

Study of Clear Air Turbulence over Iranian Plato

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This study was carried out using two sets of numerical weather forecast data and flight reports for Clear Air Turbulence (CAT) over Iranian Plato to find atmospheric flow patterns favorable to the formation of CAT. The numerical data include five months of AVN analysis with horizontal resolution of 1 degree(about 100 km) and four months forecast data of MM5 model with resolution of 50 km. Important indices associated with conditions favorable for CAT formation include wind shear, flow deformation fields and atmospheric static stability (or joint effects such as Richardson number). These indices were estimated and also some algorithms such as Dutton to find their threshold values for CAT occurrence. The best selected algorithms are determined in the 5 months period using AVN outputs. In another 4 months period the MM5 outputs were used for the forecast of CAT using the POD method. The results for these periods indicated that severe CAT rarely occurred over Iran and condition for moderate CAT are often in upstream of troughs and ridges, near straight jet stream with cold advection and upper region of ridges. The POD method shows Dutton and Ellrod-2 are the best selected algorithms for CAT forecasting and have good consistency with observations. Their typical threshold values (associated with wind shears) are about $3 \, s^{-2}$ and 20 for moderated CAT respectively.

INTRODUCTION

Clear Air Turbulence (CAT) occurs in the free atmosphere (about >10000 m AGL) and far away from the visible convection activities [1]. CAT was first recognized in 1946 by Bachman and in 1960s it was considered as a potential danger for flying aircraft [2].

CAT is either known as upper air turbulence, upper front turbulence, jet stream turbulence or all of them. It usually occurs in the lower stratosphere and upper troposphere. The four atmospheric factors having important role in CAT production are large scale waves, tropopause, jet streams and jet streams front. Two general mechanisms are considered as CAT producers: standing waves in lee mountains and sheared gravity waves in the stable layers. Both of these mechanisms are strongest during winter when wind speed and horizontal temperature gradient are

maximal. CAT occurs more often over land than over

DATA

Two sets of data from the numerical weather predictions model outputs from two different models which

the seas and its intensity increases in mountainous areas. CAT frequency is usually maximal in 30 to 45 thousands feet (about 9000 to 13600 meters AGL) aloft in the troposphere and then decreases as altitude increases. Considering the effects of CAT in aeronautics related to flight safety, regional studies of meteorological conditions favorable for its production are important. In certain conditions of clear air turbulence or mountain waves reported by pilots, some limitation is enforced regarding air traffic control. For example, under these conditions aircraft vertical separation is increased from 1000 to 2000 feet (330 to 660 meters) In this paper, CAT algorithms are estimated using numerical weather forecast data and pilot reports from the flight routes over Iranian Plato. We use different algorithms to find the best CAT algorithms for its prediction over the Iranian Plato which is a mountainous area.

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are validated by observations are used in this study to estimate the CAT algorithms. The first set is from the AVN model outputs for a 5 month (Jan. to May 2004) period. The horizontal resolution in this data is 1 degree, and is available at 00:00, 06, 12 and 18 UTC. The second set is the forecast data of MM5 model with a finer resolution of 50 km at 6 h intervals that are more suited for the calculations of CAT algorithms. These data sets are considered for the study of CAT algorithms and the large scale atmosphere patterns associated with CAT formation.

