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Midcourse Trajectory Shaping for Air and Ballistic Defence Guidance, Using Bezier Curves

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A near-optimal midcourse trajectory shaping guidance algorithm is proposed for both air and ballistic target engagement mission attributes for a generic long range interceptor missile. This guidance methodology is based on the maximum final velocity as the objective function and maximum permissible flight altitude as the in-flight state constraint as well as the head-on orientation as the terminal state constraint for anti-ballistic trajectory. The guidance algorithm utilizes a combination of the Generalized Vector Explicit Guidance or GENEX guidance with the Bezier curve-generating functions. Nominal Bezier curves are fitted by choosing control points intuitively. Waypoints are then selected on the curve to divide it to suitable portions according to the curve's length and curvature. These waypoints are then fed into the GENEX guidance law. To avoid acceleration command jumps, an algorithm is designed to switch to the next waypoint at a distance from the current currently-approaching waypoint. To provide near-optimality and meet the in-flight and terminal constraints, all the guidance algorithm parameters including Bezier control points, waypoints, switching distances and the GENEX law gain are optimized using Genetic Algorithm by setting the mentioned cost function and constraints. Simulation results show better performance compared to nominal trajectories while ensuring the flight altitude constraint for air target and head-on orientation for ballistic target.

NOMENCLATURE

$\hat{e}$	unit vector
h	altitude
$k_1,\ k_2$	GENEX guidance law gains
n	Bezier curve's order
R	position vector
r_s	waypoint switching range
s	Bezier independent variable
t	time
t_g	time to go
U	acceleration command vector
V	velocity vector
v	velocity magnitude
x	downrange
γ	flight path angle
η	GENEX guidance law independent constant

Subscripts:

0	initial
CP	control point
f	final
i	stage
M	Missile
$nLOS$	normal to Line of Sight
PIP	Predicted Intercept Point
T	Target
TM	Target-Missile
WP	waypoint

Superscripts:

$\bar{(\,)}$	average value

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$(\dot{\ })$ time derivative

INTRODUCTION

Various guidance design techniques have been proposed in the literature for the implementation of optimal or near-optimal midcourse guidance laws. In [1], an open-loop solution to midcourse guidance is presented. In [2], an analytical closed-loop explicit guidance is formulated with the definition of flight curvature as the cost function, and optimal gains are derived through simplified optimal control problem solution called Kappa guidance. In [3], a singular perturbation theory is used to extract a near-optimal midcourse guidance. This technique divides the original flight dynamics into fast and slow dynamics which prompts real-time solution but affects the optimality due to the dynamic degradation. In [4], Neural Networks are utilized to obtain midcourse guidance. Numerical solution results are used to train the network which provides a near-optimal guidance. Its weakness is that this method may fail due to extrapolations. In [5], Virtual Sliding Target (VST) approach is proposed. In this method, a proportional Navigation (PN) guidance is used to guide the missile to a virtual moving target that is approaching the real target at a constant pace. In [6] and [7], waypoint guidance algorithms are used for the midcourse phase of cruise missiles. In [6], the guidance algorithm consists of waypoints, line-following guidance and switching points. By setting the desired waypoints and drawing the connecting lines, a line-following Linear Quadratic Regulator (LQR) guidance pursues these connecting lines. Switching points from the current line to the next are then achieved by a minimum acceleration optimal control problem solution. In [7], waypoint guidance is based on an impact-angle-constraint optimal guidance law which is an LQ optimal control problem with minimum control effort cost function and terminal impact angle constraint. Other methods in this research area are modified PN [8], modified Generalized Collision Course (GCC) with super elevation consideration [9] and PN with gravity bias [10] are also discussed.

Traditionally, midcourse guidance was formulated as an optimal control Two Point Boundary Value Problem (TPBVP) to shape the trajectory for these 3 objectives: maximum final velocity, minimum flight time or maximum range, adequately discussed in [11]. The most applied one among these objectives is the maximum final velocity which is also of concern in this work. Most of the mentioned works, though different in technique, have the same general performance: propelling the missile to higher altitudes in the first portion of the flight and then diving back to a desired final point which is often the Predicted Intercept Point or PIP. The principle behind this performance is that in higher altitudes where air is thinner, the missile faces less drag force than in lower altitudes and can save its kinetic energy (or equivalently its velocity) at the end of the midcourse phase [12]. The deficiency of these techniques appears in their limited down range since, for longer down ranges, they force the missile to reach higher altitudes where aerodynamic surfaces cannot steer the missiles properly due to very low air density.

