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Extracting Dynamics Matrix of Alignment Process for a Gimbaled Inertial Navigation System Using Heuristic Dynamic Programming Method

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In this paper, with the aim of estimatinginternal dynamics matrix of a gimbaled Inertial Navigation System (as a discrete linear system), the discrete-time Hamilton-Jacobi-Bellman (HJB) equation for optimal control has been extracted. Heuristic Dynamic Programming algorithm (HDP) for solving the equation has been presented and then a neural network approximation for cost function and control input has been extracted to simplify the solutionof HJB. Design process of the optimal controller shows that, we do not need to know the system matrix. This important issue and the convergence of the HDP algorithm to the optimal control policy makes possible the estimation of the internal dynamics matrix.

Keywords: optimal control, Heuristic Dynamic Programming, neural network approximation, gimbaled Inertial Navigation System, Initial Alignment

INTRODUCTION

Initial Alignment in gimbaled Inertial Navigation Systems (INS) is a complicated experimental process, which needs a multivariable control system [1, 2]. Because of uncertainties in the mechanical and electrical parts of INS, sometimes we need to use certainidentification methods to determine system parameters [3-6]. As shown in Figure 1, a gimbaled INS consists of a three-axial suspension mechanism, with an assembly of inertial sensors (3 gyroscopes and 3 accelerometers) on it, and some necessary actuators. The design of a control system for this purpose, needs the internal dynamics matrix of the platform. In the present article, we use a new identification method to determine this internal dynamics matrix based on a HDP algorithm.

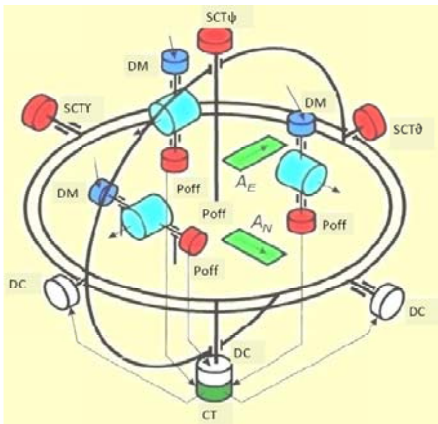

Figure 1. a gimbaled INS [1]

System identification is the process of developing or improving the mathematical representation of a physical system using experimental data. There are three types of identification techniques: modal parameter identification,structural model identification and controlmodel identification [7].The main aim of system identification is to determine a mathematical

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model of a physical/dynamic system from the observed data. Six key steps are involved in system identification [7]; (1) Developing an approximate analytical model of structure, (2) Establishingthe levels of structural dynamic response which are likely to occur using the analytical model as well as thecharacteristics of anticipated excitation sources, (3) Determining the instrumentation requirements needed to sense the motion with prescribed accuracy and spatial resolution, (4) Performing experiments and recording the data, (5) Applying system identification techniques to identify the dynamic characteristics such as system matrixes, modal parameters, as well asexcitation and input/output noise characteristics, and (6)Refiningthe analytical model based on the identified results. The traditional identification techniques extracting modal parameters from input and output data have been well-developed and widely used in engineering. However, it is often a demandingtask to carry out excitation in the field testing of large engineering structures. To obviate such difficulties of the traditional techniques, methods of extracting modal parameters from structural response data have only been deeply investigated during the lastfew decades [8].Approximate dynamic programming (ADP) is a very effective method for the solution of Discrete-Time Nonlinear HJB [9]. There are many techniques of ADP to solve the cost function and hence the optimal control policy [10]. In [11], ADP techniques are classified into: heuristic dynamic programming (HDP), dual heuristic dynamic programming (DHP), action dependent heuristic dynamic programming (ADHDP), and action dependent dual heuristic dynamic programming (ADDHP).In [12], Liu and Li used optimal control for discrete-time HJB.

In the present studybased onthe idea of HDP, we extract a new internal dynamics matrix identification method. At first, we introduce the discrete time HJB, then, we useheuristic dynamic programming algorithm for solving HJB online, and finally introduce two neural network parametric structures to approximate the optimal cost function and policy. At last, using the main results obtained, we extract an estimator for the internal dynamics estimation of a discrete linear system.

DISCRETE-TIME HJB EQUATIONS

Consider an affine dynamic system of the form:

$$x_{k+1} = f(x_k) + g(x_k)u(x_k) \quad (1)$$

where $x \in R^n$ ،$f(x) \in R^n$ ، $g(x) \in R^{n\times m}$and the input$u \in R^m$.

