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Development of a Design Procedure for Aeronautical Structures Based on Reliability

F. Javidrad[1], F. Dabirian[2]

In this paper, an approach to aeronautical-structural design based on reliability analysis is presented. In this way, the concept of the level of safety is discussed and methods of its calculation are described using statistical data. Based on the concept of the level of safety, a design procedure is proposed. In order to validate this design procedure, two design cases are studied. In the first case study, using the finite element method, a sandwich plate containing a circular disbond between the upper faceplate and core subjected to compressive in-plane load is examined. The calculated critical buckling load and its residual strength versus disbond diameter and design load versus level of safety are compared to the existing data and a good agreement is achieved. In the second case study, a delaminated sandwich beam under bending load is considered and the design load (obtained from the strain energy release rate analysis) versus level of safety is determined. The results of the study show that for the adopted visual inspection technique and a high probability of damage occurrence, the design load should be about 25% of the beam strength to achieve a reliability higher than 0.99.

INTRODUCTION

Currently, traditional aeronautical-structural design procedures are mainly based on using factor of safety (FS). In most practical cases, a combination of FS for loading and FS for material properties is employed. The values of these factors are basically relied on the aircraft mission and its structural airworthiness. These factors are commonly obtained from the past experience on similar aircraft structural performance. It is widely believed that using FS is a straightforward method for strengthening design. But, there are two shortcomings to this procedure. First, this procedure cannot be easily extended to new unconventional aeronautical structures and new materials. Second, measures of safety and reliability cannot be evaluated with an engineering precision. As a result, it is not possible to determine the relative importance of different design possibilities on structural safety and airworthiness using traditional design procedures.

Reliability-based design methods are able to provide sufficient safety for aeronautical structures. These methods can also be more efficient in terms of structural weight and cost. In addition, sensitivity of structural safety to design variables can be measured quantitatively.

In this article, the concept of the previously-developed level of safety (LoS) is extended to aeronautical structures and a design procedure is developed on this basis [1]. This procedure makes practical use of the LoS definition proposed by Lin *et.al.* [2].

In order to evaluate the proposed procedure, two design cases were studied. In the first case study, using the finite element (FE) method, a sandwich plate containing a circular disbond between the upper laminated composite faceplate and the sandwich core subjected to compressive in-plane load was examined. To verify the results of the proposed design procedure, the variation of design loads versus LoS was calculated and compared to the existing data [2]. In the second case study, a delaminated sandwich beam under bending load was considered and the design load calculated by the strain energy release rate analysis (SERR) for a prescribed LoS was determined numerically.

1. *Associate Professor, Dept. of Postgraduate Studies, Aeronautical Univ. of Sci. and Tech., Tehran, Iran.*
2. *M.Sc. Graduate, Dept. of Postgraduate Studies, Aeronautical Univ. of Sci. and Tech., Tehran, Iran.*

LITURATURE SURVEY

It has already been observed that many properties of composite materials such as strength, stiffness, damage formation and its growth are stochastic in nature. Therefore, current deterministic design approaches are unable to take into account all the variabilities that characterize aeronautical composite materials without oversizing structures or assuming a too pessimistic view of the actual material properties and the operating conditions. In recent years, there have been many applications of stochastic design approach to composite materials (see for example [3-5]).

A probabilistic design procedure for composite materials was introduced in [6]. In this procedure, the physical and mechanical properties of the composite material constituents were treated as random variables. A probabilistic lamination theory has then been established using a micromachanics approach. This has been followed by a FE structural analysis for failure prediction and structural sizing.

In [7], the history and evolution of the probabilistic methods in composite structures have been presented. It is pointed out in this article that there are still many problems that must be resolved before the probabilistic composite material design procedures become practical. However, in this report, damage events which are an important issue in reliability analysis of composite structures have not been discussed. The certification process of the composite materials for use in aircraft structures has been discussed in [8-9]. In these articles, impact damage on composite aircraft structures was introduced as a major concern. It is also emphasized that reliability analysis based on stochastic methods is a practical method to characterize impact damage in composite structures. These articles are centered on the airworthiness requirements of aircraft structures and they are not directly discussing design problems.