Since clear air turbulence is a meso to micro scale phenomenon, the data resolutions would not suit the calculation of certain CAT properties such as turbulence intensities, but are well suited for the calculations of large synoptic scale atmospheric motion associated with CATs which are reported by pilots. These data are routinely available (especially for MM5) used operationally in the Weather Forecast Center of IRIMO). Usually higher resolution numerical forecast data (say 10 km or less) are required for a detailed CAT study [4]. Today CATs are reported by AMDAR [5]. Since most of the domestic flights over Iran are not usually equipped with this system and the aircraft with AMDAR data system are restricted only to 3 or 4 routes, the CAT information are not available on a routine bases in all air routes. Also, air reports often lack detailed turbulence information. Hence, a table was prepared specifically for this study to collect CAT reports by asking pilots through the air traffic controllers of Iran CAO. The table covers all the flight levels from 28 to 39 thousand feet (about 8400 to 11800 meters). This table contains information such as CAT intensity, altitude, position and time of CAT report including type of the airplane from which the report is acquired. Hence, there is a reference for comparing calculations and observations. The times of the reports and the time of CAT algorithm calculations are almost the same within ± 30 minutes. The area of the study is 20 to 45° North and 30 to 70° East, which covers the whole Iranian Plato. The used data sets (MM5 and AVN) include air temperature, wind (direction and speed) and geopotential height in 200, 250, 300 and 350 hPa levels (with 50 hPa intervals). At these levels, geopotential height, temperature, wind patterns and wind shear, static stability and CAT algorithms such as Dutton, Ellrod-2, Brown and Ri are analyzed. To show and explain position of CAT, abbreviation of some Iranian cities are inserted in the maps used such

TRN, Tehran; TBZ, Tabriz; MSD, Mashhad; ISN, Isfahan; AWZ, Ahwaz; SYZ, Shiraz; KER, Kerman; KIS, Kish Island; ZDN, Zahedan; KRD, Khorramabad; SNJ, Sanandaj and SBM, Chabahar. Then the algorithms and relevant parameters are calculated. These parameters are introduced in the next section.

CAT ALGORITHMS

The CAT algorithms estimated from the meteorological data are those used to find the conditions in which the onset of hydrodynamics instability in the atmospheric flow may occur and is maintained to sustain clear air turbulence production.

Stratified shear flows as in the atmosphere is prone to instability when the flow Richardson number, Ri, becomes usually less than 0.5. Ri is the ratio of static stability to the vertical wind shear squared as [6]:

$$Ri = \frac{g}{\overline{\theta}} \frac{\frac{\partial \theta}{\partial z}}{\left(\frac{\partial V}{\partial z}\right)^2},\tag{1}$$

where g is the gravitational acceleration, $\overline{\theta}$ is the average potential temperature of the environment in the layer, $\frac{\partial \theta}{\partial z}$ is the vertical potential temperature gradient between two levels 50 hPa apart, and $\frac{\partial V}{\partial z}$ is the vertical shear of horizontal wind component. The other algorithm is the Brown algorithm, which can be obtained from Roach parameter ($\Phi \equiv -(D/Dt) \ln Ri$). With some modifications due to the directional perturbations effects of vertical wind shear vector this leads to the Brown algorithm defined as [8]:

$$\Phi_m = (0.3\xi_a^2 + D_s^2 + D_T^2)^{\frac{1}{2}},\tag{2}$$

where $\xi_a = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f$ is the vertical component of absolute vorticity, $D_S = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$ is the deformation field of horizontal wind shear and $D_T = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$ is the deformation flow field of stretching mechanism [9, 8]. Brown has shown that the threshold value of this algorithm for moderate turbulence level is about $10 \times 10^{-5} s^{-1}$ [8]. Dutton algorithm is calculated according to the linear regression of turbulence intensity from flight reports over northern Atlantic Ocean and northern Europe in 1976 and the horizontal and vertical wind shears in the atmospheric flow. Following the Roach study in 1970[10], the empirical Dutton algorithm (E) is introduced for CAT forecasting as the combination of vertical and horizontal wind shears as:

$$E = 1.25S_H + 0.2S_V^2 + 10.5, (3)$$

where S_H and S_V are the horizontal and vertical wind shears respectively, and are defined as:

$$S_{H} = \frac{1}{V^{2}} \left(\overline{uv} \frac{\partial u}{\partial x} - \overline{u^{2}} \frac{\partial u}{\partial y} + \overline{v^{2}} \frac{\partial v}{\partial x} - \overline{uv} \frac{\partial v}{\partial y} \right), \tag{4}$$

$$S_V = \frac{\partial V}{\partial p} \cdot \frac{\partial p}{\partial z},\tag{5}$$

Here u and v are the horizontal wind components in x and y coordinate axes respectively, and uv is their covariance, and V is the horizontal wind speed. The

experimental threshold of this algorithm for moderate CAT is about 20 [11]. The Ellrod algorithm is obtained from Petersen frontogenesis equation, which is given by $I_f = |\nabla \theta| (0.5) [DEF(\cos \beta) + CVG]$ [12].

