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A Three Stage Terminal Fuzzy Guidance Law for Reentry Vehicles

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An advanced guidance law is developed for reentry phase of a reentry vehicle. It can achieve small miss distance and desired impact attitude angle, simultaneously. To meet this requirement, a guidance law based on the fuzzy logic control approach is developed. It is partitioned into three stages. This guidance law does not require linearization of missile engagement model. Line-of-sight angle and flight path angle are used to constitute the rule antecedent of the guidance law to shape an appropriate flight trajectory for engagement. Numerical simulation results and comparison with an existing algorithm demonstrated that the proposed guidance law offers satisfactory performance and robustness, fulfilling its design goals.

Keywords: Fuzzy guidance low, Reentry, Vehicles, Stage terminal

INTRODUCTION

Guidance system is one of the most important and sophisticated parts of a flying vehicle. The vast majority of guidance laws, especially for a reentry vehicle, have one objective, i.e., to converge the distance between the vehicle and the target to zero. This is not always sufficient. In certain scenarios, especially for missiles, the mission requirements call for the vehicle to impact the target location from a specific direction with high speed [1].

While, a number of guidance methods can guide the vehicle toward the target, not many of them have addressed the unique need for impact from a specific direction. A pioneer works were done by Ref. [1][2][3] and [4]. These guidance laws applied the linear quadratic optimization technique, so the missile model had to be linearized. In addition, they are sensitive to the accuracy in estimating the time-to-go parameter. A biased proportional navigation guidance law was proposed for the impact angle control problem by introducing a time-varying bias term to classical proportional navigation guidance [5]. The key assumptions necessary for the analytical solution obtained therein are constant velocity and small error angles. Impact angle control navigation guidance law (IACNG) [6] attains a perfect intercept (zero miss-distance (MD) and zero impact angle error) against non-maneuvering targets under certain ideal conditions, but it is sensitive to disturbances. Because missile dynamic models are usually highly non-linear and uncertain, in recent years, researchers have considered the possibility of introducing the concept of fuzzy logic theory in missile guidance system design. Consideration of robust performance in varying environments is increasingly receiving attention in the modern control system design. Lin et.al [7][8][9][10] presented fuzzy guidance laws against high speed targets. Chabra et.al [11] proposed terminal fuzzy guidance law which controls the impact attitude angle with minimizing the MD for a reentry vehicle. The result was 90 m in MD. Wang et al [12] have proposed a longitudinal predictive re-entry guidance law based on variable universe fuzzy-PI composite control. They have used the longitudinal point mass model of the reentry vehicle.

In this paper, an advanced guidance law is developed for reentry phase of a flying vehicle. It achieves negligible MD and impact attitude angle error, simultaneously. This guidance law is based on the fuzzy logic control approach. Inspired by typical trajectory of a reentry vehicle with terminal guidance,

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the proposed guidance law is partitioned into three stages. A switching strategy is developed to smoothly switch between three stage fuzzy guidance laws. It also does not require linearization of missile engagement model. Line-of-sight (LOS) angle and flight path angle are used to constitute the rule antecedent of the guidance law.

The structure of current paper is as follows. Dynamic model of missile is presented in the next section. Terminal fuzzy guidance law (FGL) is derived in section 3. Numerical simulation results are provided in section 4. Finally, this paper ends with the conclusions.

MISSILE DNAMIC MODEL

For problem formulation, as shown in Figure 1, the point mass model of a reentry vehicle in planar motion [13][14][15] is used. The equations of motion are described by:

$$\dot{V} = \frac{-\mathrm{D} - mg\sin\gamma}{m} \tag{1}$$

$$\dot{\gamma} = \frac{-\mathrm{L} - mg\cos\gamma}{mV} \tag{2}$$

$$\dot{r} = V\sin\gamma \tag{3}$$

$$\dot{\theta} = \frac{Vos\gamma}{r} \tag{4}$$

here, variables V, D, L, m, r and γ are the vehicle velocity, drag, lift, mass, distance from the Earth's center and flight path angle, respectively. The gravity acceleration is given as:

$$g = \mathrm{g}_0\left(\frac{\mathrm{r}_0}{\mathrm{r}_0 + h}\right)^2 \tag{5}$$

Where

$$h = r - \mathrm{r}_0 \tag{6}$$

Here, r_0 is Earth's radius.

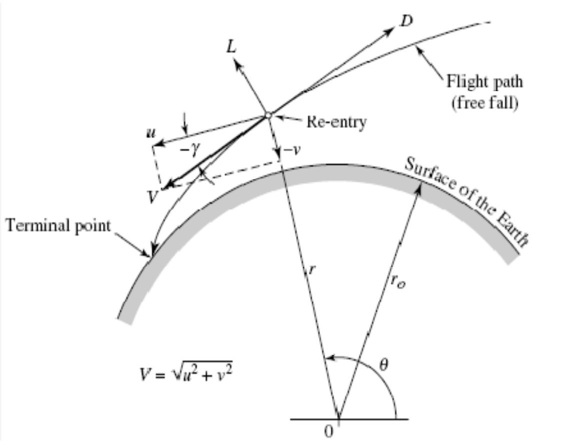


Figure 2. Geometry of reentry [16]

It is assumed that:
- The Earth is a sphere with uniform mass distribution everywhere without gravity anomaly.
- The Earth is non-rotating.
- The trajectory is planar within the launch plane.
- Angle of attack is small outside the atmosphere boundary.

