

Gravity–Compensated Robust Control for Micro–Macro Space Manipulators During a Rest to Rest Maneuver

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Many space applications require robotic manipulators that have large workspace and are capable of precise motion. Micro-macro manipulators are considered as the best solution to this demand. Such systems consist of a long flexible arm and a short rigid arm. Kinematic redundancy and presence of unactuated flexible degrees of freedom makes it difficult to control micro-macro manipulators. This paper presents a closed-loop control based on artificial constrained motion which helps to keep the end-effector on the prescribed trajectory, while the flexible arm can freely vibrate. The robustness of algorithm is checked against uncertainties of the system, which shows good performance except in the presence of gravity. A gravity compensating term is added to eliminate the drift due to gravity.

INTRODUCTION

Large work space and precise tracking capabilities are two main advantages of micro-macro robots. Macro robot is a large, light weight manipulator which carries micro manipulator as payload. Where macro manipulators are flexible; micro manipulators are small and rigid. Due to flexibility of the macro manipulator, the whole system is under-actuated, which means the number of independent actuators is smaller than the number of degrees of freedom. Despite its good motion characteristics, control of micro-macro manipulators is quite complicated because, the system is under-actuated due to the flexibility of the large manipulator. And, at the same time, the kinematic redundancy of the system. This makes it impossible to use conventional controllers designed for rigid manipulators. On the other hand, flexibility of macro manipulator results in uncertainties due to inaccurate dynamic models, which could be regarded as another obstacle to precise tracking control tasks.

Many control algorithms which are presented in literature are based on active damping control of the

flexible-link manipulator, which focuses on compensating for vibration of macro-manipulator by modulating its own actuator. This, however, forces the manipulator to practice slow and smooth motion. On the other hand, one might think of a controller which tries to keep the end-effector on the prescribed trajectory while the flexible arm is freely fluctuating around the desired path.

A vast number of research works are conducted on the subject of dynamics and control of flexible manipulators. Three major categories of solutions to control can be recognized among them. Some of the researchers on the motion control of flexible manipulators are focused on input-command shaping to prevent exciting higher modes of flexible links, see [1-4]. The second solution is based on using piezoelectric actuators, which may not perform very well for large manipulators, see [5]. The third method is based on using a micro-macro manipulator. This method provides both fast and precise motion at the cost of using complicated system. Mingli et. al. applied an adaptive controller to the motion of planar two-link flexible manipulators [6]. Xu et. al. developed a controller based on rigid body dynamics of a micro-macro robot. The basic idea was to move the macro robot close to the desired path and employ the micro robot to eliminate tracking errors [7]. Yu and Loyd studied direct and indirect adaptive control of constrained manipulators,

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based on which they proposed a control scheme for constrained manipulators with uncertainties in mass parameters [8]. Yim and Singh proposed an approach to end point trajectory control of flexible macro-micro manipulator based on nonlinear inversion and predictive control technique [9]. Yoshikawa et.al., developed a concept called compensability and introduced the measure of compensability index [10]. They employed this measure for path planning to keep the measure as large as possible, and proposed a quasi-static trajectory tracking control scheme. Cheng and Patel employed a two-layer neural network to approximate the dynamics of the system and a learning algorithm for the neural network using Lyapunov stability theory to control a micro-macro manipulator [11]. Sadigh and Misra presented a method for deriving the minimum-order set of equations of motion for dynamic systems subject to artificial constraints. They employed the method to design an open-loop controller for under-actuated manipulators [12]. Sadigh and Zamani developed a similar method based on Lagrange dynamics. The respective open-loop controller keeps the end-effector on the prescribed trajectory; while the flexible arm is free to experience elastic motion. However, the method does not account for the effect of uncertainties of the system [13]. Thus, it was further developed by Sadigh and Salehi. Taking advantage of a computed torque method-like algorithm based on an artificially constrained motion, a closed loop control was proposed. Although the proposed control shows good robustness in the absence of gravity, a steady-state error remains in presence of gravity [14]. This paper presents a robust control based on gravity compensation to eliminate this drift.

DYNAMICS

Consider an under-actuated system of micro-macro robot consisting of ν rigid and elastic links, as shown in Figure 1.

