JAST, Vol. 8, No. 2, pp 69–75

Nonlinear Guidance Law with Finite-Time Convergence Considering Control Loop Dynamics

I. Mohammadzaman[1] and H. Momeni [2]

In this paper, a new nonlinear guidance law with finite- time convergence is proposed. The second order integrated guidance and control loop is formulated considering a first order control loop dynamics. By transforming the state equations to the normal form, a finite-time stabilizer feedback linearization technique is proposed to guarantee the finite-time convergence of the system states to zero or a small neighborhood of zero before the final time of the guidance process. However, some feedback quantities such as high order LOS rate derivatives are not directly measurable; therefore, a finite-time observer is proposed to have a finite- time output feedback guidance law for more practical applications. Simulation results show the effectiveness of the proposed guidance law.

NOMENCLATURE

Variables:

a	acceleration
R	relative range of target and interceptor
T	Time
V	velocity
λ	LOS angle
τ	Time constant of autopilot

Subscripts:

M	interceptor
T	target
0	initial condition

INTRODUCTION

Proportional navigation (PN) guidance has been widely used for decades because of its implementation simplicity and its effectiveness in guiding interceptors to hit the nonmaneuvering targets [1,2]. The basic philosophy behind PN guidance is that interceptor acceleration should nullify the line-of-sight (LOS) rate.

Recently, new guidance laws have been proposed based mainly on H_∞ guidance law [3], Lyapunov-based nonlinear guidance law [4], geometric approach [5], and variable structure control [6] to meet the challenges posed by ever more agile targets. These approaches were all designed with Lyapunov theorems on asymptotic stability or exponential stability such that they had not been proved to guarantee finite-time convergence. The theoretical results only indicated that the LOS rate under the aforementioned guidance laws will converge to zero or a small neighbourhood of zero as time approaches infinity. These theoretical findings are inconsistent with practical observations. In terminal guidance the time of termination is really quite short and the guidance, law is required to ensure finite-time convergence of the LOS rate.

Recently, finite-time stability and finite-time stabilization via continuous time invariant feedback have been studied and finite-time controllers involving terms containing fractional powers have been constructed for some systems [7-9]. Finite-time stable systems might enjoy not only faster convergence but also better robustness and disturbance rejection properties. In [9], a Lyapunov stability theorem has been presented for testing finite-time stability of a double integrator system by a continuous, unbounded or bounded state feedback control law. Later, finite-time output feedback stabilizers were also derived for the double integrator using a class of nonsmooth finite-time observers [10]. For higher dimensional nonlinear systems,

1. *Ph.D., Dep't. of Electrical & Computer Eng., Tarbiat Modarres University, Tehran, Iran, Email: mohammadzaman@modares.ac.ir.*
2. *Associated Professor, Dep't. of Electrical & Computer Eng., Tarbiat Modarres University, Tehran, Iran.*

[11] proposed a continuous state feedback control law achieving local finite-time stabilization for triangular systems. [12] shows that for a lower triangular system, it is possible to achieve global finite-time stabilization by non-Lipschitez continuous state feedback.

Finite-time stability of guidance systems has been demonstrated in [13]-[15] in the sense of analyzing the stability of PN guidance law considering autopilot dynamics in a short-time. These papers cannot guarantee the convergence of the LOS rate to zero in the finite time and only analyze the stability of the guidance system with a short time stability criterion which is defined over a specified time interval. In [16], a nonlinear guidance scheme with finite- time convergence based on Lyapunov scalar differential inequality was developed for the case of ideal dynamics, *i.e.* no delays exist between the LOS rate and the applied acceleration. In an actual situation, due to flight conditions and unexpected environment, we cannot expect the ideal performance of the control system. When actual dynamics are considered, there is no guarantee for the finite-time convergence of the LOS rate and it might diverge. This divergence may severely affect the miss distance and can lead to unsatisfactory performance. To improve performance, the simultaneous design of the guidance and control loop can be used.

In this paper, a new guidance law is proposed to guarantee the finite time convergence of the LOS rate considering control loop dynamics. The integrated guidance and control loop is formulated by a nonlinear state. This equation is further changed into a normal form by coordinate transformation with respect to available states, where unavailable information is modelled as bounded uncertainties in parametric form equation [17]. For the normal form equation, a non-Lipscitz continuous state feedback guidance law is designed to guarantee the finite-time stability of the system states. Despite the assumption that some of the information in connection with the guidance law is available, higher order LOS rate derivatives are not readily measurable and they need to be estimated for more practical applications. Thus, in this paper, a finite-time observer is proposed to estimate the states in the integrated guidance and control model. In conjunction with the state feedback finite-time stabilization guidance law, our finite-time observer leads to an output feedback stabilization guidance law.