PROPOSED FUZZY GIODANCE LAW

The fuzzy logic approach is employed to formulate the terminal guidance law for the point mass model of a reentry vehicle moving in vertical plane. The problem being considered is to provide a desirable miss distance and have the vehicle-approach target as close as possible to a specified impact angle. The proposed guidance law activates when the vehicle reentries the altitude of 60 km from the Earth surface. At this altitude, the dynamic pressure affects the vehicle motion. Inspired by the natural trajectory of reentry vehicles with terminal guidance, Figure 3, and in order to reach the target point with a desirable accuracy, the trajectory of the reentry vehicle with terminal guidance is divided into three stages. First stage modulates trajectory to make the vehicle more ability to control in flight path angle. Second stage reorients vehicle toward the target, while the third stage guarantees the terminal accuracy in miss distance and desired impact angle. Each of stages uses different inputs and works at different periods of flight time. Therefore, it is needed to switch between them.

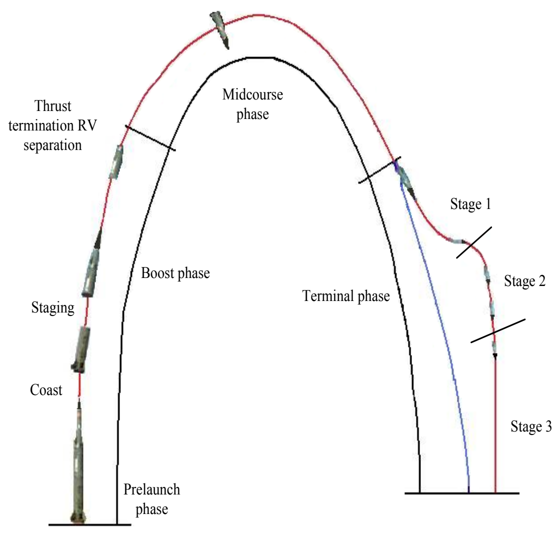


Figure 3. Typical trajectory for a reentry vehicle with terminal guidance [18]

The aerodynamic lift and drag forces depend on the angle of attack. Therefore, angle of attack is chosen as the guidance system output. Negative (or positive) angle of attack leads to negative (or positive) lift force.

The commanded angle of attack is directly entered the missile point mass model described in section 0.

The fuzzy system inputs are velocity angle error (δ), heading error angle (σ) and LOS angle error (η), defined as follows:

$$\delta = \gamma_F - \gamma \tag{7}$$

$$\sigma = \gamma - \lambda \tag{8}$$

$$\eta = \gamma_F - \lambda \tag{9}$$

here, γ is flight path angle, γ_F is final (desired) flight path angle and λ is LOS angle and given as:

$$\lambda = \tan^{-1}\left(\frac{h_t - h}{S}\right) \tag{10}$$

Here, h_t, h and S are target's altitude, vehicle's altitude and downrange, respectively. See Figure 4 and Figure 5 for geometric representation of angles λ, σ, δ and η. Three stages of the proposed FGL and the switching strategy are described in next sections.

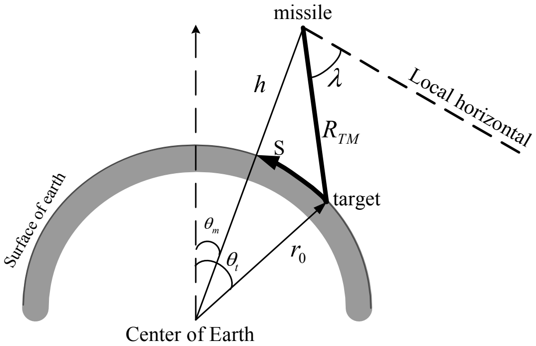


Figure 4. Definition of LOS angle [16].

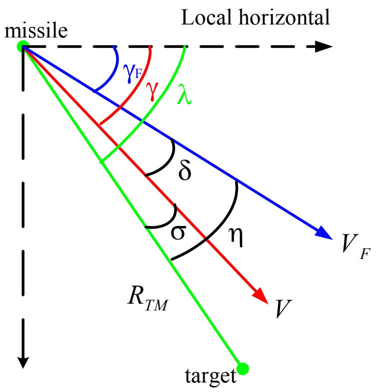


Figure 5. Definition of σ, δ and η.

TRAJECTORY MODULATION STAGE

Some of the reentry vehicles have no terminal guidance system and only use a guidance method like the Lambert guidance. This may result in an undesirable MD. In order to steer them towards the target point, a trajectory modulation is needed at the first stage of the terminal guidance. Therefore, here, the FGL is set to be:

$$\alpha_1 = f_1\left(\frac{\delta}{\eta}\right) \tag{11}$$

Where, $f_1(.)$ is an input-output mapping of fuzzy logic system.

The universe of discourse of the linguistic input variables is supposed to be $[0, 4]$ for $\left(\frac{\delta}{\eta}\right)$ ratio and $[-25^\circ, 25^\circ]$ for angle of attack. The input linguistic variable $\left(\frac{\delta}{\eta}\right)$ is assumed to take five linguistic sets and output linguistic variable (α_1) is assumed to take four linguistic sets. In order to simplify the computation in operational situations, triangular and trapezoid membership functions are utilized. It has been found that using complex forms of membership functions cannot bring any advantage over the triangular ones [11]. The linguistic sets are described by their membership functions as shown in Figure 6 and Figure 7 for input and output, respectively.

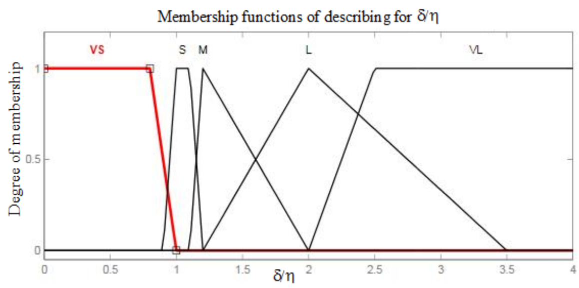


Figure 6. Membership functions of first stage input $\left(\frac{\delta}{\eta}\right)$.