The system has n number of degrees of freedom with m actuators ($m < n$). The system has m rigid degrees of freedom - the same number as the number of actuators - and n_e elastic degrees of freedom. Also, consider that ν_e link of the system are elastic. The equations of motion for this system can be obtained using Lagrange method. The elastic motion of flexible links is modeled using assumed mode method with one elastic degree of freedom for each link. With these in mind, one might derive the equations of motion of the system as:

$$\mathbf{M}_{n \times n}(\mathbf{q}, t)\ddot{\mathbf{q}}_n + \mathbf{h}_n(\dot{\mathbf{q}}, \mathbf{q}, t) = \mathbf{B}_{n \times m}(\mathbf{q}, t)\boldsymbol{\tau}_m \quad (1)$$

see [15] for more details. Now, let us consider that the end-effector is to move on a trajectory modeled as:

$$g_i(\mathbf{q}, t) = 0 \quad i = 1, 2, \dots, p \quad (2)$$

These equations, which describe the end-effector path, could be regarded as artificial constraints for the system motion. We may now partition the actuator array as $\boldsymbol{\tau}_m = [\mathbf{U}^T \ \mathbf{C}^T]^T$, in which \mathbf{C} depicts the array of actuator torques which are to act as constraint forces and \mathbf{U} depicts the remainder of the actuators which can be employed to control the constrained system. Now the task is to divide the set of equations of motion into two sets of equations as follow:

$$\begin{aligned} \bar{\mathbf{M}}_{(n-p) \times n} \ddot{\mathbf{q}}_n + \bar{\mathbf{h}}_{n-p} &= \bar{\mathbf{B}}_{(n-p) \times (m-p)} \mathbf{U}_{m-p} \\ \mathbf{C}_p &= \mathbf{C}_p(\dot{\mathbf{q}}, \mathbf{q}, t) \end{aligned} \quad (3)$$

in which, the first set of equations is the minimum-order set of equations governing the constrained system. These equations are independent of constraint forces - joint torques needed to enforce the constraints (*i.e.* \mathbf{C}). The second set of equations renders an explicit set of equations, based on which the actuator effort necessary to enforce the artificial constraints could be calculated. To obtain equations of motion in the above form, let us rewrite Eq. (1) in the partitioned form as:

$$\begin{bmatrix} \mathbf{M}_1 \\ \mathbf{M}_2 \end{bmatrix} \ddot{\mathbf{q}} + \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{C} \end{bmatrix} \quad (4)$$

wherein, without loss of generality, we assumed that the last p actuators are used to act as constraint forces. Multiplying Eq. (4) by $[\mathbf{I} \ -\mathbf{B}_{12}(\mathbf{B}_{22})^{-1}]$, one gets:

$$\bar{\mathbf{M}}\ddot{\mathbf{q}} + \bar{\mathbf{h}} = \bar{\mathbf{B}}\mathbf{U} \quad (5)$$

Where

$$\begin{aligned} \bar{\mathbf{M}}_{(n-p) \times n} &= \mathbf{M}_1 - \mathbf{B}_{12}(\mathbf{B}_{22})^{-1}\mathbf{M}_2 \\ \bar{\mathbf{h}}_{n-p} &= \mathbf{h}_1 - \mathbf{B}_{12}(\mathbf{B}_{22})^{-1}\mathbf{h}_2 \\ \bar{\mathbf{B}}_{(n-p) \times (m-p)} &= \mathbf{B}_{11} - \mathbf{B}_{12}(\mathbf{B}_{22})^{-1}\mathbf{B}_{21} \end{aligned} \quad (6)$$

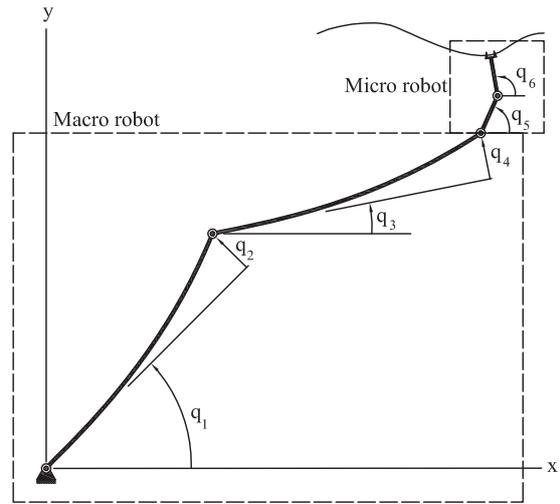


Figure 1. A schematic diagram of a micro-macro manipulator.

