

Design of Nonlinear Robust Controller and Observer for Control of a Flexible Spacecraft

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Two robust nonlinear controllers along with a nonlinear observer have been developed in this study to control a 1D nonlinear flexible spacecraft. The first controller is based on dynamic inversion, while the second one is composed of dynamic inversion and \mathcal{D} -synthesis controllers. The extension of dynamic inversion approach to flexible spacecraft is impeded by the non-minimum phase characteristics when the panel tip position is taken as the output of the system. To overcome this problem, the controllers are designed by utilizing the modified output re-definition approach. It is assumed that only one torque on the hub is used. In particular, the assumption that all state variables are measurable is not realistic; hence, sliding mode observers are used to estimate states. Actuator saturation is considered in the design of controllers. The performances of the proposed controllers are compared in terms of nominal performance, robustness to uncertainties, vibration suppression of panel, sensitivity to measurement noise, environment disturbance and nonlinearity in large maneuvers. To evaluate the performance of the proposed controllers, an extensive number of simulations on a nonlinear model of the spacecraft are performed. Simulation results show the ability of the proposed controller in tracking the attitude trajectory and damping panel vibration. It is also verified that the perturbations, environment disturbance and measurement errors have only slight effects on the tracking and damping responses.

INTRODUCTION

Flexible-body attitude control is one of the most widely studied application areas within nonlinear control theory, largely because of its importance in robotics and spacecraft applications. The equations that govern attitude maneuvers and attitude tracking are nonlinear and coupled; thus, the attitude control system must consider these nonlinear dynamics.

A common method to control space vehicles is to use a linear controller calculated for the linear approximation of the nonlinear system around an operating point. This method is largely used due to the fact that for linear systems, there are plenty of well

established control techniques and the design can be done in a more systematic way than in the nonlinear case. Nevertheless, this kind of control technique works, in general, only in a small neighborhood of the operating point where the linear approximation is valid. Thus, when the system is far from this point, the linear controller will not behave as desired.

In the context of nonlinear systems, the feedback linearization seems to be a viable choice since the nonlinear system is exactly transformed into a linear system (valid for the entire operating region) and only then the linear controller is applied. Therefore, the dynamic range of the closed-loop system is increased. However, the classical feedback linearization, suffer from the lack of robustness in the presence of uncertainties, disturbances and noise.

In [1], the problem of attitude recovery of flexible spacecraft with the plate type appendages using feedback linearization approach is investigated. The controller ability is shown in the recovery maneuver and panel vibration suppression. However, the

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performance is only tested for impulse disturbance (thruster effect) linearization is robust against impulse disturbance but is very weak against constant disturbances. Although, this method achieves good vibration suppression, it does not address the issue of robustness to combined uncertain conditions (several uncertain conditions, *i.e.* environment disturbance, sensor noise and uncertain parameters, exist together or one uncertain condition exists with large variations). Moreover, the selected controller bound is large as if actuator saturation has not been considered.

Recently, considerable efforts have been made to design robust control systems for simultaneous attitude control and vibration suppression of flexible spacecraft. However, most of them are based on linear control approach which results in a poor performance for large maneuvers. For instance, in [2], an experimental flexible arm serves as test bed to investigate the efficiency of the Δ -synthesis design technique in controlling flexible manipulators. In [3], the active optimal attitude control of a three-axis stabilized spacecraft by flywheels is studied. The corresponding time-varying linear quadratic regulators (LQR) are designed for an approximate system.

Various nonlinear robust control algorithms have been proposed on rigid spacecraft such as a mixed H_2/H_∞ controller incorporating a cerebellar model articulation controller learning method [4], adaptive fuzzy mixed H_2/H_∞ [5], adaptive mixed H_2/H_∞ [6] and LMI [7], where the neural networks, fuzzy or adaptive methods are employed to approximate the unknown nonlinear characteristics at the system dynamics. However, they didn't consider other uncertainties such as sensor noise and environment disturbances.

Although nonlinear robust control methods, such as nonlinear H_∞ control can be applied to address these issues, solving the associated Hamilton-Jacobi equation is often extremely complicated and the resulting controller is not easy to implement. Consequently, a robust feedback linearization strategy seems promising.

In [8], an adaptive feedback linearizing control law is derived for the trajectory control of the pitch angle. Unmodeled parameters appearing in the inverse feedback linearization control law are estimated using a high gain observer. However, other uncertainties such as sensor noise and environment disturbances have not been considered. In [9], a hybrid control scheme with variable structure and intelligent adaptive control method are used to control flexible space structures.

The most common approach to compensate for the nonlinear dynamics of a rigid spacecraft is the so-called inverse dynamics strategy. However, the extension of this approach to flexible spacecraft is impeded by the non-minimum phase characteristics when the panel tip position is taken as the output of the system. To overcome this problem, in [10], a re-defined

output on the flexible-link manipulators between the joint and the tip was suggested. The new output is defined so that the zero dynamics related to this output are stable. However, this method has never been used on spacecraft. In [11], the performance of neural network-based controllers is presented for tip position tracking of flexible-link manipulators. The controller is designed by utilizing the modified output re-definition approach. The scheme is developed by using a modified version of the [feedback-error-learning] approach to learn inverse dynamics of the flexible manipulator.

The combination of dynamic inversion and μ -synthesis controller is used in high-angle-of-attack control of a super maneuverable vehicle [12], aircraft flight control over large ranges of angle of attack [13], a modern fighter aircraft incorporating thrust vectoring [14], and the lateral acceleration control of a tactical missile model [15].

The combination of dynamic inversion controller and a robust controller have been applied to the non-minimum phase system. H_∞ input/output linearization formulation is applied to a nonlinear ship course-keeping control problem [16] and in the lateral acceleration control of a tactical missile model [17]. However, this composite control method has never been used on spacecraft.