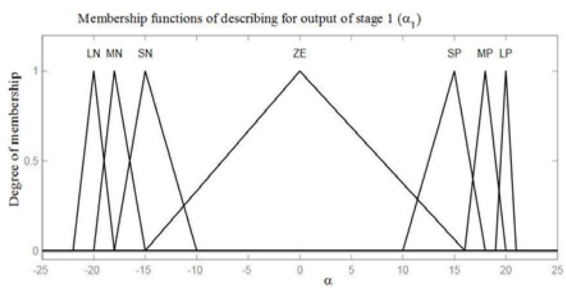


Figure 7. Membership functions of first stage output.

The rule base contains a collection of rules and forms an integral part of the total knowledge embedded in the guidance computer. If $\left(\frac{\delta}{\eta}\right)$ is very small (VS), it means the current flight path angle (γ) is closed to desired flight path angle (γ_F). In this case there is no need to modulate the trajectory and the other stages are sufficient to achieve the guidance goals. When $\left(\frac{\delta}{\eta}\right)$ is small (S), it means that both of γ and λ are far from γ_F. Modulating trajectory is needed by applying high positive lift force, which means positive angle of attack. By increasing $\left(\frac{\delta}{\eta}\right)$ ratio, the control action should be decreased to avoid the vehicle going away from the target. Table 4 shows the rule-bases used in this stage.

Table 4. Fuzzy rule-base for trajectory modulation stage.

δ/η	VS	S	M	L	VL
α_1	ZE	LP	MP	SP	ZE

The max–min (Mamdani type) inference is used to generate the best possible conclusions. In this inference mechanism, the min and max operations are, respectively, used for the *AND*/*OR* operations. This type of inference is computationally easy and effective. Thus, it is appropriate for real-time applications. The fuzzified inputs are fired individually according to each rule. The clipped membership functions of the individual rules are then merged to produce the final fuzzy set. The max operation is used to merge overlapping regions.

The outputs of the linguistic rules are fuzzy, but the guidance command must be crisp. Therefore, the outputs of the linguistic rules must be defuzzified before feeding into the plant. The crisp control action is calculated by the center-of-gravity (COG) defuzzification procedure. This criterion provides defuzzified output with better continuity. For a plant that is sensitive to the command quality such as missile autopilot, this criterion will be more appropriate than other defuzzification methods [9].

REOIENTAION STAGE

First stage leads the vehicle to go away from the target. Thus, second stage reorients vehicle toward the target. It is activated when the output of first stage decreases. For this purpose, the heading angle error σ and velocity angle error δ are used as the input variables:

$$\alpha_2 = f_2(\sigma,\delta) \tag{12}$$

Where, $\alpha_2 = f_2(.,.)$ is an input-output mapping of fuzzy logic system.

The universe of discourse of the linguistic input variables σ,δ is supposed to be $\left[-40^\circ,40^\circ\right]$ and angle of attack range is $\left[-30^\circ,30^\circ\right]$. The linguistic values taken by these variables are expressed by linguistic sets. Each of the linguistic variables is assumed to take five linguistic sets defined as large negative (LN), small negative (SN), zero (ZE), small positive (SP) and large positive (LP). To simplify the computation in the actual operation, triangular and trapezoid membership functions are suggested. The linguistic sets are described by their membership functions as shown in Figure 8 and Figure 9 for inputs and output, respectively.

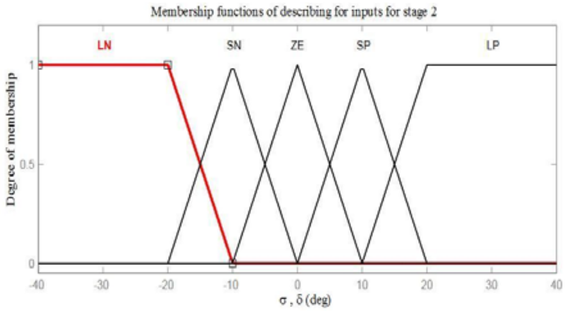


Figure 8. Membership functions for inputs σ and δ at 2nd stage

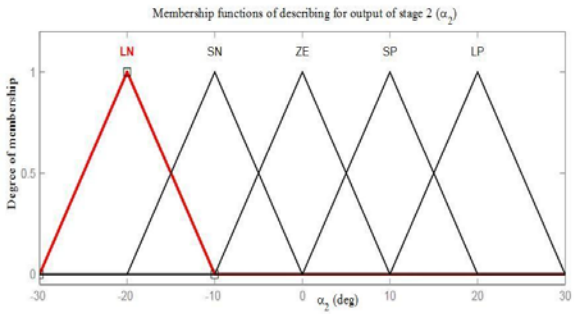


Figure 9. Membership functions for output of 2nd stage.

In reorientation stage, it is hoped that the reentry vehicle becomes toward the target with higher speed as large as possible. Therefore, the velocity error angle (δ) is considered more important than the heading error angle (σ). As a result, one can expect that the velocity error angle would vanish quicker than the heading error angle during reorientation stage. A complete set of 25 guidance rules, listed in Table 5, has been applied to meet our purpose. Referring to Table 5, the guidance rules can be divided into five distinctive groups:

- Velocity angle error (δ) is closed to zero (ZE): This means that the current flight path angle (γ) is consistent with the desired flight path angle (γ_F). The control action is thus intended to correct the heading error angle (σ). If δ is small or closed to zero the current δ will not be altered. According to Figure 10, when σ is negative then the command must lead to lift up the vehicle, that means α_2 must be positive and vice versa.
- Velocity angle error (δ) is large negative (LN): The control action is intended to significantly reverse this trend, that means α_2 must be large negative (see Figure 11)

Velocity angle error (δ) is small negative (SN): For the situation of σ being negative or closed to zero, the control action is impossible to simultaneously compensate for both of δ and σ. Therefore, design for the control action is dedicated to compensate for δ since it is considered to be more important. For σ being positive, the control action is intended to compensate for both δ and σ.