Another issue of concern in midcourse guidance is the interception of ballistic targets. In Anti-Ballistic Missiles or ABM guidance laws, it is often desired that the terminal phase start in a head-on or near head-on orientation which brings about flight path angle constraint at the end of the midcourse phase [13,14,15]. It means that the midcourse phase must end in specified target-missile geometry as well as the missile's maximum final velocity. Modern air defence systems are capable of engaging both air and ballistic targets, which enforces the modern midcourse guidance laws to be able to meet both missions.

In this paper, we first introduce GENEX guidance and Bezier curves briefly, and then combine these two elements to generate our new guidance technique. Different Bezier curve fitting constraints for both air and ballistic targets are discussed. Finally, the parameters of the proposed guidance law are optimized using Genetic Algorithm (GA) Toolbox of MATLAB software.

GENEX GUIDANCE LAW

The base of our guidance technique is Generalized vector Explicit guidance (GENEX). This guidance law can simultaneously control the final position and velocity vectors. The GENEX guidance is parameterized in terms of a design constant that controls the curvature of the flight trajectory. GENEX is not originally designed for midcourse guidance application, but its trajectory shaping characteristics are utilized for following the desired flight curves. The vector form of the guidance law may be written as [16]:

$$U = \frac{1}{t_g^2}[k_1(R_f - R_M - V_M t_g) + k_2(V_f - V_M)t_g] \quad (1)$$

where U is acceleration command, t_g is time-to-go and k_1 and k_2 are defined as:

$$k_1 = (\eta + 2)(\eta + 3)$$

$$k_2 = -(\eta + 1)(\eta + 2) \quad (2)$$

where η is the mentioned design constant. Since there is no control on the final velocity of rocket-propelled missiles, the second term of Eq. 1 must be modified as:

$$U = \frac{1}{t_g^2}[k_1(R_f - R_M - V_M t_g) + k_2 v_M(\hat{e}_f - \hat{e}_M)t_g] \quad (3)$$

BEZIER CURVES

Definition of Bezier function

Bezier Curves are commonly used in generating smooth flexible curves. However, its applications in midcourse trajectory shaping are scant. In addition to flexibility and simplicity, Bezier Curves' special characteristic is that they can be expressed in a single function as compared to Spline Curves [15]. A Bezier function may be expressed as:

$$\begin{aligned} t(s) &= \sum_{i=0}^{n} \binom{n}{i} (1-s)^{n-i} s^i t_i = (1-s)^n t_0 + \binom{n}{1}(1-s)^{n-1} s t_1 + \cdots + s^n t_n \\ R(s) &= \sum_{i=0}^{n} \binom{n}{i} (1-s)^{n-i} s^i R_i = (1-s)^n R_0 + \binom{n}{1}(1-s)^{n-1} s R_1 + \cdots + s^n R_n \\ s &\in [0,\, 1] \end{aligned} \quad (4)$$

where s is the Bezier independent variable, t is time, n is the Bezier curve's order and R is the position vector. In order to combine the Bezier function into the GENEX guidance law, velocity vector is needed. Differentiation of Eq. 3 with respect to time yields:

$$\begin{aligned} V(t) &= \dot{R}(t) = \tfrac{A}{B} \\ A &= \sum_{i=0}^{n} \binom{n}{i} (1-s)^{n-i-1} s^{i-1} [i - sn] t_i \\ B &= \sum_{i=0}^{n} \binom{n}{i} (1-s)^{n-i-1} s^{i-1} [i - sn] R_i \end{aligned} \quad (5)$$

To fit the Bezier curve, we need the initial and final points of the curve and some Control Points (CP) to shape the curve. The initial point is at the origin (launch point) and the control points are defined by the designer. Assuming the final point to be at [100000, 5000]m, a typical 3^{rd} order curve with 2 arbitrary control points can be fitted as shown in Figure 1.

A typical 4^{th} order Bezier curve with a final point located at [25000, 17000]m and some 3 arbitrary control points are shown in Figure 2.

We will use the 3^{rd} order curve for air target and the 4^{th} order for ABM guidance strategies. Dealing with more complex, long-distance or curved flight paths, one can divide the flight path into several portions. Bezier curves can then be fitted to each of these portions with tangency condition at the junctions.