The objective of this paper is to propose this composite approach for robust attitude control and vibration suppression of flexible spacecraft. μ -synthesis control law is formulated so that an outer-loop linear controller can be constructed to provide robust stability/performance against the inexact dynamic cancellation arising in the inner-loop feedback linearization design. It is notable that the proposed composite controller has not been applied to spacecraft yet.

In this paper, attitude control of a 1D flexible spacecraft is considered using two approaches: dynamic inversion and composition of dynamic inversion and μ -synthesis. The goal is attitude control and panel vibration suppression in the absence of damping and actuators on panels.

The controllers are designed by utilizing the modified output re-definition approach. It is well known that the zero dynamics of a flexible spacecraft associated with the panel deflection is unstable. Hence, the sum of the attitude angles and a scaling of the tip elastic deformation are chosen as the output. This scale is obtained by stabilizing zero dynamics of the system.

In the design of dynamic inversion controller, this summation is considered as the output. To enforce the position and rate saturation limit, a feedback controller structure is used in the inner loop. Moreover, it is often the case that the linearized model is different from the linear model. Hence, choosing weighting functions is very challenging. Another important issue in designing the μ -synthesis controller is bounding

the linear controller term which is different from the bound for the actual control signal u . Hence, it is crucial to find an appropriate weighting function for the linear controller. To evaluate the performance of the proposed controllers, a set of simulations are performed on a 1D flexible spacecraft. It was our intention that the sensors noises, disturbances and uncertainty be as close as possible to practical situations.

Up to now, we have assumed that all the states are available. However, in general, not all of the states are measurable and the feedback control scheme should be implemented via the estimated states.

FLEXIBLE SPACECRAFT EQUATION

The system under investigation consists of a rigid hub and 2 appendages attached to it. According to Figure 1, each appendage has linear density (mass per unit length) ρ , length l each, and is attached at a distance r from the hub.

The kinetic energy of the system is composed of kinetic energies of the hub, and the appendages. This kinetic energy can be written in the form of:

$$T = 1/2 J_h \dot{\theta}^2 + \int \rho [(r+x)^2 \dot{\theta}^2 + 2(r+x)\dot{\theta}\dot{y} + \dot{y}^2 + y^2 \dot{\theta}^2] dx \quad (1)$$

The potential energy does not include a gravity term and is just the usual potential energy of beam bending deformation of the form:

$$V = \int EI y''^2 dx \quad (2)$$

To derive the dynamic model of the described system, the assumed modes formulation of the flexible appendage dynamics is used. Flexible deflection of the appendages along the body axis is of the form:

$$y = \sum_{i=1}^N \varphi_i q_i \quad (3)$$

where q_i are modal coordinates, N is the number of the assumed modes considered, and φ_i are shape functions of the appendage deformation. The following shape function is an acceptable candidate for clamped beam:

$$\varphi_i = 1 - \cos(i\pi x/l) + 1/2(-1)^{(i+1)}(i\pi x/l)^2 \quad (4)$$

The vibration equations of motion are obtained by using the conventional form of Lagrange's equation. Substituting the kinetic and potential energy equations in the Lagrange's equation, the final form of the vibration equation is obtained:

$$J_h \ddot{\theta} + 2 \int \rho(r+x)^2 dx \ddot{\theta} + \int \rho \varphi_i q_i \sum \varphi_i q_i \ddot{\theta} + 2 \int \rho(r+x) \sum \varphi_i \ddot{q}_i + 2 \int \rho \varphi_i q_i^T \sum \varphi_i q_i \dot{\theta} = \tau \quad (5)$$

$$2 \int \rho(r+x) \sum \varphi_i dx \ddot{\theta} + 2 \int \rho \varphi_i \sum \varphi_i \ddot{q}_i + 2 \int EI \varphi_i'' \sum \varphi_i'' \ddot{q}_i = \tau \quad (6)$$

The above equation may be rewritten in a simple form:

$$\begin{bmatrix} J + q^T M_{qq} q & M_{\theta q} \\ M_{\theta q} & M_{qq} \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{q} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_{qq} - \dot{\theta}^2 M_{qq} \end{bmatrix} \begin{bmatrix} \theta \\ q \end{bmatrix} + \begin{bmatrix} 2\dot{\theta} q^T M_{qq} \\ 0 \end{bmatrix} = \begin{bmatrix} \tau \\ 0 \end{bmatrix} \quad (7)$$

The modal cross-inertia vector $M_{\theta q}$, modal inertia matrix M_{qq} and modal stiffness matrix K_{qq} are defined through the shape functions.

Regarding small q , by neglecting high order term of q , this equation can be linearized as:

$$\begin{bmatrix} J & M_{\theta q} \\ M_{\theta q} & M_{qq} \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{q} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_{qq} \end{bmatrix} \begin{bmatrix} \theta \\ q \end{bmatrix} = \begin{bmatrix} \tau \\ 0 \end{bmatrix} \quad (8)$$

To include structural damping, a viscous damping term is added to equation (7-2) which results in a diagonal damping matrix D , with entries γ_1 and γ_2 for the damping parameters.

$$D = \gamma_1 M_{qq} + \gamma_2 K_{qq} \quad (9)$$

$$M_{\theta q} \ddot{\theta} + M_{qq} \ddot{q} + D \dot{q} + (K_{qq} - \dot{\theta}^2 M_{qq}) q = 0 \quad (10)$$

OUTPUT REDEFINITION APPROACH

It is well known that the zero dynamics of a flexible spacecraft associated with the panel deflection are unstable. In other words, the system is non-minimum phase and is very difficult to control using panel deflection output for feedback.

In [10], the sum of the joint angle and a scaling of the tip elastic deformation is chosen as the output for control of a flexible link manipulator, namely $y_{ai} = \theta_i + \alpha_i q_i$, where $-1 < \alpha_i < 1$. For the choice of $\alpha_i = 1$, the output becomes the tip angular position and for $\alpha_i = 0$ the output becomes the joint angle.