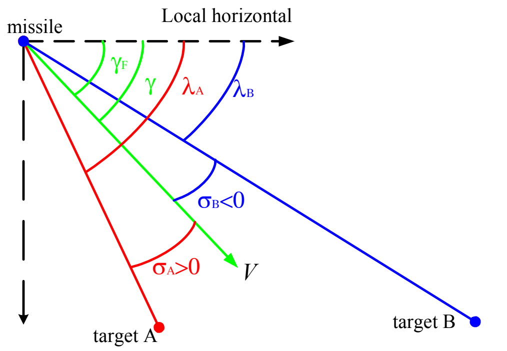


Figure 10. Reorientation stage: Heading error angle when velocity angle error is closed to zero

- Velocity angle error (δ) is small positive (SP) or *large positive* (*LP*): These cases equivalent to the opposite situations of the above situations and their corresponding fuzzy rules are developed in a same fashion.

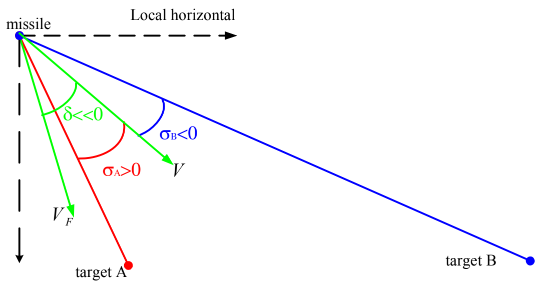


Figure 11. Reorientation stage: δ is large negative

The max–min (Mamdani type) inference and COG defuzzification are used. This stage ends when the inputs σ and δ reach to small values. This means the vehicle approaches to target position with a small velocity angle error.

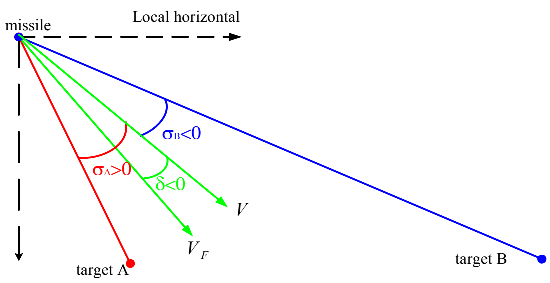


Figure 12. δ is small negative

Table 5. Fuzzy rule-base for reorientation stage.

α_2		δ				
		LN	SN	ZE	SP	LP
σ	LN	LN	SN	SP	LP	LP
	SN	LN	SN	SP	SP	LP
	ZE	LN	SN	ZE	SP	LP
	SP	LN	SN	SN	SP	LP
	LP	LN	LN	SN	SP	LP

FINE TUNNING STAGE

When the vehicle approaches homing at the end of second stage, minimizing the MD for good accuracy becomes more important. As the vehicle enters the third stage, the role of velocity error becomes minor and it would be preferable to use the heading error angle (σ), and LOS angle error (η), that is:

$$\alpha_3 = f_3(\sigma, \eta) \tag{13}$$

Where, $f_3(.,.)$ is the input-output mapping of fuzzy logic system. This function leads both of LOS angle and flight path angle to be the desired final flight path angle, which means zero MD.

The input variables of third stage, also called the linguistic variables, are σ, η and the output variable is α_3. The universe of discourse of the linguistic input variables is supposed to be $[-10°, +10°]$, and angle of attack is $[-15°, +15°]$. The linguistic sets are described by their membership functions as shown in Figure 13 and Figure 14 for inputs and output respectively.

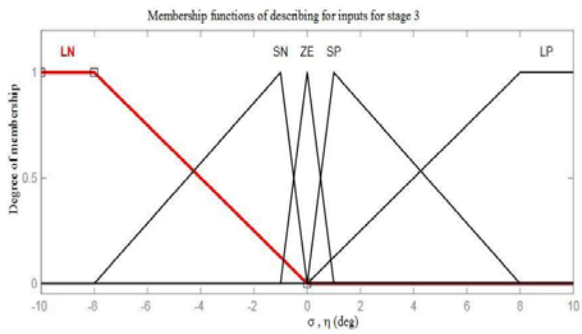


Figure 13. Membership functions of stage 3 for inputs σ and η

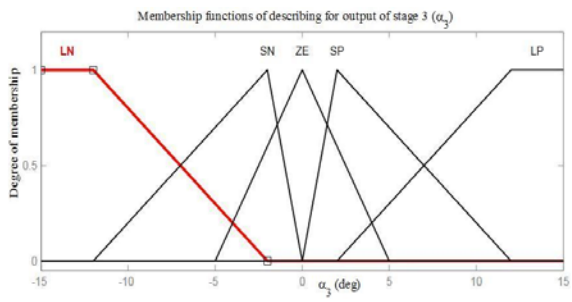


Figure 14. Membership functions of stage 3 output α_3.

In fine tuning stage, it is hoped that the RV achieves zero MD and zero velocity error angle. Therefore, if the heading error angle (σ) is large, it will be considered more important than the LOS error angle (η). A complete set of 25 guidance rules, listed in Table 6, has been applied to meet our purpose. Referring to Table 6, the guidance rules can be divided into five distinctive groups:

- *Heading angle error* (σ) *is closed to zero*: This means that the current flight path angle (γ) is closed to the LOS angle (λ). The control action is thus intended to correct the LOS angle error (η). According to Figure 15, when η is negative, the command must lead to lift up the vehicle, that means α_3 must be positive and vice versa.

