Application of a Simulation Algorithm for Dynamic analysis of a Liquid Propellant Engine

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In this paper, application of a simulation algorithm to dynamic and nonlinear analysis of a specific liquid propellant engine is presented. The mathematical model of the engine includes a set of nonlinear algebraic equations coupled with a set of time varying differential equations. In contrast to the existing liquid propellant simulation algorithms, the presented work does not depend on the method of modeling. The simulation algorithm is composed of six primary steps. Comparison of the nominal values obtained from simulation with actual designed values is presented. Typical simulation outputs of primary engine variables are also given. The results of this study are used in the initial and conceptual design stages in order to advance to other design stages.

INTRODUCTION

Liquid propellant engines (LPE) are complicated. They form an important part of aerospace systems. If the LPE simulation software were not developed, the experts would have to rely on a number of real experiments. These tests are very expensive, and are also dangerous. In contrast, if the simulation software is available, the experts may easily optimize the engine.

The primary objective of this paper is to report on the application of an existing simulation algorithm [1] to a specific LPE. The number of papers that discuss simulation of LPEs as a whole is limited [1-7]. The majority of such papers on the LPE simulation subject assume that the engine is modeled in such a way that the application of an integration routine would be adequate [5,6]. In [8-14], the general relations for simulation of the combustion chamber are given. In [15], the simulation results of a specific gas generator are discussed. The important point is that, in these and similar references, the simulation of the complete LPE as a whole are not discussed. In [16], the simulation of a specific LPE is considered, and the governing linear equations along with the software for solving these equations are presented; however, the simulation results are not given. In [17], the governing nonlinear equations for the regulator valve of a specific LPE are presented, and the simulation results are compared with experiments; in continuation, the linear governing equations of the entire LPE are developed and stability concerns are addressed. Static models for all engine components are presented in [18]. Analysis of LPE turbo-machinery is covered in [19,20]. Finally, presentation of engine feed systems is given in [6,21].

The problem statement is as follows: Given the governing equations of motion of an LPE, what is an efficient simulation method? The main contribution of this paper is the application of a developed algorithm for solving the system of equations of a single combustion chamber engine [1] to simulation and dynamic analysis of a new liquid propellant engine that is composed of 5 combustion chambers. Besides the difference in the number of combustion chambers, another primary difference between this paper and reference [1] is as follows: In this paper, the developed simulation algorithm is applied to the conceptual and initial design stages of a specific engine from the beginning in order to estimate and calculate the expected values of operational parameters that are also expected to be obtained in engine real experiments. Presenting a detailed development of the governing equations is beyond the scope of this paper; the reader is referred to reference [1] for sample governing equations that are used for the engine system and its subsystems.

The verification of the obtained nominal results is executed via analytical techniques (see Table 1). Verification of time histories of parameters in this
The organization of the rest of this paper is as follows: Contrary to the common trend where a theory is presented followed by applications of the theory to real systems, in this paper, for clarification purposes, the example problem is discussed first as frequent references are made to a real problem during presentation of the basis. After the LPE description, the logic used for solving a hydraulic circuit is discussed followed by a description of the simulation algorithm. Then, application of the simulation algorithm to the described LPE is presented. the result of the simulation code is analyzed next. Finally, summary and conclusion sections are presented.

DESCRIPTION OF THE SYSTEM

The schematic model of the engine under consideration is shown in Figure 1. This engine is composed of the following elements: (1) combustion chamber, (2) gas generator, (3) starter, (4) turbine, (5) fuel pump, (6) oxidizer pump, (7) fuel diaphragm valve, (8) stabilizer control valve, (9) oxidizer shut-down valve of the gas generator, (10) oxidizer diaphragm valve, (11) oxidizer shut-down valve of the combustion chamber, (12) regulator control valve, (13) fuel discharge valve, (14) fuel shut-down valve of the gas generator, (15) exhaust, (16)-(19) thrusters (or verniers), (20) fuel shut-down valve of thrusters, (21) oxidizer shut-down valve of the thrusters, (22) staged start valve, (23) valve for simultaneous discharge of the tanks, (24)-(35) orifices, (36)-(49) pipes, and (50) non-return valve.

The LPE operates in the following manner. First, the pressures inside the fuel and oxidizer tanks reach to their specified values. Then, electrical commands are received by starter (st-3), fuel diaphragm valve (vdfu7), and oxidizer diaphragm valve (vdox10). In continuation, turbine starts rotating, and fuel and oxidizer are permitted to enter the system. As the exit pressurized gas of the starter turns the turbine, the shaft of the turbo-pump starts rotating, and the pumps are commissioned. As a result, the fuel and the oxidizer are pumped.