Also, it was shown that a critical value α_i^* , $0 < \alpha_i^* < 1$ exists such that the zero dynamics related to the new output y_a are unstable for all $\alpha_i > \alpha_i^*$ and are stable for $-1 < \alpha_i < \alpha_i^*$. Our objective in this section is to show that by using the new output y , the dynamics of the flexible spacecraft may be expressed in such a way that the feedback linearization method is applicable for controlling the system.

The flexible spacecraft with one panel and considering one elastic mode has the order of 4. Let us

define the output as $y = \theta + \alpha q$. Now, two times differentiation of y , an explicit relationship between the output y and controller input τ would be obtained. Hence, it is apparent that the system relative degree is $r = 2 < n = 4$.

Therefore, parts of the system dynamics have been rendered unobservable in this input-output linearization, the so called internal dynamics of the system, since it cannot be seen from the external input-output relationship.

Consider the dynamics of the spacecraft as given by the (7) expressed in standard state space form:

$$\dot{x} = f(x) + g(x)u \quad (11)$$

The new set of states can be defined by $X = [\theta \ \dot{\theta} \ q \ \dot{q}]$. Choosing the state vector as X , the corresponding vector fields f and g can be written as:

$$f(x) = \begin{bmatrix} \dot{\theta} \\ S \\ \dot{q} \\ J \end{bmatrix}$$

$$\begin{aligned} S &= A_{\theta q}(M_{\theta q}M_{qq}^{-1}(K_{qq} - \dot{\theta}^2 M_{qq})q - 2\dot{\theta}q^T M_{qq}q) \\ J &= -M_{qq}^{-1}(M_{\theta q}A_{\theta q}((M_{\theta q})M_{qq}^{-1} + 1)(K_{qq} - \dot{\theta}^2 M_{qq})q \\ &\quad + D\dot{q} - 2\dot{\theta}q^T M_{qq}q)) \end{aligned} \quad (12)$$

$$g(x) = \begin{bmatrix} 0 & A_{\theta q} & 0 & -M_{qq}^{-1}M_{\theta q}A_{\theta q} \end{bmatrix} \quad (13)$$

$$A_{\theta q} = (J + q^T M_{qq}q - M_{\theta q}M_{qq}^{-1}M_{\theta q})^{-1} \quad (14)$$

The new output can be expressed as:

$$y = \theta + \alpha q$$

To find the external dynamics related to this new output, take $\mu_1 = \theta + \alpha q$ and $\mu_2 = \dot{\theta} + \alpha \dot{q}$. From Eq. (11), we can write:

$$\begin{aligned} \mu_2 &= \dot{\mu}_1 \\ \dot{\mu}_2 &= (1 - \alpha M_{qq}^{-1}M_{\theta q})A_{\theta q}(\tau + (K_{qq} - \dot{\theta}^2 M_{qq})q \\ &\quad - 2\dot{\theta}q^T M_{qq}q) - \alpha M_{qq}^{-1}(K_{qq} - \dot{\theta}^2 M_{qq})q \end{aligned} \quad (15)$$

The third function $\psi(x)$ is required to complete the transformation, *i.e.* to bring the dynamics to its normal form. It should satisfy the following equation:

$$\begin{aligned} L_{g_j} \psi_k &= \left(\frac{\partial \psi_k}{\partial x} \right) g_j = 0 \\ \left(\frac{\partial \psi_i}{\partial \theta} \right) A_{\theta q} + \left(\frac{\partial \psi_i}{\partial \dot{\theta}} \right) (-M_{qq}^{-1}M_{\theta q}A_{\theta q}) &= 0 \end{aligned} \quad (16)$$

One solution of this equation is:

$$\begin{aligned} \psi_1 &= q \\ \psi_2 &= M_{qq}^{-1}M_{\theta q}\dot{\theta} + \dot{q} \end{aligned} \quad (17)$$

By differentiating these functions and by using system dynamics, the internal dynamics is obtained as:

$$\begin{aligned} \dot{\psi}_1 &= \dot{q} \\ \dot{\psi}_2 &= M_{qq}^{-1}(K_{qq} - \mu_2^2 M_{qq})\psi_1 - M_{qq}^{-1}D\dot{q} \end{aligned} \quad (18)$$

It is shown that local asymptotic stability of zero-dynamics is enough to guarantee the local asymptotic stability of the internal dynamics [18]. The zero dynamics is defined to be the internal dynamics of the system when the system output is kept at zero by the input.

$$\dot{\psi}(0, \psi) = w(0, \psi) \quad (19)$$

$$y = 0 \rightarrow \dot{\theta} = -\alpha \dot{q} \quad (20)$$

Using Eqs. (17-18) and Eq. (20), it follows that:

$$(1 - \alpha M_{qq}^{-1}M_{\theta q})\ddot{\psi}_1 + M_{qq}^{-1}\psi_1 + M_{qq}^{-1}(K_{qq} - \alpha^2 \dot{\psi}_1^2)\psi_1 = 0 \quad (21)$$

By ignoring the term $M_{qq}^{-1}\dot{\psi}_1^2\psi_1$, according to small q and \dot{q} , Eq. (21) can be written as:

$$(1 - \alpha M_{qq}^{-1}M_{\theta q})\ddot{\psi}_1 + M_{qq}^{-1}\xi\dot{\psi}_1 + M_{qq}^{-1}K_{qq}\psi_1 = 0 \quad (22)$$

Since $1 - \alpha M_{qq}^{-1}M_{\theta q} > 0$, we can conclude that zero dynamics is asymptotically stable; hence, one can find the value of α .

FEEDBACK LINEARIZATION DESIGN

It is assumed that no actuator is available on the flexible beam-type appendages. It is well known that in such cases flexible beam is not linearizable and we must turn to the input-output feedback linearization (or the so-called dynamic inversion) control technique, (see Figure 2).