It is desired to find the control input which minimizes the cost function given as (2):

$$V(x_k) = \sum_{n=k}^{\infty} x_n{}^T Q\, x_n + u_n{}^{\mathrm{T}} \mathrm{R}\, u_n \quad (2)$$

where $Q > 0$ and $R > 0$.

From Bellman's optimality theory and [13]:

$$u^*(x_k) = \frac{1}{2} R^{-1} g(x_k)^T \frac{\partial V^*(x_{k+1})}{\partial x_{k+1}} \quad (3)$$

and:

$$V^*(x_k) = x_k{}^T Q x_k \frac{1}{4} \frac{\partial V^{*T}(x_{k+1})}{\partial x_{k+1}} g(x_k)$$
$$R^{-1} g(x_k)^T \frac{\partial V^*(x_{k+1})}{\partial x_{k+1}} + V^*(x_{k+1}) \quad (4)$$

This equation reduces to the Ricatti equation in the linear quadratic regulator (LQR) instance, which can be solved with precision. In the general nonlinear case, the HJB cannotbe solved exactly.

From [9], the value iteration HDPalgorithm for solving it is proposed as:

$$u_i(x_k) = \arg \quad min_u \begin{Bmatrix} x_k{}^T Q x_k + u^T R\, u + \\ V_i(x_{k+1}) \end{Bmatrix}$$
$$= \arg \quad min \quad {}_u\{x_k{}^T Q x_k + u^T R\, u + V_i(f(x_k) + g(x_k)u)\}$$
$$= \frac{1}{2} R^{-1} g(x_k)^T \frac{\partial V_i(x_{k+1})}{\partial x_{k+1}} \quad (5)$$
$$V_{i+1}(x_k) = min_u\{x_k{}^T Q x_k + u^T R\, u + V_i(x_{k+1})\} = x_k{}^T Q x_k +$$

$$u_i(x_k)^T R\, u_i(x_k) + V_i(f(x_k) + g(x_k)u_i(x_k)) \quad (6)$$

subsequently, for solving the equations (5), (6), neural network estimation has been proposed as follows:

$$\hat{V}_i(x) = \sum_{j=1}^{L} w_{vi}{}^j \, \emptyset_j(x) = W_{vi}{}^T \emptyset(\boldsymbol{x}) \quad (7)$$

$$\hat{u}_i(x) = \sum_{j=1}^{M} w_{ui}{}^j \, \sigma_j(x) = W_{ui}{}^T \boldsymbol{\sigma}(\boldsymbol{x}) \quad (8)$$

where $\emptyset(\boldsymbol{x})$ and $\boldsymbol{\sigma}(\boldsymbol{x})$ are the activation functions and W_{vi} and W_{ui} are the network weights.

The updatedlawfor W_{vi} and $W_{ui}$is obtained from (9), (10) as:

$$W_{v_{i+1}} = \left(\int_\Omega \phi(x_k)\phi(x_k)^T \, dx\right)^{-1} \times \int_\Omega \phi(x_k)\hat{V}_i\,(\phi(x_k), W_{vi})\, dx \quad (9)$$

$$W_{ui}|_{m+1} = W_{ui}|_m - \alpha\, \sigma(x_k)(2R\, \hat{u}_i(x_k, w_{ui}|_m) + g(x_k)^T \frac{\partial \emptyset(x_{k+1})}{\partial (x_{k+1})} W_{Vi})^T \quad (10)$$

where Ω is the training set,α is a positive step size, and m is the iteration number of the LMS algorithm.

Assuming the convergence of HDP algorithm for a linear system, the last relations show, $V_i(x_k)$ and $u_i(x_k)$ converge to the cost function of the optimal control problem and to the corresponding optimal control input,respectively.

The most important feature which is seen in (9) and (10) is that $f(x_k)$isn't is notneeded to update the critic neural network weights and this issue makes possible the estimation of system inner dynamics matrix using composition of heuristic dynamic programming control algorithm and neural network.

