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Airfoil Shape Optimization with Adaptive Mutation Genetic Algorithm

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An efficient method for scattering Genetic Algorithm (GA) individuals in the design space is proposed to accelerate airfoil shape optimization. The method used here is based on the variation of the mutation rate for each gene of the chromosomes by taking feedback from the current population. An adaptive method for airfoil shape parameterization is also applied and its impact on the optimum design and convergence of the optimization process isinvestigated. In order to demonstrate the efficiency of the proposed method, a geometric inverse design using Genetic Algorithm is carried out and the capability of the method for producing airfoil shapes is assessed. The performance of the method is further evaluated by an aerodynamic shape optimization. Results indicate the merits of the method in increasing the maximum objective value about7percent as well as decreasing the total computational time up to28 percent.

Keywords: Genetic Algorithm; Shape Optimization; Adaptive Mutation; Computational Fluid Dynamics

Introduction

Optimization problems with non-differentiable, highly nonlinear, and many local minima cost functions are commonly encountered in many engineering applications including aerodynamic shape optimization. Among different methods for aerodynamic shape optimization, Genetic Algorithms (GAs), as a popular evolutionary technique, have been widely used by researchers since they are very efficient in finding the global optimum for complex functions. One of the key features of GA is that it searches the design space from a population of points rather than as pecificone, resulting in a greater likelihood of finding the globally optimized point [1]. Additionally, it uses only the objective function and does not require its derivatives. Such features make GA attractive forpractical engineering applications like aerodynamic shape optimization [2,3]. However, GA has the disadvantage of being computationally time-consuming in aerodynamic optimization problems where Computational Fluid Dynamics (CFD) methods are used for fitness function computation. In addition, the process of shape optimization usually starts from a specific airfoil and not from a population of airfoils, which may consequently increase the number of generations and also the probability of falling into the local extremum. One can improve the optimum design and enhance the convergence rate of the optimization process by applying proper population dispersion methods. Several studies have been carried out maintaining the diversity that allows a rapid convergence and still avoids premature convergence [4, 5].Among all these methods, those that use mutation adaptation are known as the most powerful methods. The Mutation operator is used to keep the diversity of the individuals by changing chromosomes with a probability between 0 to 100%. In different studies, various mutation probabilities for GAs

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are suggested [6, 7]. These values are mostly derived by trial-and-error or from experience. According to Schwefelstudy, mutation rate should be set between 0.001 to 0.005for numerical optimization of the computer Models [8]. Jong and Grefenstette concluded that the best values for mutation rate respectively as 0.001 and 0.01, [9, 10]. Schaffer proposed that the mutation nrateshould be calculated by $1.75/(N*L^{0.5})$ where N and L are the population size and the length of individuals, respectively [11].

In the field of aerodynamic shape optimization, generally the mutation rate is set high because the domain usually includes a wide range of data and the proper population dispersion could increase the chances of relocating the peak after a change in the landscape, simply because it allows the algorithm to discover a larger part of the search space. For instance, Shahrokhi and Jahangirian applied the value of 10% for the mutation rate [12] and Ebrahimi and Jahangirian set this value 7% [13].

To improve the previous research in the field of evolutionary aerodynamic shape optimization, as a first tool, a Mutation Based Population Dispersion (MBPD) method is proposed. Mutation as a robust strategy against permanent loss of genes is considered to be the most sensitive of the required GA parameters. The purpose of mutation in GAs is preserving as well as introducing diversity. Mutation should let the algorithm avoid local minima through preventing the population of chromosomes from becoming too similar to each other; other wise, the evolution either slows or even stops. A genetic drift may occur by a very small mutation rate, whileloss of good solutions may be caused by a high one. In this work, instead of using a fixed optimum value for a mutation rate, an online adaptive valueis applied providing more flexibility. Different from other known mutation adaptation strategies [14-16], MBPD has its own mutation rate value for each gene. An adaptive approach for adjusting mutation rates for the gene locations based on the feedback is used which is obtained by observing the relative success or failure of the individuals in the population.

The main difference between the conventional GAs and the proposed method liesin the introductionof the adaptation technique. In this method, the individuals are reinitialized in each generation so that the search domain and the mutation values are advanced toward promising regions. The basis of the proposed technique is the same as the Adaptive Range Genetic Algorithms, but a straightforward extension may cause a problem in the diversity of the individuals (especially in aerodynamic shape optimization where small changes in the airfoil shape parameters could significantly alter the aerodynamic characters of the flow). To have a better diversity, by partitioning the searching region into six parts, the normal distribution for encoding is applied.

As a second tool, a robust airfoil shape parameterization method [7] is conjugated with the proposed adaptive mutation rate method. The importance of this method as a key factor in the optimization design is not only due to efficiently finding the geometry, but also because ofincreasing the convergence rate of the optimization process to the target airfoil. In order to find the best possible design variables for airfoil shape optimization, airfoil definition must incorporate a large number of degrees of freedom. One of the most popular methods for airfoil shape representation is the Bezier curve method that introduces control points around the geometry [17]. These points are then used to define the airfoil shape. However, the number of design variables is often so high that the computational time of the whole process becomes unaffordable. In addition, this method does not provide a full control over the slope of the fitted curve and in some cases may lead to impractical shapes [18]. Another common method for airfoil shape parameterization is PARSEC, which has been successfully applied to many airfoil design problems [19-21]. This technique has been developed to control important aerodynamic features by using the finite number of design parameters.

Despite its benefits over the Bezier curve, PARSEC does not provide enough geometrical flexibility at the rear part of the airfoil. Castonguay and Nadarajah studied the effect of PARSEC in aerodynamic inverse design of transonic airfoils and showed that despite its fast convergence, PARSEC is not capable of converging to the desired airfoil shape [22]. An alternative way of increasing the efficiency of the PARSEC method is proposed by Sobieczky for Divergent Trailing Edge (DTE) modeling [23]. The practical outcome of using this method is a concave surface shaping with curvature increasing towards the trailing edge of the airfoil at both upper and lower surfaces. However, Sobieczky method may increase the upper surface unfavorable pressure gradient in the viscous flow. In addition, no mathematical base is presented for the choice of function coefficients especially when the design conditions such as initial airfoil shape, Reynolds number, angle of attack and similar input parameters change. The main objective of the present study in this regard is to develop a robust method in order to overcome the deficiencies of the previous methods.

