1}^{3}z_2^i u_{Ri} - \frac{1}{\gamma_w}\sum_{i=1}^{3}\tilde{\mathbf{w}}_i^T\dot{\mathbf{w}}_i - \frac{1}{\gamma_V}\sum_{i=1}^{3}\mathrm{tr}\left(\tilde{\mathbf{V}}_i^T\dot{\mathbf{V}}_i\right) - \frac{1}{\gamma_\varphi}\sum_{i=1}^{3}\tilde{\varphi}_i\dot{\varphi}_i \quad (33)$$

Substituting $\Delta_i - u_{NNi}$ from (23), gives:

$$\dot{V} \le -k_1\left\|\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1\right\|^2 - \lambda_{\min}(\mathbf{k}_2)\|\mathbf{z}_2\|^2 + \sum_{i=1}^{3}\tilde{\mathbf{w}}_i^T\left(z_2^i\left(\boldsymbol{\sigma}_i - \dot{\boldsymbol{\sigma}}_i\mathbf{V}_i^T\boldsymbol{\zeta}\right) - \frac{1}{\gamma_w}\dot{\mathbf{w}}_i\right) + \sum_{i=1}^{3}\mathrm{tr}\left(\tilde{\mathbf{V}}_i^T\left(z_2^i\boldsymbol{\zeta}\mathbf{w}_i^T\dot{\boldsymbol{\sigma}}_i - \frac{1}{\gamma_V}\dot{\mathbf{V}}\right)\right) - \sum_{i=1}^{3}z_2^i\left(\delta_i - u_{Ri}\right) - \frac{1}{\gamma_\varphi}\sum_{i=1}^{3}\tilde{\varphi}_i\dot{\varphi}_i \quad (34)$$

Applying the adaptation rules (25), yields:

$$\dot{V} \le -k_1\left\|\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1\right\|^2 - \lambda_{\min}(\mathbf{k}_2)\|\mathbf{z}_2\|^2 + \sum_{i=1}^{3}z_2^i\left(\delta_i - u_{Ri}\right) - \frac{1}{\gamma_\varphi}\sum_{i=1}^{3}\tilde{\varphi}_i\dot{\varphi}_i \quad (35)$$

Now, from the bound(26),the robustifying control term (27) and considering the fact that $-x\tanh(x/\mu_x) \le -|x| + k\mu_x$ with $k = 0.2785$, then,the time derivative of V satisfies the following inequality:

$$\begin{aligned}\dot{V} &\le -k_1\left\|\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1\right\|^2 - \lambda_{\min}(\mathbf{k}_2)\|\mathbf{z}_2\|^2 + \sum_{i=1}^{3}\left(|z_2^i|\varphi_i^*\chi - z_2^i\chi_i\,\varphi_i\tanh\left(z_2^i/\mu_R\right)\right) - \frac{1}{\gamma_\varphi}\sum_{i=1}^{3}\tilde{\varphi}_i\dot{\varphi}_i \\ &\le -k_1\left\|\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1\right\|^2 - \lambda_{\min}(\mathbf{k}_2)\|\mathbf{z}_2\|^2 + \sum_{i=1}^{3}\left(|z_2^i|\chi_i\left(\varphi_i^* - \varphi_i\right) - \frac{1}{\gamma_\varphi}\tilde{\varphi}_i\dot{\varphi}_i\right) + k\mu_R U\end{aligned} \quad (36)$$

Where $U = \sum_{i=1}^{3}U_{Mi}$. Applying the adaptation rule (28) gives:

$$\dot{V} \le -k_1\left\|\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1\right\|^2 - \lambda_{\min}(\mathbf{k}_2)\|\mathbf{z}_2\|^2 + k\mu_R U \quad (37)$$

Now, letsdefine the following compact set around the origin:

$$\Omega := \left\{ (\mathbf{x}_1,\mathbf{z}_2) \,\middle|\, \left\|\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1\right\|^2 + \|\mathbf{z}_2\|^2 \le \frac{k\mu_R U}{\max(k_1,\lambda_{\min}(\mathbf{k}_2))} \right\} \quad (38)$$