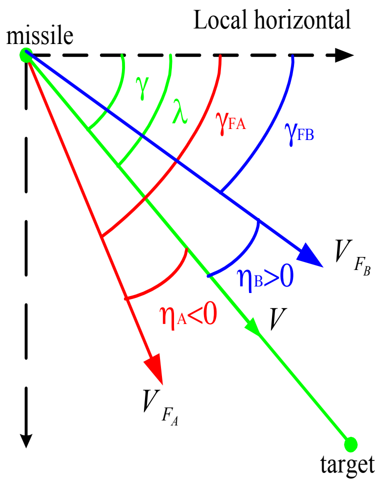


Figure 15. Fine tuning stage: LOS angle error when heading error angle is closed to zero.

- *Heading angle error* (σ) *is large positive or small positive and* η *is large*: The current flight path angle (γ) is greater than the LOS angle (λ). It means that the vehicle's flight direction is moving away from the target. The control action is thus intended to significantly reverse this trend, that means α_3 must be large negative. Figure 16 illustrates this case.
- *Heading angle error* (σ) *is small positive and* η *is small or closed to zero* (Figure 17): If η is positive or zero the control action which intends to lead σ and η to zero, is SN. But, if η is negative, the control action, which leads σ to zero, is SP.
- *The other cases*: These cases are equivalent to opposite situations of the above cases and their corresponding fuzzy rules are developed in a same manner.

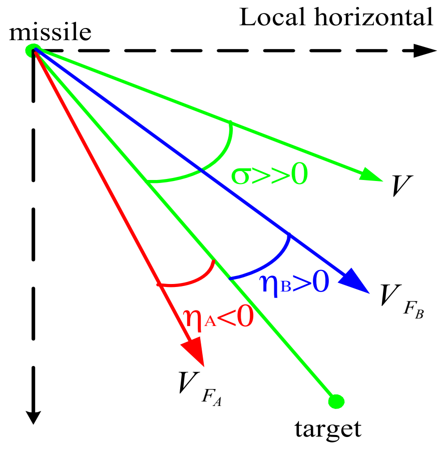


Figure 16. Fine tuning stage: σ is large positive.

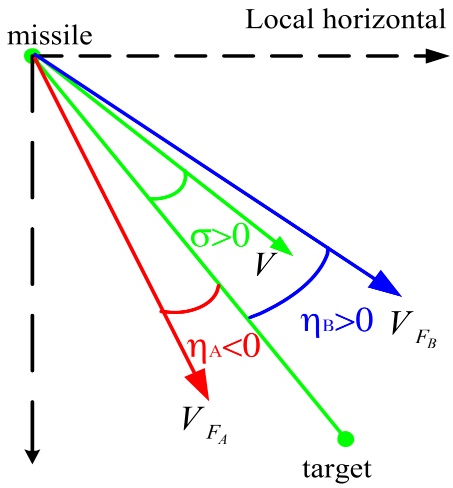


Figure 17. Fine tuning stage: σ is small positive.

In summary, it can be mentioned that in all three stages of fuzzy system, the max–min (Mamdani type) inference engine and COG defuzzification are used. In addition the fuzzy rules are developed via human expert and are ended by a trial and error manner.

Table 6. Fuzzy rule-base for fine tuning stage.

α_3		η					
		LN	SN	ZE	SP	LP	
	LN	LP	LP	LP	LP	LP	Group 5
	SN	LP	SP	SP	SN	LP	Group 4
σ	ZE	LP	SP	**ZE**	SN	LN	Group 1
	SP	LN	SP	SN	SN	LN	Group 3
	LP	LN	LN	LN	LN	LN	Group 2

SWITCHING STRATEGY BETWEEN STAGES

As previously mentioned, the complete guidance law consists of three stages. Each of stages works at different periods of flight time. Therefore, a switching strategy is required to avoid sudden big change in guidance command. First switch is between the first and second stages and the other is between second and third stages.

First stage starts from beginning of guidance with a positive high value of guidance command and continues until the angle of attack, guidance command, is vanished and heading error angle (σ) becomes less than 8°.The third stage starts when the heading error angle becomes less than 12°. Defining parameters $\left|\frac{\alpha_1}{20}\right|$ and $\left|\frac{\alpha_2}{20}\right|$ as the weighting factors, the guidance command in intersection periods is formulated according to the following equation:

$$\alpha_{1,2} = \left|\frac{\alpha_1}{20}\right|\alpha_1 + \left(1 - \left|\frac{\alpha_1}{20}\right|\right)\alpha_2 \tag{14}$$

$$\alpha_{2,3} = \left|\frac{\alpha_2}{20}\right|\alpha_2 + \left(1 - \left|\frac{\alpha_2}{20}\right|\right)\alpha_3 \tag{15}$$

Finally, total guidance command is:

$$\alpha_{com} = \alpha_1 + \alpha_{1,2} + \alpha_2 + \alpha_{2,3} + \alpha_3 \tag{16}$$

Figure 18 illustrates the general architecture of the proposed guidance law. Here, no dynamic is considered for the autopilot block and the reentry vehicle dynamic is the point mass planner model previously described in section 0.

