

# An Introduction to a New Criterion Proposed for Stopping GA Optimization Process of a Laminated Composite Plate

A. Abedian<sup>1</sup>, M. H. Ghiasi<sup>2</sup>, B. Dehghan-Manshadi<sup>3</sup>

*Several traditional stopping criteria in Genetic Algorithms (GAs) are applied to the optimization process of a typical laminated composite plate. The results show that neither the criteria of the type of statistical parameters, nor those of the kinds of theoretical models perform satisfactorily in determining the interruption point for the GA process. Here, considering the configuration of the history curve of the maximum objective or fitness value with generation number, a Logarithmic Performance Criterion (LPC) is introduced as a stopping criterion for optimization of composite plates. The results highlight the advantages of LPC in performing sufficiently smooth (no noise), requiring reasonable number of generations, less parameter dependency, less need for conservative assumption, on-line controllability, wide scope applicability, and reasonably easy application in engineering decision-making problems.*

## NOMENCLATURE

<i>GA</i>	Genetic Algorithm	<i>w</i>	Weight
<i>LPC</i>	Logarithmic Performance Criterion	$w_{\max}$	Maximum weight
<i>SC</i>	Different GA runs	$\delta_{\max}$	Minimum weight
<i>SD</i>	Standard Deviation	$\bar{w}$	Dimensionless weight
<i>S.F.</i>	Safety Factor	$\bar{\delta}$	Dimensionless displacement
<i>L</i>	String length	$\bar{x}$	Chromosomes in each population
<i>N</i>	Generation number	$F(\bar{x})$	Fitness function
<i>n</i>	Main population size	$P(\bar{x})$	Penalty function
$t(\zeta)$	The smallest number of iterations required to guarantee the observation of an optimal solution with probability of $\zeta$	$\varphi(\bar{x})$	Objective function
$\zeta$	Probability of finding the optimal solution	$K$	Positive constant
$\mu$	Uniform mutation rate $\mu \in (0, 1]$	$\alpha$	Penalty factor
<i>P</i>	Central load	$x_1$	Design variable
$\delta$	Deflection	$x_2$	Design variable
		$g(x_1, x_2)$	Objective function
		<i>A</i>	constant number derived from logarithmic fitting
		<i>B</i>	constant number derived from logarithmic fitting

1. Assistant Professor, Dept. of Aerospace Eng., Sharif Univ. of Tech., Tehran, Iran, Email: abedian@sharif.edu.
2. Ph.D Candidate, Dept. of Mech. Eng., McGill Univ., Canada.
3. Aerospace Engineer, MSC., Dept. of Aerospace Eng., Sharif Univ. of Tech., Tehran, Iran.

## INTRODUCTION

Using composite materials as the basis for manufacturing structural components has significantly increased the complexity of the design process. Additionally,

the problem of optimum design of laminated composite plates (that are widely used in aerospace and marine applications) involves large, non-convex, and integer-programming that is discrete in nature, and includes a substantial number of design variables such as geometrical parameters, ply composition, materials of the composite constituents, stacking sequence of the layers, and so on. Due to the nature of the issue (especially its discrete variables), the Genetic Algorithm for optimization of such problems, despite its disadvantages like being such a time consuming approach, has been justified and used by several researchers [1-8].

A genetic search normally starts with a random initial population. The subsequent populations are created from previous populations utilizing the genetic operators and a selection schema in a manner that probabilistic survival of better individuals (i.e. having higher fitness values) is ensured. Thus, in this process, after a large number of iterations, the population should consist of many individuals who are closely placed at the optimum design point. Now, the question is how many iterations are required to obtain the global optimum, where there is no specific suggestion for making such a prediction. Although the GA has been successfully applied to a wide variety of problems, there are just a few theoretical guidelines for how to terminate the searching process. Perhaps, due to its versatility, heuristic basis, and weakness of available mathematical models, determination of the convergence rate and the interruption point for the process is still one of the ambiguous points of the GA method. Effective bounding of the process appears to be more difficult when one faces noisy and discrete engineering applications such as the problems that involve the design of composite materials.

Generally, to investigate this key point, many practitioners have tried to define some statistical performance parameters such as "stop when there is no significant improvement during the last ten iterations" or "stop after N number of generations" or other similar statistical approaches [9], while others have tried to develop analytical models that can mathematically find the minimum required number of generations to guarantee the finding of the global optimum solution with a predefined probability.

In this research, some of these different conventional mathematical and statistical stopping criteria in genetic algorithm are applied to the optimization of a typical laminated composite plate to study their performance and applicability in such problems. Here, the GA is implemented several times with a large number of iterations in order to find the best solution. The investigation is performed under two subdivisions of the statistical parameters and theoretical approaches. It is, however, worthy to note that tracking down the effectiveness of the applied criteria shows that none of

them could serve as an ideal tool for defining the cut-off point for the evolutionary process. Subsequently, here, some new statistical parameters for stopping the GA process are introduced and examined although none of them seems to have the capability of serving in the field of composite design in a credible manner. Finally, considering the nature of the curve passing through the best objective value found in each iteration as opposed to the number of iterations, a new Logarithmic Performance Criterion (LPC), which can act as an effective online parameter for stopping the GA process, is defined and tested.