Inequality (37) indicates that when the error signals are outside the compact set Ω, then, $\dot{V} < 0$ [23]. Hence, based on the extension of the standard Lyapunov theorem, the error trajectories $\mathbf{z}_2$ and $\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1$ and consequently, $\mathbf{x}_1$ are ultimately bounded, where the radius of bound can be reduced by increasing the gains $k_1$and $\lambda_{\min}(\mathbf{k}_2)$. Also, bysetting $\mu_R = 0$, which means substituting $tanh(\cdot)$ with $sign(\cdot)$ in (27), the result would be:

$$\dot{V} \le -k_1\left\|\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1\right\|^2 - \lambda_{\min}(\mathbf{k}_2)\|\mathbf{z}_2\|^2 \quad (39)$$

Integrating of both sides of later inequality gives:

$$\int_0^\infty\left(\left\|\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1\right\|^2 + \|\mathbf{z}_2\|^2\right) \le \frac{V(0) - V(\infty)}{\max\left(k_1,\lambda_{\min}(\mathbf{k}_2)\right)} \quad (40)$$

Since V is a positive function and $\dot{V} \le 0$;so, the right hand side of (40) is bounded, therefore, according to Barbalet's lemma, $\lim_{t\to\infty}\left(\left\|\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1\right\| + \|\mathbf{z}_2\|^2\right) = 0$ and consequently,

$\lim_{t\to\infty}\left\|\mathbf{Q}^T(\mathbf{x}_1)\mathbf{C}^T\mathbf{C}\mathbf{x}_1\right\|=0$ and $\lim_{t\to\infty}\left\|\mathbf{z}_2\right\|^2=0$ or equivalently from (18), $\lim_{t\to\infty}\mathbf{q}=0$ and $\lim_{t\to\infty}\mathbf{z}_2=0$. In addition, from (13) and (14), it can be concluded that $\lim_{t\to\infty}\mathbf{x}_2=0$, so the asymptotic stability of the overall system is guaranteed.

Remark: Attitude controller design is usually performed in some fixed operation points of nominal translational trajectory. Here, the angular motion dynamic (eq. 3) depends on velocity; V and altitude; h (through $q=\frac{1}{2}\rho(h)V^2$) of operation points. So, in each point, the nominal fixed values h^*,V^* are assigned to these parameters. Since the *adaptive* controller (19) senses the variations of operation points through matrixes $\mathbf{A}_{\boldsymbol{\omega}}(V,h),\mathbf{A}_{\boldsymbol{\alpha}}(V,h)$ and $\mathbf{A}_{\boldsymbol{\delta}}(V,h)$, the acceptable performance of closed loop system is guaranteed as it is proved in theorem 1. Moreover, since the real motion trajectory may be a little different from the nominal one, this difference is considered as part of model uncertainties in controller design through $\Delta\mathbf{A}_{\boldsymbol{\omega}}$, $\Delta\mathbf{A}_{\boldsymbol{\alpha}}$ and $\Delta\mathbf{A}_{\boldsymbol{\delta}}$, which are compensated using the neural network part of control law ; $\mathbf{u}_{ad}$(eqs. 19 and 22).

Simulation results

The simulations are performed using the following parameters:

$$\begin{cases} c_{l\delta}=0.2, c_{n\delta}=0.5, c_{m\delta}=0.5 \\ c_{lp}=-0.2, c_{mr}=-0.5, c_{mq}=-0.5, \text{ and} \\ c_{m\beta}=-0.1, c_{m\alpha}=-0.1 \end{cases}$$

$$\mathbf{J}=diag(40,400,400)$$

Three neural networksof MLP type are applied to construct u_{adi} $i=1,2,3$ where each one has 5 neurons in one hidden layer. Also, the tangent hyperbolic activation function is applied in hidden layer neurons. The weights are initialized randomly with small numbers, the learning coefficients are selected as $\gamma_{\mathbf{W}}=\gamma_{\mathbf{V}}=3000$ and the controller gains as $k_1=2$, $\mathbf{k}_2=diag(300,3000,3000)$. Moreover, the following relations are applied to convert the Euler angles reference command to quaternion command [24].

