JAST. Vol. 9, No. 1, pp 49-58

JAST Journal of Aerospace Science and Technology

Practical Evaluation of EKF[1] and UKF[2] Filters for Terrain Aided Navigation

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This article deals with batch and recursive methods used in terrain navigation systems. These systems have a lot of disadvantages. That is why researchers have studied various methods of aided navigation for many years and have introduced different types of aided navigation systems with practical and theoretical advantages and disadvantages. One of the main ideas related to aided navigation is integration of extended Kalman filter and INS[3]. This integration method has a significant weakness in practice that has caused it not to be highly valued as an aided navigation method. Hence, in this article, the authors introduce a more accurate filter (UKF) to be integrated with INS and other sensors such as barometers and radar systems. Then, the use of the aided EKF and UKF navigation schemes are justified, their respective algorithms are developed and performed for the needed applications and the simulation results are presented and compared. Finally, the advantages of the proposed method are compared with those of other batch and recursive methods. The most significant idea of the present article is related to its practical application on a UAV that was tested in 2010.

***Keywords**: Inertial Navigation System, Dynamic Model, Extended Kalman Filter, Unscented Kalman Filter*

1 Introduction

Navigation is generally defined as estimation of position, speed and attitude vectors of a moving object. This would generally occur in flying objects through integrating Inertia Navigation Systems (INSs) with other systems and upgrading the position.

In recent years, the civil and military INS applications in different air, land and sea transportation systems have highly increased. However, because of initial errors and measurement errors, there is a drift from the position estimated by the INS to the real position. This error increases with an increase in intervals from the moment the system initializes. Thus, nowadays, precise and modern navigation systems [1] utilize various techniques to reduce INS drift. These methods are generally based on using aided navigation systems such as Doppler aid, astral, stage matching and chiefly GPS. If these techniques cannot be used, for example, when the GPS signal under the sea is off or because of security issues in military applications, other navigation schemes should be used to help increase the accuracy and integrity of navigation. For decades, nominal unity uneven path profile and using it as a technique for updating INSs, under general Terrain Aided Navigation (TAN), have been considered. The main idea in TAN is integration of four categories of information systems: terrain clearance, absolute height, INS data and Digital Elevation Model (DEM). The goal is to match terrain height variations profile with the flight route map reference (DEM) and estimation of the accurate position and azimuth of the plane. [2, 3, 4, 6, 11]

In inertial navigation systems, the position vector is calculated by using a set of relative motion measurements based on the first location of the plane. These systems need to be regularly initialized because of incremental errors. In this case, using the systems that calculate their instantaneous position, regardless of the previous status (e.g. GPS), is a suitable method widely used to reinitialize the inertial system.

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1. Extended Kalman Filter
2 . Unscented Kalman Filter
3 . Inertial Navigation System

2 Digital Elevation Model (DEM)

Collecting terrain elevation data is relatively time-consuming and expensive because it is usually performed manually by an operator with photogrammetry devices. The operator remodels the space geometry of the terrain surface through overlapping two satellite or air photos and measures 3 dimensional positions of the points.

Thus, using photogrammetry equipment, an experienced operator can measure 6 to 10 points in a minute. This process is performed manually using analogue pictures. However, DTEDs are provided with higher speed and precision using digitalized pictures and image processing.

The heights of the terrains measured by the photogrammetry method is not the average position in the region (pixels). Hence, the operator should measure the height in the nearest visible intersection position. This will lead to errors in terrain clearance data in both the real terrain height and position of network crossing points. Today, the manufacturers of DTEDs claim to provide an accuracy almost equivalent to 2.5 meters in the at the global level.

It should be emphasised that, except for height and position which are measured on basis the meter unit in simulink section, X and Y axes do not have any special units. They are pixel points of DEMs representing longitude and latitude within UTM transformation. The pixel points can be converted to a geographical position only on the digital plan saved on the plane computer memory.

3 Terrain Aided Navigation System Inputs

Terrain navigation algorithms combine four categories of information sources to estimate the object position. These four measurements are speed vector, altitude, flying height (ground clearance) and base map (reference matrix).