Before applying the two groups of available criteria (the statistical parameters and the theoretical models) to the composite problem at hand, it will be useful to present a brief review of them. In the statistical parameters group, stopping after a large number of generations is the most fundamental approach for terminating a GA process. A slightly more sophisticated approach has been put forward by de Jong in 1975 [10], where two flags are defined as the on-line and off-line performances of the process. The on-line performance (describing the on-going performance) corresponds to the average of all fitness functions evaluated up to and including the current trial, while the off-line performance (describing convergence) is the average of the best performance values of the runs up to a particular time. Based on this approach, the GA process stops when either the on-line or, preferably, the off-line performance appears to be stabilized [11]. A large number of such heuristic performance parameters have been defined and used in different applications of genetic search method, which are briefly reviewed below. In fact, by this group of criteria, the termination process occurs:

1. when in a specific number of iterations no improvement in the optimum design occurs [12-16];
2. when the average fitness value appears to be stabilized [13];
3. when the change in the exponential average of the fitness values of a population is small [13];
4. when a given number of function evaluations, user's patience limit, or CPU resource limit is met [14, 16-20];
5. When the required optimum appears (if known) [14];
6. when the ratio of average fitness value of the current population to the score of the best individual of the population exceeds a specified threshold [18];
7. when the diversity of the fitness values of the current population drops below a specified threshold [18];

8. when the current improvement in the fitness value in comparison with its initial improvement is ignorable [18];
9. when the maximum number of iterations is limited to 100 times the number of independent variables (by default) [18];
10. when the tolerance is limited, this specifies the minimum tolerance for the distance of the current point from the boundary of the feasible region [20];
11. when the distance between two consecutive points is less than X tolerance which specifies the minimum distance between the current points at two consecutive iterations [20];
12. when the change in the value of the objective function at the current point is less than a specified tolerance [20];

As for the theoretical models, it can be said that genetic algorithms, which are considered as stochastic search algorithms, rarely guarantee the finding of a global optimum by a fixed number of generations. In 1991, Vose and Liepins [21] showed that genetic algorithms which might be expected to display punctuated equilibrium, often appear to converge to false suboptimal solutions for a relatively large number of generations before moving on to the next solutions. Thus, convergence criteria such as de Jong's on-line and off-line performance can often be misleading. However, one can mathematically find the minimum number of generations required for reaching an optimal solution ( $t(\zeta)$ ) with the probability of  $\zeta$  ( $0 < \zeta < 1$ ), which is defined by the user [11].

Some previous works have resulted in deriving bounds for the quantity  $t(\zeta)$  by modeling genetic algorithms as Markov Chains (MC). This was first done by Nix and Vose (1992) [22]. They showed that if the state variable is taken to be the current population, then MC is periodic if and only if the mutation probability  $\mu$  is strictly positive. Hence, a steady state behavior will exist under which the possibility for indefinite trapping of MC at a suboptimal solution point will vanish [11].

In another attempt, Aytug and Koehler [9] derived the worst case bound on a GA's running time when choosing a confidence probability (i.e.  $\zeta \in [0,1]$ ) by the user is necessary. They showed that the minimum number of GA generations required for pushing the total populations toward the probability of  $\zeta$  could be calculated as follows:

$$t(\zeta) = \left\lceil \frac{\ln(1 - \zeta)}{\ln(1 - \min(1 - \mu)^{Ln}, \mu^{Ln})} \right\rceil \quad (1)$$

where,  $\mu$  is the uniform mutation rate and L and n are string length and population size, respectively. Note that for each bit, if the randomly generated number (0,1) is smaller than  $\mu$ , it will be replaced by its

complement. Aytug et al. [9] also developed a tighter bound and defined their new worst bound as follows;

$$t(\zeta) = \left\lceil \frac{\ln(1 - \zeta)}{n \ln(1 - \min(1 - \mu)^L, \mu^L)} \right\rceil \quad (2)$$

where parameters  $\zeta$ , n, L, and  $\mu$  are defined as in Equation (1).

### DETAILS OF THE TEST PROBLEM, THE GA MODELING AND RUNS

As was discussed before, for testing the performance of existing stopping criteria (i.e. both statistical parameters and the theoretical models) in the field of design and optimization of composite plates some of the above mentioned criteria are applied to a test problem, of which the details are given in this section. The test problem is optimized repeatedly under different conditions, where each repetition consists of a large number of generations (i.e. 300 to 500). Then, the results obtained by application of different criteria are compared together, and finally the new criterion (LPC) is introduced and its superiorities and advantages over other existing criteria are discussed.

The plate under consideration is made up of a Glass-Epoxy laminated composite plate (1 m long  $\times$  0.5 m width), subjected to a line distributed load

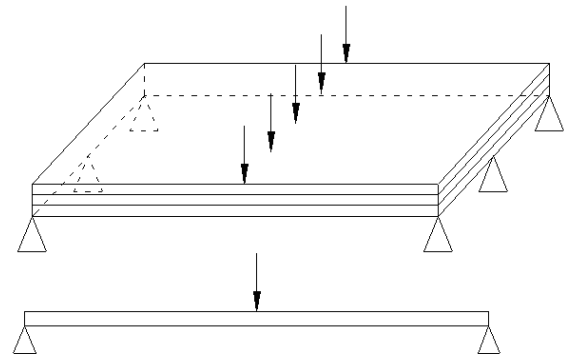


Figure 1. Description of the physical problem.

Table 1. Mechanical properties of the Glass-Epoxy layers.

