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Research Note

Viscous Nutation Damper, Modeling and Analysis

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In some aerospace vehicles, the tracking sensors which act as stabilizers in a tracking loop are mounted on a two degree of freedom gyro. The gyro must align its rotor axis with the line of sight in order to remove tracking errors. The tracking precision and sensitivity are functions of the gyros performance. One of the main factors in reducing the precision and producing instabilities is nutation vibration. This fluctuating motion, which is a dynamical inherent property of the system, is related to the gyro lateral moment of inertia, the length of gyro and its rotating speed. In order to investigate the capabilities of nutation damper and removing the wobble motion of a freely precessing body, this paper we analyze a ring damper partially filled with viscous liquid by taking into account the behavior of the damper and its subsystems. The equations of motion for the dynamical motion of gyro are obtained using Lagrangian approach, taking into account the friction of dampers and interaction of the liquid with the system equations of motion.

INTRODUCTION

Homing aerospace vehicles follow radiated target signatures. In these aerospace vehicles, the sensitive sensors are located on a two degree of freedom gyroscope. The optical set has the duty of filtering, concentrating, separating and finally signal-processing of received waves. The mechanical set of gyro consists of inner and outer gimbals, which can sensitively rotate about two orthogonal axes. The magnetic rotor of gyro surrounds these two axes. In homing aerospace vehicles, the gyroptic set is called the seeker head. To increase accuracy, seeke sensitivity and follow the target, all factors of the accuracy loss should be removed. One of the main factors which reduces the accuracy and even makes the gyro unstable is the dynamical inherent property of the system called nutation.When a moment-free inertial symmetric spinning body is subjected to an impulsive torque, *i.e.*, a suddenly applied torque with short duration, it will result in coning (or precession) motion of the spin axis about the angular momentum vector (which is fixed in space in the absence of subsequent external torques). This paper investigates a viscous damper for removing this nutation vibration [1], [2]. The ring damper, partially filled with fluid and mounted on the spinning body (or the rotor of a gyroscope), has the effect of reducing the cone angle (or nutation angle), so it has been extensively used in satellites to keep their orientation and in gyroscopic seeker to confirm precise tracking [3], [4]. In the present paper, the nutation of a gyroscopic seeker, which carries a ring damper partially filled with fluid, and which spins at a high speed of 60 Hz, is analyzed. When the optical detector inside the rotor detects the deviation of the target, the rotor is driven immediately to lock it by an impulsive torque generated by the coil surrounding the rotor. From some experimental observation, the shape of fluid in the ring looks like a crescent, so the fluid in the ring is modelled as a rigid slug in our analysis [5-8].

VISCOUS DAMPER, DYNAMIC ANALYSIS

The idealized rotor and fluid-filled ring are shown in Figure 1. Let H denote the height of the ring damper to the point O which is center of mass of the gyroscope, ΔR and D the width and depth of the rectangular cross section of the ring, respectively, γ the angle of fill of the fluid in the ring, and R the mean radius of the ring. The angular momentum vector of the gyroscope h about point O is fixed in space since the gyroscope is free after application of an impulsive torque. Let

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the Cartesian coordinate system X, Y, Z be the inertial reference frame with the origin coinciding with the mass center of the gyroscope and the Z axis parallel to h. The Cartesian coordinate system x, y, z is fixed on the rotor with the origin coinciding with the center of the ring o' and xy plane lying on the plane of the ring damper. The system u, v, z is fixed on the slug with the u axis passing through its center of mass, the angle β measured from the x axis to the u axis is the angular displacement of the slug relative to the rotor.