The impact threat to aircraft structures was characterized using a Weibull probability distribution of impact energy in [10]. A damage detection threshold of impact damage has also been set in this paper for thin laminates. In this article, a simplified probabilistic model was presented for damage tolerance evaluation, in which post-impact residual strength data were combined with the stochastic assessment of impact damage. Certification and compliance philosophy as well as probabilistic inspection for fleet reliability have been discussed in [11-12]. Material qualification, simulation of environmental effects and damage tolerance demonstration for accidental impact damage were introduced as major areas of concern for reliability analysis of composite aircraft structures.

A Monte Carlo-based simulation for probability distribution of operating stress and material strength was given in [13]. In this model, historical data on operational damage in composite structures were used to develop a stochastic method for reliability analysis. A deficiency of this model is that the inspection method and its probability of damage detection was not included.

A design process using a definition of structural level of safety that incorporates past service experience has been reported in [2]. This design method consists of collecting in-service damage data from existing fleet, establishing the baseline safety level for an existing structural component, conducting damage tolerant analyses for residual strength of the new design and sizing structural configuration for a given load and required safety level. Although the method was shown to have good performance in some circumstances, a large amount of past data is necessary to achieve a reasonable accuracy.

As it can be seen from the above brief literature review on the subject, probabilistic design issues have received considerable attention so far. Most of the work reported in literature focuses on specific areas of applications, and does not take a broad overview of the structural design process for reliability. In the present work, a probabilistic structural design procedure is introduced that makes practical use of LoS as described by Lin *et.al.* [2]. This procedure is basically independent of specific material system and component configuration and is extendable for entire life cycle of a component or structure. The only limitation of this procedure is the need for a reasonable amount of in-service and experimental data to generate probability functions as required for this analysis.

RELIABILITY FORMULATION

Damage tolerant philosophy, a requirement for airworthiness analysis of aircraft structures, is a well-known guideline for aeronautical-structural analysis and design. The purpose of damage tolerance for aircraft structures is to provide an inspection plan for each principal, structural element so that damage (initiated by accident, fatigue, corrosion cracking, ...) will never propagate to failure before detection [14]. The definition of LoS presented in [2], is relatively compatible with this purpose. Therefore, in the present research, reliability analysis is established on this definition.

Definition of Structural LoS

If we designate P as the probability of damage (or crack) that is greater than a critical value and is not detected in a non-destructive inspection (NDI) procedure, the structural LoS will be defined as the compliment of P.

$$LoS = 1 - P \tag{1}$$

This definition assumes that in a single NDI event, there is only one single defect that is stationary with respect to time. In Eq. (1), P is in fact the probability of failure. Therefore, its compliment is the probability of safety. If LoS is near 1, the probability of structural failure by an undetected damage will be near zero. The probability of structural failure can be expressed as a combination of the damage occurrence probability density function ($p(a)$) and the probability of damage detection for a defect with size a ($p_D(a)$). Using statistical rules for joint probability distribution functions, P can be written as:

$$P = \int_{a_c}^{\infty} p(a)(1 - p_D(a))da \tag{2}$$

Damage probability density function depends mainly on the adopted NDI method (through the probability of damage detection) and the probability of detected damage. Using Baye's law in statistics [15], the probability of structural failure may be written as:

$$P = \frac{\int_{a_c}^{\infty} \frac{p_o(a)}{p_D(a)}(1 - p_D(a))da}{\int_{0}^{\infty} \frac{p_o(a)}{p_D(a)}da} \tag{3}$$

where $p_o(a)$ is the detected damage probability density function. The full derivation method of this equation has appeared in [1-2]. In Eq. (3), both geometry and damage dependent factors are implicitly included. Therefore, this equation gives the probability of structural failure independent of geometry and damage characteristics. The accuracy of the calculated P directly relies on the adopted $p_o(a)$ and $p_D(a)$. Gamma distribution has been recommended for $p_o(a)$ while log-normal distribution has been suggested for expressing $p_D(a)$ [16-17].

As it has already been mentioned, Eq. (3) is valid for only one single damage with its own $p_o(a)$ and $p_D(a)$ functions. If various defects exist in a structure, each with its own specifications and LoS, the total LoS for the structure can be determined by multiplication of each LoS. Eq. (4) gives the cumulative LoS for detected defects in a single inspection event.

$$LoS = \prod_{j=1}^{N_L} \prod_{i=1}^{N_{T_j}} (1 - P_{ij})^{n_{ij}} \tag{4}$$

where i denotes damage type, j denotes damage location, N_{Tj} is the total number of damage types in location j, n_{ij} is the mean number of damages in location j and N_L is the total number of damaged locations in the structure. It is noted that in Eq. (4) the interaction between damages is neglected. It is seen that the multiple damage occurrence in a structure can have a significant effect on the structural LoS.