Property	Value
Young's Modulus, E1	38.61 GPa
Young's Modulus, E2	13.79 GPa
Shear Modulus, G12	4.83 GPa
Poisson Ratio, $\delta_{12}$	0.20
Shear Strength, S	103.42 MPa
Tensile Strength, Xt	206.84 MPa
Tensile Strength, Yt	27.58 MPa
Compressive Strength, Xc	241.32 MPa
Compressive Strength, Yc	62.05 MPa
Density	1800 kg/m <sup>3</sup>

of  $P=1000$  N at the center of the plate as depicted in Figure 1. As for the stacking sequence, each ply of the laminate is allowed to be oriented at any angle between  $0^\circ$  and  $90^\circ$  with the increments of 15 degrees. The properties of the plate layers are given in Table 1. Here, the number of layers, the thickness of each layer, and fiber orientations are considered as design variables. It is to be mentioned that genetic algorithm assumes a number of random solutions for the problem under consideration, called the main population. Each population member is a string of binary digits corresponding to a natural chromosome. Each particular element of a chromosome is then called a gene, which represents the assumed value for a specific design variable. For the problem at hand, Table 2 represents the design variables along with their allowable range and the assigned binary length in a chromosome. These variables are then used to evaluate the corresponding fitness function which is related to the objective and penalty functions defined subsequently. Here, simultaneous minimization of the weight ( $w$ ) and deflection ( $\delta$ ) of the plate is considered as the objective function and is defined as follows:

$$\varphi(\bar{X}) = K - (0.8\bar{W} + 0.2\bar{\delta});$$

$$\text{where } \bar{W} = \frac{W}{W_{\max}} = \frac{W}{30(kg)} \text{ and } \bar{\delta} = \frac{\delta}{\delta_{\max}} = \frac{\delta}{10(mm)} \quad (3)$$

"K" is a positive constant that is used to convert the minimization problem into an ever-positive function for the whole design space. The amount of this term would not affect the selection schema performed by the linear scaling operator of the GA process. In this process, a particular selection method based on the value of the fitness for each chromosome is utilized to select the

next population. The selection schema serves the role of natural events in this algorithm attributing higher selection probability values to better chromosomes. In fact, this operator boosts the outliving probability of the best chromosome while declining that of the worst chromosome in the same generation. This reproduction and selection process will be carried on until one or a set of stopping criteria are met and an optimum or a near-optimum solution is found. The optimization problem and its constraints are given below:

$$\begin{aligned} \text{Max} \rightarrow \varphi(\bar{x}) &= K - (0.8\bar{W} + 0.2\bar{\delta}) \\ \text{S.T.} \left\{ \begin{array}{l} \text{S.F.} > 1.2 \\ W < 30 \text{ kg} \\ \delta < 10 \text{ mm} \end{array} \right. & \quad (4) \end{aligned}$$

The above-mentioned constraints are then handled by a penalty term and the modified (or penalized) objective function, which is called the fitness function ( $F(\bar{x})$ ), is then reshaped as;

$$F(\bar{x}) = \varphi(\bar{x}) - \alpha \times P(\bar{x}); \quad (5)$$

$$P(\bar{x}) = 0.72(\bar{w} - 1) - 0.23(1/\text{S.F.} - 0.833) - 0.05(\bar{\delta} - 1) \quad (6)$$

The weighting parameters for three terms of the penalty function ( $P(\bar{x})$ ) are chosen regarding the significance of each term in achieving the optimum objective value. This could be attributed to the negative and positive effects (Cons. and Pros.) caused by changing one constraint on the rest of the related constraints. For instance, in this example, increasing the panel weight is a disadvantage; however, it could positively affect the process by causing the respective advantage of decreasing the deflection. Since these state variables

**Table 2.** Meaning, allotted values, and the length of the binary code in a typical chromosome.

Gene No.	Meaning	Allowable Range			Binary Length
		Lower Limit	Increment	Upper Limit	
1	Number of Layers	1	1	16	4
2	Layer Thickness (mm)	0.1	0.1	1.6	4
3	Material	1	-	1	0
14-20	Fiber Orientation in Layers 1 through 16	$0^\circ$	$15^\circ$	$90^\circ$	3

**Table 3.** Defining weighting parameters for  $P(x)$ .

Violating Const.	Cons	Pros	Cons/Pros	Weighting Coef.
Weight	Weight $\uparrow$ (max. 80)	Deflection $\downarrow$ (max. 20)	80/20	$\approx 0.72$
Safety Factor	Failure $\uparrow$ (max. 100) Deflection $\uparrow$ (max. 20)	Weight $\downarrow$ (max. 80)	120/80	0.23
Deflection	Deflection $\uparrow$ (max. 20)	Weight $\downarrow$ (max. 80)	20/80	$\approx 0.05$
			Total	1

(i.e. weight and deflection) appear to be 80% and 20% weighing factors in the objective function ( $\varphi(\bar{x})$ ), it is assumed that any increase in the panel weight could produce a maximum drawback of 80% for the weight and a maximum benefit of 20% for the deflection (see Table 3). However, in general, selection of the weighting factors for such a multi-variable penalty function is often arbitrary and a definite procedure has not yet been set [10]. It is also noted that since the objective function ( $\varphi(\bar{x})$ ) appears as the weight of unity in  $F(\bar{x})$ , the weighing factors for the terms of the penalty function ( $P(\bar{x})$ ) are also calculated such that they add up to unity. Therefore, the factor  $\alpha$  (see Equation (5)) plays a major role in penalizing the infeasible chromosomes. In fact, the relationship between the weighting factors of the two parts of the fitness function ( $F(\bar{x})$ ) accounts for the amount of penalty. It is also worth to note that there is no clear way for defining a suitable value for the ratio of the weighing factors. However, available studies show that the factors should be kept as low as possible, just above the limit below which infeasible solutions turn to be optimal [23].