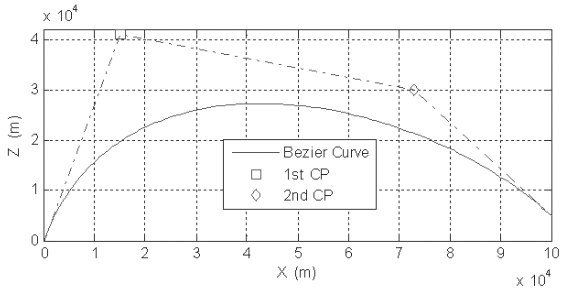

Figure 1. 3^{rd} order Bezier curves with Control Points.

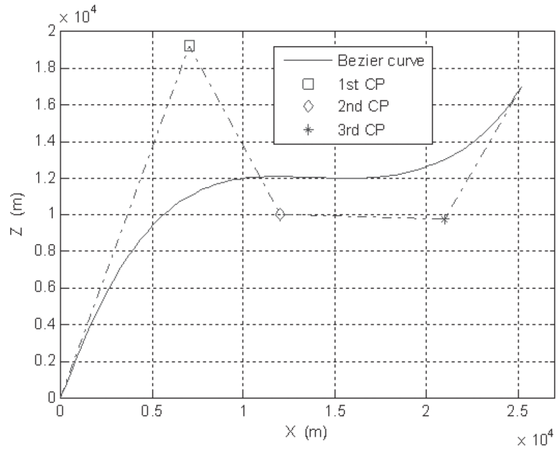

Figure 2. 4^{th} order Bezier curves with Control Points.

Therefore, multiple-tangent Bezier curves can bring forth much more flexible and curved flight paths such as flat cruise, dive-attack, agile turn, terrain following and avoidance for cruise missiles, smart munitions and UAVs and orbit injection for launch vehicles. The last flight path has been proposed in the literature [17].

GENEX GUIDANCE BASED ON BEZIER CURVE

General Guidance Law Formulation

To combine GENEX guidance with Bezier curve, we must first generate the curve based on target-missile relative information. Assuming the final point of the midcourse trajectory to be PIP, one can derive this point by the following expression:

$$R_{PIP} = R_T + V_T t_g \quad (6)$$

where R_T and V_T are target position and velocity vectors, respectively, and R_{PIP} is the PIP position vector. The following expression is suggested in [9] for t_g:

$$t_g = \frac{|R_{TM}|}{-V_T \cdot \hat{e}_{TM} + \sqrt{\bar{v}_M^2 - v_{T_{nLOS}}^2}} \quad (7)$$

where R_{TM} is target-missile vector, $\hat{e}_{TM}$ is the target-missile unit vector, $\vec{v}_M$ is average missile velocity and

$v_{T_{nLOS}}$ is target velocity component normal to target-missile line of sight. Now with the definition of initial and final points of Bezier curve and choosing some control points, we can fit a Bezier curve. We next need to choose some points on the generated curve as midpoints or waypoints. We will use these waypoints to keep the missile on the curve. We choose 3 waypoints with approximately equal distances to divide the whole curve to 4 quarters as shown in Figure 3.

GENEX law now is used to guide the missile on each curve quarter. The 4-stage guidance law based on Bezier curve is as follows:

$$U_i = \frac{1}{t^2_{g_{WP_i}}} \left[k_1(R_{WP_i} - R_M - V_M t_{g_{WP_i}}) \right. \\ \left. + k_2 v_M (\hat{e}_{\dot{R}_{WP_i}} - \hat{e}_M) t_{g_{WP_i}}\right] \\ i = 1,\ 2,\ 3,\ 4 \tag{8}$$

where U_i is the acceleration command for i^{th} stage, R_{WP_i} and $\dot{R}_{WP_i}$ are position and velocity vectors of i^{th} waypoint and $t_{g_{WP_i}}$ is the time-to-go to the next waypoint. For each quarter, the missile is guided to the next waypoint. The first term of the guidance rule weighted with k_1 carries the missile to the next waypoint while the second weighted with k_2 gives the desired flight trajectory. The guidance law will switch to the next command stage when the missile reaches the incoming waypoint or a proper distance before reaching it to prevent command jump. Note that choosing more waypoints and consequently more command stages may mean more accurate curve tracking. However, simulation results show 3 waypoints, *i.e.* 4 command stages have enough accuracy while also avoiding numerous command discontinuities which degrade missile performance. Missile velocity and waypoint velocity may not be of the same magnitude since there is no axial acceleration control in solid rocket missiles. Therefore, assuming waypoints' velocity vectors to be the same as those of the missile in magnitude, the guidance law can be rewritten as:

$$U_i = \frac{1}{t^2_{g_{WP_i}}} \left[k_1(R_{WP_i} - R_M - V_M t_{g_{WP_i}}) \right. \\ \left. + k_2 v_M (\hat{e}_{WP_i} - \hat{e}_M) t_{g_{WP_i}}\right] \\ i = 1,\ 2,\ 3,\ 4 \tag{9}$$

where U_i is missile velocity magnitude and $\hat{e}_{WP_i}$ and $\hat{e}_M$ are velocity unit vectors of waypoint and missile, respectively. Final guidance rule of GENEX law based on Bezier curve is:

$$U = \begin{cases} U_1 & R_0 < R_m < R_{WP_1} \\ U_2 & R_{WP_1} < R_m < R_{WP_2} \\ U_3 & R_{WP_2} < R_m < R_{WP_3} \\ U_4 & R_{WP_3} < R_m < R_{WP_4}(R_f) \end{cases} \tag{10}$$

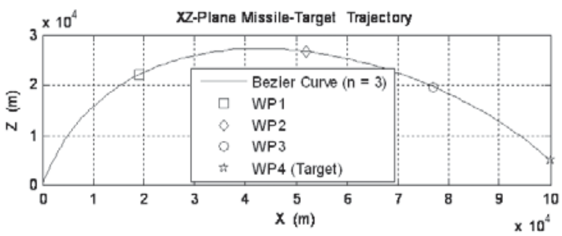

Figure 3. Chosen waypoints on a 3^{rd} order Bezier curve.

Air target midcourse strategy

In midcourse guidance strategy against distant air target, we should restrict the maximum allowable altitude as mentioned before. This altitude can be derived according to missile flight control system characteristics. We choose a maximum permitted altitude of 27500m for our typical missile. Another issue in curve-based guidance which has to be accounted for is the launch angle. Launch orientation in modern air and ballistic defence systems is vertical or highly-elevated. We choose a launch angle of 70 degrees (highly-elevated) for the missile launcher. To exert the launch angle constraint on the Bezier curve, the first control point of the curve may be chosen so that:

$$\frac{h_{CP_1} - h_{CP_0}}{x_{CP_1} - x_{CP_0}} = \tan(70 \times \frac{\pi}{180}) \tag{11}$$

where x and h are horizontal and vertical components of the control points, respectively.

Ballistic target midcourse strategy

Engagement envelope for ABM guidance is much smaller than that of air targets. This is due to limited missile performance and target's high speed. Therefore, the maximum allowable altitude limitation does not matter in this case. Instead, the head-on orientation at the end of the midcourse phase is the governing constraint in ABM midcourse guidance. Head-on constraint trajectory shaping usually results in different curves depending on the ballistic target's initial horizontal offset from the origin (missile launch point).

In this research, we focus on the case that ballistic target's predicted ground impact point is ahead of launch site and not behind it. For this case, a 4^{th} order Bezier curve can properly generate the required trajectory as shown in Figure 2. A 3^{rd} order Bezier curve cannot give the necessary curvature.

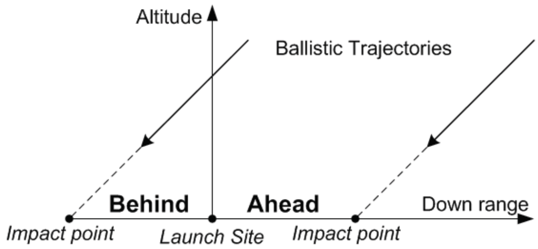

Figure 4. Ballistic trajectories relative to launch site.

After re-entry, ballistic missiles' flight path angle does not change much and can be assumed constant [14]. The re-entry angle considered by guidance designers ranges from as small as 30 degrees [18] to as large as 80 degrees in the literature [19]. A re-entry angle of 60 degrees is chosen for this research which is a compromise between these two limits. We can now exert the head-on orientation constraint by defining the last (3^{rd}) control point as:

$$\frac{h_{PIP} - h_{CP_3}}{x_{PIP} - x_{CP_3}} = \tan(60 \times \frac{\pi}{180}) \qquad (12)$$

This assures that the missile's flight path is 60 degrees at the end of the midcourse phase.