Convergence Requirements

If the damage size approaches zero, $p_o(a)$ must approach zero, too. In addition, the rate of $p_o(a)$ variation must be greater than the variation rate of $p_D(a)$. It should be noted that using numerical integration for calculating Eq. (3), the zero limit of the denominator integral must be substituted with a small number to assure numerical integration convergence is achieved. From the various cases that have been studied in [1], it is seen that 10^{-6} is a suitable number.

Structural Design Procedure Using LoS

The mathematical definition of LoS, as described earlier, is on the basis of the probabilistic damage tolerant evaluation of a structural component. This method can be applied to structures for which a sufficient amount of in-service damage data is available. This data must be integrated within a damage probability function to determine LoS. The design procedure that is developed in the course of this study is a technique by which the reliability of an existing component can be evaluated. This evaluation is performed by using in-service damage data of similar components. In addition, this technique can be used for establishing a baseline for new designs, *i.e.* the design load can be set for a required LoS and a given structural configuration. The flow chart of this procedure is given in Figure 1.

In this procedure, LoS is first determined as a function of damage size a, through selection of the probability functions $p_o(a)$ and $p_D(a)$. Then, using structural analysis methods such as FE analysis, the residual strength of the component for each damage size is evaluated. Structural residual strength can be determined as a function of LoS by erasing damage size between these two recently calculated functions. This function may directly be used for design and evaluation purposes.

CASE STUDIES

In order to assess the effectiveness of the proposed design procedure, two cases have been studied here. In the first case, a sandwich plate containing a circular disbond at the faceplate/core interface subject to in-plane compressive load was examined. The obtained results were compared to the data reported in [2]. In the second case, delaminated sandwich beams with different material systems under bending load were considered and the variation of their residual strengths versus LoS were determined.

Sandwich plate under in-plane compressive load

Stability of partially-delaminated composite and sandwich plates and shells under in-plane compressive loads is an important issue in design of aeronautical structures. Specifically, the post-buckling behavior of