Euler's parameters $\mathbf{p} = (e_0, e_1, e_2, e_3)^T$, which are a quaternion, are introduced instead of the Eulerian angles here to describe the orientation of the rotor with respect to the system X, Y, Z. The reason is that quaternion have no inherent geometrical similarities and no singularities in kinematic differential equations, but the Eulerian angles have these two characteristics. In terms of Euler's parameters, the rotational transformation matrix A from system X, Y, Z to system x, y, z can be expressed as the product of two matrices as:

$$\mathbf{A} = \mathbf{E}\mathbf{G}^{\mathbf{T}} =$$

$$\begin{bmatrix} e_0^2+e_1^2-e_2^2-e_3^2 & 2(e_1e_2-e_0e_3) & 2(e_0e_2+e_1e_3) \\ 2(e_1e_2+e_0e_3) & e_0^2-e_1^2+e_2^2-e_3^2 & 2(e_2e_3-e_0e_1) \\ 2(e_1e_3-e_0e_2) & 2(e_0e_1+e_2e_3) & e_0^2-e_1^2-e_2^2+e_3^2 \end{bmatrix} \tag{1}$$

where:

$$\mathbf{E} = \begin{bmatrix} -e_1 & e_0 & -e_3 & e_2 \\ -e_2 & e_3 & e_0 & -e_1 \\ -e_3 & -e_2 & e_1 & e_0 \end{bmatrix} \tag{2}$$

and,

$$\mathbf{G} = \begin{bmatrix} -e_1 & e_0 & e_3 & -e_2 \\ -e_2 & -e_3 & e_0 & e_1 \\ -e_3 & e_2 & -e_1 & e_0 \end{bmatrix}$$

These Four Euler's parameters are not independent; they satisfy the following constraint equation:

$$\mathbf{P}.\mathbf{P}^{\mathbf{T}} = 1 \tag{3}$$

Let $\boldsymbol{\omega} = (\omega_X, \omega_Y, \omega_Z)^T$ and $\boldsymbol{\omega}' = (\omega_X, \omega_Y, \omega_Z)^T$ denote the angular velocity of the rotor in the X, Y, Z system and x, y, z system, respectively. They satisfy the following kinematic equations:

$$\boldsymbol{\omega} = \mathbf{A}\boldsymbol{\omega}' = 2\mathbf{E}\dot{P} = -2\dot{\mathbf{E}}\mathbf{P} \tag{4}$$

$$\boldsymbol{\omega}' = 2\mathbf{G}\dot{P} = -2\dot{\mathbf{G}}\mathbf{P} \tag{5}$$

Multiplying both sides of Eq. (5) by $\mathbf{G}^{\mathbf{T}}$, and using Eq. (3) and the following identity,

$$\mathbf{G}^{\mathbf{T}}\mathbf{G} = -\mathbf{p}\mathbf{p}^{\mathbf{T}} + \mathbf{I}_{(4\times 4)} \tag{6}$$

one obtains:

$$\dot{\mathbf{p}} = \frac{1}{2}\mathbf{G}^{\mathbf{T}}\boldsymbol{\omega}' \tag{7}$$