To investigate the efficiency, flexibility and generality of the proposed methods, a geometric inverse design and an aerodynamic optimization are applied. Results obtained from the proposed methods using Navier-Stokes flow solver are compared with those of the alternative methods.

Aerodynamic Shape Optimization with Genetic Algorithm

Genetic Algorithm is a search algorithm based on natural selection and genetics. It uses three operators of mutation, cross-over and reproduction [1]. A wide range of researches in aerodynamic shape optimization is performed using GA as an optimizer [24]. In this work, GA is applied to the optimization problem of airfoil shape and fitness, genes and chromosomes are corresponding to the objective function, design variables and design candidates respectively. The tournament operator is utilized with an elitist strategy, where the best chromosomes in each generation are transferred into the next generation without any change [25]. Selected airfoil shapes comprise the initial population for comparison purposes. The objective function is evaluated using the numerical method (CFD). Then, the population is optimized according to the objective function value through GA. A simple one-point crossover operator [25]is used in this work with an 80 percent probability of combination, as the use of smaller values was observed to deteriorate GA performance [24]. The crossover operator randomly exchanges the chromosomes of the selected parents. To have a proper population diversity, an adaptive mutation rate is applied where probability of mutation is adjusted dynamically based on population fitness value. More details about this approach is presented in the following sections.

CFD Evaluation of Fitness Function

SinceCFD solver consumes most of the computational time needed for the optimization process, it must possess high efficiency and convergence rate. In this work, following the work of Jahangirian and Hadidoolabi [26] for unstructured grids, the Reynolds-averaged Navier-Stokes equations are solved using a finite volume cell-centered implicit scheme. A two-equation k-ε turbulence model is utilized in conjunction with the governing flow equations (For validation and verification of the CFD solver see [26]). Based on the fact that the flow solver performs several hundreds of times in the optimization process, the generation of high-quality grids is vital in this work. Therefore, the successive refinement approach presented by Jahangirian and Johnston [27]is used in this research. The method has the capability of producing high quality stretched cells within the shear layers and boundary as well as isotropic cells outside these regions. During the optimization process, the airfoil boundaries are changing. Consequently, the existing grid has tobemodified so asto adapt to the changing domain. Modification of the grid is carried out automatically using tension–spring analogy so that the need for the user to interrupt the computations is eliminated [28].

AirfoilShape Parameterization

Among all shape optimization methods, the challenging subjects concern the creation of wide variety of possible options and the mathematical representation of the airfoil profiles. Smoothing algorithms, which are based on the splines and polynomials, are employed by a large number of the optimization techniques for shape representation. PARSEC is one of the most effective and common methods for airfoil representation in the field of design optimization. Figure 1 shows the eleven basic parameters of this method, which are the leading edge radius (r_{LE}), upper and lower crest location (X_{UP}, Y_{UP}, X_{LO}, Y_{LO}) and curvature (Y_{xxUP},Y_{xxLO}), trailing edge coordinate and direction (Y_{TE}, α_{TE}), trailing edge wedge angle (β_{TE}) and thickness (ΔY_{TE}). Therefore, eleven design parameters are required to completely define an airfoil shape. To present the airfoil shape in this method, a linear combination of shape functions is used which is as following:

$$Y_k = \sum_{n=1}^{6} a_{n,k} X_k^{\,n-\frac{1}{2}}, \; k = 1,2 \quad (1)$$

The airfoil is divided into upper and lower surfaces and by using the information of the points in each section, the coefficients are determined. The subscript k changes from 1 to 2 for considering the length on the upper and lower surfaces, respectively. Using the above parameters, the maximum curvature of the upper and lower surfaces as well as their location are effectively controllable which is very useful in reducing the shock wave strength or delaying its occurrence. However, PARSEC fits a smooth curve between the trailing edge and the maximum thickness point, which in turn disables the necessary changes in the curvature beside the trailing edge. Therefore, despite its advantages in controlling the important parameters on the upper and lower surfaces, this method does not provide sufficient control over the trailing edge shape where significant flow phenomena can occur.

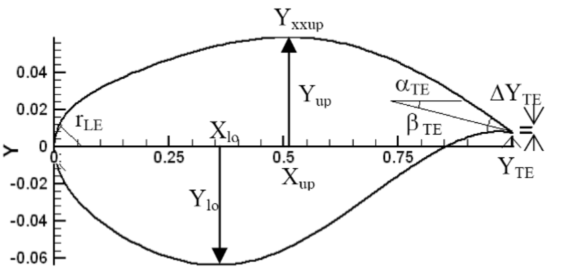


Figure 1. PARSEC method for airfoil parameterization

One of the techniques to overcome the shortcomings of PARSEC is Sobieczky method for trailing edge modeling [23]. The practical consequence of applying this method is a concave surface shape with curvature increasing towards the trailing edge at upper and lower surfaces. Such types of airfoils are known as Divergent Trailing Edge (DTE). This method is mainly based on viscous flow control near the trailing edge that has a strong influence on aerodynamic efficiency. In this method, to make the airfoil surface a divergent trailing edge, the parameters L_1, L_2 and Δα that control the increment in the trailing edge thickness (ΔY) are added(Equation 2). The parameter Δα controls the camber added to both the upper and the lower surfaces creating a DTE. L_1 and L_2 are the chord lengths measured from trailing edge, whichare modified in the Sobieczky method. The function considered for ΔY is:

$$\Delta Y_k = \frac{L_k . \tan \Delta\alpha}{\mu . n}[1 - \mu . \xi_k{}^n - (1 - \xi_k^n)^\mu] \quad , k = 1,2 \quad (2)$$

In Equation 2,ξ_k is the x-coordinate variable and the subscript k,as to consider the length on the upper and lower surfaces changing from 1 to 2. Different values are possible for parameters n and μ. The trailing edge coordinate (Y_{TE}) and thickness parameters of PARSEC may be assumedaszero; hence, they can be eliminated from the list of design variables. Consequently, the total number of design variables is increased to 12 parameters.