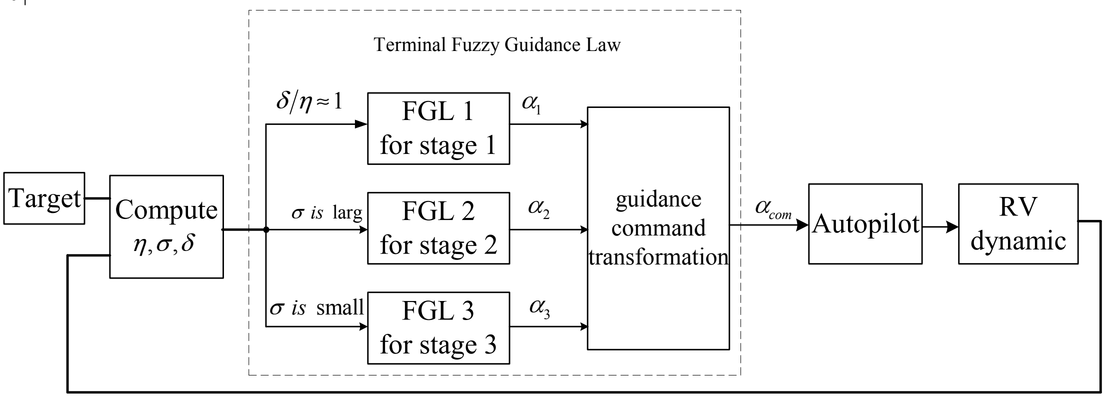
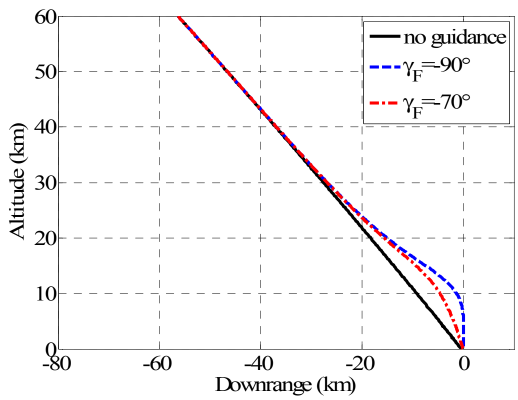


Figure 18. The reentry phase FGL architecture

EVALUATION OF PROPOSED ALGORITHM

The proposed FGL is applied to a typical reentry vehicle. It is assumed that the control system is ideal and now dynamic is considered for it. The point mass model is considered as the reentry vehicle dynamic model. The nonlinear aerodynamic model of the vehicle is a function of Mach number and angle of attack. In order to perform the simulations, the full trajectory is simulated. A pre-programmed guidance law is utilized in launch phase and the Lambert guidance method [16][17] is used up to the burnout phase. Then, the ballistic trajectory is started. In this study, the reentry fuzzy guidance starts from the 60km altitude and continues until the vehicle reaches the Earth surface. The initial conditions for the terminal phase flight are those at the end of simulated reentry trajectory of the vehicle: $V_0 = 2952.7\, m/s$, $\gamma_0 = -45.24^\circ$, $\theta_0 = 0.5063^\circ$, $h_0 = 60\, km$ and $S_0 = 56.36km$. The missile without guidance has a MD about 1km and final flight path angle is 49.78°. The simulation is carried out for two desired final impact angles of 70° and 90°. Fie 19-23 show performance of FGL. Fie 19 shows variation of altitude versus downrange. Figure 20 -23 draw time histories of flight path angle, Mach number and guidance command during the terminal guidance, respectively. It can be seen that MD is vanished to zero and impact angle error is less than 0.05°.



Fie 19. Variation of altitude versus downrange.

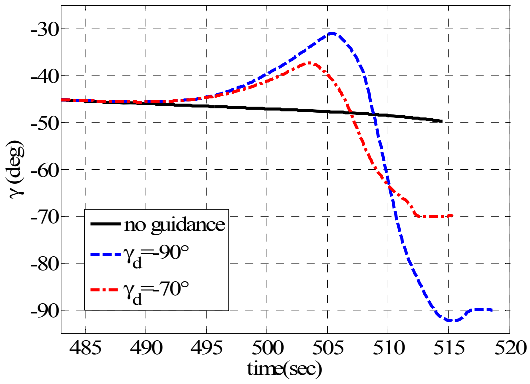


Figure 20. Time history of flight path angle during the terminal guidance

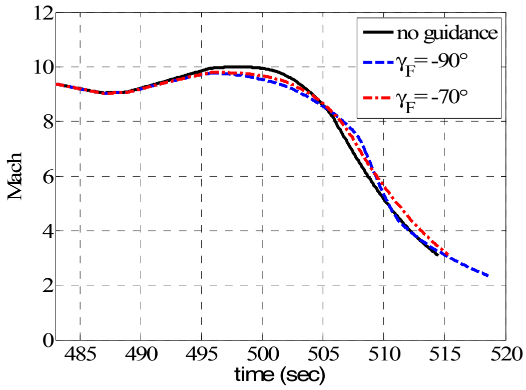


Figure 21. Time history of Mach number during the terminal guidance

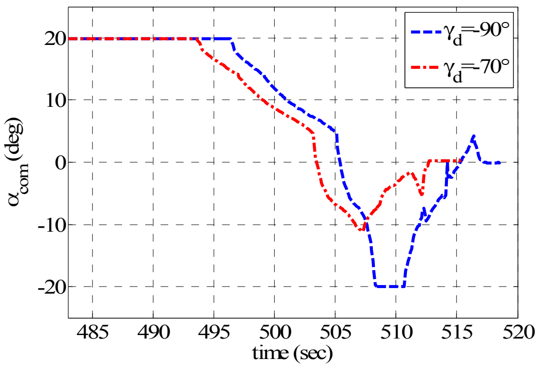


Figure 22. Time history of guidance command during the terminal guidance

ROBUSTNESS EVALUATION of THE PROPOSED ALGORITHM

In this regard, the input signal of the fuzzy system is considered to be noisy. Let, signal-to-noise ratio (SNR) is defined as the power ratio between a signal and the background noise:

$$SNR = \frac{P_{signal}}{P_{noise}} \tag{17}$$

where P is average power of the signal. The simulation is carried out for three SNR ratios: 50, 25, and 10 with variance 0.1. The noise is applied on the guidance inputs γ and λ. Figure 23- 27 show the performance of proposed guidance law for different SNR at $\gamma_F = 90^\circ$. Numerical results are shown in Table 7 for MD and Table 5 for impact velocity error angle at $\gamma_F = 70^\circ$, $\gamma_F = 80^\circ$ and $\gamma_F = 90^\circ$.