 I_f shows the frontogenesis intensity, $|\nabla \theta|$ is the magnitude of potential temperature gradient, DEF is the magnitude of the deformation field, CVG is the size of flow convergence and β is the angle between the expanding axis and the isotherms. Using thermal wind equation and maximum frontogenesis intensity, the Petersen equation will be:

$$A_T = 0.5(f\frac{T}{g})(VWS)[DEF + CVG], \tag{6}$$

where VWS is the vertical wind shear, f is Coriolis parameter and T is Temperature. Thus, the frontogenesis is associated with the increase of vertical wind shear, hence the probability of CAT occurrence. It is also related to the magnitude of potential temperature gradient, deformation fields of shearing and stretching and also convergence factors. Ellrod in 1992[7] introduced another turbulence algorithm given by:

$$TI1 = DEF \times VWS, \tag{7}$$

where VWS is the vertical wind shear and DEF is the deformation field. It has been observed that convergence parameter (CVG) has a considerable effect on CAT formation in some case studies [13, 14]. Therefore, Ellrod has found a better algorithm for CAT, which is defined as:

$$TI2 = VWS \times (DEF + CVG). \tag{8}$$

The threshold value of this algorithm for moderate CAT is about as $2\text{-}4 \times 10^{-7} \, \text{s}^{-2}$ [7]. In the present study the calculations of all CAT algorithms have been done for the layers of 300-350, 250-300 and 200-250 hPa. Then the POD method is used to estimate the correctness of CAT forecasting for the cases considered. Weise in 1977[15, 16] introduced the POD method as:

$$POD = \frac{x}{x+y},\tag{9}$$

where x is the number of correct forecasts and y is the number of wrong forecasts for CAT, hence, the POD method shows the percentage of turbulent cases which are forecast correctly. The POD can be shown as the POD (yes) or the POD (no). In this study the POD (yes) is calculated only for Dutton and Ellrod algorithms.

SYNOPTIC ANALYSIS OF THE CASE STUDIES IN THE 5 MONTH PERIOD (JAN. TO MAY 2004)

In cases during the 5 month period all pilot reports that indicate turbulent conditions are 93 days. There are

about 417 reported cases, of which 180 are classified as moderate and 237 as the light to moderate turbulence. For all cases, geopotential heights, temperatures and wind fields are analyzed. Also, static stability, wind shear and some turbulence algorithms such as Brown, Dutton, Ellrod algorithms and Richardson number are calculated using AVN model outputs. Hence, firstly the most common atmospheric flow patterns and the most convenient algorithms which are associated with CAT reports are determined over Iran for the period of this study and secondly, the threshold values of the best algorithms are determined. One example of the flow pattern from these case studies is presented in Figures 1 to 3 (below). It should be mentioned that these patterns are analyzed for all reports. After determining the best CAT algorithms and their thresholds for all case studies, the MM5 weather forecast output model was used for CAT forecasting using the best algorithms.