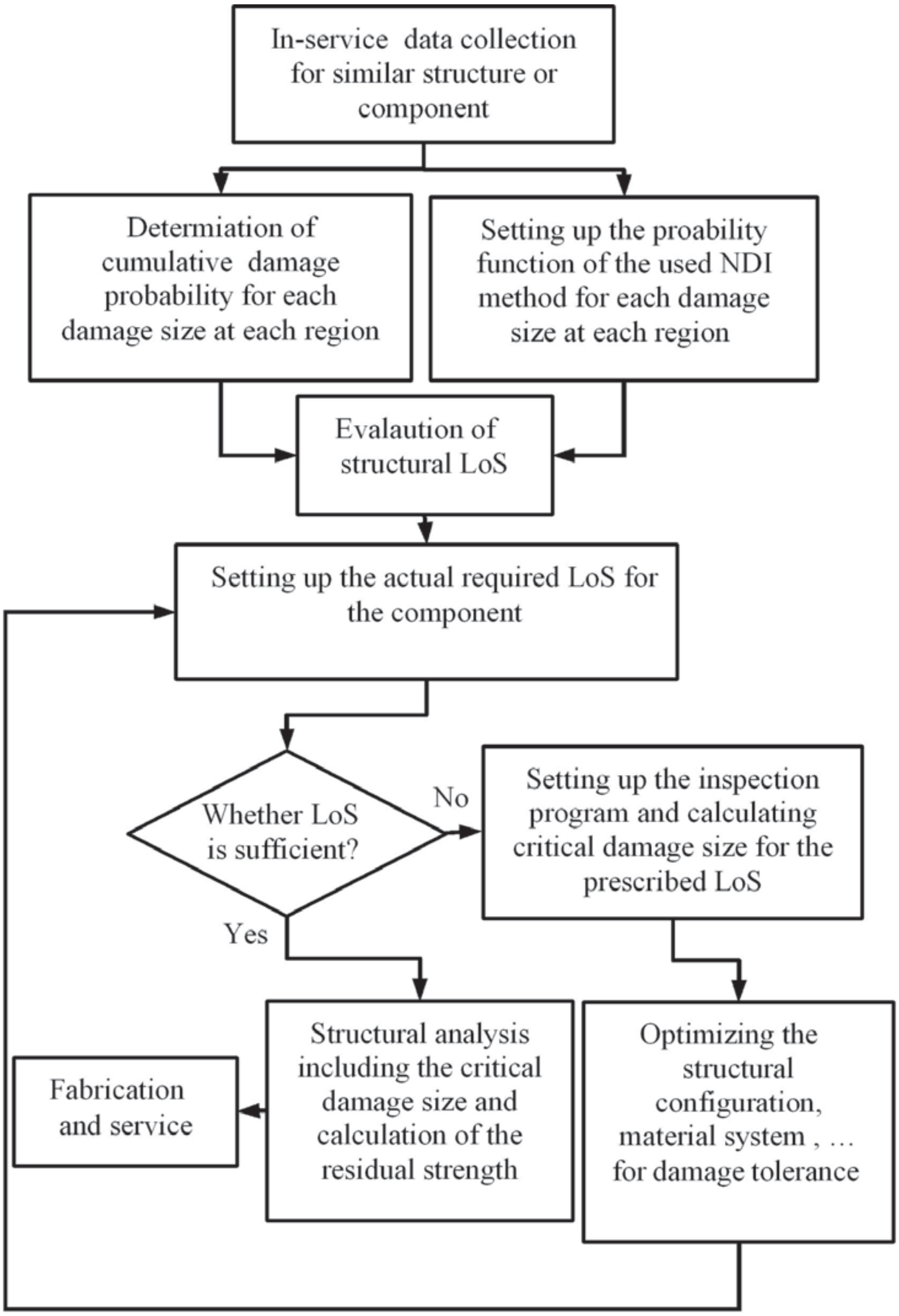

Figure 1. The proposed design flow based on the reliability analysis.

aircraft panels after low velocity impact plays an important role in strength analysis of aircraft composite structures [18].

A sandwich plate made of laminated composite faceplates and Nomex honeycomb core was considered here. Faceplates were made up of 10 layers of carbon fiber/epoxy prepreg with the lay-up sequence of (-45/0/45/0/90)$_s$. The geometry and stiffness properties of the material system are given in Figure 2 and Table 1, respectively.

Number of defects that are detected by a specific investigation technique depends on the inherent accuracy and capability of the technique. For example, a defect with the size of 1 mm may be captured by an ultrasonic NDI technique, while this defect may remain undetected by using X-ray or visual inspection

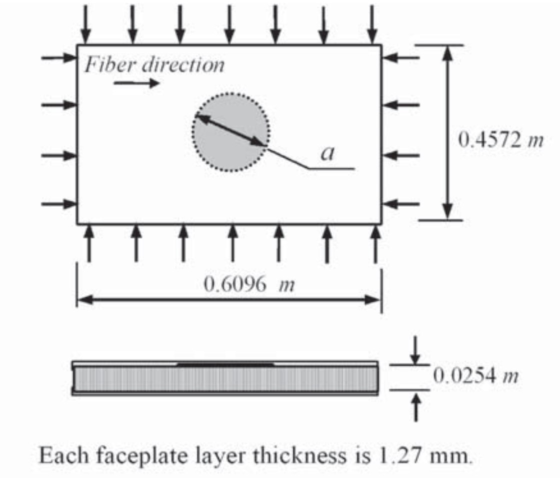

Figure 2. Sandwich plate containing a circular disbond at the faceplate/core interface.

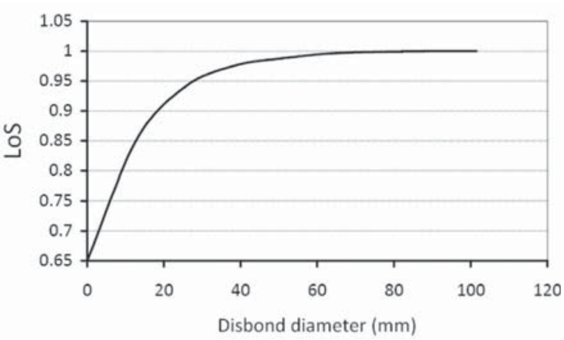

Figure 3. The variation of LoS with respect to disbond diameter.

NDI techniques. Thus, depending on the defect size, every inspection method has a probability of detection. Of course, the probability of detection is implicitly relevant to the configuration and material system of the structure under consideration. Here, it is assumed that for the sandwich plate under consideration the probability of damage detection can be expressed by $P_D(a) = 1 - e^{-a^{1.4}/1.2}$ [2]. (In practice, such an expression can be derived by curve fitting to structural maintenance data as well as experimental evaluation [19].)