On the other hand, solving the second row of Eq. (4) for \mathbf{C} , we get:

$$\mathbf{C} = \mathbf{B}_{22}^{-1}(\mathbf{M}_2\ddot{\mathbf{q}} + \mathbf{h}_2 - \mathbf{B}_{21}\mathbf{U}) \quad (7)$$

Equations (5) and (7) are in the form of Eq. (3). In conjunction with the second time derivative of Eq. (2), Eq. (5) gives n equations, which independently of the choice of \mathbf{C} and \mathbf{U} , always satisfies the constraint equations. On the other hand, one might use Eq. (7) to calculate the joint torques \mathbf{C} needed to enforce the artificial constraints.

CONTROL

To design a controller for the system one needs to devise a model for calculating the joint torques such that the desired trajectory is followed. In order to do that, one might design \mathbf{U} such that the tip of macro manipulator moves closer to the desired path. It can simply be done using the PD controller for $(n-p)$ joint torques, *i.e.*, macro manipulator joints or by controlling the macro manipulator in a master-slave mode as if it were a rigid manipulator. Because of the flexibility, this does not guarantee precise tracking which, of course, is left to be accomplished by the micro manipulator.

The main objective, in this paper, is to design \mathbf{C} such that the end-effector remains on the path during the motion. Now, let us consider that during motion, end-effector leaves the desired path such that $g_i(\mathbf{q}, t)$ become equal to δ_i , instead of $g_i(\mathbf{q}, t) = 0$. In this case, differentiating the constraint equations twice with respect to time gives:

$$\mathbf{A}_{p \times n}\ddot{\mathbf{q}}_n = -\dot{\mathbf{A}}_{p \times n}\dot{\mathbf{q}}_n + \dot{\mathbf{E}}_p + \ddot{\delta}_p \quad (8)$$

where matrices \mathbf{A} and \mathbf{E} are functions of \mathbf{q} and t . Appending Eq. (8) to Eq. (5) gives:

$$\tilde{\mathbf{M}}\ddot{\mathbf{q}} + \tilde{\mathbf{h}} = \tilde{\mathbf{B}}\mathbf{U} + \mathbf{D}\ddot{\delta} \quad (9)$$

in which:

$$\tilde{\mathbf{M}}_{n \times n} = \begin{bmatrix} \tilde{\mathbf{M}} \\ \mathbf{A} \end{bmatrix}, \quad \tilde{\mathbf{h}}_n = \begin{bmatrix} \tilde{\mathbf{h}} \\ \dot{\mathbf{A}}\dot{\mathbf{q}} - \dot{\mathbf{E}} \end{bmatrix} \quad (10)$$

$$\tilde{\mathbf{B}}_{n \times (n-p)} = \begin{bmatrix} \tilde{\mathbf{B}} \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{D}_{n \times p} = \begin{bmatrix} \mathbf{0}_{(n-p) \times p} \\ \mathbf{I}_{p \times p} \end{bmatrix}$$

Plugging the value of $\ddot{\mathbf{q}}$ from Eq. (9) into Eq. (7) one gets:

$$\mathbf{M}'\ddot{\delta} + \mathbf{h}' = \mathbf{B}_{22}\mathbf{C} \quad (11)$$

where:

$$\mathbf{M}' = \mathbf{M}_2\tilde{\mathbf{M}}^{-1}\mathbf{D} \quad (12)$$

$$\mathbf{h}' = \mathbf{M}_2\tilde{\mathbf{M}}^{-1}(\tilde{\mathbf{B}}\mathbf{U} - \tilde{\mathbf{h}}) + \mathbf{h}_2 - \mathbf{B}_{21}\mathbf{U}$$

Equation (11) represents the differential equation for end-effector tracking error. Now considering:

$$\mathbf{C} = \mathbf{B}_{22}^{-1} \left[\mathbf{M}'(-\mathbf{k}_v\dot{\delta} - \mathbf{k}_p\delta) + \mathbf{h}' \right] \quad (13)$$

reduces Eq. (11) to:

$$\ddot{\delta} + \mathbf{k}_v\dot{\delta} + \mathbf{k}_p\delta = \mathbf{0} \quad (14)$$

this shows that choosing \mathbf{C} as in Eq. (13) will asymptotically drive the tracking error to zero in the absence of uncertainty. This means that the actuator torques \mathbf{C} defined by Eq. (13) moves the micro manipulator in such a way as to compensate for the small vibrations caused by flexibility of large manipulator. As a result, the tracking error of the micro manipulator tip asymptotically approaches zero.