200-250 hPa layer 00:00UTC 16 Jan. 2004

On January 16^{th} , 5 CAT were reported by pilots at 00:00 UTC all from heavy airplanes. Four reports were received from altitudes of 36 to 38 thousand feet (about 10900 to 11500 meters) with light to moderate intensity in Tehran and Anarak (north of Yazd) areas. One report was received from 36 thousand feet (about 10900 meters) in Zahedan area with moderate intensity. Thus, the 200-250 hPa layer weather patterns were analyzed to see the cause of these CATs. A 140 kt jet streak can be seen over Oman Sea and 100 to 120 kt jet streams are located over Zahedan area. Tehran and Anarak zones are on the 80kt jet streak domain (Figures 1-a and 1-b). During this day a deep trough is observed over the south east of Iran and towards the western part of Iran where there is a geopotential ridge. Also a warm pool is formed in the center of 200 hPa geopotential trough. Tehran and Anarak areas are located downstream of a trough where there is a cold advection. The position of Zahedan area is exactly on the trough line (Figures 1-c and 1-d). Also north westerly winds are dominant in these areas.

Vertical and horizontal wind shear patterns show values of $10 \times 10^{-3} s^{-1}$ and $6 \times 10^{-5} s^{-1}$ in the Zahedan area respectively as can be seen in the Figures 2-b and 2-c. Static stability parameter has a value of about $3.5 \times 10^{-4} \ s^{-2}$ in the same area (Figure 2-a). Considering the ranges of the Dutton, Ellrod, Brown and Richardson algorithms for this case (Figures 3-a to 3-d), there are no turbulent conditions over Tehran and Anarak areas. This may be as a result of the observed weak jet stream over these areas. However, over Zahedan area, as can be seen in the Figures 3-a to 3-d, maximum values of turbulence algorithms are found on the trough line over this area which is extended toward Turkey. The CAT algorithm values over Zahedan for Dutton algorithm is about 45, for

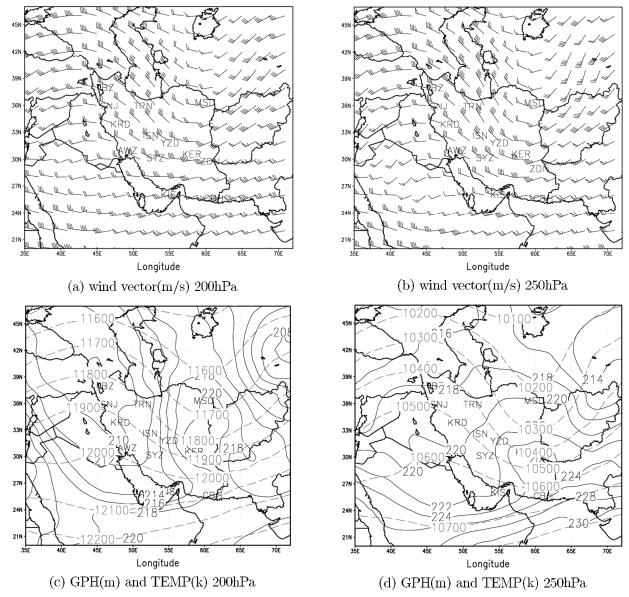


Figure 1. Wind, Temperature and Geopotential height patterns for 5 Feb. 2004, at 00:00UTC; (a) and (b) wind patterns in 250 and 300 hPa, (c) and (d) geopotential height (solid lines) and temperature (dash lines) patterns in 250 and 300 hPa. Solid lines are geopotential height lines and dash lines are isotherms.

Ellrod algorithm about $12 \times 10^{-7} s^{-2}$, and for Brown algorithm is about $10 \times 10^{-5} s^{-1}$, and the Richardson number is between 0.3 to 0.5 (Figures 3-a to 3-d). For these algorithms similar turbulence intensities have been observed by others (e.g 19), who have also noticed the same vertical wind shear pattern as observed in Figure 2-b. This shows that there is a direct relationship between clear air turbulence and vertical wind shear. In this case study, the turbulent algorithm values are consistent with observational data and reports. The same analyses are carried out for other studies of 417 cases with pilot reports.