It is assumed that full state measurement of the system is available through attitude (*e.g.* sun sensors and gyros) and structural (*e.g.* strain gauges) sensors.

The successive differentiation process is done on the output (attitude angle) until the control signal appears:

$$y = \theta + \alpha q \rightarrow \ddot{y} = \ddot{\theta} + \alpha \ddot{q} = \nu \quad (23)$$

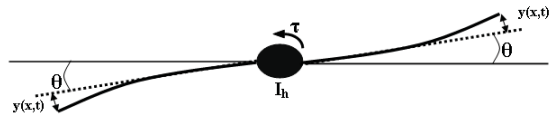


Figure 1. Flexible spacecraft model.

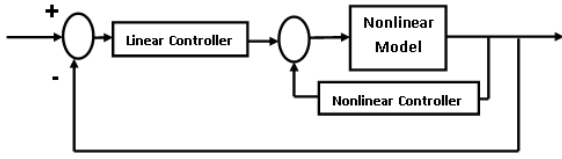


Figure 2. Feedback linearization method.

Using dynamic equations of spacecraft (7), it follows that:

$$y = (1 - \alpha M_{qq}^{-1} M_{\theta q}) A_{\theta q} (\tau + (K_{qq} - \dot{\theta}^2 M_{qq}) q - 2\dot{\theta} q^T M_{qq} q) - \alpha M_{qq}^{-1} (K_{qq} - \dot{\theta}^2 M_{qq}) q \quad (24)$$

The coefficient of τ , $A_{\theta q}$, in special case ($q = 0$) is equal to J ; in other cases, it can be shown that this term is also invertible. Hence, the signal ν should be constructed to control the linearized system. The system can be controlled by introducing linear controller of the form:

$$v = \omega_y^2 y_e - 2\zeta_y \omega_\theta \dot{y}_e \quad (25)$$

In most modern spacecraft, momentum exchange devices are used as actuators. Due to saturation effect in these actuators, taking note of saturation is very important. It has been shown by several authors that enforcing actuator constraints for input-output linearization can result in poor closed-loop performance (when compared to unconstrained closed loop performance) [19]. Different methods have been successfully used to assist in preventing the destabilizing effects of control saturations in feedback linearization method. In most cases, saturation is considered by designing a special outer loop (linear) controller; hence, these methods cannot be used in this paper. To enforce the position and rate saturation limits, feedback controller structures are used in [20-22]. Most of these structures filter the peak of the response. Simulation studies show considering the saturation in inner and outer loop together is more effective. In this paper, the structure shown in Figure 3 is used [21].

The gain can be chosen depending on the bounds of output response. In appropriate scaling, \tanh can be used to represent saturation behavior:

$$u_{sat} = \tanh\left(\frac{u}{u_{max}}\right) u_{max} \quad (26)$$

Let us define the following parameters:

$$A_1 = (1 - \alpha M_{qq}^{-1} M_{\theta q}) A_{\theta q} \quad (27)$$

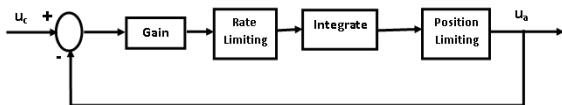


Figure 3. Enforcing Control saturation limits.

$$A_2 = (1 - \alpha M_{qq}^{-1}) M_{\theta q} A_{\theta q} (M_{\theta q} M_{qq}^{-1} (K_{qq} - \dot{\theta}^2 M_{qq}) q - 2\dot{\theta} q^T M_{qq} q) - \alpha M_{qq}^{-1} (K_{qq} - \dot{\theta}^2 M_{qq}) q \quad (28)$$

Equation (24) can be written as:

$$\ddot{y} = A_1 \tau + A_2 \quad (29)$$

Let δ be the difference between the computed control and the applied control:

$$\delta = u_c - u_a \quad (30)$$

From Eq. (29) and Eq. (30), we have:

$$\ddot{y} = A_1 (\tau + \delta) + A_2 \quad (31)$$

Then, the linearized model takes the following form:

$$\ddot{y} = v + A_1 \delta \quad (32)$$

As shown in Eq. (32), the hedge signal $A_1 \delta$ acts as a disturbance.

COMPOSITE CONTROLLER DESIGN (FEEDBACK LINEARIZATION + μ -SYNTHESIS)

The performance of feedback linearization is rather poor in the presence of uncertainty, disturbance and noise. Due to the uncertainty, inexact dynamic cancellation arises in the inner-loop feedback linearization design. Hence, a Δ -synthesis control law is added as an outer-loop linear controller. Dynamic inversion and structured singular value synthesis are combined to achieve robust control of flexible spacecraft. The controller structure is shown in Figure 2. In this method, nonlinear dynamics is linearized by input-output feedback linearization method. By definition output as $y = \theta + \alpha q$, new linear system is in the form of $\ddot{y} = \nu$, so new control signal ν should be designed.

The advantage of μ -synthesis method is that it allows the direct inclusion of modeling errors or uncertainties, measurement and control inaccuracies and performance requirements into a common control problem formulation.

Defining the two parameters A_1 and A_2 , as expressed in relations (27-29), we have:

$$\ddot{y} = A_1 \tau + A_2 = v_{real} \quad (33)$$

By considering uncertainty on parameters such as J , Eq. (33) can be written as:

$$\ddot{y} = (A_1 + \Delta A_1) \tau + (A_2 + \Delta A_2) = A_1 \tau + A_2 + \Delta A_1 \tau + \Delta A_2 = v_{real} + \Delta v \quad (34)$$

$$\Delta v = \Delta A_1 \tau + \Delta A_2 = \Delta A_1 A_1^{-1} (v_{real} - A_2) + \Delta A_2 \quad (35)$$

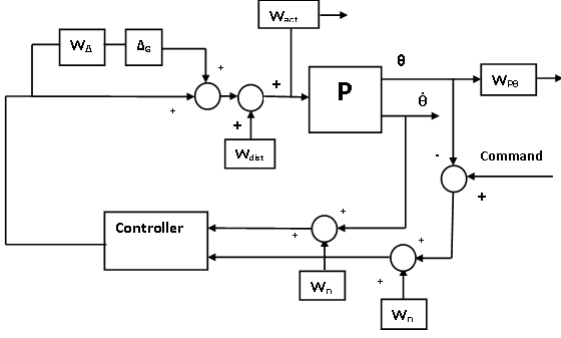


Figure 4. μ -Synthesis arrangement block diagram.