Speed vector is typically received from the inertial navigation system. Absolute height is measured by barometer and inertial data integration and high-level terrain is measured by the radar altimeter installed under the plane body [5]:

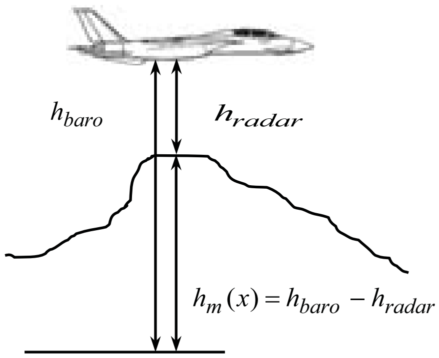


Figure 1. Height measured for ground effects and output DEM

The reference map is known as a network of terrain elevation values in a coordinate system whose flatness position is presented using latitude and longitude or Universal Transverse Mercator. Thus, the main purpose of TAN is the integration of these four types of information to produce the best possible estimate of the accurate position using sensors.

4 Recursive Algorithm for Aided Navigation

The defect of the batch method is that if the terrain is repeated or the region is flat, several positions are available in DTEDs that can adapt to measurements. Thus, there is no consistent way to find the exact number of measurements needed to find a unique position in the database. In addition, the batch method provides unknown outputs that cannot be used in the Kalman filter (for integrating navigation systems).

The recursive method is attractive because there is no prejudice about the number of points needed to estimate a unique position. The idea of this method is based on continuing processing measurements to achieve a unique estimate.

Repetitive and flat surfaces require recursive methods more than the other ones. For each new measurement, there are a number of locations on a map matching to the measured surface height, which indicates some points through which the flying object has probably passed. Depending on the surface, such regions can have any number of surfaces or any arbitrary shape.

Therefore, after each new measurement, the recursive method should deal with some estimated points in a parallel manner. Finding the proper algorithm for solving recursive methods is not as easy as the batch method. Solution should be achieved by modelling the problem while this is not true about batch methods and it can be implemented without any specific modelling.

Recursive methods, although appeared later than batch methods, are of a great diversity. Also, recursive methods have a more complex nature but, due to the variety of their algorithms, they include a wider range of topics.

Generally, nonlinear estimation of state space can be classified under three general categories:

1. Taylor series estimation of the input distribution function (EKF),
2. Unscented transformation of the input distribution function (UKF), and
3. Monte Carlo transformation of the input distribution function (PF[1]).

---

1. *Particle Filter*

Transformation of the estimation based on Taylor series, leading to relationships for the EKF and the UKF, is as follows:

$$x \approx N(m,P) \ ,$$
$$y = g(x) \ ,$$
$$\begin{pmatrix} x \\ y \end{pmatrix} \approx N\left(\begin{pmatrix} m \\ m_L \end{pmatrix}, \begin{pmatrix} P & C_L \\ C_L^T & S_L \end{pmatrix}\right) \ ,$$
$$m_L = g(m)$$
$$S_L = G_x(m)PG_x(m)^T \ ,$$
$$C_L = PG_x^T(m)\,,$$
$$\left[G_x(m)\right]_{j,j'} = \frac{\partial g_j(x)}{\partial x_{j'}}\bigg|_{x=m}\,,$$

The μ_L is output average, S_L is output variance and C_L is input-output covariance [16-19].

Please note that, due to the complexity of calculation in the relations of the EKF and the UKF and because of the three-dimensional DEM that needs curve fitting, a simple model of plane is selected for simulink in MATLAB.

Also, we should again emphasize that x and y axes on DEM do not have any dimensions and are digital pixels.

4.1 Extended Kalman Filter Method (EKF)

In this section, a state space presentation of time invariant nonlinear system has been considered:

$$\begin{aligned} x(n+1) &= \phi(x(n)) + \Gamma w(n), \\ z(n) &= \gamma(x(n)) + v(n)\,, \end{aligned} \qquad (4.1)$$

where, Γ is a matrix, input dynamics are linear and independent of $x(n)$ and operators $\phi(x)$ and $\gamma(x)$ are non-linear functions of x and n, as:

$$\ddot{o}(x) = \begin{bmatrix} \ddot{o}_1(x) \\ \ddot{o}_2(x) \\ \vdots \\ \ddot{o}_m(x) \end{bmatrix} \qquad \tilde{a}(x) = \begin{bmatrix} \tilde{a}_1(x) \\ \tilde{a}_2(x) \\ \vdots \\ \tilde{a}_p(x) \end{bmatrix}$$

Also, assume that x has m state and z has p state. Kalman filter considers a linear SMM; therefore, the first step is linearization of SMM which has been previously discussed. Also, we have assumed that $\phi(x)$ and $\gamma(x)$ are smooth enough in x. Thus, each of them has a valid Taylor expansion.