An important point to note is that, if the vibration of macro manipulator is so large that the desired path falls out of the working space of the micro manipulator, no choice of \mathbf{C} can eliminate the induced error. Therefore, when designing the control \mathbf{U} , one only needs to ensure that elastic vibrations of the macro manipulator do not get large enough to get the desired path out of the reach of the micro manipulator. Clearly this may not be a restriction, for the size of micro manipulator is normally larger than the permissible elastic deflection of the macro manipulator.

On the other hand, if there are uncertainties in the system, Eq. (11) can be written in the form of:

$$\mathbf{M}'\ddot{\delta} + \mathbf{h}' = \mathbf{B}_{22}\mathbf{C} + \mathbf{G} \quad (15)$$

In which the function \mathbf{G} represents the system uncertainties and can be calculated in terms of the matrices \mathbf{M} , \mathbf{h} and \mathbf{B} . In this case, applying the control as defined in Eq. (13) leads to:

$$\ddot{\delta} + \mathbf{k}_v\dot{\delta} + \mathbf{k}_p\delta = \mathbf{M}'^{-1}\mathbf{G} \quad (16)$$

which shows that one may not expect the tracking error to vanish unless \mathbf{G} becomes zero. Also, it can be shown that \mathbf{G} vanishes in the absence of gravity while \mathbf{q} is constant, see [16]. In the case of set point tracking (constant \mathbf{q}) in the presence of gravity, \mathbf{G} is no longer zero, and we need to modify the control to account for the effect of uncertainties. To this end, we choose \mathbf{C} as:

$$\mathbf{C} = \mathbf{B}_{22}^{-1} \left[\mathbf{M}'(-\mathbf{k}_v\dot{\delta} - \mathbf{k}_p\delta) - \mathbf{G} + \mathbf{h}' \right] \quad (17)$$

Substitution of \mathbf{C} from Eq. (17) into Eq. (15) leads to the same equation as Eq. (14), which guarantees asymptotic convergence of the tracking error to zero in the presence of uncertainties.

Calculation of \mathbf{G} may not be a good approach; for, it is numerically expensive, and in most cases the

exact values of uncertain parameters are not known. Thus, we may, instead, try to evaluate \mathbf{G} . The most important portion of \mathbf{G} , which causes steady-state error, is the one due to gravity, see [16]. Clearly this portion of \mathbf{G} is independent of $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$. Evaluation and compensation for this portion of \mathbf{G} makes the situation similar to the case of uncertainties in absence of gravity - in which it is shown that the feedback control itself is good enough to compensate for uncertainties of the system, see [14]. Considering the system to be at rest, Eq. (16) reduces to:

$$\bar{\mathbf{G}} = \mathbf{M}'\mathbf{k}_p\delta_{st} \quad (18)$$

The amount of δ_{st} can be measured on actual system. Although considering \mathbf{G} as defined in Eq. (18) considerably reduces the tracking error, it does not make the steady-state error equal to zero; for, the amount of δ_{st} depends on the position. Substituting $\mathbf{M}'\mathbf{K}_p\delta_{st}$ for \mathbf{G} in Eq. (17) turns the error equation to:

$$\ddot{\delta} + \mathbf{k}_v\dot{\delta} + \mathbf{k}_p\delta = \mathbf{G} - \mathbf{M}'\mathbf{k}_p\delta_{st} = \hat{\mathbf{G}} \quad (19)$$

Now, if we consider that at the final point where $\ddot{\delta} = \dot{\delta} = \mathbf{0}$, the steady-state error is shown by δ_{ss} , then we may have $\hat{\mathbf{G}} = \mathbf{M}'\mathbf{k}_p\delta_{ss}$. The amount of δ_{ss}

Table 1. The nominal values of physical parameters of the system.