ESTIMATION of INNER DYNAMICS MATRIX of DISCRETE-TIME LINEAR SYSTEM

Consider the linear discrete system (for example with five states), described as:

$$x(k+1) = Gx(k) + B\,u(k) \tag{11}$$

From LQR method:

$$u^*(x) = -Lx \tag{12}$$

where:

$$L = (R + B^T P B)^{-1} B^T P G \tag{13}$$

And P is the symmetric positive definite of below Ricatti equation:

$$P = G^T P G + Q - G^T P B (R + B^T P B)^{-1} \times B^T P G \; : \tag{14}$$

and

$$V^*(x) = x^T P x \tag{15}$$

A reasonable selection of activation functions vector for cost and control input would be as:

$$\boldsymbol{\emptyset}(\boldsymbol{x}) = [x_1^{\,2} x_1x_2 x_1x_3 \;\; x_1x_4 \;\; x_1x_5\, x_2^{\,2} x_2x_3\; x_2x_4\, x_2x_5\;\, x_3^{\,2} x_3x_4 \;\; x_3x_5\;\, x_4^{\,2} x_4x_5 \;\; x_5^{\,2}]^T \tag{16}$$

$$\boldsymbol{\sigma}(x) = [x_1x_2x_3x_4x_5]^T$$

Then, converged weight vectors have the form of:

$$W_v = [w_v^{\,1} w_v^{\,2} w_v^{\,3} w_v^{\,4} w_v^{\,5} w_v^{\,6} w_v^{\,7}\; w_v^{\,8} w_v^{\,9} w_v^{\,10} w_v^{\,11} w_v^{\,12} w_v^{\,13} w_v^{\,14} w_v^{\,15}]^T$$

$$W_u = \begin{pmatrix} w_u^{1.1} & w_u^{1.2} & w_u^{1.3} & w_u^{1.4} & w_u^{1.5} \\ w_u^{2.1} & w_u^{2.2} & w_u^{2.3} & w_u^{2.4} & w_u^{2.5} \\ w_u^{3.1} & w_u^{3.2} & w_u^{3.3} & w_u^{3.4} & w_u^{3.5} \\ w_u^{4.1} & w_u^{4.2} & w_u^{4.3} & w_u^{4.4} & w_u^{4.5} \\ w_u^{5.1} & w_u^{5.2} & w_u^{5.3} & w_u^{5.4} & w_u^{5.5} \end{pmatrix} \tag{17}$$

and:

$$\widehat{V^*}(x) = W_v^{\,T} \emptyset(\boldsymbol{x}), \qquad \widehat{u^*}(x) = W_u^{\,T} \boldsymbol{\sigma}(\boldsymbol{x}) \tag{18}$$

As explained in section 3, HDP algorithm converges to the cost function of the optimal control problem and to the corresponding optimal control input, respectively. i.e.

$$\widehat{V^*}(x) = V^*(x), \qquad \widehat{u^*}(x) = u^*(x) \tag{19}$$

Therefore:

$$W_v^{\,T} \emptyset(\boldsymbol{x}) = x^T P x, \qquad W_u^{\,T} \boldsymbol{\sigma}(\boldsymbol{x}) = -Lx \tag{20}$$

We define:

$$W = \begin{pmatrix} w_v^{\,1} & 0.5w_v^{\,2} & 0.5w_v^{\,3} & 0.5w_v^{\,4} & 0.5w_v^{\,5} \\ 0.5w_v^{\,2} & w_v^{\,6} & 0.5w_v^{\,7} & 0.5w_v^{\,8} & 0.5w_v^{\,9} \\ 0.5w_v^{\,3} & 0.5w_v^{\,7} & w_v^{\,10} & 0.5w_v^{\,11} & 0.5w_v^{\,12} \\ 0.5w_v^{\,4} & 0.5w_v^{\,8} & 0.5w_v^{\,11} & w_v^{\,13} & 0.5w_v^{\,14} \\ 0.5w_v^{\,5} & 0.5w_v^{\,9} & 0.5w_v^{\,12} & 0.5w_v^{\,14} & w_v^{\,15} \end{pmatrix} \tag{21}$$

Then, (16-21) results:

$$P = W, \qquad L = -W_u^{\,T} \tag{22}$$

Using (13), (22):

$$-W_u^{\,T} = \{R + B^T W B\}^{-1} \mathrm{B^T} W G \tag{23}$$

If the number of states is equal to control inputs, i.e. size(G)=size(B) and B is nonsingular, (constrains of proposed estimation algorithm), then, there is a unique solution for G from (23):

$$G = \{W^{-1} {B^T}^{-1} R + B\} W_u^{\,T} \tag{24}$$

That can be simplified to:

$$G = \{W^{-1} {B^T}^{-1} R + B\} W_u^{\,T} \tag{25}$$

A block-diagram of the proposed estimation method is shown in Fig. 2.