Table 1. Stiffness properties of carbon fiber/epoxy lamina and honeycomb core [2]. (subscript 1 represents fiber direction.)

	Unidirectional Lamina	Nomex Honeycomb Core
E_1 (MPa)	151.7×10^3	4.14
E_2 (MPa)	9.2×10^3	4.14
E_3 (MPa)	——-	0.413
G_{12} (MPa)	2×10^3	0.413
G_{13} (MPa)	2×10^3	89.63
G_{23} (MPa)	2×10^3	41.37
υ_{12}	0.34	0.5
υ_{13}	0.01	0.01
υ_{23}	0.01	0.01

To derive an expression for the detected damage probability density function, it is customary to determine the ratio of the measured no. of defects with the size a from a specimen ($n(a)$) to the total no. of detected defects ($n(s)$) in that specimen. This ratio can be treated as the probability of detected damage for defects with the size a (Eq. (5)).

$$p_o(a) = \frac{n(a)}{n(s)} \tag{5}$$

Once the variation of the probability of detected damage versus defect size has been determined, a prescribed distribution may be fitted to this variation using mathematical tools. Experience shows that Weibull and Gamma distribution would be appropriate in such cases [2, 7]. To make a baseline for comparison between the results obtained in the present study with those given in [2], Gamma distribution (Eq. (6)) with κ=2.1 and θ=1.3 was adopted in this case study.

$$P_o(a) = \frac{1}{\theta^{\kappa}\Gamma(\kappa)} a^{\kappa-1} e^{-\frac{a}{\theta}} \tag{6}$$

where Γ represents the Gamma function. Now, to determine LoS for each damage size, the integrals of Eq. (3) must be calculated. A numerical integration method was utilized for this calculation. The calculated LoS is presented in Figure 3.

ANSYS general purpose FE program was utilized for non-linear buckling analysis of the plate. The critical buckling load (P_{cr}) was determined for various disbond diameter, a. Due to symmetry, only one-quarter of the plate was modeled. Faceplates were modeled using composite shell elements while the core of the sandwich was modeled by 3-D brick anisotropic elements. Constraints in the form of rigid links were imposed on the faceplates/core interface except at the disbond area. A sample of the FE mesh with its exaggerated deformation is exhibited in Figure 4.

A simple equation has been given in [20] to determine Post-buckling loads for a similar structure.

$$P_{post} = P_{cr} + \left[\frac{\epsilon_{\max}}{\epsilon_0} - P_{cr}\right]\frac{1}{k} \tag{7}$$

where ϵ_{max} is the maximum material strain (=13500 $\mu\epsilon$), ϵ_0 is the uniform strain produced in layers due to application of a unit load in the direction of loading and k is the stress concentration factor at the disbond profile. The stress concentration factor is the ratio of the extrapolated strain at the crack profile to the uniform strain at far distance [14] that may be calculated using FE analysis. ϵ_0 can also be determined from FE analysis. For the considered geometry and

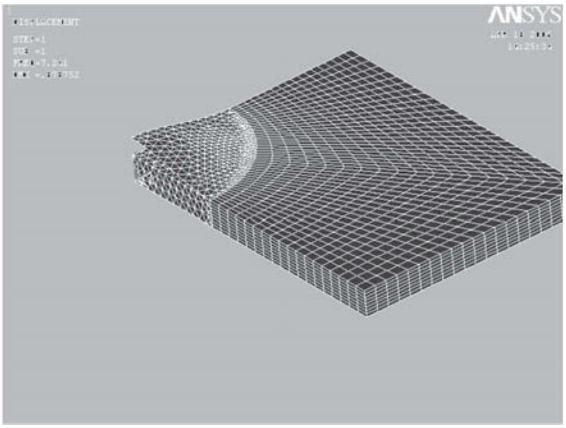

Figure 4. FE model for one-quarter of the sandwich plate with a circular disbond.