Links				
	Mass (Kg)	Length (m)	$EI_A(N.m^2)$	$I(Kg.m^2)$
Link 1	10	1	1120	-
Link 2	10	1	560	-
Link 3	2.5	0.25	-	0.005
Link 4	2.5	0.25	-	0.13
Motors and Payload				
	Motor 2	Motor 3	Motor 4	Payload
Mass (Kg)	5	3	2	10

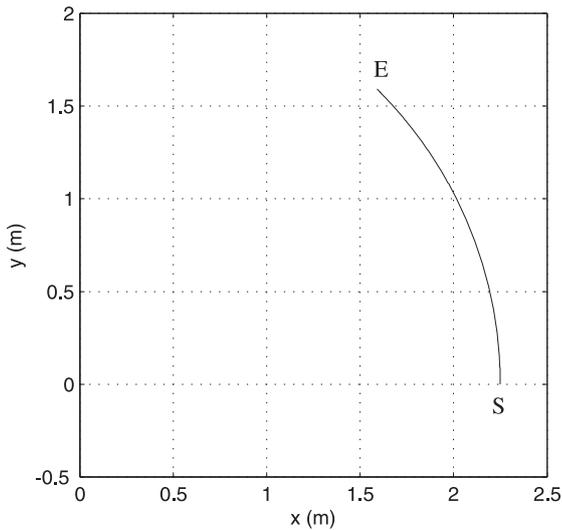


Figure 2. Desired path in work space.

is obtained by measuring δ when four times of time constant of the linear system is passed. So, substitution of $\mathbf{M}'\mathbf{K}_p(\delta_{st} + \delta_{ss})$ for \mathbf{G} in Eq. (17) effectively reduces the steady-state error to zero.

ILLUSTRATIVE EXAMPLE

A micro-macro robot with two large link and two small links are considered as shown in Figure 1. The system has four rigid Degrees of freedom and to describe elastic motion, one elastic degree of freedom is used for each flexible link. Therefore, the system is assumed to have six degrees of freedom. The nominal values of the parameters of the system is given in Table 1. End-effector of the system is to move from rest to rest on a circular path of radius 2.25(m) from zero to $\frac{\pi}{4}(rad)$. The desired path of the end-effector is defined by:

$$\begin{cases} x = r \cos\left(\frac{\theta_f}{2}(1 - \cos(\omega t))\right) \\ y = r \sin\left(\frac{\theta_f}{2}(1 - \cos(\omega t))\right) \end{cases} \quad 0 \leq t \leq \frac{\pi}{\omega}$$

$$\begin{cases} x = r \cos(\theta_f) \\ y = r \sin(\theta_f) \end{cases} \quad t \geq \frac{\pi}{\omega} \quad (20)$$