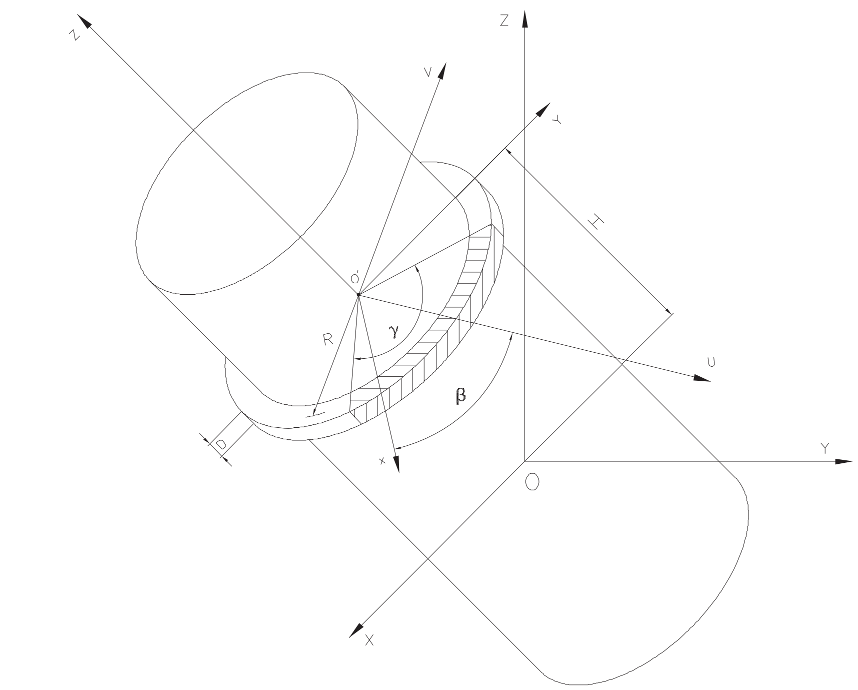

Figure 1. Idealized rotor and fluid-filled ring.

EQUATIONS OF MOTION

The inertia matrix $\mathbf{I_g}$ of the rotor about the point O in the x, y, z system is defined as $\mathbf{I_g} = diag(J, J, J_3)$, Where J and J_3 are the moments of inertia of the rotor about the x, y, z axes. The inertia matrix $\mathbf{I_m}$ of the mercury about the point o in the u, v, z system is:

$$\mathbf{I_m} = \begin{bmatrix} I_1 & 0 & -I_4 \\ 0 & I_2 & 0 \\ -I_4 & 0 & I_3 \end{bmatrix}$$

where the values of I_1, I_2, I_3 and I_4 are:

$$I_1 = I_{uu} = \int (v^2 + z^2) dm$$
$$= 2\int_0^{\frac{\gamma}{2}} (R^2 \sin^2\theta + H^2) \frac{m}{\gamma} d\theta = m\left[H^2 + \frac{R^2}{2}(1-K)\right],$$

$$I_2 = I_{vv} = \int (u^2 + z^2) dm$$
$$= 2\int_0^{\frac{\gamma}{2}} (R^2 \cos^2\theta + H^2) \frac{m}{\gamma} d\theta = m\left[H^2 + \frac{R^2}{2}(1+K)\right],$$

$$I_3 = I_{zz} = \int (u^2 + v^2) dm$$
$$= 2\int_0^{\frac{\gamma}{2}} R^2 (\sin^2\theta + \cos^2\theta) \frac{m}{\gamma} d\theta = mR^2,$$

$$I_4 = I_{uz} = \int uz dm$$
$$= 2\int_0^{\frac{\gamma}{2}} \frac{mHR}{\gamma} \cos\theta \, d\theta = mRHK',$$

$$K = \frac{\sin\gamma}{\gamma}, \quad K' = \frac{\sin\gamma/2}{\gamma/2}, \quad m = \gamma RD\rho\Delta R.$$

The kinetic energy T_g of the rotor is:

$$T_g = \frac{1}{2}\boldsymbol{\omega}'^{\mathbf{T}} \mathbf{I_g} \boldsymbol{\omega}' = \frac{1}{2}\left[J\left(\omega_x^2 + \omega_y^2\right) + J_3\,\omega_z^2\right]$$

The kinetic energy T_m of the fluid is:

$$T_m = \frac{1}{2}\boldsymbol{\omega}_{\mathbf{s}}^{\mathbf{T}} \mathbf{B^T I_m B} \boldsymbol{\omega}_{\mathbf{s}}$$
$$= \frac{1}{2}\{(I_1\cos^2\beta + I_2\sin^2\beta)\,\omega_x^2$$
$$+ (I_2\cos^2\beta + I_1\sin^2\beta)\,\omega_y^2 + I_3\omega_z^2$$
$$+ I_3\dot{\beta}^2 + (I_1 - I_2)\sin 2\beta\,\omega_x\omega_y$$
$$- 2I_4\cos\beta\,\omega_x\omega_z - 2I_4\sin\beta\,\omega_y\omega_z$$
$$- 2I_4\cos\beta\dot{\beta}\,\omega_x - 2I_4\sin\beta\dot{\beta}\,\omega_y + 2I_3\dot{\beta}\,\omega_z$$

where $\mathbf{B}$ is the transformation matrix from system x, y, z to system u, v, z:

$$\mathbf{B} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$\boldsymbol{\omega}_{\mathbf{s}} = (\omega_x, \omega_y, \omega_z + \dot{\beta})^T$$