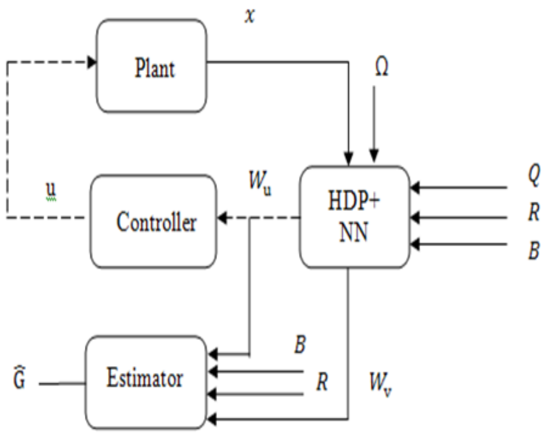

Figure 2. Block-diagram of the proposed estimation method

ESTIMATION of INNER DYNAMICS MATRIX of GIMBALED INERTIAL NAVIGATION SYSTEM

Gyroscopes random error is defined as a stable stochastic process with autocorrelation function as follows [1]:

$$G = \{W^{-1} {B^T}^{-1} R + B\} W_u^{\,T} \tag{26}$$

where σ_i is the standard deviation of random error of gyroscopes, which is known, and μ_i is the damping coefficient.

Differential equation of gyroscope error is as:

$$\dot{D}_i + \mu_i D_i = \sqrt{2\sigma_i^2 \mu_i}.w \tag{27}$$

Where w is a white noise with $\mathcal{N}(0,1)$.

Dynamic equations of inertial navigation system considering gyroscopes disturbances are [14]:

$$\dot{x}(t) = Gx(t) + Dw(t) \qquad (28)$$

where:

$$G = \begin{pmatrix} 0 & \Omega_d & 0 & 0 & 0 \\ -\Omega_d & 0 & \Omega_n & 0 & 0 \\ 0 & -\Omega_n & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mu_E & 0 \\ 0 & 0 & 0 & 0 & -\mu_N \end{pmatrix}$$
$$D = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \sqrt{2\sigma_E^2\mu_E} & 0 \\ 0 & \sqrt{2\sigma_N^2\mu_N} \end{pmatrix} \qquad (29)$$

Inpractice two-channel control for horizontal axes is used. While in this paper, for extracting matrix A, because of constrains in the algorithm, a five-channel virtual control is utilized. Therefore:

$$\dot{x}(t) = Gx(t) + Bu(t) + Dw(t) \qquad (30)$$

By assumption of matching condition ($Dw(t) = Bw_1(t)$), the equation (30) can be rewritten as:

$$\begin{aligned} \dot{x}(t) &= Gx(t) + Bu(t) + Dw(t) \\ &= Gx(t) + Bu(t) + \frac{B}{k}Dw(t) \\ &= Gx(t) + B\tilde{u}(t) \end{aligned} \qquad (31)$$

where:

$$\tilde{u}(t) = u(t) + w_1(t) \qquad (32)$$

For implementing the proposed method, we need thediscrete model of the system. For this aim, we use the approximation:

$$\dot{x}(t) \cong \frac{x(k+1) - x(k)}{\Delta t} \qquad (33)$$

Therefore:

$$x(k+1) = \tilde{G}x(k) + \tilde{B}\tilde{u}(k) \qquad (34)$$

where:

$$\tilde{G} = I + G\Delta t \quad , \quad \tilde{B} = B\Delta t \qquad (35)$$

If we choosethe following parameters:

$$\Omega_d = \Omega_e * \sin(\varphi)\left(\frac{rad}{s}\right)$$

$$\Omega_n = \Omega_e \cos(\varphi)\left(\frac{rad}{s}\right)$$

$$\Omega_e = 7.29 * 10^{-5} \quad \left(\frac{rad}{s}\right)$$

$$\varphi = 35.5^{\circ} \quad , \quad \mu_E = 3 * 10^{-3}$$

$$\mu_N = 10^{-3} \quad , \quad \Delta t = 0.01\ (s)$$

With substituting in (29), (35):

$$\tilde{G} = \begin{pmatrix} 1 & 4.23e-7 & 0 & 0 & 0 \\ -4.23e-7 & 1 & 5.93e-7 & 0 & 0 \\ 0 & -5.93e-7 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.999 & 0 \\ 0 & 0 & 0 & 0 & 0.999 \end{pmatrix} \qquad (36)$$

With applying HDP+NN, W_u and W_v converge to:

$$W_u = \begin{pmatrix} -1.0000 & 0.0000 & 0.0002 & 0 & 0 \\ -0.0002 & -1.0003 & 0.0000 & 0 & 0 \\ 0.0002 & -0.0000 & -1.0002 & 0 & 0 \\ 0 & 0 & 0 & -0.00001 & 0 \\ 0 & 0 & 0 & 0 & -0.00003 \end{pmatrix}$$
$$W_v = \begin{pmatrix} 1562640609 & 273 & -646052 & 0 & 0 \\ 1563094323 & 460 & 0 & 0 & 1562864141 \\ 0 & 0 & 16666 & 0 & 5 \end{pmatrix} \qquad (37)$$

The convergence of the two weights (for example$W_u(3,3)$and$W_v(1,2)$) is shown in Fig . 2.