The outcomes show that using a proper parameterization method can significantly increase the convergence rate of the optimization process [23]. Therefore, it decreases the total time of calculations, especially in aerodynamic shape optimization where several time-consuming CFD simulations are applied. However, for different applications no mathematical base for the optimum selection of parameters is proposed. In addition, the Sobieczky method might lead to the overlap of the upper and lower surfaces, e.g. it does not guarantee a physically acceptable trailing edge [2]. An other significant problem related to the Sobieczky method is that it mainly tends to pull the trailing edge downward for increasing the curvature at the rear part of the airfoil. However, the upper surface unfavorable pressure gradient in the viscous flow may be increased by such changes. On the other hand, because of the formation of negative curvature over the airfoil, negative values for $\Delta\alpha_{TE}$ will make the shape of the trailing edge worse.

One way to reduce the pressure drag is flattening the upper surface of the airfoil, which creates a weaker shock wave on the airfoil. Therefore, the Sobieczky formulation is changed to create a smoother upper surface. Thus,instead of Eq. (2) the following function is proposed for ΔY:

$$\Delta Y = \frac{\tan \Delta\alpha}{b_1 . b_2}[1 - b_3 . \xi^{b_2} - (1 - \xi^{b_2})^{b_1}] \quad (3)$$

Where ξ and Δα are the same as Equ. 2and $b_1$to b_3 are the coefficients.In this formula, in fact, on the contrary to other methods such as Sobieczky which utilize fixed a parameterization function, in this, work two different types of parameters are used. The first type is responsible for creating airfoil shapes including leading edge radius (rle), upper and lower crest location (X_{UP}, Y_{UP}, X_{LO}, Y_{LO}) and curvature (Y_{xxUP}, Y_{xxLO}), trailing edge direction (α_{TE}), wedge angle (β_{TE}) and $\Delta\alpha_{TE}$. However b_1, b_2 and b_3 in equation 3 which alter independently forthe upper and lower surfaces, are responsible for the creation of a proper trailing edge. In other words, the process of optimization in the proposed method is carried out for two groups of parameters. The first group, common among all methods, is the PARSEC parameters. While, the second group including b_1, b_2 and b_3 coefficients alter equation 3 to achieve the optimum shape particularly near the trailing edge of the airfoil. This increases the flexibility of the parameterization function in order to obtain the optimum airfoil with less number of generations. Acording to the work of Ebrahimi and Jahangirian [7] the ranges of the above parameters are considered as shown in Table 1 against the fixed values used by Sobieczky and reference 2.

In addition to the mentioned constraints for the proposed coefficients, some geometrical constraints according to the optimization problem have to be considered. Otherwise, after a number of optimization descent steps, very thin and physically unrealizable airfoils start to appear. In this work, the following constraints are applied for PARSEC parameters.

Table 1. Comparison of parameterization coefficients

Parameterization method		b1	b2	b3
Upper Surface	Present Method	0.4<Var.< 8.5	0.5< Var.< 16.0	-5.5< Var.<5.5
	Sobieczky	1.3< Fixed <1.8	Fixed =3	=b1
Lower Surface	Present Method	0.8< Var.< 5.7	0.5< Var.< 14.1	0.6< Var.<5.3
	Sobieczky	1.3< Fixed <1.8	Fixed =3	=b1

$0.008 \leq r_{le} \leq 0.016$	$0.3 \leq X_{Up} \leq 0.55$	$0.050 \leq Y_{Up} \leq 0.065$	$-0.48 \leq Y_{xxUp} \leq -0.3$
$0 \leq \alpha \leq 20$	$0 \leq \beta \leq 16$	$0.25 \leq X_{Lo} \leq 0.40$	$-0.07 \leq Y_{Lo} \leq -0.04$
$0.3 \leq Y_{xxLo} \leq 0.9$	$0.0 \leq \Delta\alpha \leq 20$		

Population Dispersion with Adaptive Mutation Rate

To treat an optimization problem with a large design space, Adaptive Range GA (ARGA) was proposed by Arakawa and Hagiwara for binary genetic algorithms [29]. The idea behind this method is to adapt the population toward promising design regions according to the distribution of the design variables. It uses the statistics of the top half of the population to adapt the genes in the search space. Hence, the adapted population is distributed in the hopeful search region. The method was further extended to real coded applications of single and multiple objective function optimization problems [30]. However, the crucial point for the successful application of this method is to maintain the gene diversity, longer than the Simple Genetic Algorithm (SGA). While the initial gene diversity contributes to the robustness of the method, the adaptive feature improves its local search capability.