As for the GA runs, here, the optimization process is repeated three times with the same characteristics for the GA operators (i.e. 100% crossover, 100% mutation, and population size of 25). These runs are marked as SC1, SC2, and SC3 in Table 4. The only difference among these three runs is in their first population, which for each run is generated randomly. The rest of the runs (i.e. SC4, SC5, and SC6) differ either in their crossover and mutation percentages or in the population size. The obtained results from these runs enable one to understand the effects of the chosen rates for the GA operators and the number of chromosomes of the population on the optimization process.

**RESULTS AND DISCUSSION**

As explained before, the two sets of the conventional stopping criteria in the GA optimization process are tested here, first. It will be shown that none of these criteria are performing successfully when applied to the optimization of composite material problems. The results obtained by each set of the criteria will be presented and discussed in this subsection, separately.

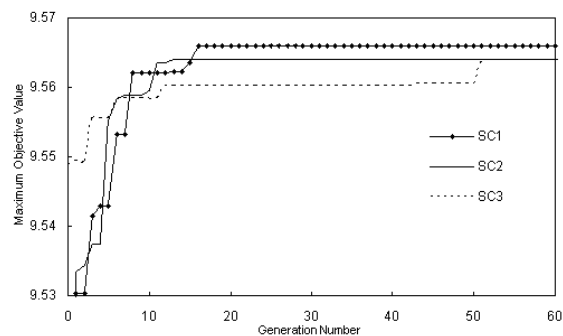
**Table 4.** Description of the runs.

Run	Crossover 1&2 points	Mutation 1&2 points	Population size
SC1	100	100	25
SC2	100	100	25
SC3	100	100	25
SC4	100	100	50
SC5	100	50	25
SC6	50	100	25

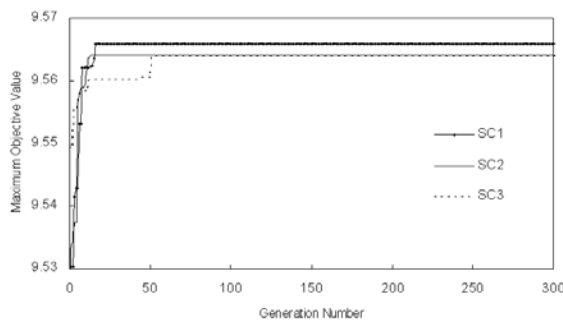
In the last part of this subsection, after presenting the complete definition and the fundamentals of the suggested LPC criterion, the results obtained by its application to the test problem will be shown and discussed in detail as well.

**The Statistical Parameters**

As mentioned before, the stopping point suggested by the first series of criteria, for the GA runs of the test problem called statistical parameters are studied here. First of all, the stagnation of maximum objective value (the most popular stopping criterion as explained in item 1, section 1) is studied by monitoring the values obtained for runs SC1, SC2, and SC3 for only 60 generations as depicted in Figure 2. This is because the example is too large and so many calculations are necessary in each generation. The figure shows that the maximum objective occurs at generation 16 for SC1, while it happens at generations 13 and 51 for SC2 and SC3, respectively. Based on Figure 3, by continuing the process even up to 300 generations for all three cases, now better results is achieved. Therefore, if one looks at any of the three runs separately, based on the stagnation of the maximum objective value, stopping the process at generation 300 would be a reasonable decision. However, since all these three runs belong to a single problem optimized under similar GA conditions but different first population members (due to random generation of the first population), the stopping point suggested by SC2 and SC3 in comparison to SC1 would



**Figure 2.** Maximum objective value for SC1, SC2, and SC3.



**Figure 3.** Maximum objective value in the first 60 generations.

be incorrect. This is because a much better result is offered by SC1. It means that if either one of the SC2 or SC3 runs continue for higher number of generations, it is definitely possible for them to find a similar result as that of the SC1. This may even be true for SC1, i.e. the solution obtained by SC1 may only be a local optimum brought about by premature convergence. In fact, continuation of any one of them mentioned may produce a better result. Therefore, this stopping criterion may not be as successful as suggested in the literature [9, 11-14, 23] though it is the most reliable criterion as far as regularity and ease of traceability is concerned.

The results of Figure 5 also show that the stopping criteria (as those in item 4, section 1), which suggest termination of the process after a given number of evaluations, or patience limit, etc. are not reliable enough for cutting off the GA process for a problem as the one at hand. In fact, it is not sufficient to set a limit for the number of generations because the best solution may occur in early generations as for the SC1, while it may take longer in the case of the SC2 and SC3 runs.

Using the stagnation of mean objective value as the stopping criterion (item 2, section 1), as was used in the previous research work [13] may not be useful for this sort of problems. As Figure 4 depicts, no acceptable stagnation in the mean objective value for the SC1 run during 300 generations is observed. Similar trends for the mentioned quantity are detected for the SC2 and SC3, which are not shown here. Some of the researchers have suggested that such a behavior may happen either due to the values considered for the rate of GA operators or due to the type of the applied penalty functions. However, as the effects of the penalty functions appear as a smooth reduction in the fitness value of the infeasible chromosomes during the reproduction process, it could not incur such a noisy effect. Therefore, it must mainly occur due to the probabilistically based operators of the GA process.