SIMULATION

To evaluate the proposed guidance law, we perform computer simulation for both scenarios. For air target simulation, we suppose a stationary target at [100000, 5000]m which means the target and PIP are at the same position during the flight simulation. We implement the guidance law on a 6-Degree-Of-Feedom missile simulation for better evaluation. The same typical 3^{rd} order Bezier curve of Figure 1 is used for simulation. As shown in Figure 5, the missile has accurately followed the typical Bezier curve.

For ballistic target simulation, we consider a re-entry target with a constant speed of 1100m/s at [50000, 60000]m. The equivalent equation for this

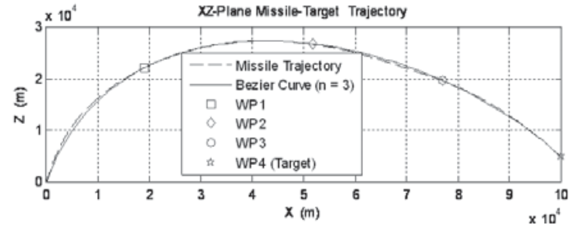

Figure 5. Flight trajectory and Bezier curve.

Figure 6. Command and achieved acceleration time histories.

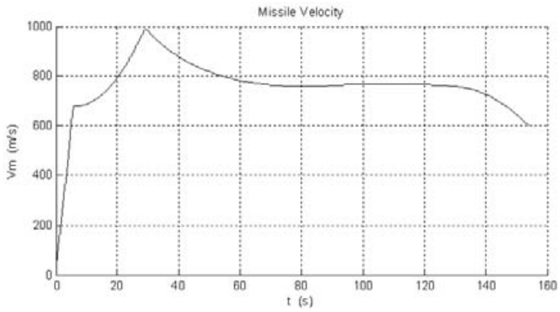

Figure 7. Velocity time history.

assumption can be simply written as:

$$R_T - R_{T0} = \int_{t_0}^{t_f} V_T \, dt$$

$$where \begin{cases} V_T = cte = [-v_T \cos\gamma_T, -v_T \sin\gamma_T] \ (m/s) \\ v_T = 1100 \ (m/s), \ \gamma_T = 60 \ (\text{deg}) \\ R_{T0} = [50000, 60000] \ (m) \end{cases} \qquad (13)$$

This speed is much greater than the missile's average speed which is about 750m/s. The same typical 4^{th} order Bezier curve of Figure 2 is used for simulation.

In this scenario, although the curve is rougher than the air target midcourse trajectory, the missile properly follows the typical 4^{th} order Bezier curve.

OPTIMIZATION

Optimization includes both Bezier curve and Guidance law parameters. These parameters are defined in Table 1.

r_s is the range-to-go to the incoming waypoint on which the command switches to the next stage.

The reason for considering a wide range of variation for Bezier curve control points is their special characteristics whose large variance may cause little change in the resulted curves. So this wide range was necessary to make the Bezier as flexible as possible. Also for r_s, large amounts were necessary for air target trajectory.

Cost function for both cases is the maximum final velocity and constraints for either scenario are

Table 1. Optimization parameters.

Type of parameter	Air Defence Guidance rule		ABM Defence Guidance rule	
	Optimization parameter	Range	Optimization parameter	Range
Bezier Curve	x_{CP_i} $i = 1, 2$	$(0\ ,\ 100000)\ m$	x_{CP_i} $i = 1, 2, 3$	$(0\ ,\ 35000)\ m$
	h_{CP_i} $i = 1, 2$	$(0\ ,\ 40000)\ m$	h_{CP_i} $i = 1, 2, 3$	$(0\ ,\ 40000)\ m$
Guidance rule	r_{s_j} $j = 1, 2, 3$	$(100\ ,\ 6000)\ m$		
	η ($GENEX$ law constant)	$(0\ ,\ 2)$		