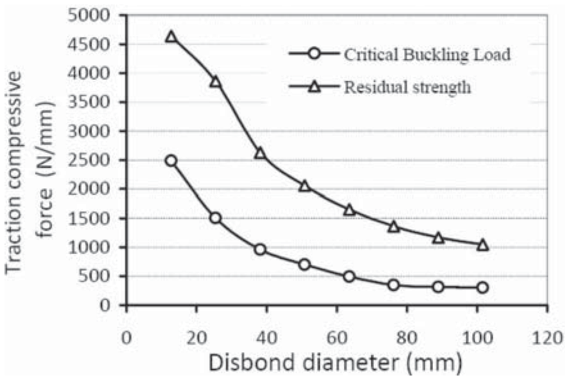

Figure 5. Critical buckling load and residual strength versus disbond area.

material system, ϵ_0 was found to be 1.8158 $\mu\epsilon$ per unit load. In Eq. (7), it is assumed that for applied loads greater than P_{cr}, the buckled area carries no more loads and, therefore, all the excessive loads are carried solely by the undamaged portion of the sandwich plate.

The critical loads and the residual strength determined for various disbond diameters are depicted in Figure 5. Compared with [2], it shows that the difference is less than 3% which is perhaps due to the difference in FE modeling between the present study and [2]. It is noted that a different FE model was used in [2] for this problem. In this reference, the sandwich plate has been modeled by two shells that are connected by rigid links.

Neglecting disbond diameter between the graphs, as depicted in Figure 3 and Figure 5, gives the critical buckling load and the residual strength with respect to LoS, as shown in Figure 6. It is seen that the design load drops drastically when LoS approaches 1.

Reliability design of a sandwich beam under bending loads

In this section, five sandwich beam specimens with different material systems were considered for the reliability design analysis discussed in this paper. The goal of this study is to generate design load curves for sandwich beams with respect to the required LoS.

Four of these sandwich beams had E-glass/vinylester faceplates and one had carbon epoxy faceplates. The sandwich beams had different density PVC plastic cores. The length and the width of all specimens were 610 mm and 305 mm, respectively. Total thickness of the specimens and their material systems are given in Table 2.

Fiber volume fraction and stiffness properties of faceplate materials, which are extracted from [21], are given in Table 3. All of the five specimens had the layup of $[0_6]_s$ where the 0° axis is set along the span of the beam. Density, thickness and stiffness properties of core materials are presented in Table 4 [21].

To evaluate the interfacial fracture of the sandwich materials, critical SERR must be determined experimentally. To meet this purpose, an artificial crack was introduced at the faceplate/core interface and the specimens had been tested with application of an opening load. This mixed-mode I/II specimen, called "Cracked Sandwich Beam" specimen (CSB) in [21], is fairly similar to the mixed-mode Double Cantilever Beam (DCB) specimen whose mode-I version is a standard specimen for interlaminar fracture testing. Figure 7 shows the configuration of a CSB specimen.

This specimen is basically used for measuring

Table 2. Thickness and material system of sandwich beam specimens.

Specimen No.	Total Thickness (mm)	Faceplates	Core
1	30.89	H80/G	H80
2	32.04	H100/G	H100
3	30.85	H100/C	H100
4	31.97	H130/G	H130
5	29.32	H200/G	H200

Table 3. Lamina properties of composite materials used as faceplates.

	H80/G	H100/G	H100/C	H130/G	H200/G
Fiber Volume Fraction	0.4	0.33	0.49	0.34	0.56
E_{xx} (GPa)	20.60	17.02	58.12	17.53	27.59
E_{yy}(GPa)	10.70	8.52	8.45	8.82	13.95
υ_{xy}	0.33	0.36	0.44	0.36	0.31
G_{xy} (GPa)	3.69	2.91	2.70	2.78	4.91

Table 4. Properties of core materials used in sandwich beam specimens.

	H80	H100	H130	H200
Density (kg/m^3)	80	100	130	200
Thickness (mm)	25.4	25.4	25.4	25.4
E (MPa)	80	105	140	230
υ_{xy}	0.29	0.31	0.35	0.35
G (MPa)	31	40	52	85

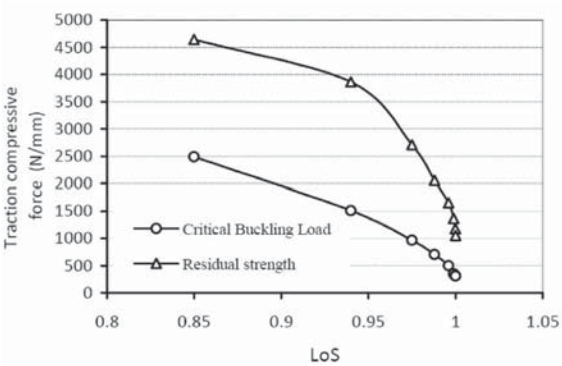

Figure 6. Critical buckling load and residual strength with respect to LoS.