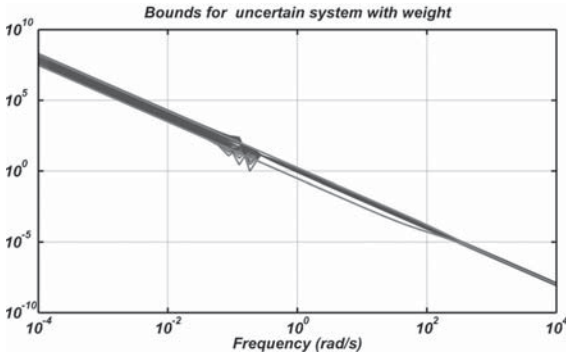


Figure 5. Bounded of uncertainties and chosen weight.

By substituting real parameters, Eq. (34) can be written as:

$$\ddot{y} = v_{real} + \Delta A_1 A_1^{-1} v_{real} - \Delta A_1 A_1^{-1} A_2 + \Delta A_2 \quad (36)$$

As Eq. (36) shows, parameter uncertainty results in a multiplicative uncertainty in controller input ($\Delta A_1 A_1^{-1}$) and a disturbance ($-\Delta A_1 A_1^{-1} A_2 + \Delta A_2$). The controller structure is shown in Figure 4.

To include the uncertainty in the model, different system parameters such as J were perturbed by 20function ($\frac{1}{s^2}$) of the system was selected as a double integrator, *i.e.* $\frac{1}{s^2}$. Then, the bode diagram of the actual system and the nominal transfer function plus the multiplicative weighting function were obtained. The weighting function were then tuned to get the best possible match which is obtained for:

$$W_\Delta = \frac{70(s+1)}{s+100} \quad (37)$$

The effect of uncertain parameters on this transfer function and uncertain plant $P(I + W_\Delta \Delta_G)$ is shown in Figure 5.

W_p puts weighting on the error between complementary sensitivity function of the closed loop system and an ideal model of system response. The performance objective can be written as $|W_p S| \leq 1$. So W_p should be selected as $W_p < \frac{1}{\|S\|}$. According to the first and third frequency of vibration modes of flexible

panel, this function is chosen as:

$$W_{p\theta} = \frac{0.1(s^2 + s + 0.25)}{(s^2 + 4s + 0.01)} \quad (38)$$

Therefore, the hub performance weight has a relatively large magnitude at low frequencies.

According to the weakness of dynamic inversion method against constant disturbance, a disturbance weight is chosen as:

$$\frac{W_{dist}}{|v|} = \frac{W_{distact}}{|\tau|} \quad (39)$$

$W_{distact}$ of constant magnitude equal to 0.001 is applied to the system. The parametric uncertainty disturbance in Eq. (36) is very small so it hasn't been considered in comparison.

To enforce the controller saturation limits in the inner loop, the feedback controller structure shown in Figure 3 is used. Also, this saturation can be considered in designing μ -synthesis controller; however, using the dynamic inversion formulation, the actuator dynamics is not directly accessible. In [19], an algorithm is derived to catch bound on v in feedback linearization outer loop according to actuator saturation limit. In this paper, this limit is approximately obtained according to the following equation by assuming small q :

$$\begin{aligned} \tau = & (J + q^T M_{qq} q - M_{\theta q} M_{qq}^{-1} M_{\theta q})(1 - \alpha M_{qq}^{-1} M_{\theta q}) \\ & (M_{\theta q} M_{qq}^{-1})(v - K_{qq} - \dot{\theta}^2 M_{qq})q - M_{\theta q} M_{qq}^{-1}) \\ & (K_{qq} - \dot{\theta}^2 M_{qq})q + 2\dot{\theta} q^T M_{qq} q \end{aligned} \quad (40)$$

According to actuator saturation limitation $|\tau| < 0.8$ N.m, it is chosen as:

$$W_{act} = 4000 \quad (41)$$

The weighting noise function W_n is used to model sensor noise since all of the feedback signals are corrupted to some extent by noise. It is assumed that the angular velocity and the pitch angle are measured by rate gyro and earth sensor that are corrupted with random measurement noise of magnitude 0.1 deg per second and 0.2 deg. W_n is a high-pass filter according to high frequency noise nature.

$$W_n = \left(\frac{0.2\pi}{180} \right) \frac{0.12s + 1}{0.001s + 1} \quad (42)$$

A concern is that as the number of states in the problem formulation increases, the accuracy of the numerical solution decreases. So, in this paper the controller is designed using attitude and rate of attitude feedback.

With regard to many signal input of controller and performance requirement (robustness against

noise, disturbance, uncertainty and actuator saturation in this complex system), the resulting controller is of high order. This is obviously not practical for application; it can be reduced tremendously without degrading much of the performance. Using balanced truncation, the order is reduced to 7 without much loss of closed-loop performance or robustness.

SLIDING MODE OBSERVER

The objective of the observer is to accurately estimate q and \dot{q} in the presence of disturbances and model uncertainties. It is designed based on the simplified version of the model, which is defined in Eq. (8).