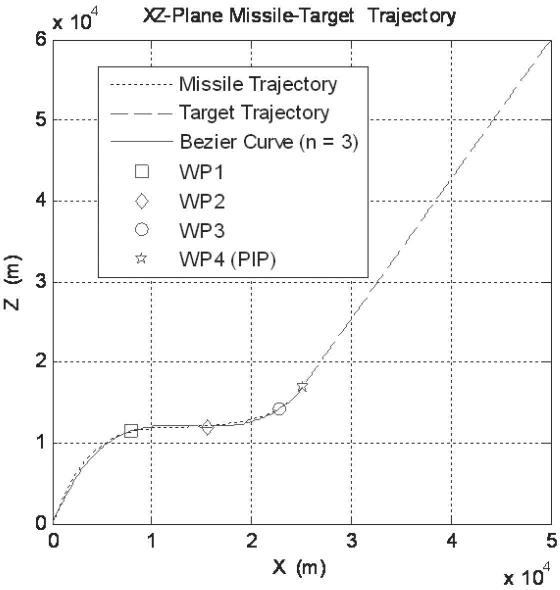

Figure 8. ABM flight trajectory and Bezier curve.

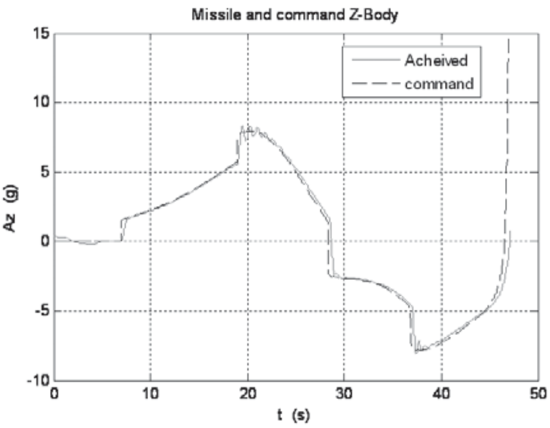

Figure 9. Command and achieved acceleration time history.

as discussed above. Optimization is performed by GA Toolbox of MATLAB Software.

Table 2 shows the optimization result for air target midcourse guidance, compared to the typical Bezier curve with non-optimal (nominal) guidance rule parameters. Note that the typical Bezier curve itself has met the altitude constraint and, therefore, is close to near-optimal characteristics due to its intuitive

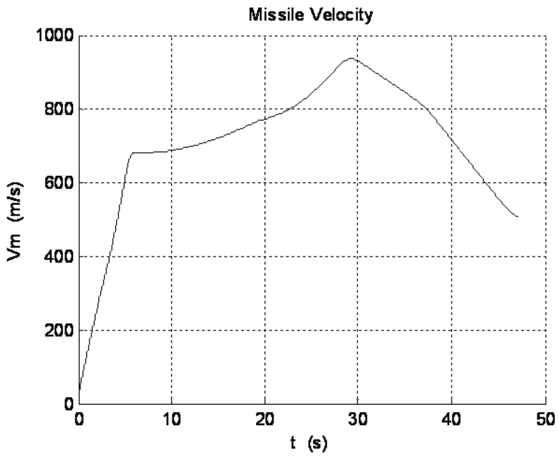

Figure 10. Velocity time history.

trajectory shaping attributes. Nevertheless, the optimization shows 7% increase in missile performance.

Table 3 shows the optimization result for ballistic target midcourse guidance, compared to the 4^{th} order typical Bezier curve with non-optimal guidance rule parameters.

Table 4 shows the optimized values for guidance parameters as well as the typical value for nominal trajectories for both air and ballistic target scenarios. Although it may be thought that zero η is optimal

Table 2. Optimization result for air target midcourse guidance.

Bezier curve	Guidance rule	$v_f\ (\frac{m}{s})$
typical	nominal	611.01
optimized	optimized	654.85

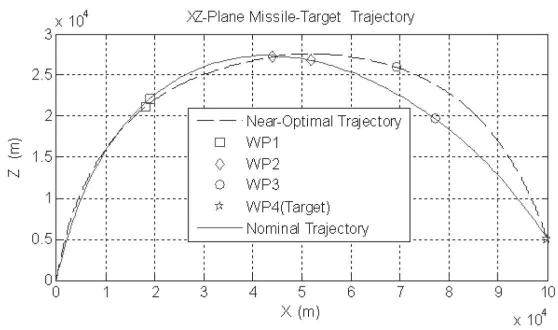

Figure 11. Comparison of near-optimal and nominal missile trajectories.