FINITE TIME STABILITY

In this section, we review some basic concepts and definitions related to the notion of finite-time stability.

Definition 2.1 [9]. Consider the system

$$\dot{x} = f(x),\ \ f(0) = 0, x \in R^n \tag{1}$$

where $f : D \to R^n$ is non-Lipschitz continuous on an open neighborhood D of the origin $x = 0$ in R^n. The equilibrium $x = 0$ of (1) is finite-time convergent if there are an open neighborhood $U \subseteq D$ of the origin and a function $T_x : U\ 0 \to (0, \infty)$, such that every solution trajectory $x(t, x_0)$ of (1) starting from the initial point $x_0 \in U\ 0$ is well-defined and unique in forward time for $t \in [0, T_x(x_0)]$, and $\lim_{t \to T_x(x_0)} x(t, x_0) = 0$. Here, $T_x(x_0)$ is called settling time (with respect to initial state x_0). The equilibrium of (1) is finite-time stable if it is Lyapunov stable and finite-time convergent. When $U = D = R^n$, the origin is a globally finite-time stable equilibrium.

Definition 2.2 [11]. Consider a controlled system

$$\dot{x} = f(x) + g(x)u, x \in R^n, u \in R^m \tag{2}$$

with $f(0) = 0$ and $g(0) \neq 0$. It is finite-time stabilizable via continuous time invariant state feedback if there is a continuous feedback law $u = \mu(x)$ such that the origin $x = 0$ of the closed loop system $\dot{x} = f(x) + g(x)\mu(x)$ is a (locally) finite-time stable equilibrium.

Theorem 2.1 [9]. Consider the nonlinear system described in (1). Suppose there are C^1 (continuously differentiable) function $V(x)$ defined in a neighborhood $\vec{U} \subset R^n$ of the origin, and real numbers $c > 0$ and $0 < \alpha < 1$, such that $V(x)$ is positive definite on $\vec{U}$ and $\dot{V}(x) + cV^{\alpha}(x)$ is negative semi-definite on $\vec{U}$. Then, the zero solution of system (1) is finite-time stable. The settling time, depending on initial state $x(0) = x_0$, is given by:

$$T_x(x_0) \leq \frac{V(x_0)^{1-\alpha}}{c(1-\alpha)} \tag{3}$$

for all x_0 in some open neighbourhood of the origin. If $\vec{U} = R^n$ and $V(x)$ is also radially unbounded (*i.e.* $V(x) \to \infty$ as $\|x\| \to \infty$), the zero solution of system (1) is globally finite time stable.

In this paper, we use the properties of homogenous systems about which some concepts will be introduced.

bfseries Definition 2.3 [7]. A function $W(x)$ is homogenous of degree $\sigma \in R$ with dilation $(r_1, ..., r_n)$, $r_i > 0$, $i = 1, ..., n$ if for all $\epsilon > 0$

$$W(\epsilon^{r_1} x_1, ..., \epsilon^{r_n} x_n) = \epsilon^{\sigma} W(x) \tag{4}$$

A vector field $f(x) = [f_1(x), ..., f_n(x)]^T$ is homogenous of degree $k \in R$ with dilation $(r_1, ..., r_n)$ if for all $\epsilon > 0$

$$f_i(\epsilon^{r_1} x_1, ..., \epsilon^{r_n} x_n) = \epsilon^{k+r_i} f_i(x), i = 1, ..., n \tag{5}$$

System (1) is called homogeneous if its vector field f is homogeneous.

Theorem 2.2 [12]. Consider the uncertain nonlinear system of the form:

$$\dot{x}_1 = x_2 + f_1(x,u,t)$$
$$\dot{x}_2 = x_3 + f_2(x,u,t)$$
$$\vdots$$
$$\dot{x}_n = u + f_n(x,u,t) \tag{6}$$

where $x = (x_1 \ldots x_n)^T \in R^n$ and $u \in R$ are the system state and input, respectively, and $f_i : R^n \times R \times R \to R$, $i = 1, ..., n$ are C^1 uncertain functions with $f_i(0,0,t) = 0$, $\forall t$. System (6) is globally finite-time stabilizable by non-Lipshcitz continuous state feedback if the following condition holds:

$$|f_i(x,u,t)| \le (|x_1| + \ldots + |x_i|)\gamma_i(x_1, \ldots, x_i), \tag{7}$$

where $\gamma_i(x_1, \ldots, x_i) \ge 0$ is a known C^1 function and there exist a set of parameters $q_1 = 1 > q_2 > \ldots > q_k := (2n+3-2k)/(2n+1) > 0$, and C^0 virtual controllers $x_1^*, \ldots, x_k^*$ and a state feedback control law of the form:

$$x_1^* = 0, \quad \varsigma_1 = x_1^{1/q_1} - x_1^{*1/q_1}$$
$$x_2^* = -\varsigma_1^{q_2}\beta_1(x_1), \quad \varsigma_2 = x_2^{1/q_2} - x_2^{*1/q_2}$$
$$\vdots$$
$$x_k^* = -\varsigma_{k-1}^{q_k}\beta_{k-1}(x_1, \ldots, x_{k-1}), \varsigma_k = x_k^{1/q_k} - x_k^{*1/q_k}$$
$$u = x_{n+1}^* = -\varsigma_n^{q_{n+1}}\beta_n(x_1, \ldots, x_n) := -\varsigma_n^{1/(2n+1)}\beta_n(.) \tag{8}$$

where $\beta_i(.) > 0, \quad i = 1, ..., n$ being C^1 functions.

Lemma 2.1 [18]. For $x \in R$, $y \in R$, and $p \ge 1$ which is an integer, the following inequalities hold:

$$(|x| + |y|)^{1/p} \le |x|^{1/p} + |y|^{1/p} \tag{9}$$

Theorem 2.3 [10]. Consider the second order system as:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u + f_0(x_1, x_2)$$
$$y = x_1 \tag{10}$$

where f_0 is any C^1 function defined in a neighbourhood of the origin and satisfies $f_0(0) = 0$ and the control law is:

$$u = \mu(\varsigma_1, \varsigma_2)$$
$$\dot{\xi}_1 = \xi_2 - k_1 |\xi_1 - y|^{\sigma_1} \operatorname{sgn}(\xi_1 - y)$$
$$\dot{\xi}_2 = v - k_2 |\xi_2 - y|^{\sigma_2} \operatorname{sgn}(\xi_1 - y) \tag{11}$$

where $\sigma_1 > 0, \sigma_2 > 0, k_1 > 0, k_2 > 0$. The zero solution of the closed loop system composed of (10) and (11) is locally finite-time stable if:

$$0 < \sigma_2 < 1, \quad 2\sigma_1 - \sigma_2 = 1 \tag{12}$$

and $\mu(x_1, x_2)$ which finite-time stabilizes the double integrator system satisfies:

$$\mu(\epsilon^{1/\sigma_1}x_1, \epsilon x_2) = \epsilon^{\sigma_2/\sigma_1}\mu(x) \tag{13}$$

MODELING OF INTEGRATED GUIDANCE AND CONTROL LOOP

In this section, an integrated model for guidance and control loop is formulated. Consider a two-dimentional interceptor and target engagement as shown in Figure 1. Proportional navigation (PN) is the guidance law that implements parallel navigation, which is defined by the rule $\dot{\lambda} = 0$ with an additional requirement $\dot{R} < 0$. The kinematic relation between the target and interceptor motion is described as:

$$R\ddot{\lambda} + 2\dot{R}\dot{\lambda} = a_t - a_m \tag{14}$$

By defining the state $X_g = \dot{\lambda}$, Eq. 14 is expressed by:

$$\dot{X}_g = a_g X_g + b_g a_m - b_g a_t \tag{15}$$

where $a_g = -2\dot{R}/R$, $b_g = -1/R$. Consider the control loop dynamic as a first order system:

$$\frac{a_m}{a_{mc}} = \frac{1}{\tau s + 1} \tag{16}$$

in which a_{mc} denotes the acceleration command from the guidance loop. Equation (16) can be expressed in a state space form as:

$$\dot{X}_c = a_c X_c + b_c u_c \tag{17}$$

where $X_c = a_m$, $u_c = a_{mc}$, $a_c = -1/\tau$, $b_c = 1/\tau$ and τ is the time constant of the control loop. The guidance loop (15) and the control loop (17) can be combined as:

$$\dot{X}_{igc} = \begin{pmatrix} a_g & b_g \\ 0 & a_c \end{pmatrix} X_{igc} + \begin{pmatrix} 0 \\ b_c \end{pmatrix} u_c + \begin{pmatrix} -b_g \\ 0 \end{pmatrix} a_t \tag{18}$$

where $X_{igc} = [X_g \ \ X_c]^T$ and $Y_{igc} = X_g$.