In the fuel path, fuel passes through connection 33. Then the fuel path is divided into three branches. In the first branch, some fuel enters the stabilizer of the combustion chamber. In the second branch, some fuel is guided towards the regulator, and finally, in the third branch, some fuel heads towards the thrusters. In the first branch, the combustion chamber stabilizer guides a controlled amount of fuel to the fuel discharge valve. Under the condition that fuel discharge valve is not activated, the fuel that enters this valve travels towards the combustion chamber after going through connection 35 and orifice 25. In the second branch, the amount of fuel that is guided towards the regulator valve passes through connection 31 and then enters the regulator valve. The regulator control valve passes a controlled amount of fuel through connection 27. The exit fuel from connection 27 is then divided into two sub-branches or parts. One part is passed through connection 28, and, after it has passed through the gas-generator fuel shut-down valve, it enters the gas-generator. The other part, after passing through connection 26, enters the gas-generator stabilizer valve, and the pressure command is issued. Finally, in the third branch, fuel enters the thrusters after passing through connection 44 and the thrusters’ fuel shut-down valve. In the oxidizer path, the oxidizer exiting the oxidizer pump is divided into two branches. In one branch, the oxidizer passes through connection 32 and orifice 24, and after passing through the staged start valve and oxidizer shut-down valve, it enters the combustion chamber. This amount of oxidizer and the amount of fuel that enters the combustion chamber from the fuel path are combined and due to the resulting combustion, the engine is turned on. The staged valve is designed to prevent excess pressure overshoot in the combustion chamber. The second branch itself is divided into two other sub-branches or parts. In the first sub-branch, some oxidizer is passed through connection 29, and it enters the gas-generator stabilizer control valve. After passing through the non-return valve, connection 30, and gas-generator shut-down valve, this stabilizer sends a controlled amount of oxidizer into the gas-generator. This amount of oxidizer and the amount of fuel that enters the gas-generator from the fuel path are combined, and combustion occurs. The exit pressurized gas of the gas-generator keeps the turbine rotating and the starter gradually leaves the circuit. In the second sub-branch, some oxidizer is passed through connection 34 and the thrusters’ oxidizer shut-down valve, and then enters the thrusters. This amount of oxidizer that enters the thrusters is combined with the amount of fuel that comes from the fuel path, and combustion takes place.

DESCRIPTION OF THE LOGIC FOR SOLVING A HYDRAULIC CIRCUIT

In a hydraulic circuit of the type used in LPEs, there are three variables. These are: (1) pressure at the beginning of the circuit, (2) pressure at the end of the circuit, and (3) the flow rate that passes through the circuit. In order to solve the circuit, two of these three variables must be known. Otherwise, it would be impossible to solve the circuit. Now consider the schematic drawing shown in Figure 1. In this LPE, the tank fluid pressure (or the pressure at the entrance of
the diaphragm valves) and the downstream pressure (e.g., the combustion chamber pressure) are known. Therefore, it would be possible to solve the hydraulic circuit; as a result, the intermediate pressures and flow rates would be determined. Consider the case in which the fuel pump is not filled with fluid. Therefore, if the fuel pump entrance pressure were known, one could solve the circuit in a component by component manner. But, the pump entrance pressure is not known, and its value must be calculated. This is a sign that equations are coupled. A new method for solving such equations is used here. In this work, the
Newton-Raphson method is adopted. This method is characterized by the following equation.

\[ y_{m+1} = y_m - \frac{g(y)}{g'(y)} \]  

(1)