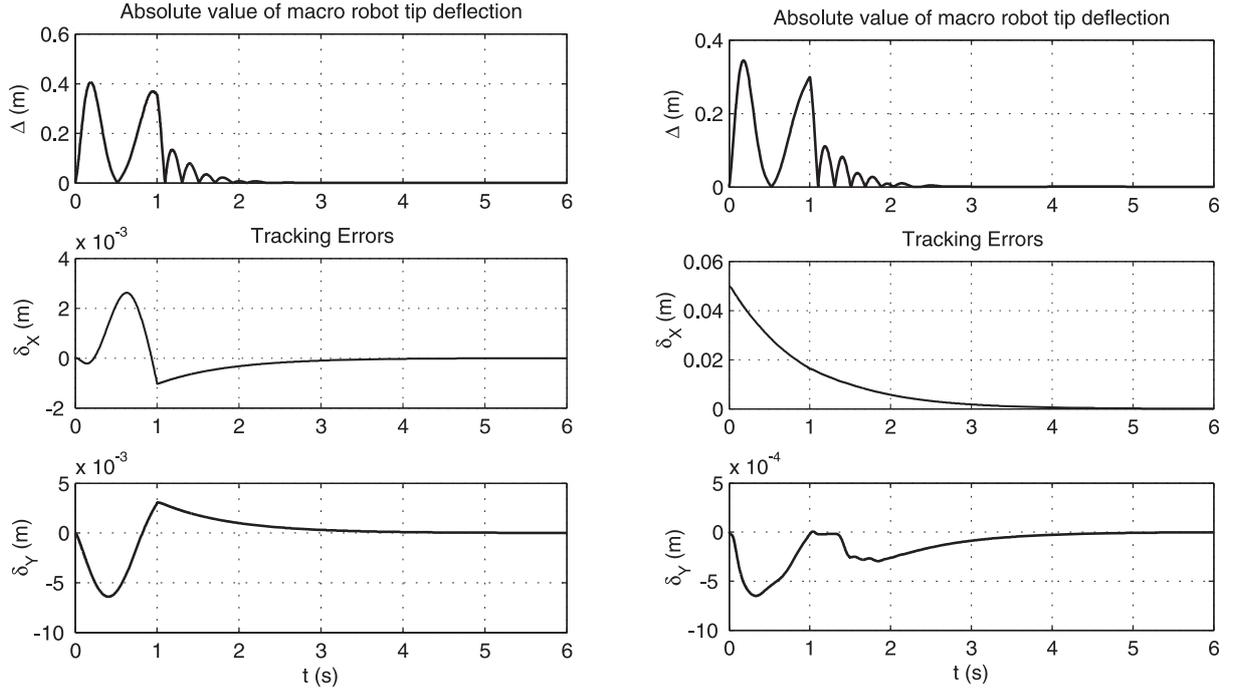
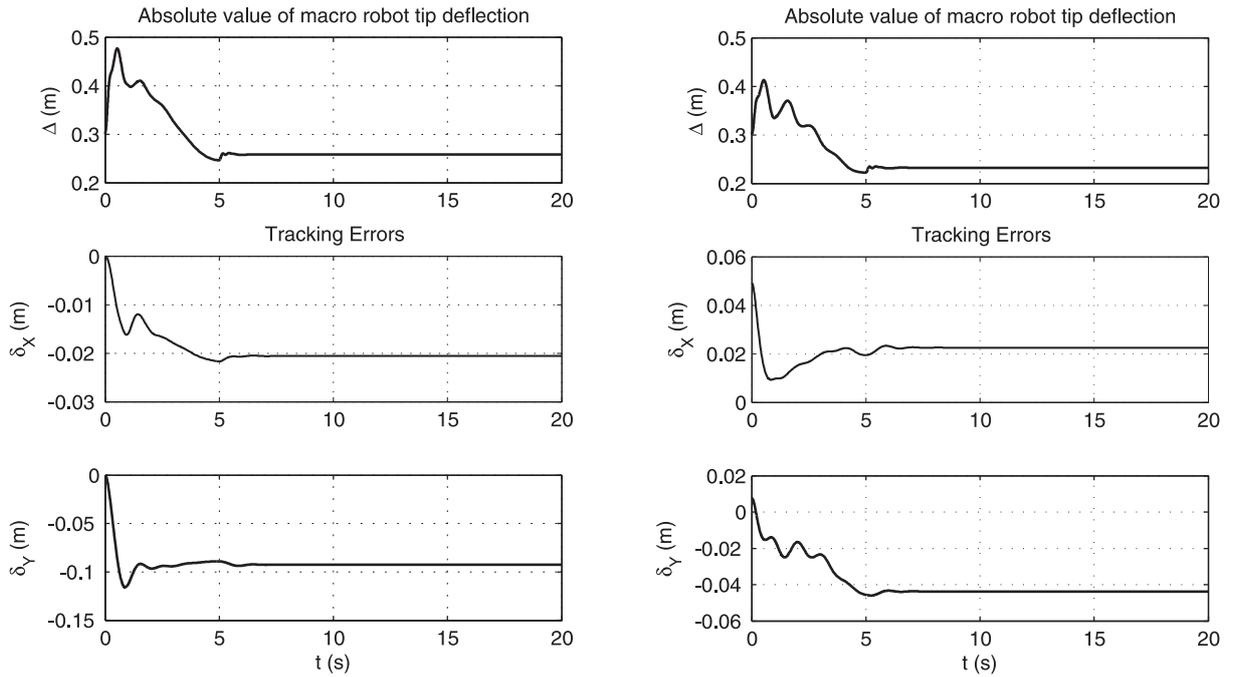
in which $\theta_f = \frac{\pi}{4}$, $r = 2.25(m)$ and $\omega = 0.2\pi(Hz)$ (Figure 2).