When SNR=50, trajectory and flight path angle schemes are not much affected and it is identical to no-noise case. Mean value of MD is still a small value (less than 0.5m) and mean impact angle error is less than 0.1°. This can be considered a good intercept for hypersonic reentry vehicle. When SNR=25, noise effect begins to appear, especially, on flight path angle. Noise effect grows as SNR decrease. The third stage guidance tries achieving zero-MD which is more important than velocity error angle. Therefore, MD remains less than 5m. First stage of guidance is not affected with noise because it depends on guidance inputs ratio δ/η that reduces noise effect. The noise more affects third stage of guidance which in inputs and output boundaries are small. This gives more stationary to guidance command. Therefore, we can say that proposed guidance law has a good noise rejection and robust performance.

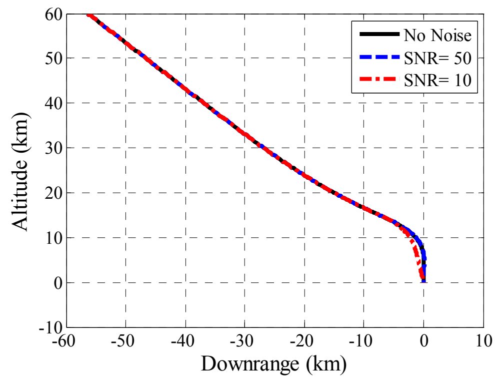


Figure 23. Variation of altitude versus downrange in presence of noise

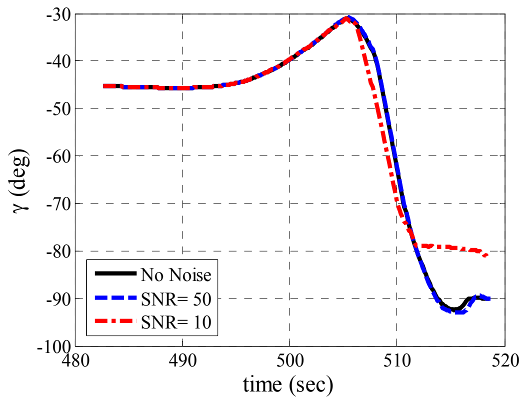


Figure 24. Time history of flight path angle during the terminal guidance in presence of noise

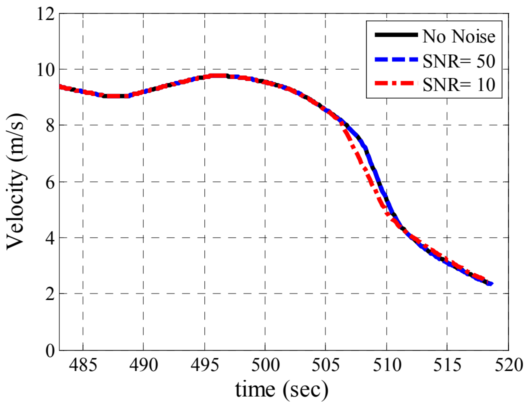


Figure 25. Time history of Mach number during the terminal guidance in presence of noise

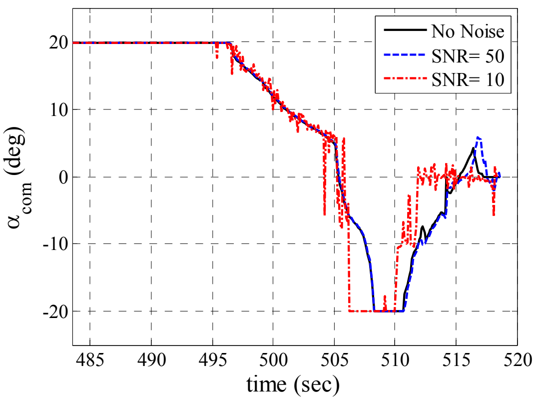


Figure 26. Time history of guidance command during the terminal guidance in presence of noise.

COMPARING THE PROPOSED ALGORITH WITH LACNGL

The impact angle control navigation guidance law (IACNG) [6] gives zero MD and zero impact velocity error angle at ideal conditions. IACNG form reveals a structure similarity to biased-proportional navigation guidance law [16] and is implied to two terms. The first term is a pure proportional navigation whose effective navigation ratio is a time-varying gain, the second term is a bias:

$$\dot{\gamma}_{com} = \frac{\delta}{\eta}\dot{\lambda} - \frac{K}{V}\eta \qquad (18)$$

Figes 26- 29 show the comparison between FGL and IACNGL for $\gamma_F = -90°$ in cases of SNR=10. Table 7 and Table 5 show MD and velocity error angle of proposed FGL and IACNGL for $\gamma_F = 70^{\circ}$, $\gamma_F = 80^{\circ}$ and $\gamma_F = 90^{\circ}$. The following points can be found from the simulation results:

- Proposed FGL is better than IACNGL in both MD and impact angle.
- MD in IACNGL is greater than 1km in cases of low SNR values, but MD in FGL is less than 5m.
- IACNGL decreases the impact velocity (less than 1 Mach in low SNR) while FGL keeps it very high (2.5~3 Mach).

The proposed FGL is more robust to reject noise than IACNGL.