The gravitational potential V_m of the fluid is:

$$V_m = -m\mathbf{g^T A B^T}(RK', 0, H)^T$$
$$= m\{gH\left(e_0^2 - e_1^2 - e_2^2 + e_3^2\right) -$$
$$RK'\left[2g\cos\beta\,(e_0e_2 - e_1e_3) - 2g\sin\beta\,(e_0e_1 - e_2e_3)\right]\}$$

where $\mathbf{g} = (0, 0, g)^T$ is the gravity with components parallel to X, Y, Z. Using the Lagrange multiplier method, the equations of motion of the gyroscope are:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{p}}}\right) - \frac{\partial L}{\partial \mathbf{p}} = \mathbf{Q_p} + \lambda\mathbf{p} \tag{8}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\beta}}\right) - \frac{\partial L}{\partial \beta} = Q_s \tag{9}$$

where $L = T_g + T_m - V_m$ is the Lagrangian of the gyroscope, λ is the Lagrange multiplier due to the constraint Eq. (3), $\mathbf{Q_P}$ is the generalized force resulting from the gravitational force of the fluid, and Q_β is the generalized force due to the frictional force between the fluid and the wall of the ring. The above equations of motion are five nonlinear second order ordinary differential equations and one nonlinear algebric equation for six unknowns, *i.e.*, $\mathbf{p}$, β, and λ. In order to avoid solving λ and the constraint equation, and to reduce the number of governing equations, the arguments $\dot{\mathbf{p}}$ in L are replaced by the quasi-coordinates ω'. Therefore, $L(\mathbf{p}, \dot{\mathbf{p}}, \beta, \dot{\beta})$ alters to $\bar{L}(\mathbf{p}, \boldsymbol{\omega}, \beta, \dot{\beta})$. Using the chain rule, one has:

$$\frac{\partial L}{\partial \dot{\mathbf{p}}} = \frac{\partial \bar{L}}{\partial \omega_j'}\frac{\partial \omega_j'}{\partial \dot{\mathbf{p}}} \tag{10}$$

$$\frac{\partial L}{\partial p_i} = \frac{\partial \bar{L}}{\partial \omega_j'}\frac{\partial \omega_j'}{\partial p_i} + \frac{\partial \bar{L}}{\partial p_i} \tag{11}$$

From Eq. (5) and the following identity,

$$\tilde{\boldsymbol{\omega}}' = \mathbf{2G\dot{G}T} = 2\dot{\mathbf{G}}\mathbf{g^T} \tag{12}$$

where

$$\tilde{\boldsymbol{\omega}}' = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

one obtains:

$$\frac{\partial L}{\partial \dot{\mathbf{p}}} = 2\mathbf{G^T}\frac{\partial \bar{L}}{\partial \boldsymbol{\omega}'} \tag{13}$$

$$\frac{\partial L}{\partial \mathbf{p}} = -2\dot{\mathbf{G}}^{\mathbf{T}}\frac{\partial \bar{L}}{\partial \boldsymbol{\omega}'} + \frac{\partial L}{\partial \mathbf{p}} \tag{14}$$

From Eqs. (13) and (14), Eq. (8) can be rewritten as:

$$2\mathbf{G^T}\frac{d}{dt}(\frac{\partial \bar{L}}{\partial \boldsymbol{\omega}'}) + 4\dot{\mathbf{G}}^{\mathbf{T}}\frac{\partial \bar{L}}{\partial \boldsymbol{\omega}} - \frac{\partial \bar{L}}{\partial \mathbf{p}} = \lambda \mathbf{p} \tag{15}$$