The guidance law should nullify the LOS rate and guarantee the finite-time convergence of the LOS rate to zero or a small neighbourhood of zero. To apply nonlinear control theory and finite time stability theorems to the integrated guidance and control loop,

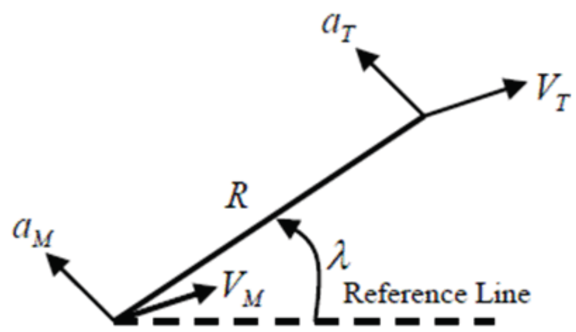

Figure 1. Interceptor and target engagement geometry.

state Eq. 18 must be transformed into a normal form. The measured output is:

$$y = Y_{igc} =: x_1 \tag{19}$$

Differentiating the output, we have:

$$\dot{x}_1 = x_2 + \Delta_1 \tag{20}$$

where

$$x_2 = a_g X_g + b_g X_c, \quad \Delta_1 = -b_g a_t \tag{21}$$

Similarly, differentiating equation (21) yields:

$$\dot{x}_2 = a_1 x_1 + a_2 x_2 + b_g b_c u_c + \Delta_2 \tag{22}$$

where

$$a_1 = \dot{a}_g - a_c a_g - \dot{b}_g a_g \Big/ b_g$$

$$a_2 = a_g + a_c - +\dot{b}_g \Big/ b_g$$

$$\Delta_2 = -a_g b_g a_t \tag{23}$$

Thus, using Eqs. (19)-(23) and $X = (x_1 \ \ \dot{x}_1)^T$, the integrated guidance and control dynamic can be presented as a normal form by:

$$\begin{array}{l} \dot{X} = \begin{pmatrix} 0 & 1 \\ a_1 & a_2 \end{pmatrix} X + \begin{pmatrix} 0 \\ +b_g b_c \end{pmatrix} u_c + \begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} \\ Y = x_1 \end{array} \tag{24}$$

Feedback linearization technique can be applied to Eq. 24 and can transform it to the standard form in which some finite- time stability theorems can be used to guarantee the finite-time convergence of the LOS rate to zero.

NONLINEAR GUIDANCE LAW

In this section, the design procedure of the nonlinear guidance law is presented for the integrated guidance and control system given by (24). Based on the preceding finite-time state feedback and output feedback stabilization theorems, we can prove sufficient conditions for the finite time stability of system (24). All parameters of Eq. 24 are not directly measurable and may have uncertainty. Therefore, we assume that we have an estimation of these parameters.

Theorem 4.1. System (24) with the non-maneuvering target ($a_T = 0$) is finite-time stable by the state feedback of the form:

$$u_c = \frac{1}{\hat{b}_g \hat{b}_c}(\hat{a}_1 x_1 + \hat{a}_2 x_2 + v(x_1, x_2)) \tag{25}$$

where $\vec{a}$, $\vec{a}$, $\vec{b}$ and $\vec{b}$ are the estimations of a_1, a_2, b_g and b_c, respectively, and $v(x_1, x_2)$ is :

$$v(x_1, x_2) = -\beta_2(x_1, x_2)\left(x_2^{1/q_2} + x_1 \beta_1(x_1)^{1/q_2}\right)^{q_3} \tag{26}$$

where $\beta_i(.) > 0, \quad i = 1, 2$ and $1 > q_2 > q_3 > 0$.

Proof. Consider that the estimation and measurement error of the integrated guidance and control dynamics parameters in equation (24) are as follows:

$$a_1 = (\hat{a}_1 \Big/ \hat{b}_g \hat{b}_c) + \tilde{a}_1$$

$$a_2 = (\hat{a}_2 \Big/ \hat{b}_g \hat{b}_c) + \tilde{a}_2$$

$$1 = (1 \Big/ \hat{b}_g \hat{b}_c) + \tilde{a}_3 \tag{27}$$

By applying the feedback linearization technique to Eq. 24 with the control input (25), the system can be transformed to Eq. 6 in theorem 2.2:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = v(x_1, x_2) + f_2(x_1, x_2) \tag{28}$$

in which the uncertain function f_2 is:

$$f_2(x_1, x_2) = \tilde{a}_1 x_1 + \tilde{a}_2 x_2 + \tilde{a}_3 v(x_1, x_2) \tag{29}$$