Determining the optimum fixed mutation rate for an aerodynamic shape optimization problem where a small change to the airfoil shape parameter can intensively change the aerodynamic character of airfoil is a very time consuming and complicated problem. The new technique developed in this paper (referred to as MBPD) provides a better diversity in the design space using Genetic Algorithm. Unlike other well-known mutation adaptation strategies, MBPD has its own mutation value for each gene where initially all mutation rates are set to an initial value in the specified boundary. Based on the feedback obtained from monitoring fitness value evaluations of individuals, an adaptive approach for adjusting mutation rates for the gene locations is proposed. This technique consists of two phases of exploration and refinement. In the exploration phase, the variation range of each gene is divided into six equal zones and the mutation rate at each one depends mainly on whether individuals located in one of the sixth zones are successful or not (Fig. 2).Then in each generation, the mutation rate (K_i) for each gene is updated based on the feedback taken from the relative fitness of individuals. If the fitness value of the individual corresponding to the gene location (P_i) is less than the average fitness value of the population within the same zone (P_{Avg}), it is assumed that the value for the corresponding gene could not generate more successful results. Therefore, the mutation rate is increased by 15% for the corresponding gene (λ). On the other hand, if P_i to P_{Avg} ratio is greater than 1, then K_i is decreased to its initial value. In the case of a minimization problem, an inverse operation should be applied.

$$K_{i,\ new} = \begin{Bmatrix} K_{i,\ old} + \lambda \ \ if \ \ \frac{P_i}{P_{avg}} < 1 \\ \\ K_{i,\ old} - \lambda \ \ if \ \ \frac{P_{ij}}{P_{avg}} \geq 1 \end{Bmatrix} \qquad (4)$$

As a result of the updates at each generation, K_i values are allowed to vary within the limits defined by the lower and upper bounds. If an update causes a parameter to exceed the limits, it is set to the corresponding boundary value. The update rule for a maximization problem can be seen in Equation 4, which is applied to each locus separately.

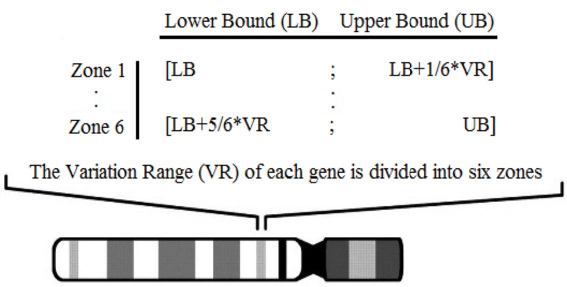


Figure 2. Upper and lower limits of each zone in the optimization process with MBPD

Based on the numerical experiments, the exploration phase consists of around 11 to 15 generations. In addition, the maximum mutation rate is limited to 35%. Once the generated data is spread out enough in the design space, the refinement phase will start. In this phase, the majority of the new GA members are concentrated in the promising region of the design space, while there are still some individuals seeking the design space that has not been fairly investigated.

In fact in this phase, the calculations are carried out for two parallel sets of populations, the first one has members with P_i>P_{Avg}, with a fixed value of mutation rate, while the second group has members with P_i<P_{Avg}, in which individuals have Variable Mutation Rate (VMR) changing in each generation (Fig. 3). Consequently, very dynamic chromosomes exist in this group. At the end of each generation, chromosomes in the second group gaining the fitness value of more than the average, migrate to the first group and an inverse procedure is carried out in the second group.

Figure 4 illustrates the flowchart of the proposed method in the refinement phase. Theoretically, MBPD is capable of escaping the local optima, which causes premature convergence problem because the second group provides proper diversity regardless of fitness.

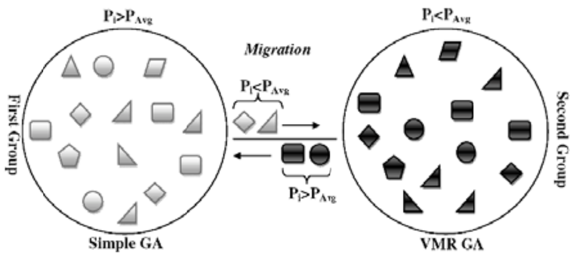


Figure 3. Substituation of chromosoms at the end of each generation in the refienement phase

Results

Results are discussed in two parts. In the first part, the efficiency of the proposed population dispersion method as well as the new parameterization method are investigated through a simple geometric reconstruction problem. In the second part, the new method is applied to the aerodynamic design of airfoils using CFD as an evaluation tool and Genetic Algorithm as an optimizer.

Geometric Reconstruction

The new technique developed in this paper referred as MBPD is used to provide a better diversity in the design space using Genetic Algorithm. This approach is expected to reduce the number of generations in order to locate an optimal individual. To investigate the efficiency of the proposed method, an inverse geometric reconstruction problem is applied.RAE2822 and NACA0012 airfoils are considered as the target and initial airfoil shapes, respectively and the objective function is considered as:

$$OF = \frac{\sum_{i=1}^{np}(Y_i - Y_{ti})^2}{2n_p} \quad (5)$$

where Y_i and Y_{ti} are the design and target coordinates of surface points with X_i fixed coordinates, and np is the number of the chosen points. The above formulation should be minimized in the optimization process. The shape variable bounds imposed in this case are the same as aerodynamic optimization. In Table 2, a comparison is carried out between the new and conventional GA with fixed mutation rates of one and three percent. For the new method, the initial mutation rate is assumedas0.03 and is allowed to change between a lower bound of 0.03 and an upper bound of 0.15. These settings are determined empirically to provide the best performance of the method. In this table, mean values (μ), standard deviation (σ) and maximum and minimum values of the upper crest location (Gene No. 3) as a sample are compared at 10^{th} generation.

The normal distributions of the upper crest location at 3^{rd},13^{th} and 35^{th}generationsareshown in Figure 5 for both the present method and conventional GA. As shown in this figure, the present method permits the search tool to seek the optimum in a wider range of the design space.