In a GA optimization process, the improvement in the fitness or objective value highly depends on the diversity of the population. When the diversity drops below a logical value the improvement in the objective value may face stagnation. This has been considered as a GA stopping criterion (item 7, section 1) by some of the researchers. Tracking down the standard deviations (SD) of the SC1, SC2, and SC3, which are depicted in Figure 5, shows that they sharply incline during the early generations. This is followed by the overall declining trend of the population diversity during the next 50 generations as shown for the SC1 in Figure 6. However, this relative stagnation in SD is highly disrupted for higher number of generations than 50. Even with a more integrated view of the standard deviation values, i.e. considering the average value of

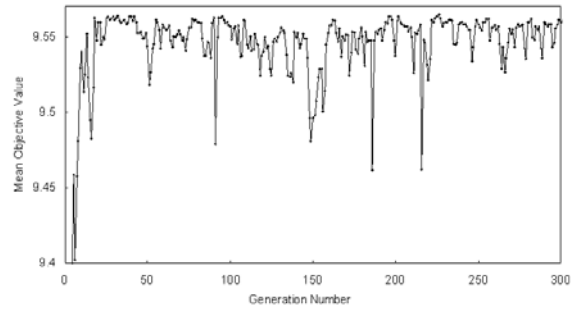


Figure 4. Mean objective value for SC1 in 300 generations.

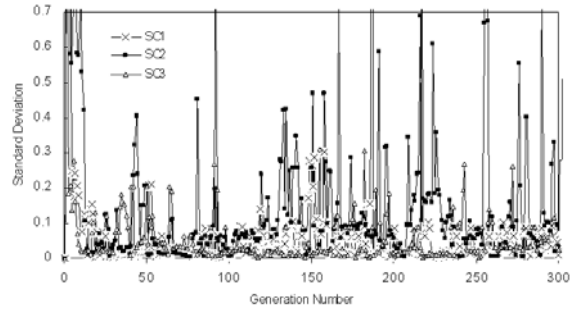


Figure 5. Standard deviations for SC1, SC2, and SC3.

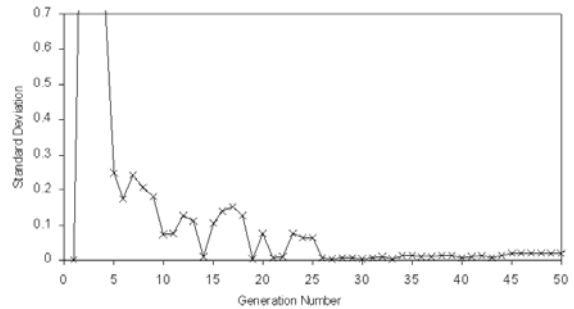


Figure 6. Trend of the standard deviations for SC1 in the first 50 generations.

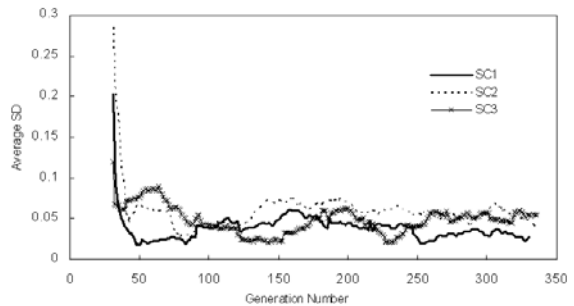


Figure 7. Average standard deviation for every 30 generations for SC1, SC2, and SC3.

the SD for every last 30 generations as shown in Figure 7, it does not seem that SD can be serve as a promising criterion. Based on the figure, the minimum value of this quantity appears at generation 50 for the SC1, but fluctuates again with increase of the generation number. The trend shows that this quantity continues to have a noisy nature in generations higher than 50,

which depicts the high unreliability of the quantity as a criterion for stopping the GA process. The same discussion is applicable when the difference between the maximum and minimum values or the difference between the maximum and mean values of the objective are considered as the stopping criteria for the GA runs. Based on Figures 8 and 9, these values also have a noisy nature despite the stagnation of the objective values for most of the 300 generations of the three runs of SC1, SC2, and SC3 depicted in Figure 3.

**The Theoretical Models**

The results obtained by applying the previously mentioned theoretical models [9] to the current test problem suggest inordinately large numbers for the minimum required generations. For the problem at hand, considering the corresponding required parameters ( $L=56, \mu=3/56, n=25$ ), even by setting a very low level of confidence (i.e.  $\zeta = 0.01\%$ ), both worst case bound formula (i.e. Equation (1)) and its modified version suggest almost infinite number of generations for obtaining the global optimum. Apparently, the obtained results show that the GA process may converge to an optimum value faster than what the theoretical models suggest. Perhaps, this is due to some of the conservative assumptions incorporated into the theoretical models such as the existence of one-max fitness function, which may not be valid here. It is also worth mentioning that the theoretical models do not pay any attention to the relationship between the convergence rate (which mainly comes from the principle of survival of the fittest) and the fitness distribution. Therefore, here, none of the theoretical models seems to be a suitable mean for stopping the GA process.

**The Logarithmic Performance Criterion (LPC)**

Since the existing stopping criteria do not sufficiently fit the present problem, a new stopping criterion, namely the Logarithmic Performance Criterion (LPC), is introduced and tested here. Based on the value of the first derivative of a logarithmic function fitted to the history of maximum objective values found up to the current generation, the LPC depicts the probability of finding a better solution within the next generations. The algorithm is halted when the LPC value at the current generation falls below a specific user-defined threshold.

As Figure 10 shows, the graphs of maximum objective values obtained from the results of all the considered runs (SC1 through SC6) are intrinsically similar and each one could be best explained by a logarithmic function. Such a behavior of the maximum objective value is widely observed in many other physical problems. For example, in addition to the intricate test case, a simple mathematical test problem

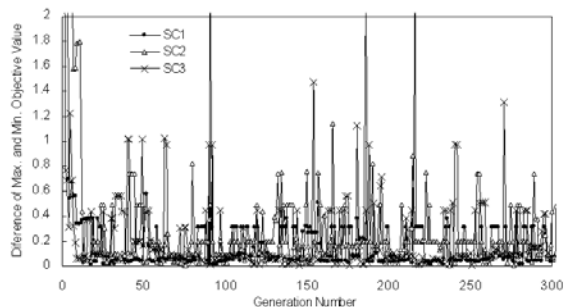


Figure 8. Difference of maximum and minimum objective values in each generation for SC1, SC2, and SC3.