Premultiplication of Eq. (15) by $\mathbf{G}$ and using two identities, *i.e.*, $\mathbf{G}\,\mathbf{G^T} = \mathbf{I}$ and $\mathbf{G}\mathbf{p} = \mathbf{0}$, one has:

$$\frac{d}{dt}(\frac{\partial \bar{L}}{\partial \boldsymbol{\omega}'}) + \tilde{\boldsymbol{\omega}}'\frac{\partial \bar{L}}{\partial \boldsymbol{\omega}} - \frac{1}{2}\mathbf{G}\frac{\partial \bar{L}}{\partial \mathbf{p}} = 0 \tag{16}$$

Subsequently, the equations of motion can be rebuilt in the following form:

$$\begin{cases} \frac{d}{dt}(\frac{\partial \bar{L}}{\partial \boldsymbol{\omega}'}) + \tilde{\boldsymbol{\omega}}'\frac{\partial \bar{L}}{\partial \boldsymbol{\omega}} - \frac{1}{2}\mathbf{G}\frac{\partial \bar{L}}{\partial \mathbf{p}} = 0 \\ \frac{d}{dt}(\frac{\partial \bar{L}}{\partial \dot{\beta}}) - \frac{\partial \bar{L}}{\partial \beta} = Q_s \\ \dot{\mathbf{p}} = \frac{1}{2}\mathbf{G^T}\boldsymbol{\omega}' \end{cases} \tag{17}$$

Frictional Force

Since the shear stress is a linear function of the gradients of velocities with respect to spatial coordinates from the viewpoint of viscous fluid mechanics, we assumed that the frictional force F_D between the mercury and the wall of the ring is proportional to their relative velocity. Using the principle of virtual work, one can obtain the generalized force resulting from F_D as:

$$Q_s = -C_d R^2 \dot{\beta} \tag{18}$$

The equivalent Reynolds number of a straight rectangular pipe is:

$$R_e = \frac{U_m D_h}{\nu} = \frac{R\dot{\beta}}{\nu}\frac{2D\,\Delta R}{(D+\Delta R)}$$

where ν is the kinematic viscosity. Since the cross-sectional area of the ring is small and the spin rate of the rotor is high in our case, the magnitude of the Reynolds number is of order 10^4 on average. So the evaluation of shear stress from turbulent flow must be considered. For the turbulent flow, the shear stress τ_0 on the wall of a straight pipe with circular cross section is:

$$\tau_0 = 0.0791\, R_e^{-1/4}(1/2\rho U_m^2) \tag{19}$$

where ρ is the density of fluid and U_m is the average flow velocity ($U_m = R\dot{\beta}$). Considering the effect of the curved pipe with circular cross section, Eq. (19) is modified as:

$$\frac{\tau}{\tau_0} = 1 + 0.075 R_e^{1/4}(D_h/2R)^{1/2} \tag{20}$$

where D_h is diameter of the circular cross section. The D_h in Eq. (20) is replaced by $\frac{2D\,\Delta R}{(D+\Delta R)}$ for the curved pipe with rectangular cross section. Equating the frictional force F_D with the shearing force, which is obtained by multiplying the shear stress by the common contact area of the fluid slug and the ring,

$$F_D = C_d R\dot{\beta} = 2(D+\Delta R)R\gamma\tau \tag{21}$$

The damping coefficient c_d can be evaluated by substituting Eqs. (19) and (20) into (21) and using $\dot{\beta} = (\sigma - 1)\omega_z$, one has,