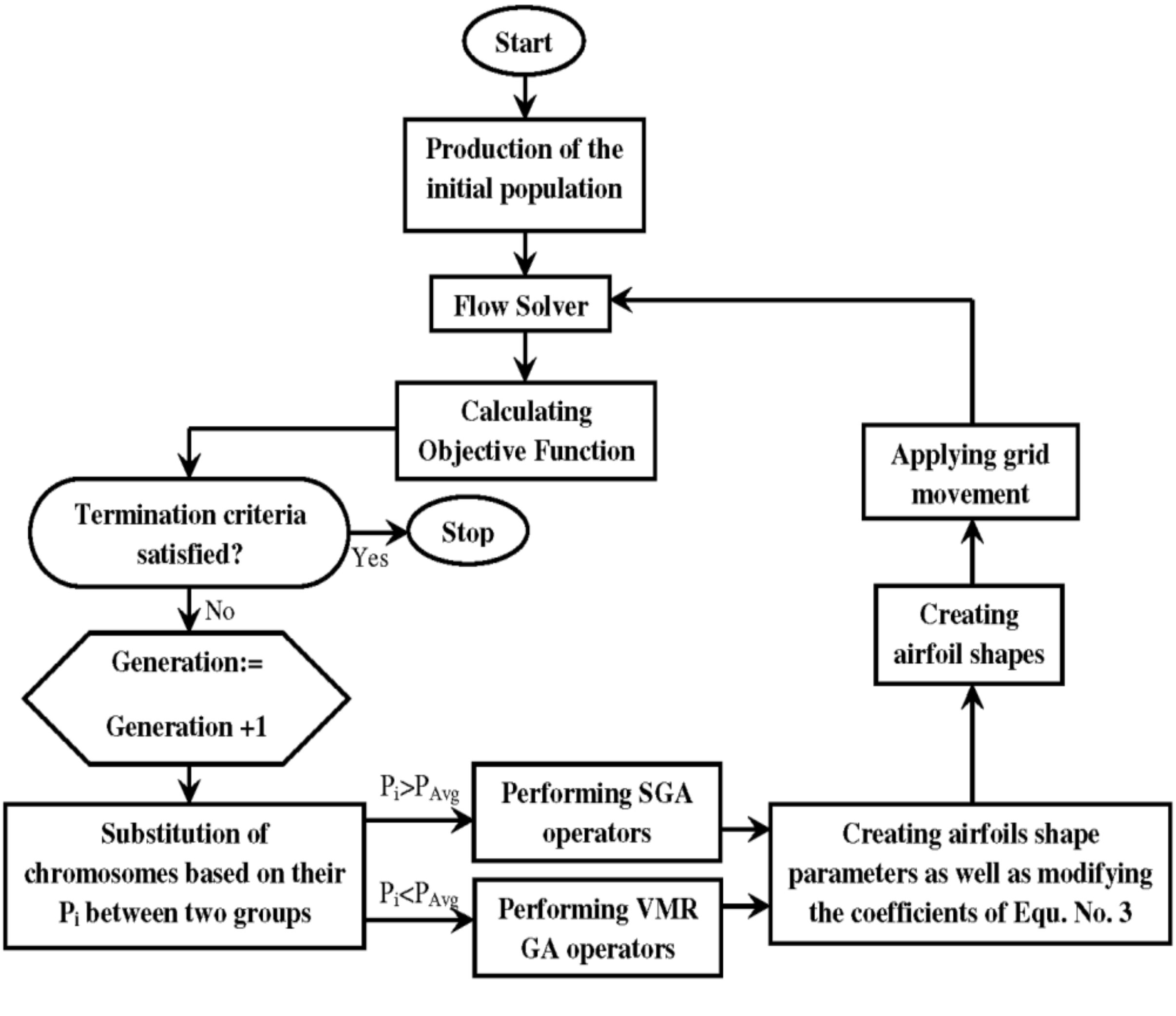


Figure 4. Flowchart of the MBPD in the refienement phase

Table 2. Statistical comparison of the upper crest location at 10th generation for inverse geometry reconstruction

Applied method	Mean values (µ)	Standard deviation (σ)	Max. Value	Min. Value
SGA with 1% mutation rate	0.0648922	3.56E-05	0.06491	0.06482
SGA with 3% mutation rate	0.0647035	7.95E-05	0.06473	0.06446
Present method	0.0605638	0.00323	0.06421	0.05313