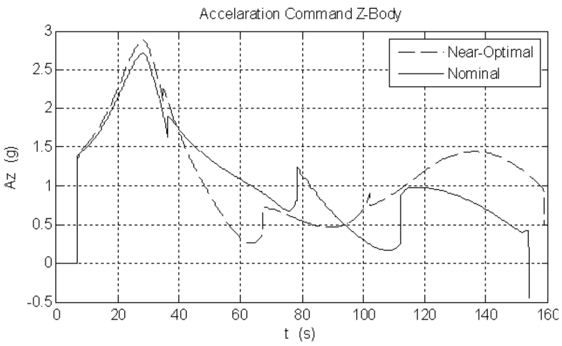

Figure 12. Comparison of near-optimal and nominal command acceleration.

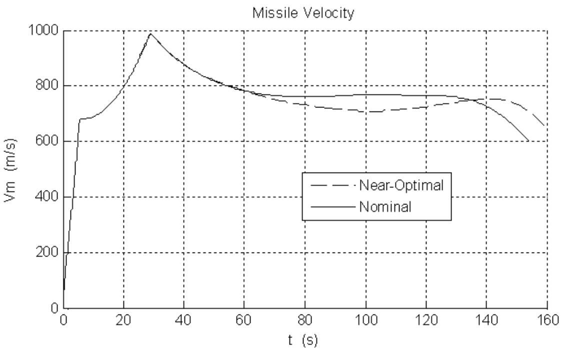

Figure 13. Comparison of near-optimal and nominal missile velocity.

in terms of control effort, extensive simulation results as well as the current optimization results of table 4 showed non-zero but small (*e.g.* less than 0.5) η gives better performance. This may be due to the variation of air density, sound speed or other parameters with altitude which affects aerodynamic coefficients (in the 6-DOF simulation) and consequently the missile performance which may cause the time-average optimal η to shift slightly from zero for the entire flight time.

Nevertheless, it was found through massive simulations and optimizations that our performance index is not very sensitive to the variation of η (at most 5%) in the sensitivity analysis point of view.

At the end, it is enlightening to mention the reason why the simulation results were not compared to PN guidance which is commonly regarded by guidance designers. The reason is that PN guidance and its simple derivatives, such as gravity compensation PN, could not reach the ranges discussed in this paper for air target due to the missile's limited aerodynamic performance, and PN's disability to meet the head-on constraints for a ballistic target. Meanwhile a combination of waypoints with PN or even other fundamental tactical laws like Line-of-Sight (LOS) guidance, by itself, is an issue of concern in both practical and theoretical research and thus comparing it with the current work is out of the scope of this paper. However, qualitatively speaking, PN guidance cannot smoothly follow the Bezier curves due to lack of trajectory shaping attributes compared to GENEX law, unless too many waypoints that will deteriorate missile performance due to consecutive command jumps and switchings are used.

Table 3. Optimization result for ballistic target midcourse guidance.

Bezier curve	Guidance rule	v_f $\left(\frac{m}{s}\right)$
typical	nominal	508.37
optimized	optimized	564.80

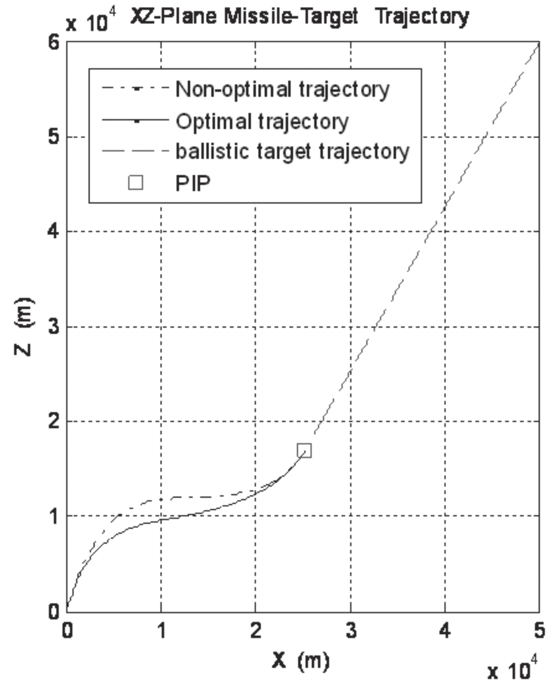

Figure 14. Comparison of near-optimal and nominal missile trajectories (ballistic target).