$$q_{c1}=\sin\left(\frac{\varphi_c}{2}\right)\cos\left(\frac{\psi_c}{2}\right)\cos\left(\frac{\theta_c}{2}\right) -\cos\left(\frac{\varphi_c}{2}\right)\sin\left(\frac{\psi_c}{2}\right)\sin\left(\frac{\theta_c}{2}\right)$$

$$q_{c2}=\cos\left(\frac{\varphi_c}{2}\right)\cos\left(\frac{\psi_c}{2}\right)\sin\left(\frac{\theta_c}{2}\right) +\sin\left(\frac{\varphi_c}{2}\right)\sin\left(\frac{\psi_c}{2}\right)\cos\left(\frac{\theta_c}{2}\right)$$

$$q_{c3}=\cos\left(\frac{\varphi_c}{2}\right)\sin\left(\frac{\psi_c}{2}\right)\cos\left(\frac{\theta_c}{2}\right) -\sin\left(\frac{\varphi_c}{2}\right)\cos\left(\frac{\psi_c}{2}\right)\sin\left(\frac{\theta_c}{2}\right)$$

$$q_{c4}=\cos\left(\frac{\varphi_c}{2}\right)\cos\left(\frac{\psi_c}{2}\right)\cos\left(\frac{\theta_c}{2}\right) +\sin\left(\frac{\varphi_c}{2}\right)\sin\left(\frac{\psi_c}{2}\right)\sin\left(\frac{\theta_c}{2}\right)$$

Simulation results are shown in Fig.s1-7. First, performance of the closed-loop system is evaluated without parameters uncertainties, namely $\Delta_i(\cdot)=0\quad i=1,2.3$. In thiscase, the controller is applied without adaptive parts u_{adi} and u_{Ri}. AsFig.1 shows, the quaternions track the desired reference commands, also, the corresponding Euler angles which are depicted in Fig. 2 track the desiered trajectory asymptotically. Fig. 3, shows that the other states of system such as angular rates and deflection commandsremain bounded.To evaluate the performance and robustness of system in the presence of uncertainties, the simulation is repeated in presence of the following parameters uncertainties,

$$\Delta\mathbf{J}=\begin{bmatrix}10 & 50 & 70\\ 50 & 60 & 60\\ 70 & 60 & 50\end{bmatrix},\ \Delta\mathbf{A}_{\boldsymbol{\omega}}=0.3\mathbf{A}_{\boldsymbol{\omega}}$$

$$,\ \Delta\mathbf{A}_{\boldsymbol{\alpha}}=0.3\mathbf{A}_{\boldsymbol{\alpha}},\ \Delta\mathbf{A}_{\boldsymbol{\delta}}=-0.3\mathbf{A}_{\boldsymbol{\delta}}$$

As Fig.4 shows, if the adaptive parts of control signals are neglected, the quaternions cannot track the desired reference signals and consequently, the Euler angels which are plotted in Fig. 5 have a large tracking error,but when the adaptive terms are added to control signal,as depicted in Fig. 6, the matched uncertainties are cancelled out adaptively and as depicted in Fig.s 4 and 5, an acceptable tracking performance is achieved.Moreover, according to stability theorem, all error signals including neural network weights error should remain bounded. As it is shown in Fig. 7, the NN's weightsare bounded and since the ideal weights in (21) are bounded so, the weights errorsremain bounded. In continuance, a sinusoidal external disturbance as defined in eq. (3) is applied to the closed loop system, as Fig. 8, shows in the absence of adaptive parts of control law, the disturbance effects on tracking performance and increases the tracking errors.The tracking errors are improved as soon as these adaptive parts are added to the control law.

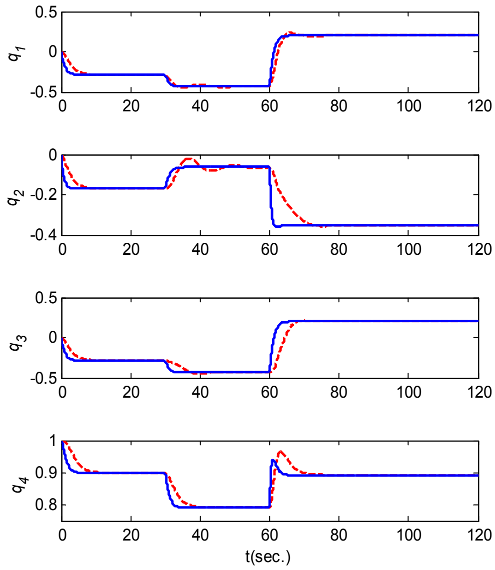

Figure 1. Quaternion trackingin the absence of uncertainties, solid line: reference signal, dashed line: quaternion