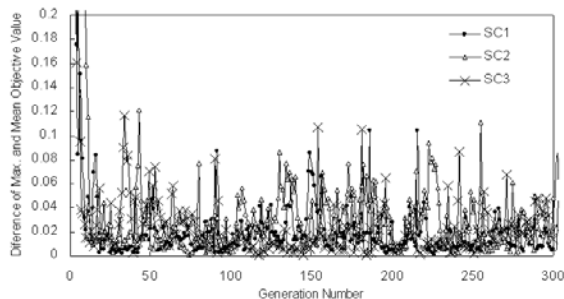


Figure 9. Difference of maximum and mean objective values in each generation for SC1, SC2, and SC3.

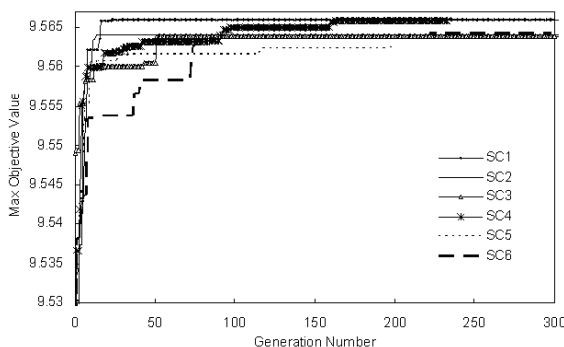


Figure 10. Maximum objective versus generation number for all cases of SC1 to SC6.

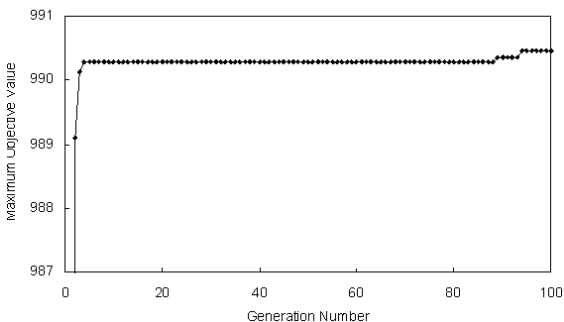


Figure 11. Maximum objective versus generation number for a simple mathematical problem.

represented by Equation (7) is also considered in this study. Where, the optimization process displays a similar behavior for the maximum value of  $g(x)$ , see

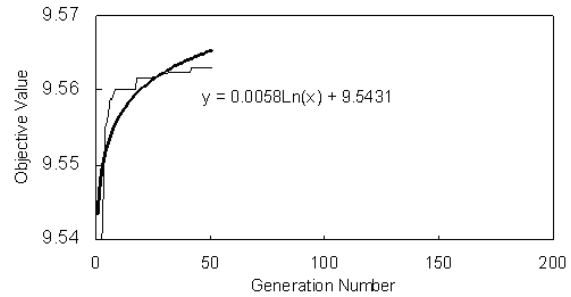
Figure 11.

$$Max : g(x_1, x_2) = 1000 - [x_1^2 + x_2^2 + 25(\sin^2 x_1 + \sin^2 x_2)]$$

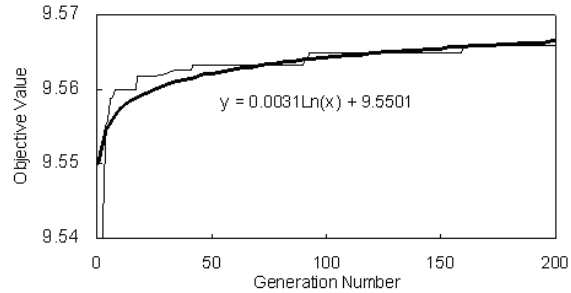
$$S.T. : x_1^2 + x_2^2 \geq 2 \quad (7)$$

Figures 12(a) and 12(b) illustrate logarithmic functions of the form  $(y=A\ln(x)+B)$  that could be fitted to the maximum objective values found in the preceding generations. The slope of the curves at any generation changes due to not only the increase in generation number but also the variations in A and B. This characteristic, i.e. the slope of the curve at the current generation, is used here as the basis for defining a new logarithmic performance criterion (LPC) that could be served as a stopping condition for the GA process. The idea is to find the constants A and B at any generation for fitting the best curve to all of the maximum objective values obtained during the preceding generations. The first derivative of the fitted function evaluated at the current generation is then referred to as the LPC value for that generation. The difference between the subsequent LPC values plays an important role in stopping the GA process. Plotting the end slope of the logarithmic functions (the LPC values) versus the generation number shows a smooth and regular declining curve during the reproduction process. This is clearly shown in Figure 13 for the SC3 and the average value for three runs of each one of the SC1, SC2, and SC3 cases. It is observed that after a large number of generations the curve asymptotes to the horizontal line, and the probability of finding a better solution by continuing the process approaches zero.

In other words, the end slope of the curve or the LPC could be used for determining the potential or capability of the GA in finding a better solution in the subsequent generations. Thus, one can easily decide on the interruption point by setting a minimum desirable value for the LPC or defining a minimum acceptable deviation in the maximum objective value per generation. The latter is controlled by the minimum admissible changes in different terms of the objective function such as the weight and deflection in the present case. It is important to note that the method provides an online performance parameter that enables the user to calculate the LPC value in the current generation and to control the stopping point by concurrently adjusting its minimum acceptable value. Additionally, due to the origin and nature of the LPC, it is implicitly influenced by the selection of the probability values for the genetic operators, population size, form of the objective function, etc. A full description of the LPC characteristics is discussed next.