The nominal state

$$\begin{cases} \dot{X}(t) = AX(t) + BU(t) + B\zeta(t, X(t), U(t)) \\ Y(t) = CX(t) \end{cases} \quad (43)$$

The upper bound of $\zeta(t, X(t), U(t))$ can be written as:

$$\|\zeta(t, X(t), U(t))\| \leq r_1 \|U(t)\| + \beta(t, X(t)) \quad (44)$$

where r_1 is a known scalar and β is a known function. Assume that the following conditions are satisfied: A1) the pair (A, B) is controllable
A2) the Markov parameter CB is full rank
A3) the invariant zeros of (A, B, C) (if any) are stable
From assumptions A2 and A3, there exists a linear change of coordinates T_0 , in which the system triple $(A, B, \text{and } C)$ has the following structure:

$$T_0 A T_0^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad (45)$$

$$T_0 B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, \quad C T_0^{-1} = [0 \quad I_P] \quad (46)$$

Equation (43) can thus be re-written as:

$$\begin{cases} \dot{X}_1(t) = A_{11}X_1(t) + A_{12}Y(t) + B_1U(t) \\ \dot{Y}(t) = A_{21}X_1(t) + A_{22}Y(t) + B_2U(t) + B_2\zeta \end{cases} \quad (47)$$

where $X_1 \in R^{(n-p)}$, $Y_1 \in R^p$ and the matrix A_{11} have stable eigen values. A state observer proposed in [16] is employed to estimate the states of (43):

$$\begin{cases} \dot{\hat{X}}(t) = A\hat{X}(t) + BU(t) - G_l e_y(t) + G_n v \\ Y(t) = C\hat{X}(t) \end{cases} \quad (48)$$

where the linear gain is:

$$G_l = T_0^{-1} \begin{bmatrix} A_{12} \\ A_{22} - A_{22}^s \end{bmatrix} \quad (49)$$

and the nonlinear gain is:

$$G_n = T_0^{-1} \begin{bmatrix} 0 \\ I_P \end{bmatrix} \quad (50)$$

A_{22}^s in (49) is a stable design matrix. If $P_2 \in R^{p \times p}$ is a symmetric positive Lyapunov matrix for A_{22}^s , the discontinuous output error injection v can be defined as:

$$v = \begin{cases} -\zeta(t, X(t), U(t)) \|B_2\| & |e_Y| \succ \varepsilon \\ 0 & |e_Y| \prec \varepsilon \end{cases} \quad (51)$$

where $e_Y = \hat{Y} - Y$ is the output estimation error. The function ζ represents an upper bound on the magnitude of $\zeta(t, X(t), U(t))$. Moreover, only pitch angle and pitch rate are considered to be known from measurements in the design of the observer.

SIMULATION RESULTS

In this section, simulation results for the closed loop system (7) with the control laws derived in the previous sections are presented using MATLAB and SIMULINK software. In the simulation, the same system parameters are chosen as those in [1].

$$\begin{aligned} EI &= 1500N/m^2, \quad \rho = 0.2kg/m, \quad l = 30m, \\ J_h &= 4000kgm^2, \quad r = 1m \end{aligned} \quad (52)$$

The control input and its rate are bounded as:

$$|u| \prec 0.8 Nm, \quad |\dot{u}| \prec 0.8 Nm/s \quad (53)$$

The environmental disturbances (*i.e.* gravity gradient, solar pressure, aerodynamic and magnetic torques) on the spacecraft are obtained from the following equation:

$$\tau_d = \left[0.005 - 0.05 \sin\left(\frac{2\pi t}{400}\right) + \delta(200, 0.2) + \nu_1 \right] \quad (54)$$

where $\delta(T, \Delta T)$ denotes an impulsive disturbance with magnitude 1, period T , and width ΔT . The terms ν_1 denotes white Gaussian noises with mean values of 0 and variances of 0.005^2 .

It is assumed that the angular velocity and the pitch angle are measured by rate gyro and earth sensor respectively, that are corrupted with random measurement noise. Earth sensor noise has Gaussian distribution, zero mean and standard deviation of 0.2 degree. The Gyro noise sources correspond to a random drift rate and a random bias rate. This model is represented by the following Laplace transformed equation:

$$\omega_M = H_{gyro}\omega + \omega_D + \omega_N \quad (55)$$

ω_M and ω are the measured and actual spacecraft angular velocity, respectively. Gyro random bias rate ω_N and Gyro random drift noise ω_D have Gaussian

distribution, with zero mean, and standard deviation of 10^{-6} rad/s. Gyro transfer function is:

$$H_{gyro} = (4469s + 89.22) / (s^3 + 89.22s^2 + 4469s + 89.22)$$

The velocity and acceleration of a point on the panel of flexible spacecraft are measured by a tachometer and accelerometers with Gaussian distribution noise, zero mean and standard deviation of 0.0001 m/s and 0.0001 m/s², respectively. The robustness specification is to account for variation on the values of J , M_{qq} , $M_{\theta q}$ and K_{qq} in (7) which would represent the model parameter uncertainties in the system up to 20

In this paper, the coefficient $M_{qq}^{-1}M_{\theta q}$, by considering one elastic mode, is equal to 5.982, and α should be chosen less than its inverse. In simulations, this constant is chosen as $\alpha = 0.14$. By considering more

elastic modes, *i.e.* three, this coefficient is $\begin{bmatrix} 12.441 \\ 2.8555 \\ -0.0669 \end{bmatrix}$;

hence, for higher elastic mode, we can choose larger α .

The observer gains are selected to be $P_2 = [-2 \ -2]$ and $\zeta = 0.00001$. It is being used herein to provide on-line estimates for tip and rate of tip position, which are needed for the computation of the control signals.

Also, the gain parameters in feedback linearization method are chosen as $\omega_y = -0.025$ and $\zeta_y = 0.2$. In this subsection, a comparison of robustness obtained for the nonlinear system with the two proposed controllers (1- feedback linearization 2-combination of feedback linearization and μ -synthesis) are presented.

A number of time and frequency domain analysis procedures are carried out on the resulting designs and their performance is tested. In all simulations, no damping is considered.