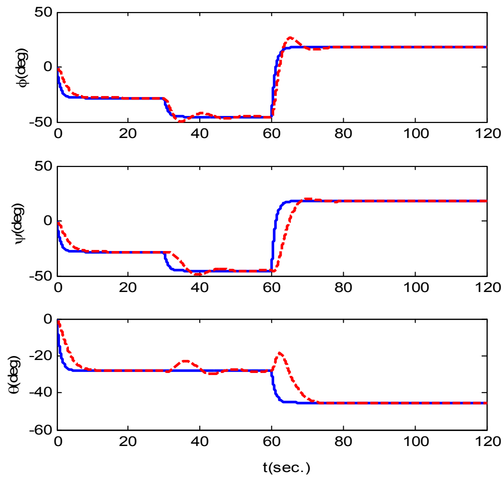

Figure 2. Corresponding Euler angles trackingin the absence of uncertainties, solid line: reference signal, dashed line: Euler angle

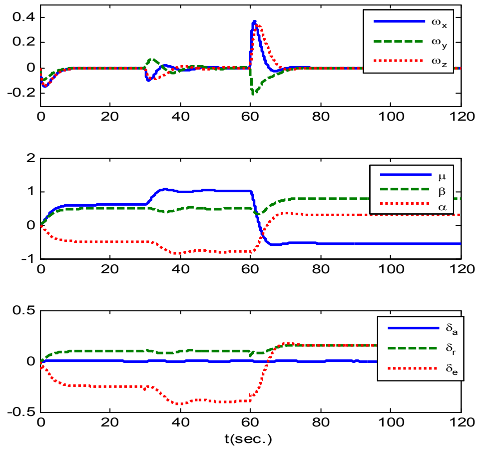

Figure 3. States trajectory in the absence of uncertainties.

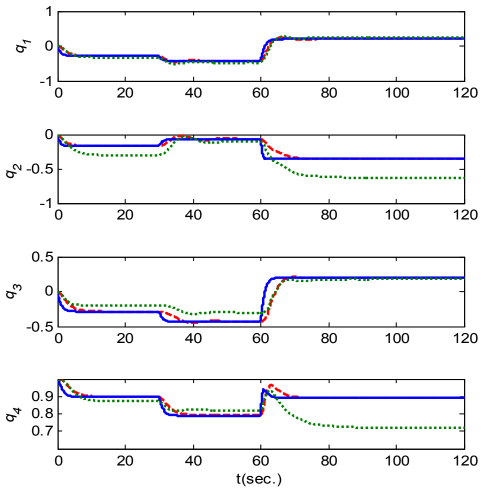

Figure 4. Quaternion tracking in the presence of uncertainty, solid line: reference, dotted line: control without adaptive parts, dashed-dotted line: control with adaptive parts.

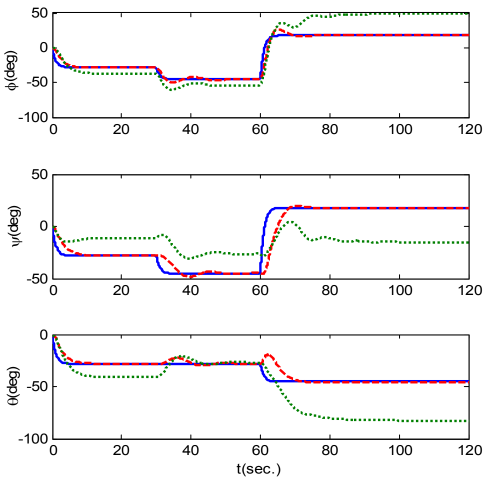

Figure 5. Corresponding Euler angle tracking in the presence of uncertainty, solid line: reference, dotted line: control without adaptive parts, dashed-dotted line: control with adaptive parts.