Table 7. MD (m) of FGL and IACNGL

	γ_F	No Noise	SNR=50	SNR=25	SNR=10
FGL	70	0.00	0.13	5.0	-1.15
IACNGL		0.00	0.19	2719	5975
FGL	80	0.00	-0.01	1.14	1.08
IACNGL		0.00	0.06	2200	6243
FGL	90	0.00	0.03	-1.51	1.12
IACNGL		0.00	-31	372	6575

Table 8. Impact velocity error angles of FGL and IACNGL.

	γ_F	No Noise	SNR=50	SNR=25	SNR=10
FGL	70	0.0126	-0.0908	-3.5	-12.7
IACNGL		0.0043	0.853	-19.1	62.6
FGL	80	-0.049	0.0829	-0.0665	-14.3
IACNGL		0.00	-0.0419	-26.5	44.0
FGL	90	0.003	-0.0149	-0.3659	-8.73
IACNGL		0.00	7.4	-31.62	28.2

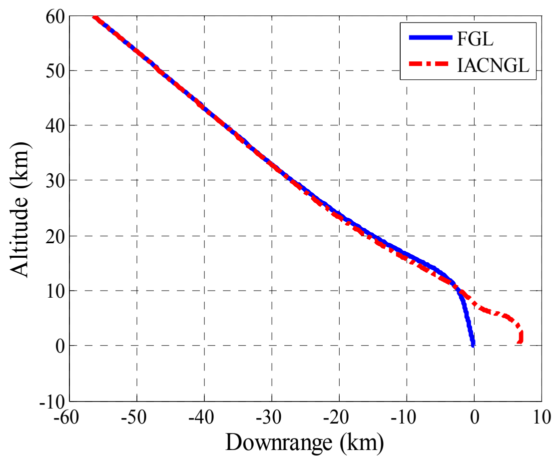


Figure 27. Reentry trajectories resulted from for FGL and IACNGL at $\gamma_F = -90°$ and SNR=10.

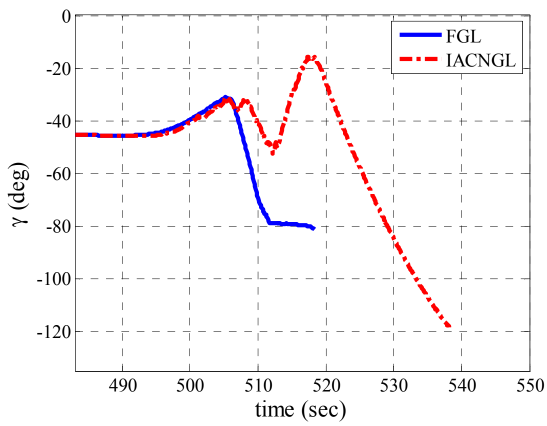


Figure 28. Flight path angle c resulted from for FGL and IACNGL at $\gamma_F = -90°$ and SNR=10.

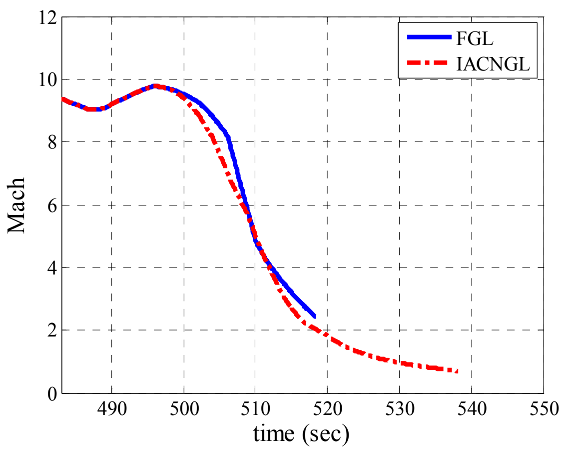


Figure 29. Velocity resulted from for FGL and IACNGL at $\gamma_F = -90°$ and SNR=10.

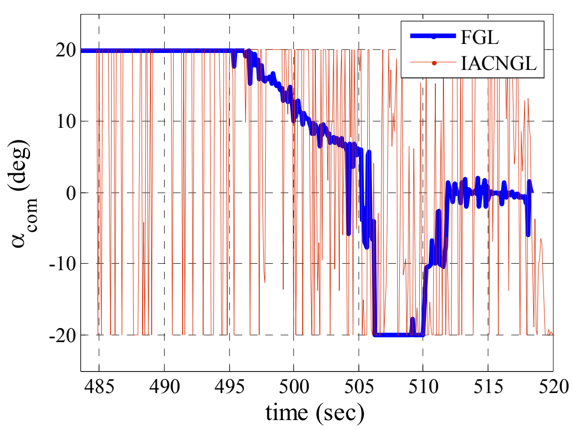


Figure 30. Guidance command resulted from for FGL&IACNGL at $\gamma_F = -90°$ and SNR=10.

CONCLUSIONS

The need to guide a reentry vehicle in terminal phase is intended to impact a ground target with stringent specified impact direction arises from the modern technology development efforts. It is found that a fuzzy guidance approach is effective for this problem and easy to implement. One superiority of the proposed algorithm is that it only needs to measure two parameters of LOS angle (λ) and flight path angle (γ). The fuzzy logic approach is employed to formulate the terminal guidance law for a point mass reentry vehicle moving in a plane. The other notable feature of the proposed FGL is that, inspired by the real flight trajectory of a reentry vehicle, during the reentry phase, the guidance law is divided into three stages. First stage modulates trajectory to make the vehicle more ability to control in flight path angle. Second stage reorients vehicle toward the target, while the third stage guarantees the terminal accuracy in MD and desired impact angle. Simulations are carried out for two desired final impact angles. The effect of noise is investigated and the results of this preliminary study indicated a satisfactory robustness capability for the proposed FGL. It is desirable to enhance the proposed scheme in three dimensional space as well as evaluating its performance in presence of the control system model.

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