Using state feedback control law (8) in theorem 2.2 in system (28), we see that $v(x_1, x_2)$ is calculated as (26). Also, using theorem 2.2 makes system (28) globally finite-time stabilizable by non-Lipshcitz continuous state feedback (26) if the uncertain function f_2 satisfies condition (7). Using Eqs. 26 and 29, f_2 we can rewrite as:

$$f_2(x_1, x_2) =$$

$$\tilde{a}_1 x_1 + \tilde{a}_2 x_2 - \tilde{a}_3 \beta_2(x_1, x_2)\left(x_2^{1/q_2} + x_1 \beta_1(x_1)^{1/q_2}\right)^{q_3} \tag{30}$$

and we can write:

$$
\begin{aligned}
|f_2| &= |\tilde{a}_1 x_1 + \tilde{a}_2 x_2 + \tilde{a}_3 v| \leq |\tilde{a}_1|\,|x_1| + |\tilde{a}_2|\,|x_2| + |\tilde{a}_3|\,|v| \\
&\leq |\tilde{a}_1|\,|x_1| + |\tilde{a}_2|\,|x_2| + |\tilde{a}_3 \beta_2(x_1, x_2)| \\
&\left|\left(x_2^{1/q_2} + x_1 \beta_1(x_1)^{1/q_2}\right)\right|^{q_3} \leq |\tilde{a}_1|\,|x_1| + |\tilde{a}_2|\,|x_2| \\
&+ |\tilde{a}_3 \beta_2(x_1, x_2)| \left(\left|x_2^{1/q_2}\right| + \left|x_1 \beta_1(x_1)^{1/q_2}\right|\right)^{q_3}
\end{aligned}
\tag{31}
$$

By lemma 2.1, inequalities (31) can be rewritten as:

$$
\begin{aligned}
|f_2| \leq\ & |\tilde{a}_1|\,|x_1| + |\tilde{a}_2|\,|x_2| + \\
& |\tilde{a}_3 \beta_2(x_1, x_2)| \left(|x_2|^{q_3/q_2} + |x_1|^{q_3}\,|\beta_1(x_1)|^{q_3/q_2}\right) \\
& \leq \gamma(x_1, x_2)\,(|x_1| + |x_2|)
\end{aligned}
\tag{32}
$$

where

$$
\gamma(x_1, x_2) > \max \begin{Bmatrix} \left(|\tilde{a}_1| + \left|\tilde{a}_3 \beta_2(x_1, x_2) \beta_1(x_1)^{q_3/q_2} x_1^{q_3 - 1}\right|\right), \\ \left(|\tilde{a}_2| + |\tilde{a}_3 \beta_2(x_1, x_2)|\, x_2^{(q_3/q_2) - 1}\right) \end{Bmatrix}
\tag{33}
$$

If we choose the parameter $\gamma(x, y)$ to satisfy inequality (33), condition (7) in theorem 2.2 holds and system (24) is finite-time stable by state feedback (25).

Guidance law (25) uses the states of the plant which contains higher order LOS rate derivatives. In real application, these derivatives are not readily measurable and need to be estimated. Now, we will construct a class of output feedback finite-time stabilizing control law by considering the output feedback control law which only needs the LOS rate measurement. Then, the estimates of states and uncertainties can be used in an observer-based nonlinear guidance law to achieve the desired guidance and control performance.

Theorem 4.2. System (24) with the non-maneuvering target ($a_T = 0$) and observer (11) is finite time stable by the output feedback

$$
u_c = \frac{1}{\hat{b}_g \hat{b}_c} (\hat{a}_1 \xi_1 + \hat{a}_2 \xi_2 + v) \tag{34}
$$

if condition (12) holds for the observer parameters and $v(x_1, x_2)$ finite time stabilizes the double integrator system and satisfies condition (13). Also, the uncertain function f_2 should be a C^1 function defined in a neighbourhood of the origin and satisfies $f_2(0) = 0$.

Proof. By theorem 4.1, guidance law (25) finite time stabilizes system (24) and system (24) is transformed to double integrator system (28). By theorem 2.3, system (28) with output feedback control law (31) is finite-time stable with observer dynamic (11) by choosing the observer parameters in a manner that satisfies conditions (12) and (13) and the uncertain function f_2 in (29) is a C^1 function and satisfies $f_0(0) = 0$.

SIMULATION RESULTS

We investigate a two-dimentional interception problem. The interceptor and target initial positions are $R_{m_0} = [0 \quad 0]^T\ m$ and $R_{t_0} = [2000 \quad 1000]^T\ m$ in cartesian coordinate, respectively. Also, their initial velocities are $v_{m_0} = [169.9 \quad 107.3]^T\ m/s$ and $v_{t_0} = [-58.13 \quad 26.83]^T\ m/s$. The lag of the control loop in Eq. (16) is considered as $\tau = 1.5$ sec.