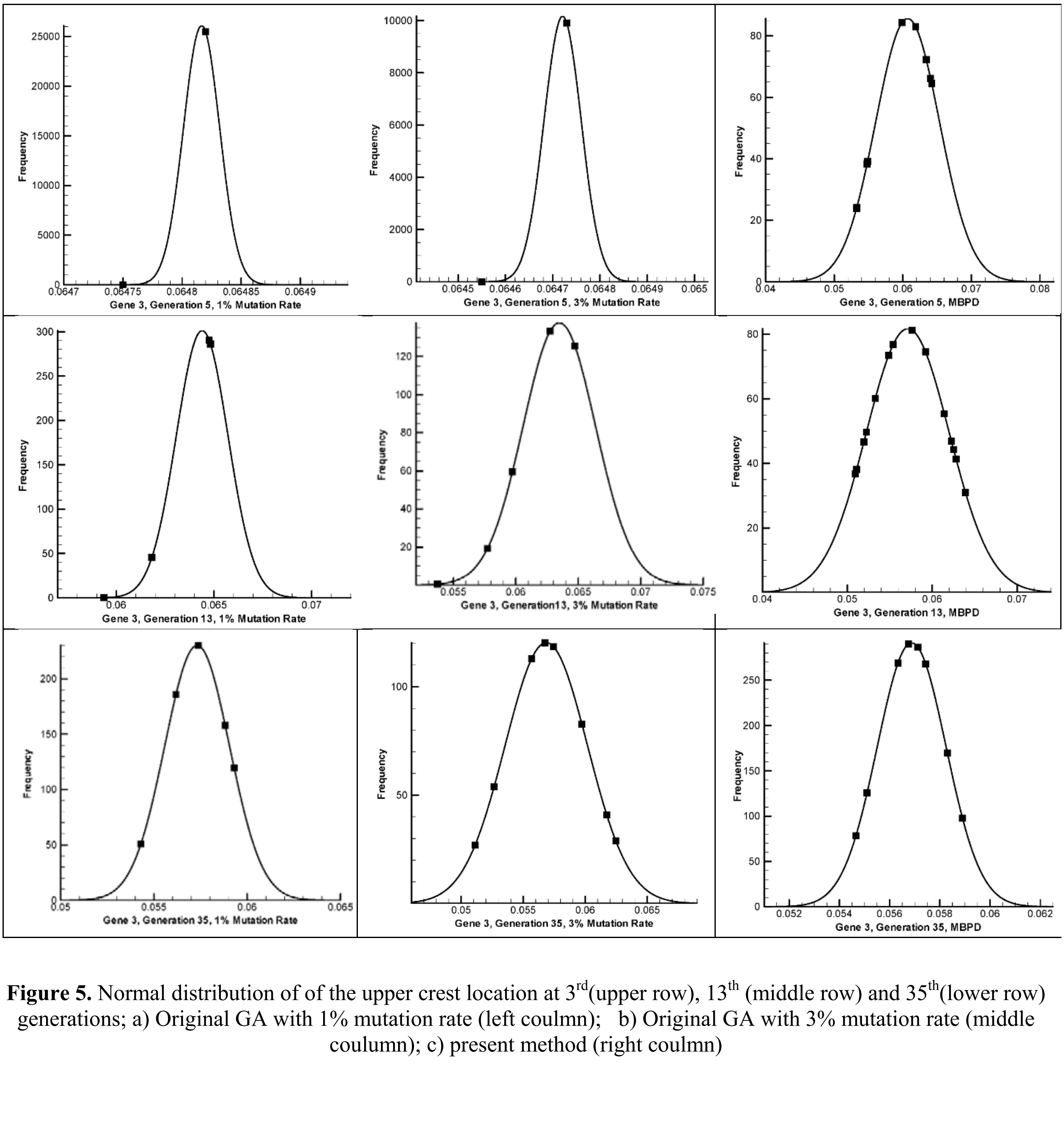


Figure 5. Normal distribution of of the upper crest location at 3rd(upper row), 13th (middle row) and 35th(lower row) generations; a) Original GA with 1% mutation rate (left coulmn); b) Original GA with 3% mutation rate (middle coulumn); c) present method (right coulmn)

Figure 6shows the variation of the shape parameter b3for the upper surface against the generation numbers for all 20 populations and the best member, which gives a better overview of the performance of the parameterization method.

With applying MBPD and the proposed airfoil parameterization method, the target, the initial and inverse constructed geometry after 200 generations are compared in Figure 7a.

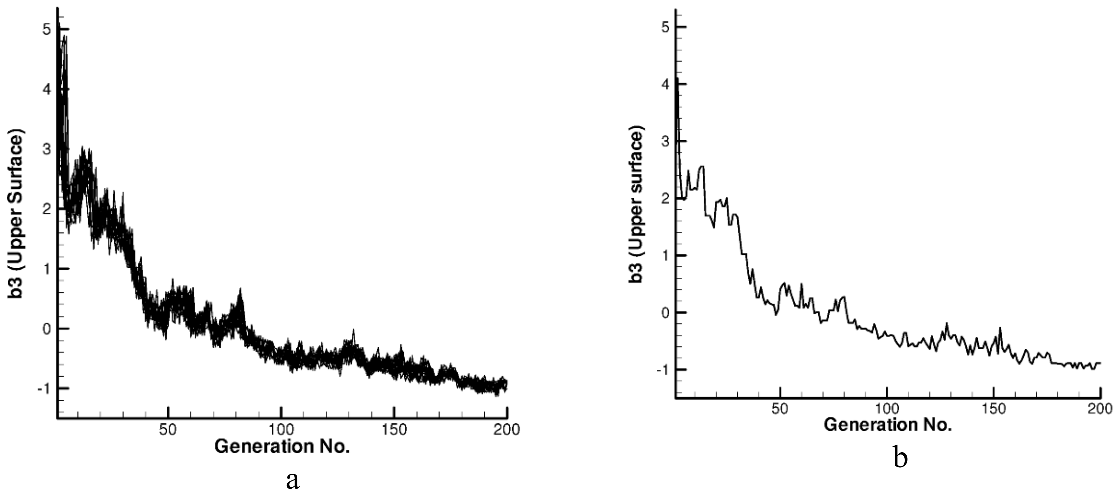


Figure 6. Variation of parameter b3 for the upper surface as the function of generation numbers for inverse geometry reconstruction; a) all populations; b) best member

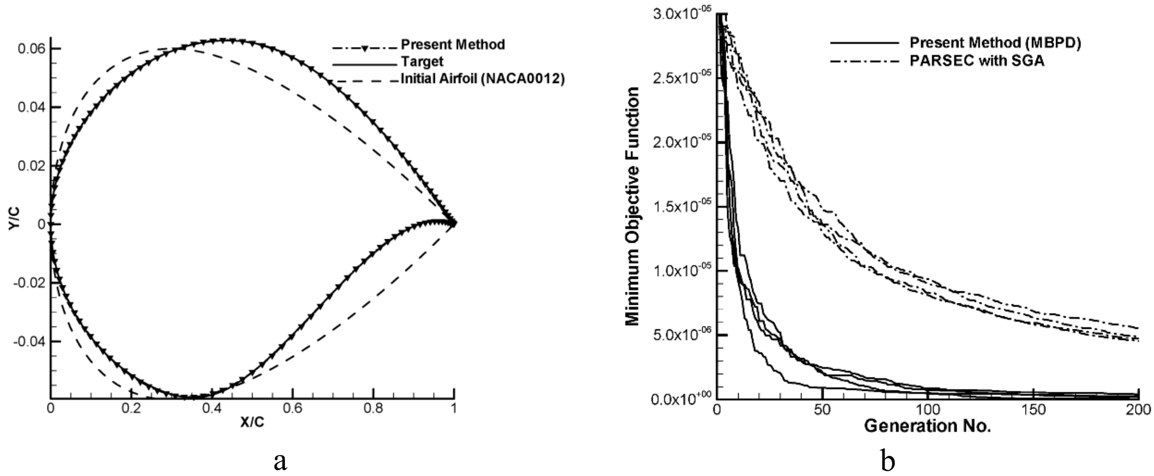


Figure 7. Inverse constructed geometry after 200 generations; a) comparison of initial, target and obtained shape by the propose method; b)comparison of the convergance history

To be able to investigate the efficiency of MBPD, the convergence rates for the proposed method in comparison to the PARSEC using SGA are illustrated in Figure 7b. To make sure that the results are not influenced by the random nature of the GA, the convergence histories are repeated using different random seed numbers and this repetition indicates the flexibility added by the proposed method is significant.