If $\hat{x}(n) = x(n)$, Taylor expansion of ϕ around $\hat{x}(n)$ would be as follows:

$$\phi(x(n)) = \phi(\hat{x}(n)) + J_\phi(\hat{x}(n))\left[x(n) - \hat{x}(n)\right] + \cdots \ , \qquad (4.2)$$

$$J_{\ddot{o}}(x) = \frac{\partial \ddot{o}}{\partial x} = \begin{bmatrix} \frac{\partial \ddot{o}_1}{\partial x_1} & \frac{\partial \ddot{o}_1}{\partial x_2} & \cdots & \frac{\partial \ddot{o}_1}{\partial x_N} \\ \frac{\partial \ddot{o}_2}{\partial x_1} & \frac{\partial \ddot{o}_2}{\partial x_2} & \cdots & \frac{\partial \ddot{o}_2}{\partial x_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \ddot{o}_m}{\partial x_1} & \frac{\partial \ddot{o}_m}{\partial x_2} & \cdots & \frac{\partial \ddot{o}_m}{\partial x_N} \end{bmatrix}.$$

Similarly, Taylor expansion of γ around $\hat{x}^-(n)$ is as follows:

$$\gamma(x(n)) = \gamma(\hat{x}^-(n)) + J_\phi(\hat{x}^-(n))\left[x(n) - \hat{x}^-(n)\right] + \cdots$$

We consider only the first two terms of expansions in the above expression. Their result will be a first order estimation of ϕ and Γ that are linear estimations of x.

So, new results for linearized SMM are:

$$\begin{aligned} x(n+1) &= \phi(\hat{x}(n)) + J_\phi(\hat{x}(n))\left[x(n) - \hat{x}(n)\right] + \Gamma w(n) \\ z(n) &= \gamma(\hat{x}^-(n)) + J_\gamma(\hat{x}^-(n))\left[x(n) - \hat{x}(n)\right] + v(n) \end{aligned}$$

(4.3)

Of course, the above SMM depends on $\hat{x}(n)$ and $\hat{x}^-(n)$ estimation that will be found in the next steps.

In short, EKF algorithm steps are as follows:

1. Prediction:

$$\begin{aligned} P^-(n+1) &= J_\phi\left(\hat{x}(n)\right)P(n)J_\phi^T\left(\hat{x}(n)\right) + \Gamma Q(n)\Gamma^T \\ \hat{x}^-(n+1) &= \phi\left(\hat{x}(n)\right) \end{aligned} \qquad (4.4)$$

2. Updating:

$$K(n) = P^{-}(n)J_{\gamma}^{T}(\hat{x}^{-}(n))\left[J_{\gamma}(\hat{x}^{-}(n))P^{-}(n)J_{\gamma}^{T}(\hat{x}^{-}(n)) + R(n)\right]^{-1}$$
$$\hat{x}(n) = \hat{x}^{-}(n) + K(n)\left[z(n) - \gamma(\hat{x}^{-}(n))\right]$$
$$P(n) = P^{-}(n) - K(n)J_{\gamma}(\hat{x}^{-}(n))P^{-}(n) \quad (4.5)$$

3. Increasing frequency:

The value of n increases and the process goes back to first step.

If we assume filtering as a transform for the distribution function of the input and output of the system that leads to the same conception of unscented transformation to achieve the UKF, we would have a new EKF relationship:

$$x_k = f(x_{k-1}, k-1) + q_{k-1} \qquad q_{k-1} \approx N(0, Q_{k-1})$$
$$y_k = h(x_k, k) + r_k \qquad r_k \approx N(0, R_k) \quad (4.6)$$

1. Prediction:

$$m_k^{-} = f(m_{k-1}, k-1)$$
$$P_k^{-} = F_X(m_{k-1}, k-1)P_{k-1}F_X^{T}(m_{k-1}, k-1) + Q_{k-1} \quad (4.7)$$

2. Updating:

$$V_k = y_k - h(m_k^{-}, k) \quad (4.8)$$
$$S_k = H_X(m_k^{-}, k)P_k^{-}H_X^{T}(m_k^{-}, k) + R_k$$
$$K_k = P_k^{-}H_X^{T}(m_k^{-}, k)S_k^{-1}$$
$$m_k = m_k^{-} + K_k V_k$$
$$P_k = P_k^{-} - K_k S_k K_k^{T}$$

$$F_X(m, k-1) = \left.\frac{\partial f_j(X, k-1)}{\partial X_{j'}}\right|_{X=m} \quad (4.9)$$
$$H_X(m, k) = \left.\frac{\partial h_j(X, k)}{\partial X_{j'}}\right|_{X=m}$$

Disadvantages of EKF Methods:

Disadvantages of EKF Methods are as follows:

1. The EKF will cause errors because of its linear nature and, without proper linearization, rapidly leads to divergence.
2. The EKF does not have any derivation for non-analytic functions and, therefore, its implementation is not feasible when the system dynamics or observation function is non-analytic and the Hessian or Jacobian matrix is not available.
3. This method is effective for flying objects with low manoeuvrability like cruise missiles but it is not effective for flying objects with higher manoeuvrability. In fact, the UKF method is better for these objects, and
4. In the EKF method, by changing the initial values of the program and measuring noise variance in different runs of the program, different estimations would result which will be discussed in the simulation section. Thus, the EKF is not a robust method for estimation and, instead, the UKF method is recommended. [13.20].