The results for the classical feedback linearization and combined controller are given in Figures (6-7), respectively.

A. Feedback linearization controller

In normal conditions or in conditions that only one finite uncertain variation (disturbance, noise and uncertainty) exists, this method responds very well. It means the feedback linearization design leads to smaller maximum overshoot and complete suppression of panel deflection. The dynamic inversion controller achieves this decoupling at the cost of larger and faster control effort.

But in a large maneuver or in combined uncertain conditions (several uncertain conditions exist together or one uncertain condition with larger variations), much larger control efforts are necessary (out of maximum acceptable control input) and the attitude rate and position cannot converge. The high peak in control torque response is due to the impulse present in the disturbance.

Simulation results show that the system is more robust against sensors noise and uncertainty than

against environment disturbance. And also, the system is more sensitive against sine term of disturbance than disturbance constant term, and it has the most robustness against impulse disturbance term. For brevity, the relevant figures are omitted.

B. Composite controller

Figure 7 shows the simulation results of the composite controller. With the designed composite algorithm, the response of pitch angle is shown in Figure 7a. We can see that the attitude angle approaches reference trajectory at time of 600 sec. Hence, fast and precise attitude control is achieved for the current design system. As compared to Figure 6a, in dynamic inversion method, the response has a steady state error and cannot converge.

Figure 7d shows low frequency oscillation of the appendage in composite method. The maximum tip deflection of the appendage is larger in dynamic inversion method and can be seen to be around 0.16 m in Figure 6d. Overall, comparing the plots 7e and 6e, we can see that the composite controller has larger tip deflection rate causing faster panel deflection damping.

The requirement for momentum of RW is illustrated in Figure 7c. As compared to Figure 6c, the composite method requires larger controller effort. Simulations show the composite control algorithm performs well in large maneuvers; however, it has larger controller order.

As shown in Figures 6-7: b, d, e, the simulation results demonstrate the capability of the nonlinear observer in accurately estimating the state variables in the presence of uncertainties.

The simulation results by the composite controller when $\alpha = 0$ (means the system output is only hub angles) show that the Euler angles cannot converge. It shows the ability of output re-definition method used.

CONCLUSION

Vibration attenuation is a difficult control problem due to the stringent requirements on performance and inherent characteristic of such structures. In this paper, flexible spacecraft attitude is controlled by two controller designs. The first controller is a dynamic inversion; the second is a combination of dynamic inversion and μ -synthesis controller. The controllers are designed by utilizing the modified output re-definition approach. In practice, the assumption that all state variables are measurable is not realistic; hence, a robust nonlinear observer has been designed based on the sliding mode methodology.

It is assumed that only one torque on the hub is used. Actuator saturation is considered in the design of controllers. At first, the performance of the two designs were compared in areas such as speed of response, damping of panel vibration modes and size of control