Aerodynamic Optimization

To show the efficiency of the proposed population dispersion as well as new parameterization methods in the optimum aerodynamic shape design and their influence on the convergence behavior of GA, an aerodynamic design case is applied. A transonic flow is considered with the Mach number of 0.75, Reynolds Number of 6.5 million and incidence angle of 2.79 degrees. RAE-2822 airfoil is considered as the initial airfoil and the objective function is the lift coefficient to the drag coefficient (C_l/C_d) which is computed by solving the Reynolds-averaged Navier–Stokes equations. The computational field is discretized using triangular unstructured grids. A successive refinement method is used for the unstructured viscous grid generation [27]. To prevent the time-consuming process of grid re-generation for every geometry, spring analogy is applied to adapt the grid to the new geometry. The unstructured grid around the final optimum airfoil containing 10651 triangular cells is shown in Figure 8a, and a comparison between initial and optimum airfoil shapes is illustrated in Figure 8b.Figure 9 also illustrates the Mach number contours for the optimum shapes besides the initial airfoil.

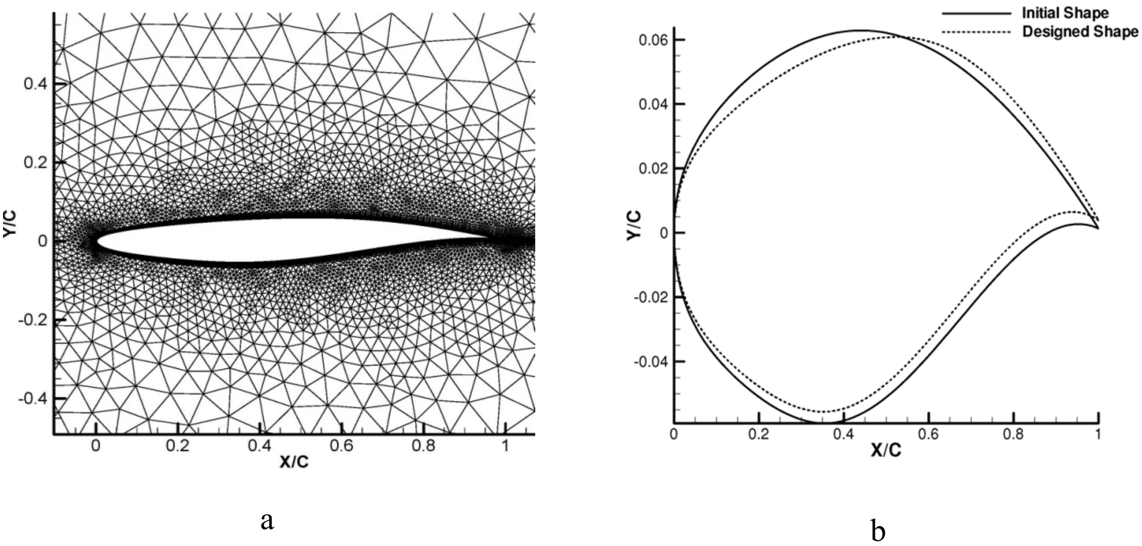


Figure 8. Designed Airfoil shape a) Surrounding unstructured grids b) Compared to the initial airfoil shape

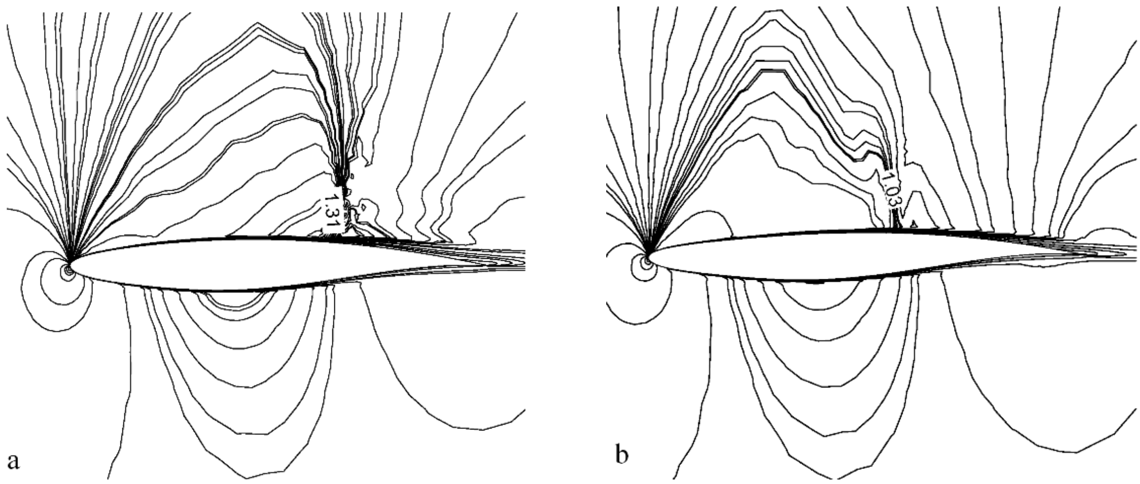


Figure 9. Mach number contours around a) Initial airfoil b) Present Method.

In Figure 10, for instance, the values of the 4^{th} and 8^{th} genes, which are the maximum thickness of the lower surface and the upper crest location, are depicted after 40 generations. The result shows the expectation that the exploration phase scatters individuals in the design space, while after this phase, members are refined around the optimum values.