where, \( y \) is the unknown variable, \( g(y) \) is an error function, \( g' \) is the derivative of the error function with respect to the unknown variable, and \( m \) is the iteration number. The use of equation (1) is described by posing an example. Consider the fuel path of Figure 1, and assume that fuel pump is being filled; in other words, fluid has not reached connection 35 yet. In this case, the pressure at the beginning of the circuit, and pressure at the end of the circuit is known; the value of the pressure at the entrance of the fuel pump and the flow rate must be calculated. First, the entrance pressure of the fuel pump is guessed (initial guess equals one atmosphere, and for subsequent iterations, use the most recent value). With this pressure value and the known pressure at the beginning of the circuit (or the known pressure at the entrance of the fuel diaphragm valve 7), the flow rate of the fuel that enters the fuel pump may be calculated. With the value of the flow rate, the value of the guessed pressure at the fuel pump entrance, and the angular speed of the turbo-pump shaft, the exit pressure of the fuel pump may be calculated. Note that the angular speed of the turbo-pump is a state variable, and its initial value would be zero, and for subsequent steps, the value from the previous integration step is used. Since the fluid has not filled the fuel pump, the exit pressure of the fuel pump must equal one atmosphere. If the calculated value of the fuel pump exit pressure equals one atmosphere, then, it is concluded that the guessed value of the fuel pump entrance pressure is correct, and the circuit is solved. Otherwise, the guessed value of the fuel pump entrance pressure is corrected by the use of equation (1). The error function value, \( g(y) \), would equal the difference between the calculated fuel pump exit pressure and one atmosphere. The derivative of the error function, \( g' \), is numerically evaluated. Finally, equation (1) is evaluated to get a new value for the fuel pump entrance pressure. The recursive application of equation (1) is continued until \( g(y) \approx 0 \); once this happens, the hydraulic circuit is solved. Then, the simulation code computes the derivatives of state variables, and the differential equations are numerically integrated. The method of integration is the Euler’s technique. It should be noted that the application of this scheme with a more complicated integration routine to other engines (e.g., a fourth-order runge-kutta technique) did not offer any increase in accuracy [1]. In short, the algebraic equations are solved implicitly first in order to determine the intermediate values of the pressures and flow rates and then to integrate the differential equations of motion.

**SIMULATION ALGORITHM**

The simulation algorithm is composed of six primary steps as described below.

**Step 1:** Define the first Newton-Raphson loop. The start of the loop would be at the exit of the diaphragm valve. Consider the case where the
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fuel path is not filled, and the first branch is not encountered. In this case, the end of the loop would be at the exit of the element that is being filled. Otherwise, the end of the loop would be at the end of the first branch. In this step, the exit pressure of the diaphragm valve must be guessed. For example, consider Figure 1. The exit pressure of the diaphragm valve (which is the same as the pump entrance pressure) is guessed. If there are no branches in the fluid path towards the combustion chamber or the gas-generator, solve the hydraulic circuit and calculate the pressure of the combustion chamber or the gas-generator; then, go to Step 5. Note that in this case, there is only one Newton-Raphson loop, and the end of the loop is inside the combustion chamber or the gas-generator.

Step 2: With the known pressure at the entrance of the diaphragm valve and the assumed fluid pump entrance pressure from the previous Step, solve the hydraulic circuit up to the first branch. With reference to Figure 1, the hydraulic circuit up to the exit of connection 33 is solved.

Step 3: For each of the branch paths, define a new Newton-Raphson loop nested with the previous Newton-Raphson loop (note that if another branch is encountered while following the original branch path, another Newton-Raphson loop nested with the previous defined Newton-Raphson loop must be defined). For example, while following the branch at the exit of connection 51 towards the thrusters 54 and 55, one must define a new Newton-Raphson loop for the branch that is encountered at the exit of connection 47 nested with the second Newton-Raphson loop that was defined for the branch at the exit of the connection 51.

Step 4: Solve the hydraulic circuits related to each branch. At the end of this Step, the pressures and flow rates of the circuit would be known.

Step 5: For the case where there are no branches, correct the pressure that was guessed in Step 1 until the calculated pressure of the combustion chamber or the gas-generator equals the corresponding integrated values. If there are any branches, the guessed pump entrance pressure is corrected until the conservation of mass at the first branch is satisfied.

Step 6: If the simulation time is ended, then stop; otherwise, integrate the equations of motion and go to Step 1. Note that the goal of Steps 1 through 5 is to calculate the pump entrance pressure. As soon as this is done, the values of intermediate pressures and flow rates would be known.

NUMERICAL EXAMPLE

As was mentioned, the entrance pressures of the pumps are not known. As an example, the fuel path solution is discussed here. The oxidizer path solution technique is similar. These two paths are coupled by the combustion in the combustion chamber and in the gas-generator; the algebraic implicit equations of each path do not depend on those of the other. The algorithm for computing the pump entrance pressure is based on following the fluid in its path. Volumes of various elements in the fluid path are known. When the circuit equation is solved, the following is used to determine whether an element is filled with fluid or not.

\[
\frac{d\tilde{V}}{dt} = \frac{1}{\rho^*} \dot{m}, \quad 0 \leq \frac{\tilde{V} - V(t)}{\tilde{V}} \leq 1
\]

where, \( V(t) \) is the instantaneous value of the amount of filled volume, \( V \) is the volume of the element, \( t \) is the time variable, \( \rho \) is the density of the fluid, and \( \dot{m} \) is the mass flow rate of the fluid that enters the element.