$$\begin{aligned} C_d &= 6.65\times 10^{-2}\rho(D+\Delta R) \\ &\times \left[\frac{(D+\Delta R)v}{D\,\Delta R}\right]^{1/4} R\gamma\left[(\sigma-1)R\omega_z\right]^{3/4} \\ &+ 5.93\times 10^{-3}\times\rho\left[(D+\Delta R)D\,\Delta R\right]^{1/2} R^{3/2}\gamma(\sigma-1)\,\omega_z \end{aligned} \tag{22}$$

where $\sigma = \frac{J_3}{J}$

State Equations

Considering state vector $\mathbf{X}$ as $\mathbf{X} = (e_0, e_1, e_2, e_3, \omega_x, \omega_y, \omega_z, \dot{\beta}, \beta)^T$, and using Eqs. (17) and (22), we can write the state form of dynamic equations in the form of $\dot{\mathbf{X}} = \mathbf{g}(\mathbf{X})$.
where:

$$\mathbf{g} = \begin{bmatrix} \frac{1}{2}\mathbf{G^T}\begin{bmatrix} X_5 \\ X_6 \\ X_7 \end{bmatrix} \\ \mathbf{M^{-1}f} \end{bmatrix} \tag{23}$$

Hence $\mathbf{M}_{5\times 5}$ is:

$$\begin{bmatrix} J+I_1\cos^2 X_9+I_2\sin^2 X_9 & \frac{I_1-I_2}{2}\sin 2X_9 & -I_4\cos X_9 & -I_4\cos X_9 & 0 \\ \frac{I_1-I_2}{2}\sin 2X_9 & J+I_2\cos^2 X_9+I_1\sin^2 X_9 & -I_4\sin X_9 & -I_4\sin X_9 & 0 \\ -I_4\cos X_9 & -I_4\sin X_9 & I_3+J_3 & I_3 & 0 \\ -I_4\cos X_9 & -I_4\sin X_9 & I_3 & I_3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{24}$$

and,

$$\mathbf{f} = [f_1\ \ f_2\ \ f_3\ \ f_4\ \ 0]^T \tag{25}$$

where,

$$f_1 = (J - I_3 - J_3 + I_1 \sin^2 X_9 + I_2 \cos^2 X_9)X_6X_7$$
$$+ \ (I_4 \cos X_9)X_5X_6 + (\frac{I_1 - I_2}{2} \sin 2X_9)X_5X_7$$
$$+ (I_4 \sin X_9)X_6^2 - (I_4 \sin X_9)X_7^2$$
$$+ [(I_1 - I_2) \sin 2X_9]\, X_5X_8 - \ (2I_4 \sin X_9)X_7X_8$$
$$+ [(I_2 - I_1) \cos 2X_9 - I_3]\, X_6X_8 - (I_4 \sin X_9)X_8^2$$
$$+ 2mgH(X_1X_2 + X_3X_4)$$
$$+ mgK'R(-X_1^2 + X_2^2 + X_3^2 - X_4^2) \sin X_9$$

$$f_2 = (I_3 - J + J_3 - I_2 \sin^2 X_9 - I_1 \cos^2 X_9)X_5X_7$$
$$+ \ (-I_4 \sin X_9)X_5X_6 + (\frac{I_2 - I_1}{2} \sin 2X_9)X_6X_7$$
$$- (I_4 \cos X_9)X_5^2 + (I_4 \cos X_9)X_7^2 + (I_4 \cos X_9)X_8^2$$
$$+ [I_3 + (I_2 - I_1) \cos 2X_9]\, X_5X_8 + (2I_4 \cos X_9)X_7X_8$$
$$+ [(I_2 - I_1) \sin 2X_9]\, X_6X_8 + 2mgH(X_1X_3 - X_2X_4)$$
$$+ mgK'R(X_1^2 - X_2^2 - X_3^2 + X_4^2) \cos X_9$$