Consider guidance law (26) with $\beta_1 = 0.5$, $\beta_2 = 2$, $q_2 = 4/7$ and $q_3 = 3/7$

$$
v(x_1, x_2) = -2(x_2^{7/4} + 0.3x_1)^{3/7} \tag{35}
$$

Also, in order to satisfy conditions (12) and (13), observer parameters are chosen as $\sigma_1 = 4/7$ and $\sigma_2 = 1/7$. By assuming $k_1 = 10$ and $k_2 = 20$, the observer dynamics in Eq. (12) are:

$$
\begin{aligned}
\dot{\xi}_1 &= \xi_2 - 10\,|\xi_1 - y|^{4/7}\,\mathrm{sgn}(\xi_1 - y) \\
\dot{\xi}_2 &= v - 20\,|\xi_2 - y|^{1/7}\,\mathrm{sgn}(\xi_1 - y)
\end{aligned}
\tag{36}
$$

Therefore, the guidance law for the integrated guidance and control system is applied as Eq. (31) in which $v(x_1, x_2)$ is calculated by Eq. (32) and estimations of the states are calculated by Eq. (33). The output of the system is LOS rate which is measurable by the seeker in interceptor. We assume that the parameters estimation of system (24) has bounded parametric uncertainties.

Case 1: Suppose that the target does not maneuver. The LOS rate is plotted in Figure 2. It is clear from Figure 2 that the LOS angular rate converges to zero in finite time. Figure 3 shows the interceptor acceleration. Closing velocity between the interceptor and target is shown in Figure 4. PN-based guidance laws should nullify the LOS rate and closing velocity should be $\dot{R}$. If the LOS rate diverges from zero, closing velocity will approach zero and cause instability of the guidance system and we cannot hit the target. Therefore, it is desirable that closing velocity has not changed from its initial value and has distance from

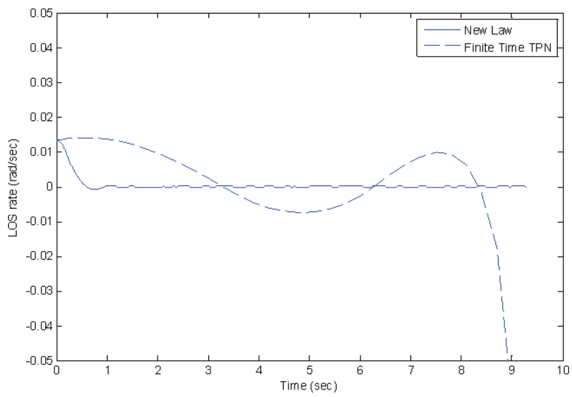

Figure 2. LOS rate in case 1.

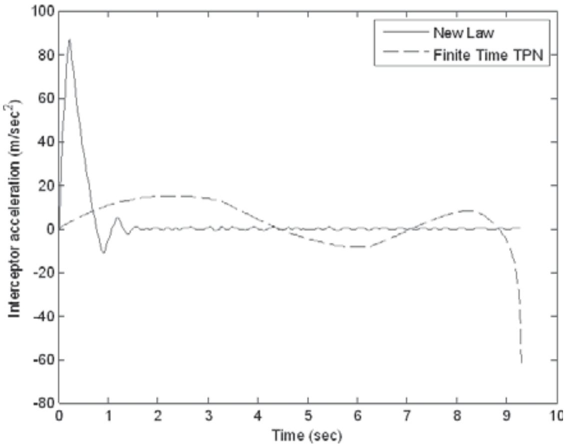

Figure 3. Interceptor acceleration in case 1.

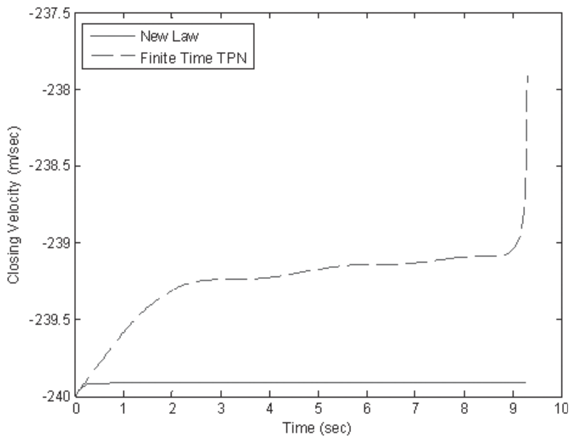

Figure 4. Closing velocity in case 1.

zero. It is clear from Figure 4 that with the new guidance law, closing velocity has not changed during the interception of the target from its initial value.