At the beginning, before the diaphragm valve 7 is opened, the pump entrance pressure equals one atmosphere. After the opening of the diaphragm valve, the fluid starts to fill valve 7, and until the value of \( V \) from equation (2) is less than 1, the fluid does not reach the pump, and the pump entrance pressure would equal one atmosphere. As soon as valve 7 is filled, the fluid reaches the pump and until the value of \( \tilde{V} \) for the pump from equation (2) is less than 1, the fluid does not reach connection 33. Now, we start applying the simulation algorithm.

Steps 1 and 2: Define the first Newton-Raphson loop. The beginning of the loop is at the entrance of the pump and the end of the loop is at the exit of the pump. As was mentioned in Section 3 of the paper, the pump entrance pressure is calculated to ensure that exit pump pressure would equal one atmosphere. As soon as the pump is filled with fluid, fluid starts filling connection 33. In this case, the exit pressure of connection 33 would equal one atmosphere. Again, as was done previously, the pump entrance pressure is computed to ensure that exit pressure of connection 33 equals one atmosphere. As soon as connection 33 is filled with fluid, part of the fluid is guided through connection 31 and the other part goes through connection 51.

Step 3: When the branch is encountered, a new Newton-Raphson loop nested with the first Newton-Raphson loop is defined. The beginning of this loop is at the first branch (or at the exit of connection 33) and the end of the loop is at the beginning of the second branch (or at the exit of connection 51). The third Newton-Raphson loop nested with the second one is because of the branch at the exit of
element 20. In the same manner, the fourth loop (nested with the third one) relates to the branch at connections 47 and 48. The fifth loop or the last loop is related to computations from the exit of connections 47 and 48 to inside the thrusters' combustion chamber.

**Step 4:** The path equation for the hydraulic circuit from the exit of connection 33 towards the gas-generator is solved (the pressure at the exit of connection 33 is known from Steps 1 and 2, and also the pressure at the end of the path like the gas-generator pressure is known; therefore, with reference to discussion in Section 3, equations of the circuit could be solved). Similarly, the hydraulic circuit equation in the exit path of connection 33 till exit of connection 51 is solved. Likewise, knowing the pressure at exit of connection 51 and pressure at the end of the path towards the combustion chamber, the circuit equation in the path of connection 51 towards the combustion chamber is solved. Note that in case the path is filled with fluid, then the combustion chamber would be the pressure at the end of the path; otherwise, the end pressure is the pressure of the element that is being filled with fluid and equals one atmosphere. Similarly, the two hydraulic circuits defined from the exit of the connection 51 till the interior point of thrusters 55 and 56 are computed.

**Step 5:** If the conservation of mass at the first branch (or at the exit of element 51) holds, it can be concluded that the pump entrance pressure is correct and the circuit is solved; otherwise, the pump entrance pressure would be fixed using the recursive Newton-Raphson method (see equation 1), and the computations would be repeated.

Finally, in this Step, the differential equations of motion are integrated.

**SIMULATION OUTPUT**
The plots of angular speed of the turbo-pump shaft, combustion chamber pressure, and gas-generator pressure are shown in Figures 2-4. The nominal values obtained from the simulation code are compared to the designed values in Table 1. Percent errors are acceptable.

**PARAMETRIC STUDY AND OPTIMIZATION CAPABILITY**
The developed simulation code for the specific liquid propellant engine under study has the capability to be used in studying the effects of various element and subsystem parameters on forecasting the performance and operation of the engine system. This code may be used as a suitable tool to optimize the behavior and operation of the engine system.

For example, the effects of changes in the regulator pressure drop (Figure 1, element number 12), nominal turbo-pump angular speed (Figure 1, element number 4), and pressure drop of combustion chamber oxidizer injectors (Figure 1, element number 1) are studied as described below.