The torques τ_3 and τ_4 are considered as constraint forces, *i.e.* $\mathbf{C} = [\tau_3 \ \tau_4]^T$. τ_1 and τ_2 are chosen as the joint torques necessary for driving a similar rigid manipulator on a similar path employing a computed torque method. The simulation results for two cases of uncertainties; *i.e.* mass and length uncertainties, in the absence of gravity are shown in Figure 3. For this simulation, ω is considered as $\omega = \pi(Hz)$. As one can see, the end-effector is precisely following the desired trajectory even if the tip of the macro robot has a considerable deviation compared to the tip of that if it were rigid.

Figure 4 shows the results for the case where there are uncertainties about the presence of gravity. In this case, the macro robot commits a steady-state error, which is reduced by the micro manipulator, but is not totally eliminated. The results obtained based on the gravity-compensation are presented in Figure 5. As expected, the steady-state error of the tip of the macro robot remains unchanged; however, the drift of the end-effector of the micro manipulator is considerably reduced even if it is not zero yet. The results based on gravity compensation with $\mathbf{G} = \mathbf{M}'\mathbf{K}_p(\delta_{st} + \delta_{ss})$ presented in Figure 6, show that the steady-state error is completely eliminated, which establishes the superiority of the proposed method.

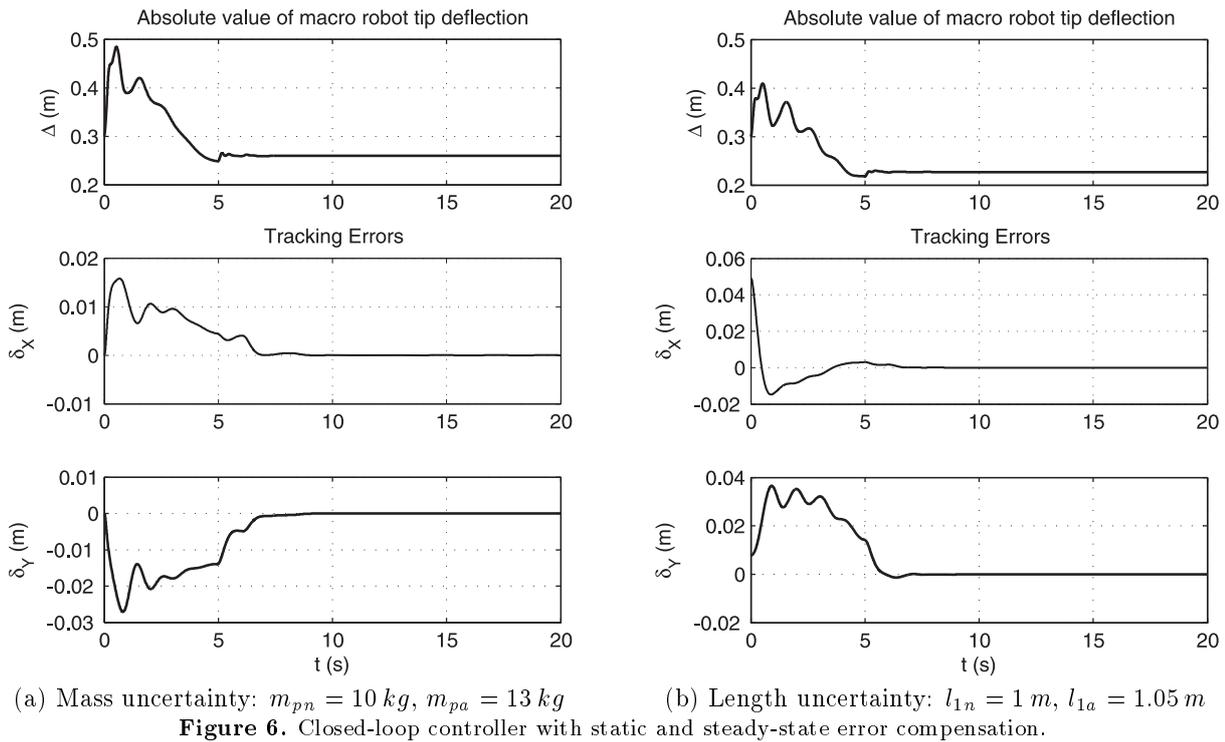
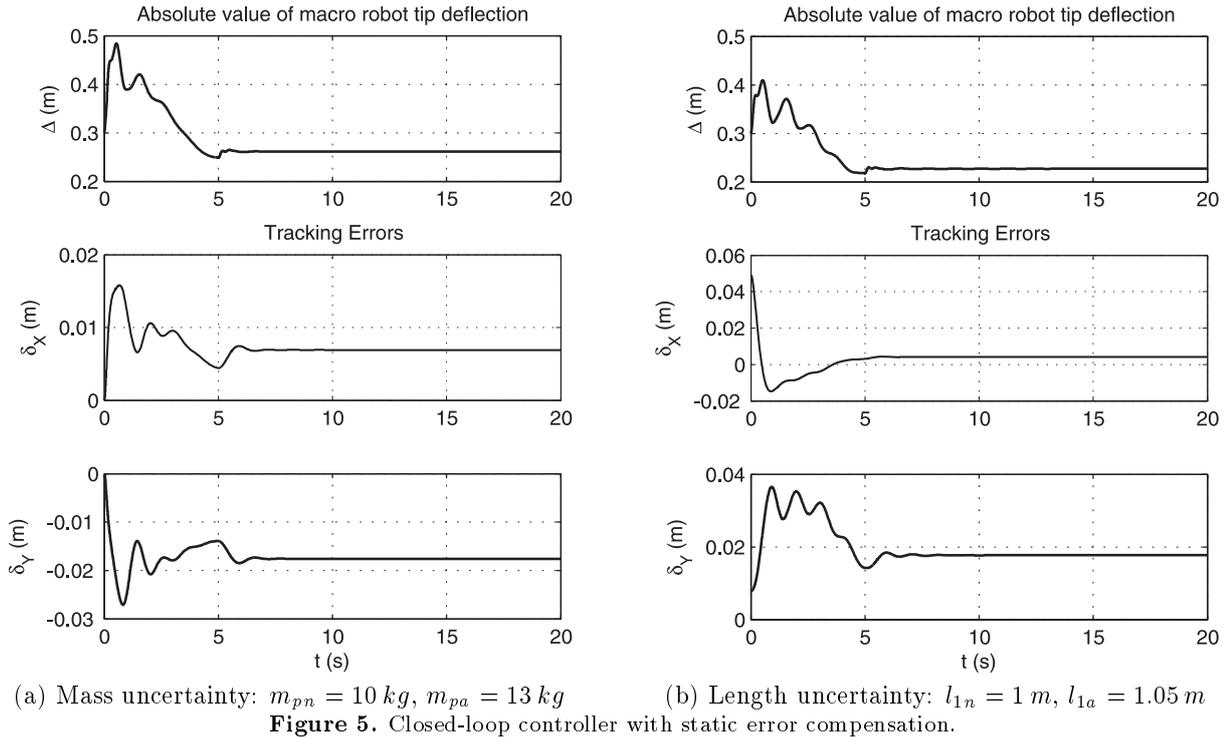
CONCLUSION

A method for control micro-macro manipulators is presented. The method splits the control problem into

(a) Mass uncertainty: $m_{pn} = 10 \text{ kg}$, $m_{pa} = 13 \text{ kg}$ (b) Length uncertainty: $l_{1n} = 1 \text{ m}$, $l_{1a} = 1.05 \text{ m}$ **Figure 3.** Closed-loop controller without gravity compensation in absence of gravity.(a) Mass uncertainty: $m_{pn} = 10 \text{ kg}$, $m_{pa} = 13 \text{ kg}$ (b) Length uncertainty: $l_{1n} = 1 \text{ m}$, $l_{1a} = 1.05 \text{ m}$ **Figure 4.** Closed-loop controller without gravity compensation in the presence of gravity.

two parts. The first part is to devise a controller which moves the macro manipulator close to the desired path, and the second part is to design a controller to converge the tracking error of the end-effector to zero. The

main advantage of the controller is that it does not prevent fast motion of the macro manipulator, which normally excites the higher modes. A close-loop control with gravity compensation is proposed. The proposed



control was applied to a micro-macro manipulator with uncertainties in the presence of gravity, and the results were quite satisfactory.

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