(a) For 50 generations



(b) For 200 generations

**Figure 12.** Fitting a logarithmic function to the maximum objective trend.

### COMPARISON OF THE LPC WITH THE CONVENTIONAL STOPPING CRITERIA

One of the most outstanding characteristics of the LPC is in its high degree of regularity and smoothness seen in the trend of the values with the generation number during the GA process. In comparison to the irregular and noisy behavior of the existing stopping criteria discussed in the previous sections, the LPC shows a smooth trend during the whole process. It starts with a high value and regularly decreases as the generation number increases (see Figure 13). The horizontal asymptotic configuration of the LPC curve in the final stages of the process could be interpreted as a very low (or zero) potential of the GA in finding better solutions in the subsequent generations. In general, this trend could also be assumed as zero performance of the GA after infinite number of generations like what is expected from the mathematical models.

The next important characteristic of the LPC could be related to its capability of providing the user with an efficient tool for defining a more reasonable cut off point for the GA process. Theoretical models usually end up with very large numbers for minimum required GA generation cycles that are not accessible in engineering applications, which are usually comprised of several design variables. However, in practice, GA processes normally converge considerably earlier (in lower number of generations) than predicted by theoretical models [11]. Since the LPC takes the history into account (by considering the trend of the maximum objective values found in all the previous



generations, as observed in practice) it can easily overcome the above mentioned disadvantage by defining a more reasonable value for the required number of generations. This advantage is mainly attributed to the less conservative assumptions considered by the LPC in comparison to the theoretical models.

In comparison to the conventional criteria, taking a wider data scope into account and less parameter dependency of the LPC could also be considered as another distinguished characteristic of the method. Since the LPC value is calculated by differentiating the best fitted logarithmic function to the maximum objective values found in all the previous generations, it does not depend on any user-defined parameter; also the use of a wide data range is ensured. This is in contrary to some of the traditional stopping criteria, such as items 5, 7, and 10 in section 1, which merely pay attention to the current population or some other criteria such as items 2, 3, and 6-8, all of which deal with two or more populations only (i.e. two

consequently found generations or the first and last generations).

The LPC can implicitly adapt itself to an unexpected jump in the GA results occurred by appearance of a better solution in a generation number like  $N=50$  in the current example. This is clearly shown in Figure 14. As explained before, the LPC value could represent the potential of GA for finding a better solution. Therefore, as the figure illustrates, a jump in the maximum objective value increases the value of LPC. In other words, the jump shows that a better point is found, and as a result, the GA process now has a higher capability for finding a better solution in the subsequent generations by using the aspects of the newly found chromosome. In such conditions, the LPC curve visibly projects out of its regular path while all the theoretical models and also most of the statistical parameters discussed earlier fail to present such a sensitivity.

Additionally, the LPC is an on-line parameter that is capable of being concurrently controlled by the user. While the theoretical and the statistical criteria such as items 1, 4, and 9 in section 1, are defined at the beginning of the process, they are not affected by the results found during the evolutionary search. The LPC value is calculated and controlled in each generation. The minimum acceptable value set for the LPC, which is the only user-defined parameter in determining the interruption point, is also capable of being concurrently trained regarding the best solution found up to the current generation. For instance, for the composite test case considered here, a designer with some sense of engineering, can set the minimum desirable change in weight and deflection of the panel (considering the best chromosome found in the current generation). Then, it is possible to calculate the minimum LPC value and subsequently use this value for determining an admissible interruption point.

Providing a better facility for optimization of commonplace engineering decision-making problems is another characteristic of the LPC in comparison to the conventional approaches. For this sort of problems, designers normally find the optimum balancing point for two or more conflicting design parameters (i.e. weight, reliability, and production cost). The most important challenging point of such problems is that a designer compares parameters that are different in nature and are implicitly inter-related. Therefore, using a specific relationship among these parameters considering their values in the current search point is unavoidable. Interestingly, the LPC gives this opportunity to a designer to correct his decision in each generation and also to train it for the best solution obtained so far.

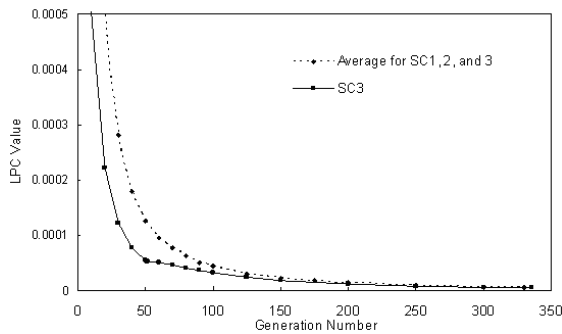


Figure 13. The LPC values versus generation number.

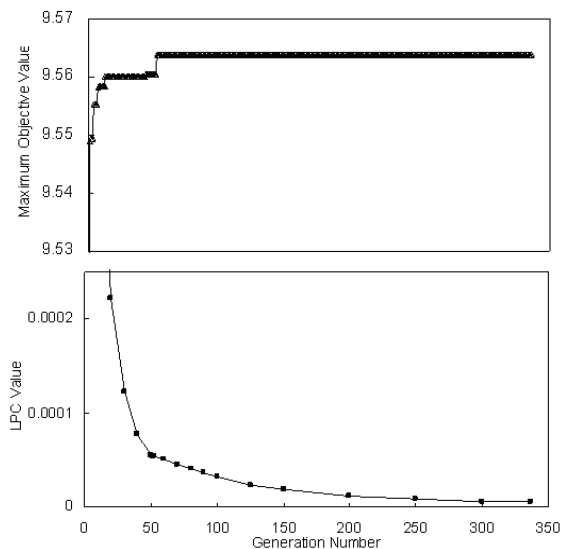


Figure 14. Self training of the LPC with a jump in maximum objective value.