1. The simulation results show that an increase in the regulator pressure drop (20% change of the

### Table 1. Comparison of Simulation and Design Values at Nominal Conditions.  

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>%Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Turbine Power</td>
<td>2.02</td>
</tr>
<tr>
<td>2</td>
<td>Fuel pump mass flow rate</td>
<td>2.59</td>
</tr>
<tr>
<td>3</td>
<td>Fuel pump head</td>
<td>1.67</td>
</tr>
<tr>
<td>4</td>
<td>Oxidizer pump mass flow rate</td>
<td>0.47</td>
</tr>
<tr>
<td>5</td>
<td>Oxidizer pump head</td>
<td>0.44</td>
</tr>
<tr>
<td>6</td>
<td>Fuel pump exit pressure</td>
<td>0.42</td>
</tr>
<tr>
<td>7</td>
<td>Oxidizer pump exit pressure</td>
<td>0.31</td>
</tr>
<tr>
<td>8</td>
<td>Fuel mass flow rate of the thruster combustion chamber</td>
<td>0.0</td>
</tr>
<tr>
<td>9</td>
<td>Oxidizer mass flow rate of the thruster combustion chamber</td>
<td>0.80</td>
</tr>
<tr>
<td>10</td>
<td>Pressure inside the thruster combustion chamber</td>
<td>0.73</td>
</tr>
<tr>
<td>11</td>
<td>Turbo-pump shaft angular speed</td>
<td>0.49</td>
</tr>
<tr>
<td>12</td>
<td>Pressure inside the gas-generator</td>
<td>1.98</td>
</tr>
<tr>
<td>13</td>
<td>Fuel mass flow rate of the gas-generator</td>
<td>2.73</td>
</tr>
<tr>
<td>14</td>
<td>Oxidizer mass flow rate of the gas-generator</td>
<td>2.50</td>
</tr>
<tr>
<td>15</td>
<td>Pressure inside the combustion chamber</td>
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<tr>
<td>16</td>
<td>Fuel mass flow rate of the combustion chamber</td>
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</tr>
<tr>
<td>17</td>
<td>Oxidizer mass flow rate of the combustion chamber</td>
<td>1.19</td>
</tr>
</tbody>
</table>
Figure 5. Effect of change in regulator pressure drop.

Figure 6. Effect of change in nominal turbo-pump angular speed.

Figure 7. Effect of change in pressure drop of oxidizer injectors.

angular displacement of the regulator control element toward reducing the cross sectional area of the fluid passage) causes a 4% increase in the combustion chamber pressure. These results are graphically demonstrated via figure 5. This amount of decrease or increase in the combustion chamber pressure causes a change in the amount of generated liquid engine thrust force. Increase or decrease of regulator pressure drop, assuming that the energy balance of the whole engine system remains constant (without changes in the turbo-pump system operational characteristics), has an adverse effect on the value of combustion chamber pressure. Therefore, increase or decrease of regulator pressure drop causes a decrease or increase in the value of engine thrust, respectively.

2. The simulation results indicate that a 20% increase in the nominal value of the turbo-pump angular speed causes a 5.6% increase in the combustion chamber pressure; a 10% decrease in the nominal value of the turbo-pump angular speed causes a noticeable decrease of 43% in the combustion chamber pressure. These results are summarized in Figure 6. It is obvious that changes in nominal value of the turbo-pump angular speed have a direct relationship with the combustion chamber pressure (the change in the mentioned angular speed implies changes in mass flow rates of fuel and oxidizer as well as changes in the discharge pressures of pumps). By an increase (decrease) in engine energy balance due to changes in the nominal turbo-pump angular speed and ultimately due to increases (decreases) in discharge pressures and discharge mass flow rates of the pumps, the amount of combustion chamber pressure, and consequently, the amount of generated thrust will increase (decrease).

3. Execution of the simulation code shows that a decrease in the pressure drop of combustion chamber oxidizer injectors (by increasing the pressure drop coefficient by a factor of 10) causes a 2.6% increase in the combustion chamber pressure. On the other hand, an increase in the pressure drop of the combustion chamber oxidizer injectors (by halving the pressure drop coefficient) causes an 8.3% decrease in the combustion chamber pressure. These results are summarized in Figure 7. The physical explanation for this phenomenon is exactly the same as that described for the change in the regulator pressure drop (see item 1 above).

SUMMARY AND CONCLUSIONS
In this paper, application of a newly developed LPE simulation algorithm to a new engine is considered. The simulation method and its application are discussed. The output values of the simulation code at nominal values are compared with the real values, and satisfactory results are obtained. A parametric study is
performed, and it is shown that simulation code may be used for engine optimization purposes. The execution time of the simulation code using a 1500 MHz computer approximately equals 29 seconds.

REFERENCES