4.2 Unscented Kalman Filter Approach (UKF)

The UKF filter, based on unscented transformation and sigma points selection, leads to fewer errors because there is no need to calculate Jacobian or Hessian matrix and also it is possible to calculate higher order moments in comparison with Taylor transformation in the EKF.

In fact, we get a lot of information by using a few points in this method which is considered another advantage for this approach. In this transformation, we consider the system as deterministic and add noise to the states of the system by an augmentation and consider them all as a new state with a known variance and mean value. Thus, the model is not stochastic anymore and there is no need for Gaussian noise to be considered a limitation to the EKF. In this method, unlike the particle filter method, samples are not selected randomly. Samples (or the sigma points) are selected so that they could have a Gaussian distribution.

In this method, also known as Sigma Point Kalman Filter (SPKF), the state variable is redefined as a combination of the main state and noise variable.

Figure 4.1 demonstrates the advantage of the UKF to the EKF in getting correct information out of the variance and the average of the output: [16, 17, 18, 19].

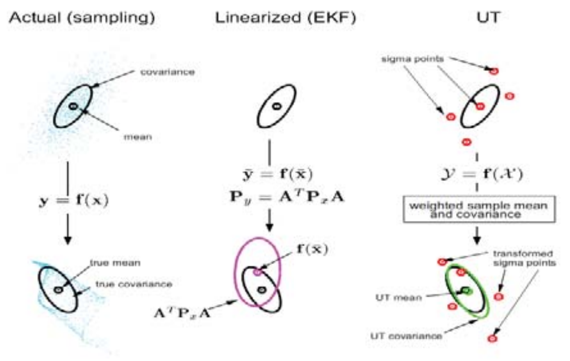


Figure 4-1. Display of unscented transformation performance

As it is clear from the above figure, the mean and variance in the EKF are not the same as what should occur under y = f (x) while this adaptation occurs exactly in the UKF. If we write the unscented transformation matrix formula, we will have:

$$
\begin{aligned}
&X = [m \;\; \ldots \;\; m] + \sqrt{C}\left[0 \;\; \sqrt{P} \;\; -\sqrt{P}\right] \qquad C = a^2(n+k)\\
&Y = g(X)\\
&m_u = YW_m\\
&S_u = YWY^T\\
&C_u = XWY^T\\
&w_m = \left[W_m^{(0)} \;\; \ldots \;\; W_m^{(2n)}\right]^T\\
&W = \left(I - [w_m \;\; \ldots \;\; w_m]\right) \times \mathrm{diag}\left(W_C^{(0)} \;\; \ldots \;\; W_C^{(2n)}\right)\\
&\times\left(I - [w_m \;\; \ldots \;\; w_m]\right)^T\\
&W_m^{(0)} = \frac{\lambda}{n+\lambda} \qquad W_m^{(i)} = \frac{\lambda}{2(n+\lambda)} \qquad i = 1 \ldots 2n\\
&W_C^{(0)} = \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta) \qquad W_m^{(i)} = \frac{1}{2(n+\lambda)}
\end{aligned}
\tag{4.10}
$$

and the UKF formula based on this transformation will be as follows:

1. Prediction:

$$
\begin{aligned}
&x_{k-1} = \left[m_{k-1} \;\; \ldots \;\; m_{k-1}\right] + \sqrt{C}\left[0 \;\; \sqrt{P_{k-1}} \;\; -\sqrt{P_{k-1}}\right]\\
&\hat{x}_k = f(x_{k-1}, k-1)\\
&m_k^- = \hat{x}_k w_m\\
&P_k^- = \hat{x}_k \; W \; \hat{x}_k^T + Q_{k-1}
\end{aligned}
\tag{4.11}
$$