After analyzing all cases and determining the most well agreed turbulence algorithms and their

threshold values regarding the moderate and light to moderate CATs, the CAT forecasting correctness was carried out using the POD method (relation (9)). For this purpose, the previous calculations are repeated again, but with MM5 model weather prediction outputs. The results show that the Dutton and Ellrod algorithms show better agreement with CAT reports than with other parameters. Hence, for the next case studies (in the 4 month period from Dec.2004 to March 2005), Dutton and Ellrod algorithms were calculated again using MM5 model outputs to find the percentage of their correctness. In these studies, the pilot reports which are about 430 cases (including 144 moderate CAT and 286 light to moderate CAT) were tested

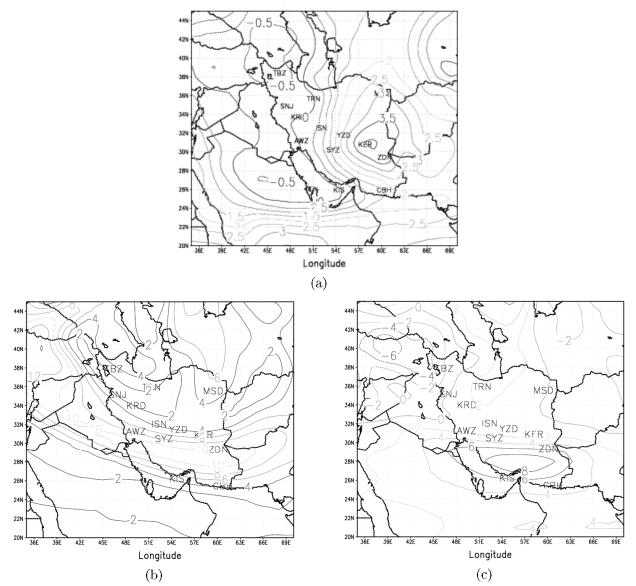


Figure 2. (a) Static stability $(\times 10^{-4}s^{-2})$, (b) vertical wind shear $(\times 10^{-3}s^{-1})$ and (c) horizontal wind shear $(\times 10^{-3}s^{-1})$ in 250-300 hPa layer for 5 Feb. 2004, 00:00UTC.

using Dutton and Ellrod algorithms, and finally the probability of detection parameters (POD) regarding the x and y values (as in 5) were calculated. The results are shown in Table 1. It can be seen that regarding all CAT reports (430 cases), the forecasts by Ellrod and Dutton algorithms are nearly 80% and 70% correct respectively.

DISCUSSION

Previous studies [17, 18] show that CAT's are associated with 100 kt or more winds in jet streak. Wind patterns show that maximum winds of jet streak are between 90 to 150 kt and rarely more than 140 kt in this study. Vertical cross section of the wind and temperature fields in 16^{th} of Jan. 2004 shows that most

of CAT reports occurred near the jet streak especially in the outer part of the jet (Figure 4).

Endlich 1964 [19] mentioned that shallow troughs and ridges are rarely associated with moderate or severe CAT's and as it can be seen in the cases of the first 5 month period of the study, geopotential height and wind patterns show that most of the CAT reports (about 80%) are associated with shallow troughs and ridges located in the cyclonic part of the jets.

This study shows that 64 CAT cases are located upstream of the troughs, 34 upstream of the ridges, 15 associated with straight jets (without deep trough or ridge), and finally 13 cases are found at the peak of the geopotential ridges. Hence, based on this study it is inferred that upstream of the troughs are the best locations for CAT formation due to the hydrodynamic