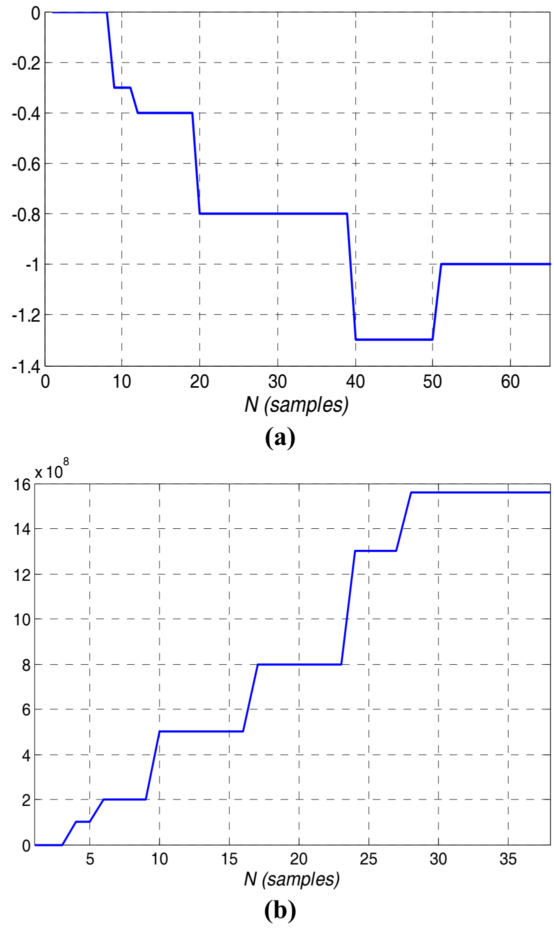

Figure 3. The convergence of (a) $W_u(3,3)$ and (b) $W_v(1,2)$

Using (25):

$$\hat{\tilde{G}} = \begin{pmatrix} 1.0000 & 0.0000 & -0.0002 & 0 & 0 \\ -0.0000 & 1.0000 & 0.0000 & 0 & 0 \\ 0.0000 & -0.0000 & 0.9999 & 0 & 0 \\ 0 & 0 & 0 & 0.9999 & 0 \\ 0 & 0 & 0 & 0 & 0.9999 \end{pmatrix} \qquad (39)$$

Comparing (36) and (39) shows that the proposed estimation method works properly. In [13], another closed-loop subspace identification method which is based on the least-square problem is presented. In this procedure,$\hat{\tilde{G}}$can be found as the solution to the problem

$$U_n(1:(f-1)n_y:)\hat{\tilde{G}} = U_n(n_y + 1:fn_y,:) \qquad (40)$$

For details see [13].

From (40):

$$\hat{\hat{G}} = \begin{pmatrix} 1.0000 & 1.22e-7 & -0.0002 & 0 & 0 \\ -1.22e-7 & 1.0000 & 0 & 0 & 0 \\ 0.0000 & -0.0000 & 0.9999 & 0 & 0 \\ 0 & 0 & 0 & 0.98 & 0 \\ 0 & 0 & 0 & 0 & 0.98 \end{pmatrix} \quad (41)$$

comparing (39) and(41) shows that the proposed method in this paper enjoys ahigher accuracy especially in estimating the drift filter of gyroscopes that is related to the last two entries on the diagonal $\hat{\hat{G}}$.

CONCLUSION

In this paper, a new method for extracting the inner dynamics matrix for a discrete time system was proposed. This idea is obtained from solving the HJB through thecomposition of Heuristic Dynamic Programming and Neural Network, in which the linear system critic network converges to the solution of ARE, the control network converges to the optimal policy and the internal dynamics did not require to implement HDP. This method was applied for estimatingtheinternal dynamics matrix of a gimbaledInertial Navigation System. furthermore, theestimation of gyroscopes random error filter was performed with precision.

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