2. Updating:

$$
\begin{aligned}
&x_k^- = \left[m_k^- \;\; \ldots \;\; m_k^-\right] + \sqrt{C}\left[0 \;\; \sqrt{P_k^-} \;\; -\sqrt{P_k^-}\right]\\
&Y_k^- = h(x_k^-, k)\\
&m_k = Y_k^- w_m\\
&S_k = Y_k^- \; W \; (Y_k^-)^T + R_k\\
&C_k = x_k^- W (Y_k^-)^T
\end{aligned}
\tag{4.12}
$$

3. Calculation to obtain K:

$$
\begin{aligned}
&K_k = C_k S_k^{-1}\\
&m_k = m_k^- + K_k(y_k - m_k)\\
&P_k = P_k^- - K_k S_k K_k^T
\end{aligned}
\tag{4.13}
$$

5 Assumptions of the Problem

Generally, the governing equations are:

$$
\begin{aligned}
&X(n+1) = A(n)X(n) + B(n)w(n)\,,\\
&z(n) = H(n)X(n) + v(n)\,,
\end{aligned}
$$

$$
\begin{array}{ll}
X(n) & \text{State}\\
z(n) & \text{Observation}\\
v(n) & \text{White Observation Uncertainty},\\
w(n) & \text{White System driving Uncertainty},\\
X(0) & \text{Initial condition},
\end{array}
\tag{5.1}
$$

$$
\begin{aligned}
&E\{v(n_1)v^T(n_2)\} = \begin{cases} R(n_1) & n_1 = n_2\\ 0 & n_1 \neq n_2 \end{cases},\\
&E\{w(n_1)w^T(n_2)\} = \begin{cases} Q(n_1) & n_1 = n_2\\ 0 & n_1 \neq n_2 \end{cases}\\
&E\{x(0)x^T(0)\} = y,\; E\{x(0)\} = 0,\; E\{w(0)\} = 0,\; E\{v(0)\} = 0
\end{aligned}
$$

For the non-manoeuvring plane, assuming linear dynamics for the plane and nonlinear observations, i.e. the mapping between measured elevation by INS and input of extended Kalman filter (vertical and horizontal position of the plane), dynamic equations of the system will be as follows:

Time-continuous dynamic equations are:

$$\dot{x}(t) = A_C x(t) + B_C u(t) + w(t)$$
$$z(t) = h(x(t)) + v(t)$$

$$A_C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B_C = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \tag{5.2}$$

$$H = \frac{\partial h(x(t))}{\partial x(t)} = \begin{bmatrix} h_1 & 0 & h_2 & 0 \end{bmatrix}.$$

Discrete-time dynamic equations are:

$$x(n+1) = Ax(n) + Bu(n) + w(n), \qquad z(n) = h(x(n)) + v(n), \tag{5.3}$$

$$\overline{x}(n) = \begin{bmatrix} x(n) & \dot{x}(n) & y(n) & \dot{y}(n) \end{bmatrix},$$

$$A = (I + DA_C) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times \Delta,$$

$$= \begin{bmatrix} 1 & \Delta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B = \int e^{A_C t} B_c dt = \int (I + DA_C) B_C dt = \int AB_C dt =$$

$$\int \begin{bmatrix} 1 & \Delta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\Delta^2}{2} & 0 \\ \Delta & 0 \\ 0 & \frac{\Delta^2}{2} \\ 0 & \Delta \end{bmatrix}.$$

where, $\omega(n)$ is a white noise with zero mean and variance matrix of Q, as:

$$Q = \begin{bmatrix} \sigma^2_{\omega_{c_x}} / T & 0 \\ 0 & \sigma^2_{\omega_{c_y}} / T \end{bmatrix}$$

where, $v(n)$ is observation noise with a mean of zero and variance of R. Here, unlike classical problems in which H is assumed to $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ show the vertical and horizontal position measurements, H is defined $\begin{bmatrix} h(1) & 0 & h(2) & 0 \end{bmatrix}$ as where h (1) and h (2) are the outputs of the linearized function h (x (t)). This is due to the type of linearization applied in this issue and the input of the algorithm in which the noise location is considered as:

$$v(n) = v_h,$$
$$R = \sigma^2_{v_{c_x}} / T.$$

The initial conditions are considered as below:

$$\hat{x}^-(0) = \begin{bmatrix} 1000 & 100 & 1000 & 100 \end{bmatrix},$$
$$P^-(0) = \mathrm{diag}(\begin{bmatrix} 1000 & 10 & 1000 & 10 \end{bmatrix}).$$

5.1 Flight Test Physical Conditions

To examine the software abilities, all features of the software were applied on a DEM model in the north of Tehran (according to a table specification).