Case 2: Suppose that the target maneuver is $a_T = [8 \quad 10]^T \ m/s^2$. We apply the proposed guidance

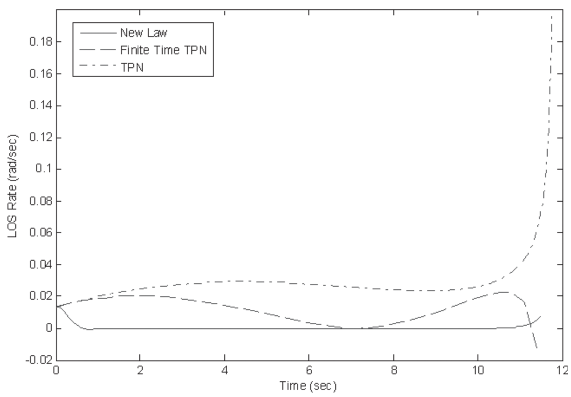

Figure 5. LOS rate with new law, finite-time TPN and TPN in case 2.

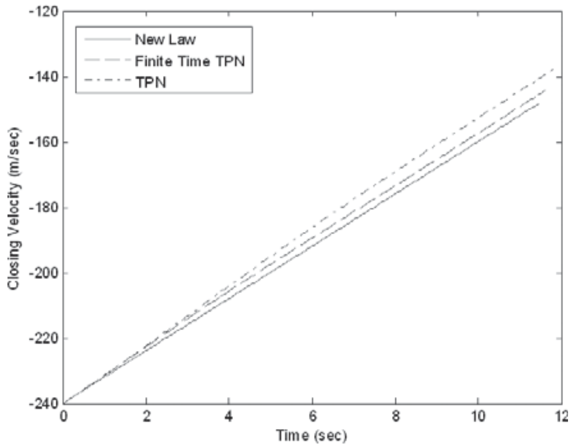

Figure 6. Closing velocity with new law, finite-time TPN and TPN in case 2.

law and compare it with true proportional navigation (TPN) guidance law, which is

$$a_{mc} = N\dot{R}\dot{\lambda} \tag{37}$$

where N is navigation constant and is chosen $N = 4$. Figure 5 shows the LOS rate between the interceptor and target with the new law and TPN. Although, we have the target maneuver, the new guidance law could keep the LOS rate near the zero and have good performance. But PN guidance laws could not nullify the LOS rate and will diverge from zero in the end of the engagement. Figure 6 shows the closing velocity between the interceptor and target.

Case 3: Suppose that the target maneuver is $a_T = [12 \quad 20]^T \ m/s^2$. Table 1 shows the result of the simulation with a proposed guidance law, finite-time TPN and TPN. The miss distance of the TPN is large and the guidance system is unstable at time 16.9 but the proposed guidance law has good performance and achieves zero miss distance. Therefore, it is clear that the proposed guidance law is robust against target maneuver. Figures 7 and 8 show the LOS rate and closing velocity between the interceptor and target with a new law and TPN.

CONCLUSIONS

In this paper, a nonlinear guidance law is proposed to improve the overall performance of guidance and control interceptor systems by applying finite-time stability theorems. The proposed guidance law guarantees the convergence of all states of the system, *i.e.* LOS

Table 1. Comparison of miss distance between new law and TPN in case 3.

	Miss Distance (m)	Time (sec)
New Law	0	13.4
Finite-Time TPN	29	18.7
TPN	140	16.9

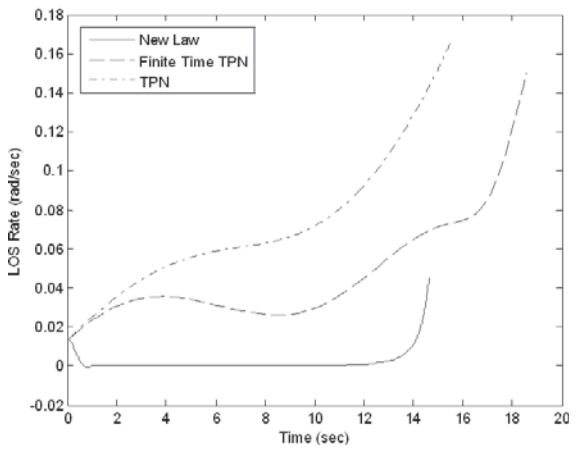

Figure 7. LOS rate with new law, finite-time TPN and TPN in case 3.