$$f_3 = [(I_1 - I_2) \cos 2X_9]\, X_5X_6 + (I_4 \sin X_9)X_5X_7$$
$$- (I_4 \cos X_9)X_6X_7 + (\frac{I_1 - I_2}{2} \sin 2X_9)(X_6^2 - X_5^2)$$
$$- 2mgK'R(X_1X_2 + X_3X_4) \cos X_9$$
$$+ 2mgK'R(X_2X_4 - X_1X_3) \sin X_9$$

$$f_4 = [(I_1 - I_2) \cos 2X_9]\, X_5X_6 - (I_4 \cos X_9)X_6X_7$$
$$+ (I_4 \sin X_9)X_5X_7 + (\frac{I_1 - I_2}{2} \sin 2X_9)(X_6^2 - X_5^2)$$
$$- 2mgK'R(X_1X_2 + X_3X_4) \cos X_9$$
$$+ 2mgK'R(X_2X_4 - X_1X_3) \sin X_9 - C_d R^2 \dot{\beta}$$

NUMERICAL RESULTS

By implementing the above equations for two types of fluid comprising mercury and oil with the following values of parameters, nutation angle variation versus time is obtained as shown in Figures 2 and 3. As demonstrated in these figures, mercury-filled ring damper has better performance in reducing nutation angle than oil-filled ring damper.

$R = 16.71\, mm$, $\Delta R = 1.4\, mm$,

$H = 30.39\, mm$, $D = 0.78\, mm$,

$\gamma = 1.33\, rad$, $J = 400\, g.cm^2$, $J_3 = 600\, g.cm^2$

$\rho_{Mercury} = 13.6\, g/cm^3$, $\nu_{Mercury} = 0.00117\, cm^2/s$

$\rho_{Oil} = 0.912\, g/cm^3$, $\nu_{Oil} = 4.2\, cm^2/s$

$$\mathbf{X_0} = \begin{bmatrix} 1 & 0 & 0 & 0 & 100 & 0 & 250\pi & 0 & 0 \end{bmatrix}^T$$

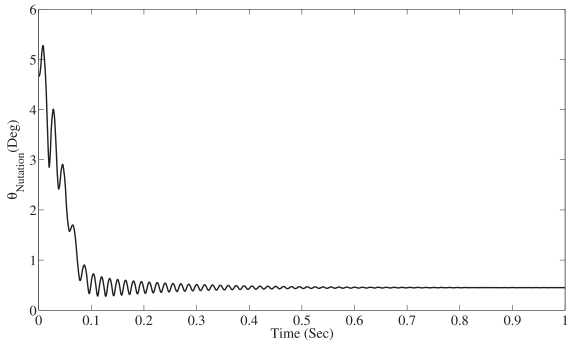

Figure 2. Nutation angle variation for mercury-filled damper.

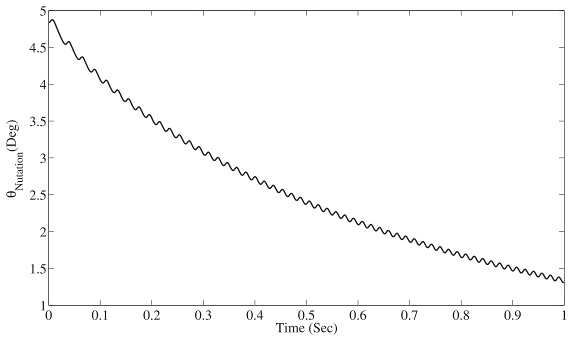

Figure 3. Nutation angle variation for oil-filled damper.

SUMMARY

In this paper, the effect of parameters of viscous damper on the decay of wobble motion of the rotor is analyzed. Complete nonlinear equations of motion are adopted here on the parameter analysis since the use of simplified equations of motion may result in losing something important. Coupled equations of motion are derived in terms of quasi-coordinates in order to reduce the number of equations of motion. The shearing force between rotor and the fluid is obtained by assuming a steady turbulent flow over a straight pipe. Finally, the effects of two different fluid types such as mercury and oil are inspected in viscous damper.

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