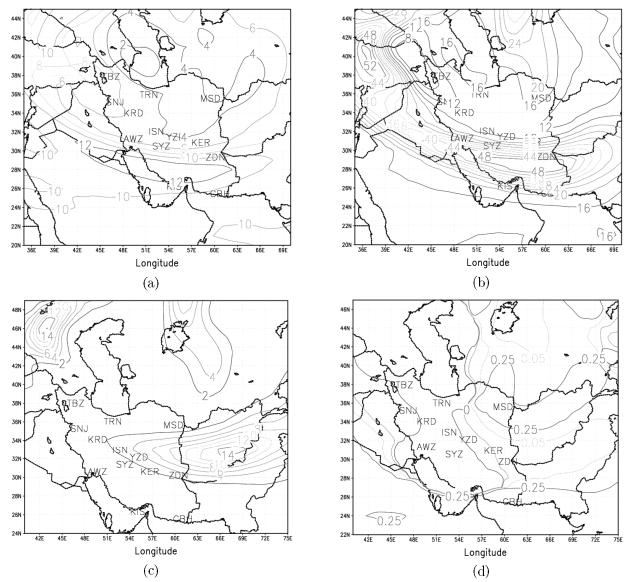


Figure 3. Clear Air Turbulence algorithms in 250-300 hPa for 5 Feb. 2004, 00:00UTC. (a) Brown algorithm ($\times 10^{-5}s^{-1}$), (b) Dutton algorithm, (c) Ellrod-2 algorithm ($\times 10^{-7}s^{-2}$) and (d) Ri algorithm.

instability; and the more stable conditions are dominant at the peak of geopotential ridges with the least number of CAT reports. Also, as the pilot reports show, only 8% of the CAT reports are reported (Figure 5) and more than half of these reports are light to moderate CAT (Figure 6). These findings correspond well with those of Endlich (19).

Figure 7 shows that in most cases the CAT occurred between 250 and 300 hPa layers. It can be related to the mechanism in which the combination of polar and subtropical jets in this layer is responsible for strengthening the vertical wind shear Verification of Dutton, Ellrod and Brown algorithms with pilot reports show that there is a correlation between algorithms and reports. This means that these algorithms are well coordinated with pilot reports

with 68%, 60% and 48% respectively. So Dutton and Ellrod are considered as the best algorithms to be used for CAT forecasting. Figures 8 and 9 show that most of the CAT reports are in the following relevant algorithms range for moderate CAT: 18<Dutton<22, $2 \times 10^{-7} s^{-2}$ <Ellrod<4 $\times 10^{-7} s^{-2}$. These values for light to moderate CATs are 15<Dutton<20, $1 \times 10^{-7} s^{-2}$ <Ellrod<3 $\times 10^{-7} s^{-2}$, which are similar to the Dutton and Ellrod values that concluded for observations from some parts of the United States and Europe especially for moderate CAT [11,7].

Vertical wind shear patterns are also similar to CAT algorithm patterns as in the previous studies [20, 11]. Verification of derived Dutton and Ellrod algorithms by MM5 outputs, using POD method shows that these algorithms are in good agreement (about

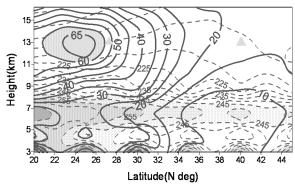


Figure 4. Vertical cross section of wind and temperature for 16 Jan. 2004. Solid and dash lines show isotach (m/s) and isotherms o (Celsius) respectively.5Symbol is the position of CAT reports.

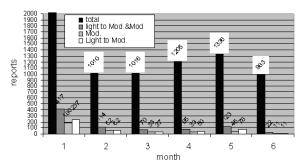


Figure 5. Comparison CATs intensity with all reports in the 5 month period as follw: 1-5 months reports totally, 2-Jan., 3-Feb., 4-Mar. and 5-May 2004.

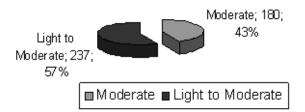


Figure 6. Comparison of light-moderate and moderate CATs in the 5 month period (Jan. to May 2004).

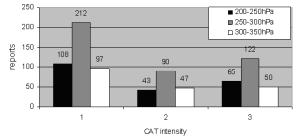


Figure 7. CAT reports related to layers as fallow: 1-total (Mod. And Light to Mod., 2-Mod. and 3-Light to Mod.