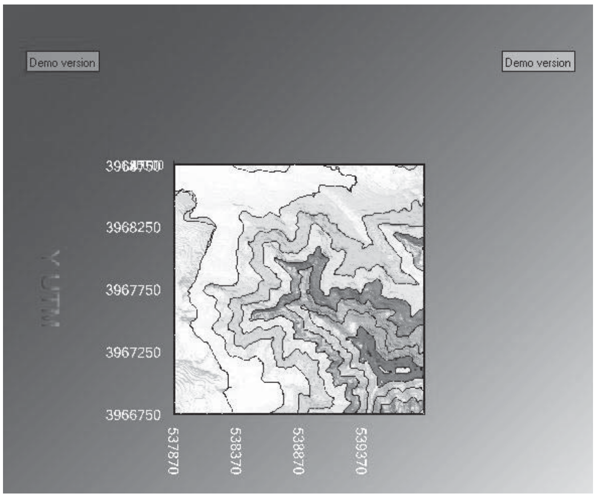


Figure 5-1. The DEM used in flight test.

Table 5.1. DEM specification.

Dimension		kilometer 2*2
Row Number		200
Column Number		200
Resolution		meter 10*10
X UTM	Southwest Coordinate of DEM	537870
Y UTM	Southwest Coordinate of DEM	3966750
ZoneUTM		North 39
Roughness(sigma-t)		107.13
region		Mountainous

Using the auto sample feature of the simulation software and applying it on the DEM sample, 199 height data related to the flight path on DEM on the reference map were produced.

Table 5.2. Sampling parameter.

Maximum Error Of INS IN 100 km	±250 m
Plane Velocity	0.8 mach
Distance of Previous Point	200 km
Distance to Next Point	100 km
Sample rate	38 ms
Flight Heading	West-East
Flight Height	100 m
Maximum DEM Error	± 7 m
Radar Altimeter Maximum Error	3% Flight Height
Barometric Altimeter Maximum Error	3% Flight Height

5.2 The Necessity and Stages of TAN Algorithm Implementation

To use the terrain navigation system, the designer should determine the flight route and reference map. Route designer can choose the map in a way that the missile moves over the maps in line with rows or columns. Also, map intervals should not be large because of the INS error.

Therefore, as shown in Fig.7-2, before the flight, a number of reference map plans (DEM) of the terrain height in certain regions are selected and stored with a proper format in the plane computer system. Then, during the flight and after the initializing, the missile is fired at point 4 and using the navigation system and TAN, it would be directed through the middle points of 5 - 12 to the final point (point 13). In fact, along this route, in some intervals between the middle points, INS and TAN data are integrated and, in this way, the drift of navigation system will be corrected.

Various kinds of maps used in TAN differ in length, width, cells and dimensions. The size of cells indicates the accuracy of TAN algorithm. Hence, the first map used in TAN is the largest plan and as the missiles come closer to the goal, sizes of maps will be smaller and the intervals will be shorter. On the other hand, decreasing the size of cells improves the accuracy of TAN algorithm.

6 Simulation Results

6.1 Effects of the EKF Method on the Mountain DEM

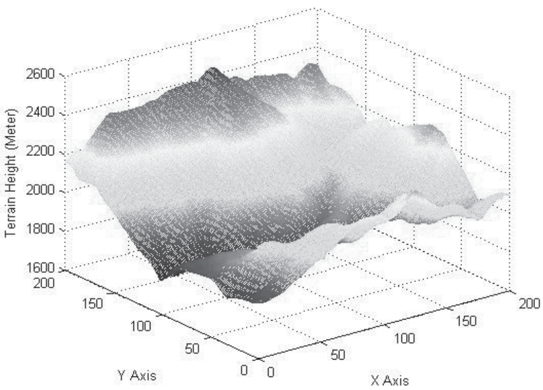


Figure 6-1. DEM mountain view .

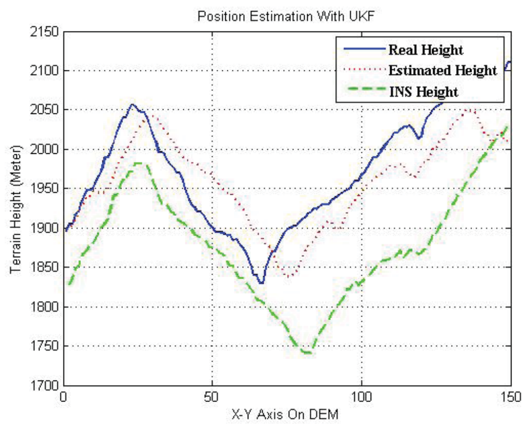


Figure 6-2. Display of actual, noisy and estimated height.