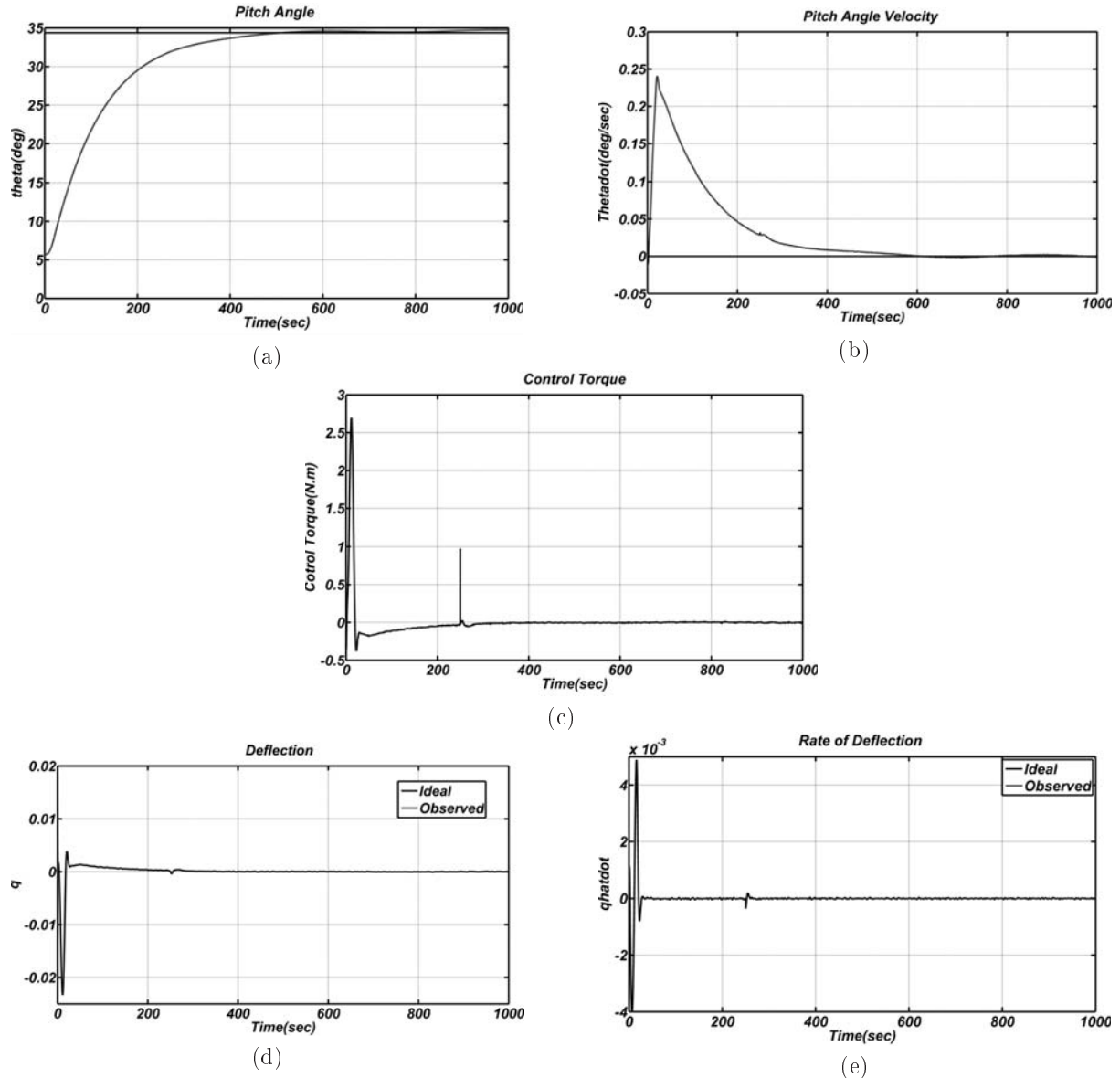


Figure 6. Dynamic inversion, (a) Euler angle (b) angular velocity (c) reaction wheel torque (d) tip deflection (e) tip rate.

deflection used. In the next step, the robustness of the two designs to uncertainty was examined. Finally, the sensitivity of the controllers to measurements noise, environment disturbance and in large maneuvers was compared.

Simulation results prove combined controller ability in controlling attitude and also suppression vibration of panels with exhibit excellent performance and robustness for a broad range of operating conditions with minimal control effort. The simulations have demonstrated the capability of the observer in yielding accurate estimates of the state variables in the presence of uncertainties.

It is important to note that these controllers damp

vibration of panels without considering damping term and without using any filter. In this paper, it is assumed that sensor noise, disturbance and uncertainty are close to real values. It is notable that this combined control method has never been used on spacecraft and, rarely, terms such as disturbance, noise, uncertainty, nonlinearity and saturation are considered in simulations of flexible spacecraft simultaneously.

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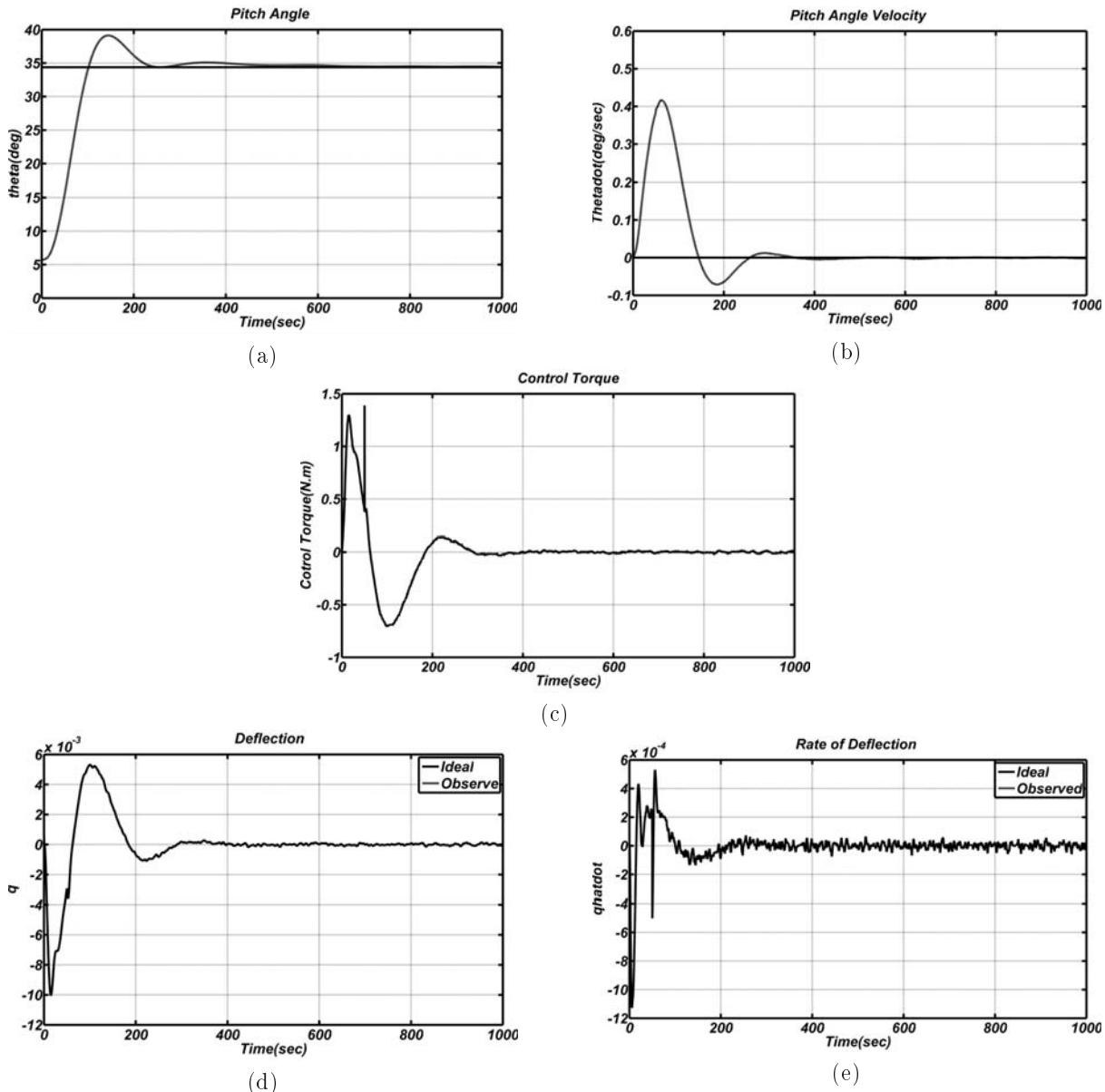


Figure 7. Composite method, (a) Euler angle (b) angular velocity (c) reaction wheel torque (d) tip deflection (e) tip rate.

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