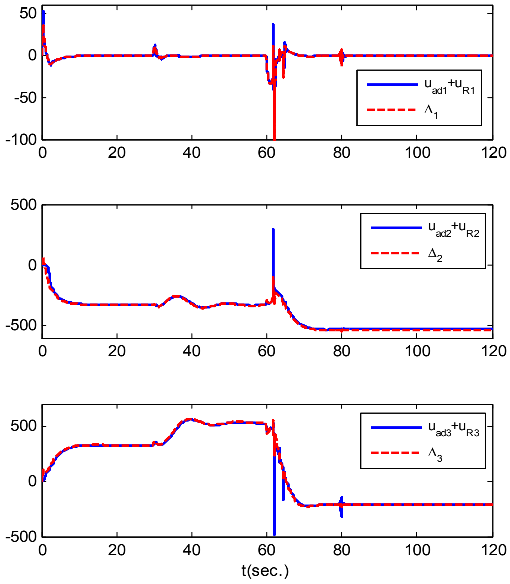

Figure 6. Uncertainty approximation with NN adaptive part of control signal.

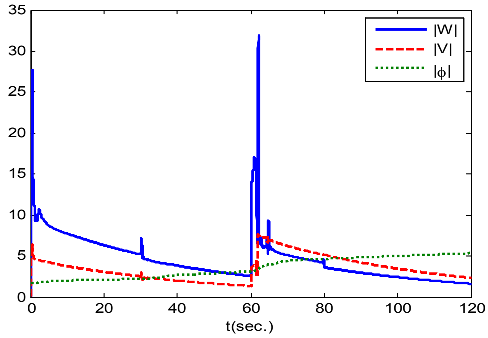

Figure 7. Norm of adaptive weights.

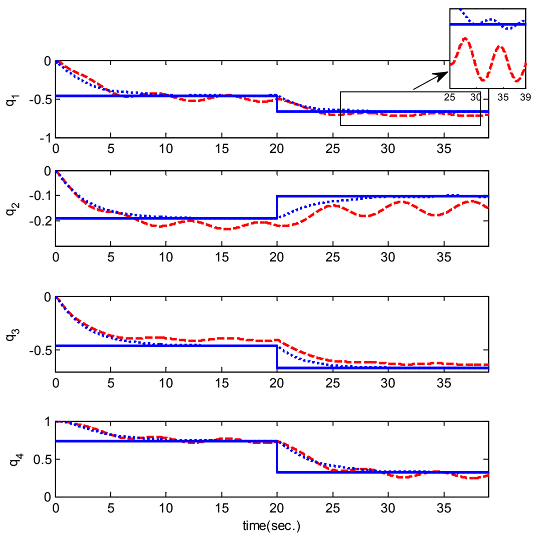

Figure 8. Quaternion tracking in the presence of externaldisturbance , solid line: reference, dotted line: control with adaptive parts, dashed-dotted line: control without adaptive parts.

CONCLUSIONS

A robust nero-adaptive quaternion controller based on back-stepping techniquehas been proposed for areodynamic control flight vehicleswhich are nonlinear in aerodynamics with inertia uncertainties, atmospheric moment uncertainties and bounded disturbances. The proposed controller includes a neuro-adaptive and an adaptive robustifying parts. The neural network is designed to approximate the matched uncertainties of the system and the adaptive robustifying control term applied to guarantee the robustness of system against approximation error of the neural network. The adaptation laws for the neural network weights and adaptive gain are obtained using the Lyapunov's direct method. The ultimate boundedness of the error signals are analytically shown using Lyapunov's method.

References

[1] Crouch, P., Spacecraft attitude control and stabilization: Applications of geometric control theory torigid body models, *IEEE Transactions on Automatic Control*, vol.29, no.4, 1984, pp.321-331.

[2] Singh, S.N. and Iyer, A., Nonlinear decoupling sliding mode control and attitude control of spacecraft, *IEEE Transactions on Aerospace and Electronic Systems*, vol.25, no.5, pp.621-633, 1989.

[3]Wei B. and Barba,P. M., Quaternion Feedback for Spacecraft Large Angle Maneuvers, *J.Guid. Contr. Dynam.*,vol. 8, no. 3, 1985, pp. 360-365.

[4]Wallsgrove, R.J. and Akella, M.R., Globally stabilizing saturated attitude control in the presence ofbounded unknown disturbances, *Journal of Guidance, Control and Dynamics*, vol.28, no.5, 2005, pp.957-963.

[5] Cai, W.C., Liao, X.H. and Song, Y.D., Indirect robust adaptive fault-tolerant control for attitudetracking of spacecraft, *Journal of Guidance, Control and Dynamics*, vol.31, no.5, 2008, pp.1465-1463.