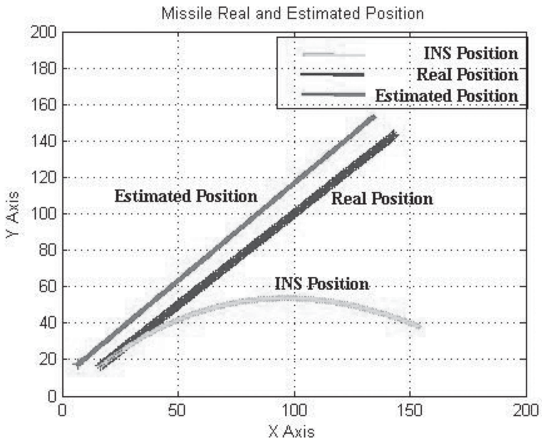
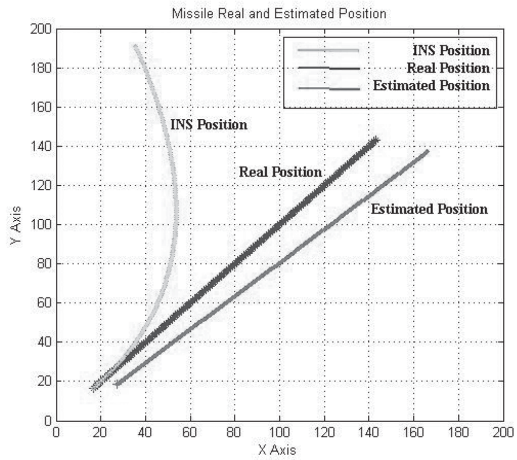


Figure 6-3. Display of the main, noisy and estimated path.



6-6. Display of the main, noisy and estimated path

6.2 Effects of the EKF Method on the Knoll DEM

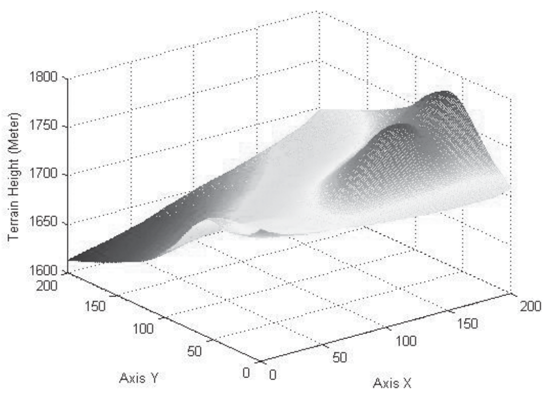


Figure 6-4. Display of Knoll DEM .

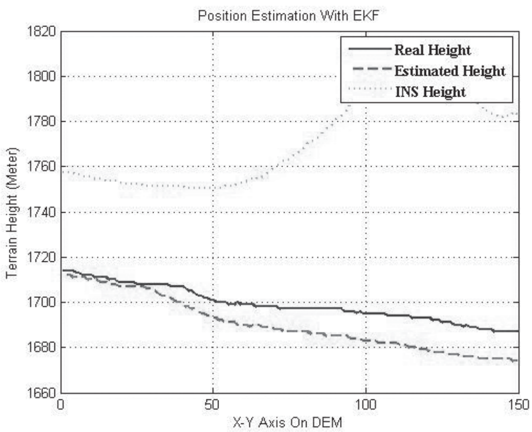


Figure 6-5. Display of actual, noise and estimated height.

6.3 Results of UKF Method on the Mountain DEM

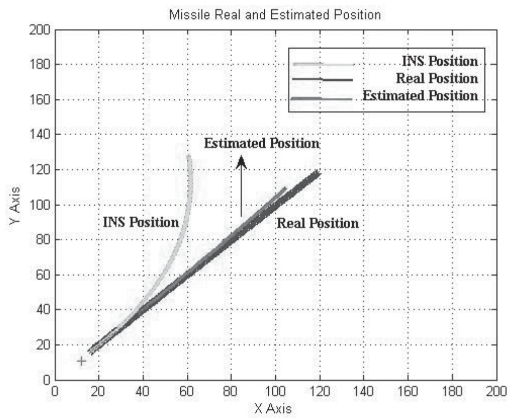


Figure 6-7. Display of the main, Figure 6-7 Display of the main,

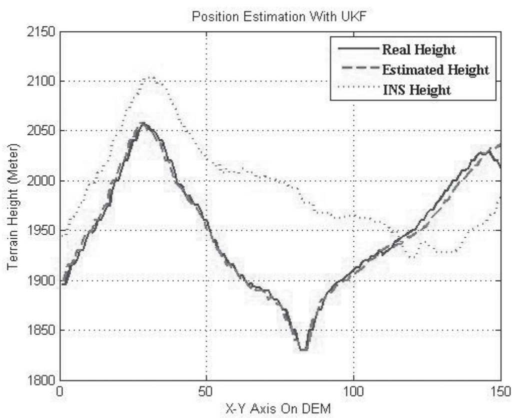


Figure 6-8. Display of actual, noisy and estimated height.