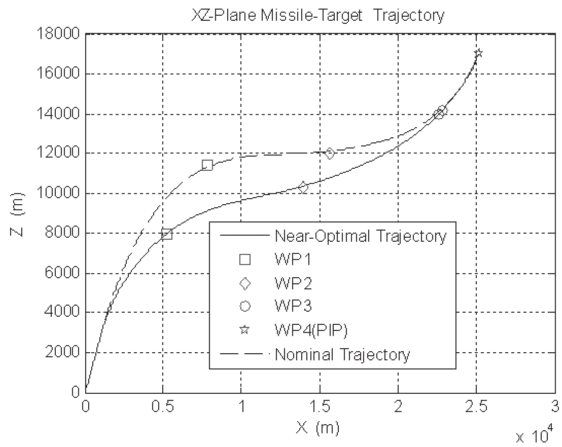

Figure 15. Larger view of near-optimal and nominal missile trajectories (ballistic target).

Table 4. Optimized and typical values.

Type of parameter	Optimization parameter	Air Defence Guidance rule		ABM Defence Guidance rule	
		Optimized	Nominal	Optimized	Nominal
Bezier Curve	x_{CP_1}	14526 m	15000 m	5237 m	7000 m
	x_{CP_2}	90454 m	73000 m	9828 m	12000 m
	x_{CP_3}	--	--	20589 m	21000 m
	h_{CP_1}	39911 m	41000 m	13738 m	19233 m
	h_{CP_2}	31529 m	30000 m	7921 m	10000 m
	h_{CP_3}	--	--	10153 m	9700 m
Guidance rule	r_{s_1}	3992 m	5000 m	2354 m	3000 m
	r_{s_2}	6452 m	5000 m	1044 m	3000 m
	r_{s_3}	3124 m	5000 m	845 m	3000 m
	η	0.15	0	0.28	0

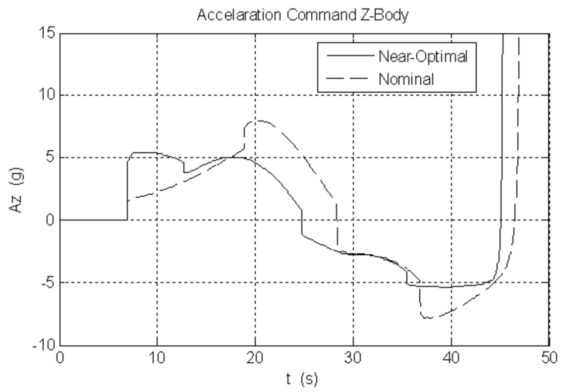

Figure 16. Comparison of near-optimal and nominal command acceleration (ballistic target).

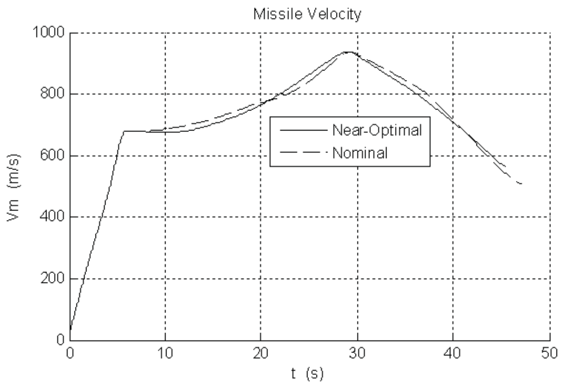

Figure 17. Comparison of near-optimal and nominal missile velocity (ballistic target).

CONCLUSION

A new midcourse guidance method is proposed for both air and ballistic target engagement. We used GENEX guidance law as the core of the guidance rule; then, we combined it with a Bezier curve to achieve trajectory shaping attributes. To improve the missile's performance, we tuned the parameters of both guidance law and Bezier curve for maximum possible final velocity in the presence of missile 6-Degree-Of-Freedom performance and the constraints. The novelty of the manuscript is the application of Bezier curves to the guidance problem. It is worth noting that there are several guidance schemes which are applicable to both air and ballistic targets. The guidance technique may also be applicable to launch vehicles, cruise missiles, UAVs, air-launched missiles and air-drop smart munitions.

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