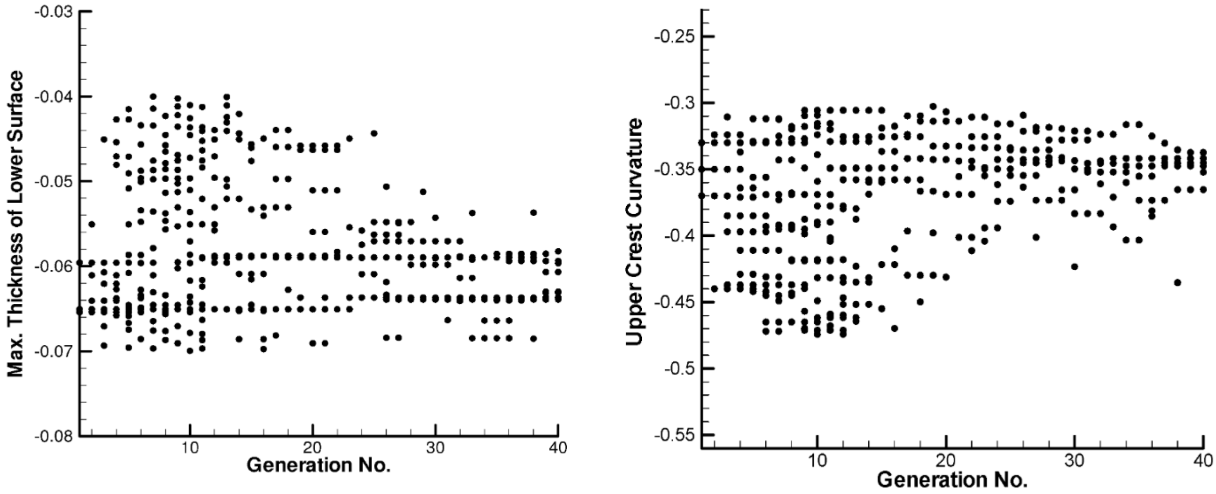
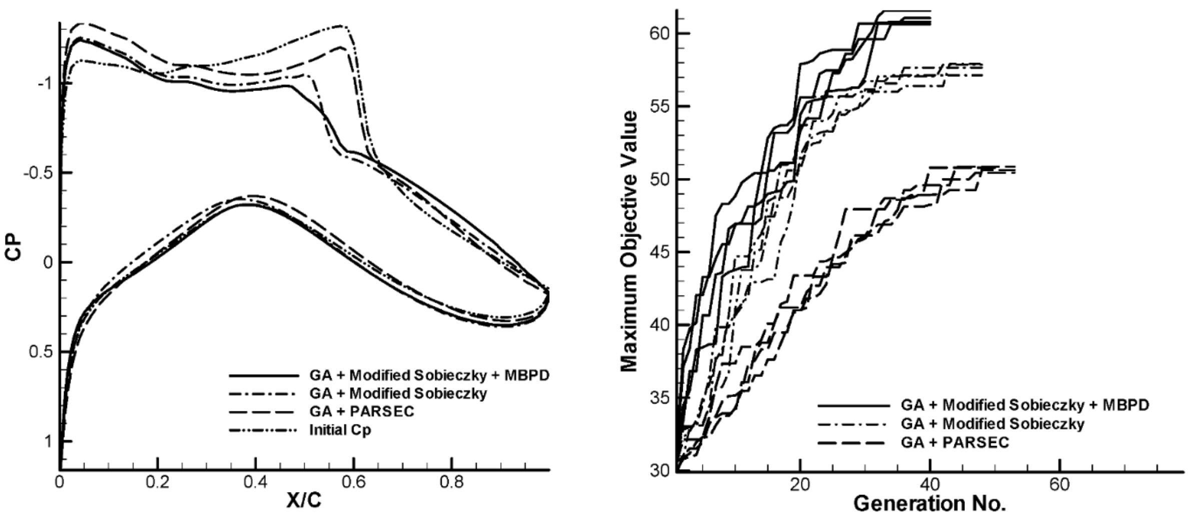


Figure 10. Values of the (left) maximum thickness of the lower surfaceand (right) upper crest location after 40 generations

To demonstrate the efficiency of the proposed method, in Figure 11a, a comparison of surface pressure coefficient distributions besides the initial airfoil results are presented for three cases; for two cases, the modified Sobieczky parameterization method is applied, while the MBPD is used only for one case, and for the last case the original PARSEC is considered.

According to these figures, there is a rather strong shock wave near the middle part of the initial airfoil upper surface. Although all methods weakened the shock wave in the process of optimization, the proposed parameterization method applying MBPD has a significant impact. In Figure 11b, a comparison of the convergence history of the maximum objective function is presentedforthe proposed parameterization method with and without applying MBPD. The optimization process is repeated for each parameterization method using four different random seed numbers. Results emphasize the superiority of the proposed methodbyincreasing the maximum objective value up to 7%.



a) b)

Figure 11. Investigating the efficiency of MBPD a) Surface pressure distributions b) Convergence history of the maximum objective value

As for the computational time to reach the optimum shape, it is evident from the figure that the required generations for gaining the maximum objective function for the proposed method is around 35;this is whileit is around 45 when the MBPD is not applied. Considering that each generation consists of 20 members, around 200 CFD evaluations areeliminated which considerably decreases the computational timerequired.

Conclusions

A new method for scattering individuals in the design space was proposed and its influences were investigated through inverse geometry construction. The result showed that the proper population dispersion was achieved. Besides, an airfoil shape parameterization was proposed and its flexibility and generality in creating different airfoil shapes were investigated using the inverse shape reconstruction. This method was then applied to the aerodynamic shape optimization and results were compared in terms of the computational efficiency and the airfoil optimum shape. The outcomes of the proposed methods indicated that not only the total number of flow solver calls are noticeably decreased, but also the optimum design is enhanced at the same time.

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