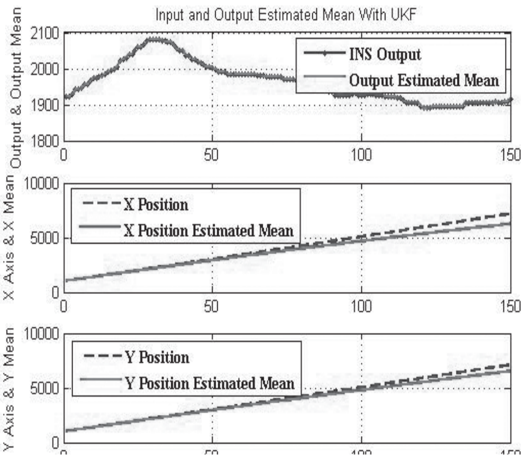


Figure 6-9. Display of actual and estimated input/output mean.

6.4 Effects of the UKF Method on the Knoll Hill DEM:

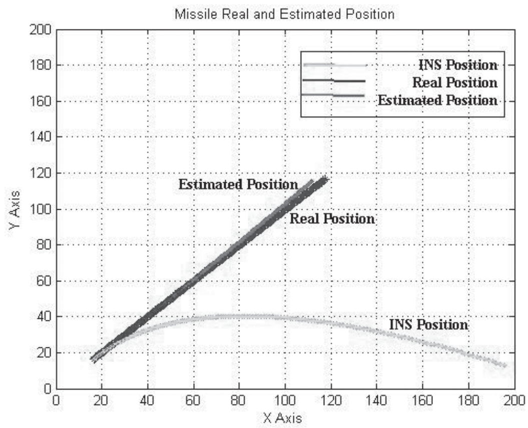


Figure 6-10. Display of the main, noisy and estimated path.

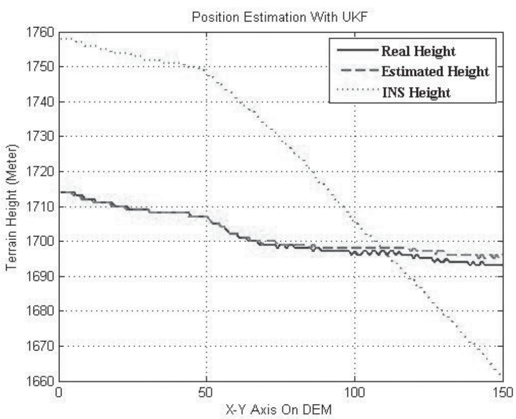


Figure 6-11. Display of actual, noisy and estimated height.

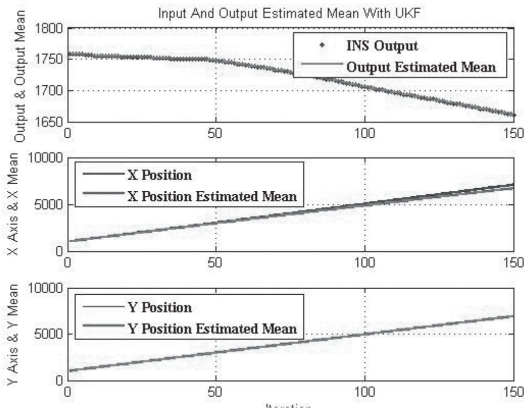


Figure 6-12. Display of actual and estimated input output mean.

9 Conclusion

From the results obtained in this research work, the superiority of the UKF to the EKF method in flying objects' position estimation becomes clear. Certainly, the EKF may be able to estimate the position of the flying objects with low manoeuvrability and is helpful to improve the INS estimates, but it does not work for the flying objects with low manoeuvrability. Meanwhile, in parts of the map where roughness is low and the terrain is flat, the EKF leads to errors due to linearization while this does not occur in the UKF.

Thus, the UKF as a recursive method with less computational complexity than the PF [9, 15, 21, 22] can be widely used as a robust aided navigation system with INSs. As a final point, it needs be noted that this article may not have a strong theoretical contribution to science but, owing to its significant practical applications in industrial manufacturing, it may be favoured over other similar research works.

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