### SUMMARY AND CONCLUSIONS

In this research, the Logarithmic Performance Criterion (LPC), which is the first derivative of a logarithmic function fitted to maximum fitness values found in all generations, is introduced as a new stopping criterion for the GA process of laminated composite plates. The results show that, contrary to the statistical parameters and theoretical models, the LPC could be assumed as an adequate parameter for defining the interruption point for the GA process considering its smooth changes during the process and its reasonable suggestion for the minimum required number of generations. The form and overall trend of this parameter (i.e. the LPC) fulfills the mathematical assumptions, and is indeed faster in convergence than other discussed methods as observed in practice. In brief, the following advantages and efficiencies were obtained by applying the LPC to the discussed examples. One may also expect such advantages when using the LPC:

- Sufficiently regular shape and alteration
- More reasonable restriction on the number of generations
- Less parameter dependency
- Requiring less conservative assumptions
- Implicitly self-tuning
- On-line and concurrently controllable by the user
- Accounting for a wider data scope by considering the previous generations
- Providing a better facility for optimization of commonplace engineering decision making problems.

### REFERENCES

1. Goldberg DE., *Genetic Algorithm in Search, Optimization, and Machine Learning*, Addison-Wesley Publishing Company Inc., (1989).
2. Nagendra S. Haftka RT. Gurdal Z., "Stacking Sequence Optimization of Simply Supported Laminates with Stability and Strain Constraints", *AIAA Journal*, **30**(8), PP 2132-2137(1992).
3. Mesquitea, L, Karmat MP., "Optimization of Stiffened Laminated Composite Plates with Frequency Constrains", *Engineering Optimization* , **11**, PP 77-86(1987).
4. Caligiana G, Cesari F, Persiana F, Seggiani GM., "Optimization of an Air Intake Ramp in Composite Material by utilizing FEM and GA Numerical Program", XII ADM International Conf., Rimini, Italy, PP 112-123(2001).
5. Liu B, Haftka RT, Akgun MA., "Composite Wing Structural Optimization Using Genetic Algorithms and Response Surfaces", *AIAA Journal*, (1998).
6. Sandrine G., "Experimental Design Optimization and Thermophysical Parameter Estimation of Composite Materials Using Genetic Algorithms", Ph.D. Thesis, Mechanical Eng. Dept. of Virginia Polytechnic Institute and State University(1999).
7. Grant AE. Soremekun, "Genetic Algorithms for Composite Laminate Design and Optimization", M.Sc. Thesis, Virginia Polytechnic Institute and State University(1997).
8. Callahan KJ, Weeks GE., "Optimum Design of Composite Laminates Using Genetic Algorithm", *Composite Engineering*, **2**(3), PP 149-160(1992).
9. Aytung H, Koehler GJ., "New Stopping Criterion for Genetic Algorithms", *European Journal of Operational Research*, **126**, PP 662-674(2000).
10. De Jong KA., "An Analysis of the Behavior of a Class of Genetic Adaptive Systems", Ph.D. Thesis, University of Michigan(1975).
11. Greenhalgh D, Marshal S., "Convergence Criteria for Genetic Algorithm", Society for Industrial and Applied Mathematics (SIAM J.C.), **30**, PP 269-282(2000).
12. Aickelin U., "An Indirect Genetic Algorithm for Set Covering Problems", *Journal of the Operational Research Society*, **53**(10), PP 1118-1126(2002).
13. Gazen C, Erosy C., "Genetic Algorithm for Designing Multihop Lightwave Network Topologies", *Artificial Intelligent in Engineering*, **13**, PP 211-221(1999).
14. Alba E, Troya JM., "A Survey of Parallel Distributed Genetic Algorithms", *Journal of Complexity*, **4**(4), PP 31-52(1999).
15. Nemes A, Lantos B., "Genetic Algorithm-Based Optimization of Fuzzy Logic Systems for Dynamic Modeling of Robots", *Periodica Polytechnica Ser. El. Eng.*, **43**(3), PP 177-187(1999).
16. Soremekun GAE., "Genetic Algorithm for Composite Laminate Design and Optimization", M.Sc. Thesis, Virginia Polytechnic Institute and State University(1997).
17. Chow TT, Lin Z, Song CL., "Applying Neural Network and Genetic Algorithm in Chiller System Optimization", Proceeding of the 7th International IBPSA Conference, Rio de Janeiro, Brazil, (2001).
18. GALib, "A C++ Library of Genetic Algorithm Components", *Version 2.4*, (1996).
19. Vertan C, Vertan CI, Busuloiu V., "Reduced Computation Genetic Algorithm for Noise Removal", *IEE Image Process*, **443**, PP 313-316(1997).
20. The MathWorks Inc., "Genetic Algorithm and Direct Search Toolbox", (1994-2004).
21. Vose MD, Liepins GE., "Punctuated Equilibria in Genetic Search", *Complex Systems*, **5**, PP 31-34(1991).
22. Nix A, Vose MD., "Modeling Genetic Algorithm with Markov Chains", *Annals of Mathematics and Artificial Intelligence*, **5**, PP 79-88(1992).
23. Coello Coello C.A., "A Comprehensive Survey of Evolutionary-Based Multiobjective Optimization Techniques", *Allel problem solving from nature*, **5**, PP 292-301(1998).