[6] Seo, D. and Akella, M.R., Separation property for the rigid-body attitude tracking control problem, *Journal of Guidance, Control and Dynamics*, vol.30, no.6, 2007, pp.1569-1576.

[7] Song, Y.D. and Cai, W.C., Quaternion observer-based model-independent tracking control of spacecraft, *Journal of Guidance, Control and Dynamics*, vol. 32, no.5, 2009, pp.1476-1482.

[8] Park, Y., Inverse optimal and robust nonlinear attitude control of rigidspacecraft, *Aerospace Science and Technology,* 2012. http://dx.doi.org/ 10.1016/j.ast.2012.11.006.

[9] Su, Y. and Zheng, C., Simplenonlinear proportional-derivative control for globalfinite- time stabilization of spacecraft, *Journal of Guidance, Control and Dynamics*, vol. 38, no.1, 2015

[10]Juang, J.C., Jan, Y.W. and Lin, C.T., quaternion feedback attitude control design, A nonlinear H_∞ approach, Asian Journal of Control, vol. 5, no. 3, pp. 406-411, 2003.

[11]Kristiansen*, R.,* Nicklasson, P.J. and Gravdahl J.T., Satellite attitude control by quaternion-based back stepping, *IEEE Trans. Control Systems Technology*, vol. 17, no. 1, 2009, pp. 277-283.

[12]Zou, A.M., Kumar, K. D. and Hou, Z. G., Quaternion-based adaptive output feedback attitude control of spacecraft using Chebyshev neural networks, *IEEE Trans. Neural Networks*, vol.21, no. 9, pp.1457-1462, 2010.

[13] Alipour, M.R., FaniSaberi, F. and Kabganian, M., Modelling, design andexperimental implementation of non-linear attitude tracking with disturbance compensation using adaptive-sliding control based on quaternion algebra, *The Aeronautical Journal*, vol. 122, 2018, pp. 148-171,.

[14] Ma, Y. Jiang, B., Tao, G. and Cheng, Y., Actuator failure compensation and attitude control for rigid satellite by adaptive control using quaternion feedback, *Journal of Francklin Institude*, vol. 351, 2014, pp.296-314.

[15] Song, C., Kim, S.J. and Kim, S.H., Robust control of the missile attitude based on quaternion feedback*, Control Engineering Practice,* vol. 14, 2006, pp. 811–818.

[16] Xia, Y., Lu, K., Zhu, Z. and Fu, M., Adaptive back-stepping sliding mode attitude control of missile systems *Int. J. Robust. Nonlinear Control,* 2013.DOI: 10.1002/rnc.2952

[17]Hoseini S.M., Farrokhi M., and Koshkouei, A.J., Robust adaptive control of nonlinear non-minimum phase systems with uncertainties, *Automatica,* vol. 47, 2011, pp. 348–357.

[18]Jie, G., Yongzhi, S. and Xiangdong, L., Finite-time sliding mode attitude control for a reentry vehicle with blended aerodynamic surfacesand a reaction control system*, Chinese, Journal of Aeronautics*, vol. 27, no.4, 2014, pp. 964–976.

[19]Song, Y.D. and Cai W.C., New Intermediate quaternion based control of spacecraft: part I-almost global attitude tracking*, International Journal of Innovative Computing, Information and Control,* vol. 8, no. 10, 2012.

[20] Hall, J. S., Analysis and experimentation of control strategies for underactuated spacecraft, Ph.D. dissertation, Dept. Mech. and Astro. Eng., Naval Postgraduate School, Monterey, CA, 2009.

[21] Hoseini, S. M., Farrokhi, M., and Koshkouei, A. J. "Adaptive neural network output feedback stabilization of nonlinear non-minimum phase systems", *Int. J. Adapt. Control Signal Process*, vol. 24, 2010, pp. 65-82.

[22] Hoseini, S. M., Havaii, M., Amelian, J. and Shahmirzai, M., Robust adaptive control of flexible manipulators using multilayer percepron, *J. Intell. Fuzzy Systems.*

[23] Farmanbordar, A., and Hoseini, S. M., "Neural network adaptive output feedback control of flexiblelink manipulators," *J. Dyn. Sys., Meas., Control*, vol. 135, no. 2, 2012.

[24] Titterton, D. H. *Strapdown Inertial Navigation Technology*, American Institute of Aeronautics and Astronautics lnc, 2004.