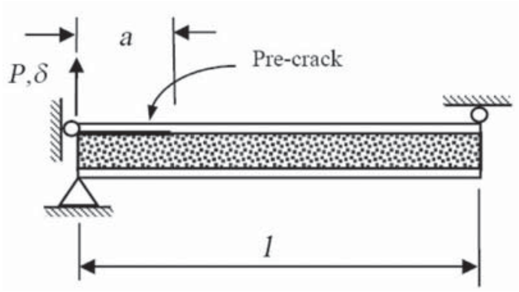

Figure 7. CSB test specimen.

load and load point opening displacement at crack extension during a test. This test demonstrates stable crack growth under displacement control, so that multiple load-displacement data at crack growth may be measured from a single specimen. Once the test has been started, the crack tip was monitored for growth. The crack was allowed to grow steadily to a prescribed length and then load was removed. Unloading data was recorded. This procedure was repeated over approximately 50% of span length.

As it has already been mentioned, this specimen exhibits a mixed mode I/II crack extension. Therefore, experimental load-displacement-crack length data may only be used for determination of the total energy release rates at crack extension. There are many data reduction methods available that can be extended to this type of mixed mode specimen. One of the methods is the area method that is expressed by:

$$G_c = \frac{1}{b}\frac{\Delta E}{\Delta a} \tag{8}$$

where G_c denotes the critical mixed mode SERR, ΔE is the area under a single load/unload cycle, Δa is the crack extension in a single load/unload cycle and b is the specimen width. The experimental load-displacement data for these specimens have been given in [21]. The G_c values have been determined using these experimental data and are given in Table 5.

The first step in our reliability design process is the selection of an inspection method. In [22], a damage detection probability density function (Eq. (9)) was suggested for delamination of sandwich panels used in aircraft aileron structures.

$$P_D(a) = \frac{e^{\alpha+\beta\ln(a)}}{1+e^{\alpha+\beta\ln(a)}} \tag{9}$$

where α and β are two constants that are dependent on the inspection method. In [22], α=4.22 and β=4.69 are suggested for visual inspection method.

Similar to the previous case study, a Gamma distribution probability function with κ=4.23 and θ=1.4 is adopted here for the detected delamination probability function which is compatible with recommendation of [23]. The graphs of the probability functions have appeared in [1]. The determined LoS (using Eq. (3)) versus crack length is shown in Figure 8. It is seen that for crack lengths less than 20 mm LoS is below 0.9. In practice, LoS less than 0.95 is useless. Thus, crack length has to be selected more than 35 mm in the design of these sandwich beams.

FE mesh using 8-node quadrilateral plane-strain elements was generated for the cross section of the beam (Figure 9). For all five specimens, the structure of the mesh was the same. Appropriate material properties were given to the model for each specimen. To model the crack edges, interface elements were inserted at the delaminated area to attribute two numbers to nodes lying along the crack length. To improve the accuracy of the calculated stress distribution at crack tip region, 6-node triangular singular elements have been used at the crack tip. Finite element models were produced for four crack lengths: 25 mm, 50 mm, 75 mm and 100 mm.

To determine the required applied displacement for crack growth initiation, a gradually increasing

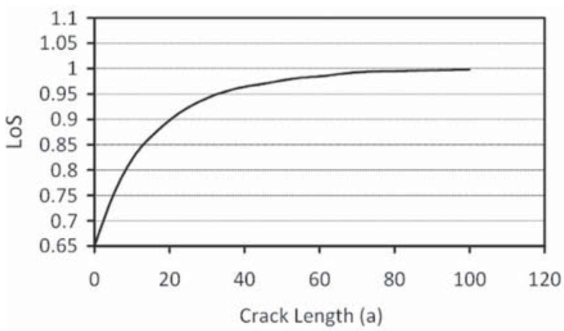

Figure 8. LoS versus crack length for sandwich beam.

Table 5. Mixed mode G_c for sandwich beam specimens.