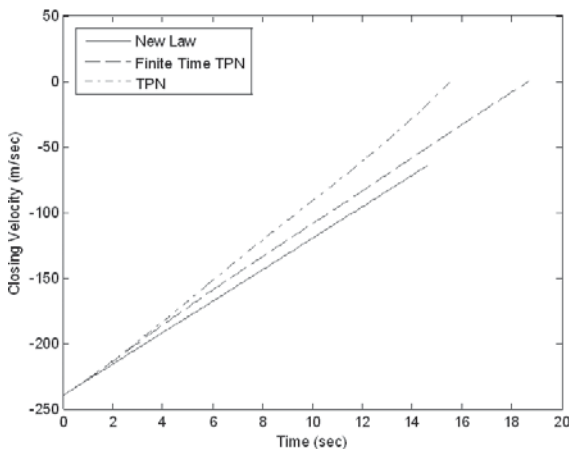

Figure 8. Closing velocity with new law, finite-time TPN and TPN in case 3.

rate, to zero or a small neighborhood of zero in finite-time. Also, a finite-time observer is proposed to estimate the high derivative of the LOS rate which cannot be measured in real applications. Therefore, a finite time output feedback guidance law is applied to a guidance and control system including parametric uncertainties. Simulation results show the effectiveness and robustness of the proposed guidance law against maneuvering targets.

REFERENCES

1. P. Zarchan, "Tactical and Strategic Missile Guidance", *AIAA Series*, **199**, (2002).
2. R. T. Yanushevsky, "Modern Missile Guidance", (2008).
3. C. D.Yang, and H. Y. Chen, "Nonlinear Hinfinity robust guidance law for homing missiles", *Journal of Guidance, Control, and Dynamics*, **21**, .
4. R. T. Yanushevsky, and W. J. Boord, "New Approach to Guidance Law Design", *Journal of Guidance, Control, and Dynamics*, **28**, PP 162-166(2005).
5. Y. C. Chiou, and C. Y. Kuo, "Geometric Approach to Three- Dimensional Missile Guidance Problem", *Journal of Guidance, Control, and Dynamics*, **21**, PP 335-341(1998).
6. J. Moon, K. Kim, and Y. Kim, "Design of Missile Guidance Law via Variable Structure Control", *Journal of Guidance, Control, and Dynamics*, **24**, PP 659-664(2001).
7. S. P. Bhat, and D. S. Bernstain, "Finite time stability of homogeneous systems", *American Control Conference*, PP 2513-2514(1997).
8. S. P. Bhat, and D. S. Bernstain, "Continues finite time stabilization of the translational and rotational double integrators", *IEEE Transaction on Automatic Control*, **43**, PP 678-682(1998).
9. S. P. Bhat, and D. S. Bernstain, "Finite time stability of continues autonomuse systems", *SIAM Journal on Control and Optimization*, **38**, PP 751-766(2000).
10. Y. Hong, J. Huang, and Y. Xu, "On an output feedback finite-time stabilization problem", *IEEE Transaction on Automatic Control*, **46**, PP 305-309(2001).
11. Hong, Y, "Finite-time stabilization and stabilizability of a class of controllable systems", *Systems & Control letters*, **46**, PP 231-236(2002).
12. X. Huang, W. Lin, and B. Yang, "Global finite-time stabilization of a class of uncertain nonlinear systems", *Automatica*, **41**, PP 881-888(2005).
13. D.Y. Rew, M.J. Tahk, and H. Cho, "Short Time Stability of Proportional Navigation Guidance Loop", *IEEE Transaction on Aerospace and Electronic Systems*, **32**, PP 1107-1115(1996).
14. P. Gurfil, M. Jodorkovsky, and M. Guelman, "Finite Time Stability Approach to Proportional Navigation Systems Analysis", *Journal of Guidance, Control, and Dynamics*, **21**, PP 853-861(1998).
15. I. Mohammadzaman, H.R. Momeni, "Short Time Stability Approach to Guidance Law Design", *Journal of Space Science and Technology*, **6**, PP 37-44(2010).
16. D. Zhou, and S. Sun, "Guidance Laws with Finite Time Convergence", *Journal of Guidance, Control, and Dynamics*, **32**, PP 1838-1846(2009).
17. D. Chwa, and J. Y. Choi, "Adaptive nonlinear guidance law considering control loop dynamics", *IEEE Transaction on Aerospace and Electronic Systems*, **39**, PP 1134-1143(2003).
18. C. Qian, and W. Lin, "Non-Lipschitz continues stabilizer for nonlinear systems with uncontrollable unstable linearization", *Systems & Control letters*, **42**, PP 185-200(2001).