80% and 70% respectively) with pilot reports (Table 1).

It is necessary to mention that Ri number can not be considered as a CAT algorithm because in

Table 1. Correctness CAT forecasting using POD method.

		X(correct forecast)	Y(incorrect forecast)	POD(yes)
	Dutton algorithm	344 cases	93 cases	≌80%
	Ellrod algorithm	310 cases	127 cases	≌71%

many cases there is no turbulent condition although the Ri values show an unstable situation. This can occurr due to large layer thickness (about 50 hPa) in this study such that the local Ri number can not be estimated in this layer. In that case it is recommended to calculate this algorithm in layers with smaller thicknesses using higher resolution models or work in isentropic coordinate to find better results from Ri values.

CONCLUSION

It appears that the jet streams with 100 kt wind or more is the most important factor for the occurrence of clear air turbulence. In this study, these jet streams are mostly straight or often associated with shallow troughs and ridges so that the curvature for the intensity of deformation field that may be weaken, number can not be estimated in the layer. In such cases it is recommendable to calculate this algorithm in layers with smaller thicknesses using higher resolution models or work in isentropic coordinates to find better results from Ri values.

As it is seen from the pilot reports, severe CAT reports are rarely observed over Iran. Vertical wind shear patterns have a good correlation with turbulence algorithm patterns such that its contours are well correlated with calculated algorithm values.

Also upstream of the troughs is the best position for clear air turbulence formation due to the strong wind shears suitable for hydrodynamic instability. It is also found that the moderate CATs occur during winter when polar jet is in its most intense condition over Iran. In this study it was shown that Ri index was not a good indicative index for occurrence of CAT condition as the present data were not enough to calculate Ri locally as required. The threshold values of other algorithms over Iran are similar to the reported values over Europe and U.S.A. although with more observational data better results will be obtained. POD method in this study shows that for forecasting CAT the Dutton and Ellrod algorithms which correspond well with pilot reports (direct observations) are more appropriate.

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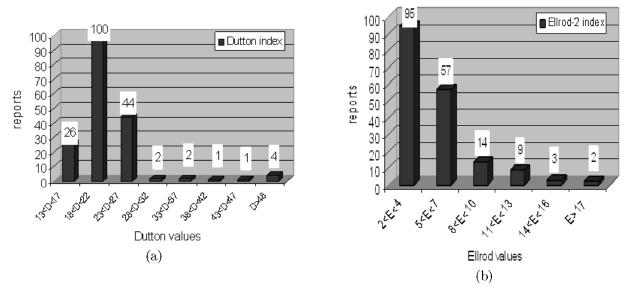


Figure 8. Abundance algorithms Moderate CAT for (a) Dutton, (b) Ellrod-2.

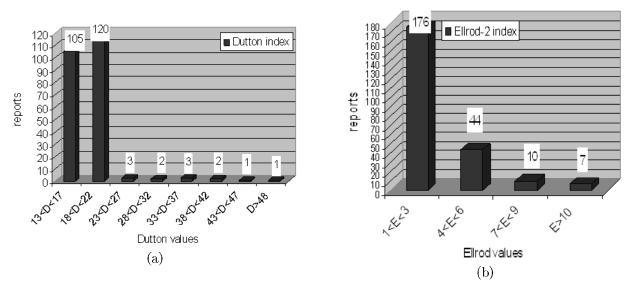


Figure 9. Abundance algorithms for Light to Moderate CAT for (a) Dutton, (b) Ellrod-2.

Table 2. Acronyms

Aviation data	AVN
Above Ground Level	AGN
Aircraft Meteorological Data Relay	AMDAR
civil Aviation Organization	CAO
Clear Air Turbulence	CAT
Islamic Republic of Iran Meteorological Organization	IRIMO
Meso-scale Model version5	MM5
Probability Of Detection	POD
Universal Time Coordinate	UTC

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