Specimen No.	G_c (N/mm)
1	0.48
2	0.56
3	0.58
4	0.88
5	1.34

displacement has been applied to the load point of the specimen (as shown in Figure 7) and the total SERR ($G_{I/II}$) has been determined using the modified version of Virtual Crack Closure Method [14,24]. Whenever the total G growth criterion (Eq. 10) is satisfied, *i.e.* the determined $G_{I/II}$ exceeds the G_c for the material system under consideration as given in Table 5, crack is extended with a specified increment. This procedure was repeated to develop the simulated load-displacement-crack length data over part of the specimen length.

$$G_c = G_{I/II} = G_I + G_{II} \tag{10}$$

where G_I and G_{II} are the partitioned SERR for modes I and II, respectively. The described modeling procedure including calculation of SERR was programmed within the FE software ANSYS, to determine the load-displacement data for the specified crack lengths. The simulation results show that the load at crack growth P_{cr}, is proportional to the square root of the mixed mode SERR. So, P_{cr} at each crack length may be determined as follows:

$$P_{cr} = \sqrt{\frac{G_c}{G_{I/II}}} \tag{11}$$

where $G_{I/II}$ is the SERR at the crack tip location when the beam is subjected to a unit load. The critical loads for all specimens at various crack lengths are shown in Figure 10.

Replacing crack lengths in Figure 10 by LoS, as given in Figure 8, demonstrates the variation of design load with respect to LoS. The results, as shown in Figure 11, represent that the residual strengths for all specimens decrease at approximately a constant rate with increasing LoS. It is also seen that increasing LoS for these specimens requires a drastic drop in the design loads. For instance, increasing LoS from 0.9 to 0.999 needs 75% decrease in design load.

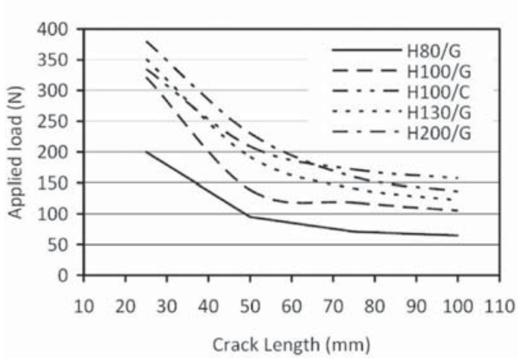

Figure 10. Applied critical loads versus crack length.

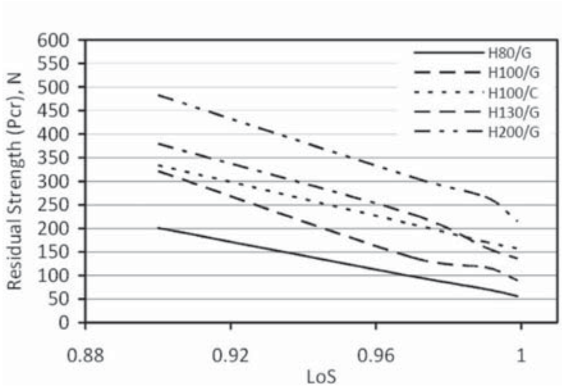

Figure 11. Variation of residual strengths of sandwich beams with LoS.

CONCLUSIONS

The reliability methodology discussed in this paper is a process that is established on analytical and empirical data associated with engineering judgment to quantify the uncertainty which exists in traditional design. It incorporates inherent accuracies of the associated inspection programs into the design process through their probabilities of defect detection. Theoretically, the method is applicable to any type of structure or material system where defects have been characterized. The method also provides a baseline for the comparison of the relative safety between dissimilar structures and materials. The main limitation of this method is that the damage detection probability and detected damage probability density functions may not be easily accessible. Basically, proper derivation of these functions needs relatively a large amount of past service experience and test data on similar structures.

In this paper, a procedure for the design of aeronautical structures based on the concepts of level of safety was introduced. To validate this procedure, two cases were studied. In the first case study, local buckling of a sandwich plate subjected to an in-plane compressive traction load was analyzed. Comparison between the obtained results and the existing data shows a decent agreement. In the second case study, five sandwich beam specimens with different materials were considered and their residual strengths verses level of safety were determined. The results show that to increase the level of safety from 0.9 to 0.999, only 25% of the residual strength must be taken into account for design purposes.

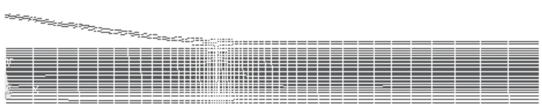

Figure 9. FE mesh structure of the sandwich beam.

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