

Effect of Electric Field on Liquid Viscosity in Co-Axial Cylindric Pipes

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Influence of an external electric field on the properties of some commercial liquids when flowing between co-axial cylindric pipes is theoretically and experimentally investigated. Strong dependence of the liquids properties on electric field parameters, characterized by increase in flowrate and drop in viscosity, is revealed. Condition for the effect's validity and hydrodynamic calculation of some appropriate pressure gains are also presented. The results of the study may be successfully used in all drilling operations and oil-and-gas production. Some areas of electric field application in aerospace science and technology are noted.

INTRODUCTION

This study is a continuation of the Kvaerner John Brown Ltd. research projects No. 5800 and 6551 dealing with the influence of electric and magnetic fields on the properties of various liquids [1], and is aimed at improving the liquids ability to flow between co-axial (concentric) cylindric pipes and, accordingly, the technological processes involved in drilling as well as oil and gas production. The advantages of using an electric field (e.f.) were clearly revealed by the author in [1]. The paper, however, presented practical benefits of applying the e.f. to transportation systems which, as is known, are assembled with cylindrical pipes. This time, we expand our research to determine fluids characteristics - first of all, viscosity drop range under the e.f. and liquid "memory" on this agent - of oil and oil products (emulsions), drilling muds and cement slurries, which are generally used between concentric pipes in the above-mentioned technological processes.

But what I would like to note in advance is that some aspects of this research are of sufficient commercial interest and not described in this paper in an explicit form.

It is worthwhile to note that there are wide references in which the effect of e.f. on liquids properties have been studied [2-11]. However, such works have mainly studied the electrorheological properties of polymers and oils suspensions [2,3,5,7]. Influence

of external electric field on liquids viscosity is weakly studied. The investigations have been mainly dedicated to electrorheological (ER) effects, e.g., Zhang and Zhu in [9], or molecular mechanism of viscosity change [4,6] without applications of this effect in technological processes. For example, in [9] it has been established that under the influence of an applied electric field, the variation of apparent viscosity of electrorheological (ER) fluid flow causes ER effects. It has also been found that according to the Bingham model, which is widely used for describing the rheological properties of ER fluids, this variation should be very weak at high shear rates.

THEORETICAL APPROACH

To simplify theoretical consideration let's study the motion of incompressible liquid in co-axial cylindrical pipe under the influence of an electric field. Non-stationary isothermal laminar motion of an incompressible liquid in horizontal cylindrical pipe is described by the following equation written in cylindrical coordinates:

$$\rho \frac{\partial \nu}{\partial t} = \eta \left(\frac{\partial^2 \nu}{\partial r^2} + \frac{1}{r} \frac{\partial \nu}{\partial r} \right) + \frac{\partial P}{\partial z} + f, \quad (1)$$

where ρ and η are the liquid's density and viscosity respectively, $\partial P / \partial z$ - time-independent pressure change along pipe length, $\nu = \nu(r, t)$ is the velocity of liquid as a function of radial coordinate and time, f is the force acting on liquid per unit volume (in our case e.f.). The newly developed capacitor [12] generates electric field

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described by the law

$$f = Ae^{-\sigma r},$$

where A and σ are constant coefficients characterizing external transverse e.f. and type of the capacitor. In practice, an e.f. can be either constant or alternating. Since the alternating one is more preferable for commercial purposes, we solve the equation (1) taking into account, that e.f. varies with time by the harmonic law

$$A = A_0 e^{-i\omega t},$$

where ω is the e.f. frequency, A_0 is the e.f. initial (amplitude) value at time $t = 0$. Therefore, the equation (1) is finally written as :

$$\rho \frac{\partial \nu}{\partial t} = \eta \left(\frac{\partial^2 \nu}{\partial r^2} + \frac{1}{r} \frac{\partial \nu}{\partial r} \right) + \frac{\partial P}{\partial z} + A_0 e^{-i\omega t} e^{-\sigma r} \quad (2)$$

with the boundary condition :

$$\nu(r = R_{in}, t) = 0; \nu(r = R_{ex}, t) = 0, \quad (2a)$$

where R_{in} and R_{ex} are radiuses of internal and external cylinders respectively, and the time condition

$$\nu(r, t = 0) = \nu_{in} \quad (2b)$$

where ν_{in} is the initial velocity of the liquid until e.f. does not effect it. To solve the problem the following substitution is first made :

$$\nu(r, t) = w(r, t) - \frac{\partial P}{\partial z} \frac{r^2}{4\eta} \quad (3)$$

From the physical viewpoint, the substitution (3) means constancy of the velocity profile with narrowing radius in cylindrical coordinates, thereto, every point of the liquid in the new system will be shifted relative to its previous position by $\frac{\partial P}{\partial z} \cdot \frac{r^2}{4\eta}$. In terms of the recent paper of the author [13], the above substitution characterizes correlation function that in this case equals to $r^2/4$. In the new coordinates, the equation (2) is transformed to

$$\rho \frac{\partial w}{\partial t} = \eta \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) + A_0 e^{-i\omega t} e^{-\sigma r} \quad (4)$$

with the changed boundary and time conditions:

$$w(r = R_1, t) = \frac{\partial P}{\partial z} \frac{R_1^2}{4\eta}, \quad w(r = R_2, t) = \frac{\partial P}{\partial z} \frac{R_2^2}{4\eta},$$

$$w(r, t = 0) = \nu_{in} + \frac{\partial P}{\partial z} \frac{r^2}{4\eta}$$

The equation (4) may be presented in the following form:

$$w(r, t) = u(r) e^{-i\omega t} e^{-\sigma r} \quad (5)$$

In this case, subsequently realizing all the $w(r, t)$ function derivatives by r and t , after reducing for exponential coefficient $e^{-\sigma r}$ which never equals zero, we obtain :

$$(-i\omega)\rho u(r) = \eta \left\{ \frac{\partial^2 u(r)}{\partial r^2} - \left(2\sigma - \frac{1}{r}\right) \frac{\partial u(r)}{\partial r} + \left(\sigma^2 - \frac{\sigma}{r}\right) u(r) \right\} + A_0; \quad (6)$$

that represents non-homogeneous Bessel equation. Solution of the equation (6) may be given as the sum of that of corresponding homogeneous equation expressed by Bessel function and partial solution of non - homogeneous equation found by using Lommel function [14] :

$$u(r) = \{C_1 J_0(\beta r) + C_2 Y_0(\beta r)\} e^{\sigma r} - \frac{A_0}{\eta} S_{1,0}(r), \quad (7)$$

where the following indications are introduced: $\beta = \sqrt{i\omega\rho/\eta}$, $S_{1,0}(r)$ — Lommel function of zero order, $J_0(\beta r)$ and $Y_0(\beta r)$ — Bessel functions of imaginary argument of zero order of the first and second kinds, which can be represented respectively as

$$J_0(\beta r) = ber_0(\beta_1 r) - bei_0(\beta_1 r) \quad , \beta_1 = \sqrt{\omega\rho/\eta}$$

$$Y_0(\beta r) = e^{i\pi/2} I_0(\beta r) - \frac{2}{\pi} K_0(\beta r),$$

and after several $Y_0(\beta r)$ transformations look as follows:

$$Y_0(\beta r) = Z_1 + iZ_2,$$

$$Z_1 = -\{bei_0(\beta_1 r) + 2ker_0(\beta_1 r)/\pi\},$$

$$Z_2 = ber_0(\beta_1 r) - 2kei_0(\beta_1 r)/\pi$$

The constants C_1 and C_2 can be found from boundary conditions (2b). To this end, the solution to the following equations is required:

$$\begin{aligned} \frac{\partial P}{\partial z} \frac{R_{in}^2}{4\eta} &= \{C_1 J_0(\beta R_{in}) + C_2 Y_0(\beta R_{in})\} e^{\sigma R_{in}} \\ &\quad - \frac{A_0}{\eta} S_{1,0}(R_{in}), \end{aligned}$$

$$\begin{aligned} \frac{\partial P}{\partial z} \frac{R_{ex}^2}{4\eta} &= \{C_1 J_0(\beta R_{ex}) + C_2 Y_0(\beta R_{ex})\} e^{\sigma R_{ex}} \\ &\quad - \frac{A_0}{\eta} S_{1,0}(R_{ex}). \end{aligned}$$

It follows that

$$C_2 = \frac{\varphi_2 - \varphi_1 + A_0 \{S_{1,0}(R_{in}) - \gamma S_{1,0}(R_{ex}) e^{-\sigma \delta}\}}{Y_0(\beta R_{in}) - \gamma Y_0(\beta R_{ex})} \frac{e^{-\sigma R_{in}}}{\eta},$$

$$C_1 = \frac{\varphi_1 + A_0 S_{1,0}(R_{ex})}{\eta J_0(\beta R_{ex})} e^{-\sigma R_{ex}} - C_2 \frac{Y_0(\beta R_{ex})}{J_0(\beta R_{ex})};$$

herein

$$\varphi_1 = \frac{\partial P}{\partial z} \frac{R_{ex}^2}{4}, \quad \varphi_2 = \frac{\partial P}{\partial z} \frac{R_{in}^2}{4},$$

$$\gamma = \frac{J_0(\beta R_{in})}{J_0(\beta R_{ex})}, \quad \delta = R_{ex} - R_{in}$$

Returning to the original $\nu(r, t)$ function:

$$\nu(r, t) = \left\{ C_1 J_0(\beta r) + C_2 Y_0(\beta r) - \frac{A_0}{\eta} S_{1,0}(r) e^{-\sigma r} \right\} e^{-i\omega t} - \frac{\partial P}{\partial z} \frac{r^2}{4\eta} \quad (8)$$

To find the flowrate Q through cross-section between the concentric cylindrical pipes, one should calculate the integral

$$Q = \int_{R_1}^{R_2} 2\pi r \nu(r, t) dr$$

The final result will be in the following form:

$$Q_{(E)} = \{a_1 \cdot \Delta J_1 + a_2 \cdot \Delta Y_1 - a_3 \cdot \Delta H\} e^{-i\omega t} - a_4 \cdot \delta R, \quad (9)$$

where

$$a_1 = 2\pi C_1 / \beta, \quad a_2 = 2\pi C_2 / \beta,$$

$$a_3 = 2\pi A_0 / \eta, \quad a_4 = (\partial P / \partial z) \cdot (\pi / 8\eta),$$

$$\Delta J_1 = J_1(\beta R_{ex}) R_{ex} - J_1(\beta R_{in}) R_{in},$$

$$\Delta Y_1 = Y_1(\beta R_{ex}) R_{ex} - Y_1(\beta R_{in}) R_{in},$$

$$\Delta H = H(S_{ex}; \sigma; R_{ex}) - H(S_{in}; \sigma; R_{in}),$$

$$\delta R = R_{ex}^4 - R_{in}^4;$$

herein, the $J_1(\beta r)$ and $Y_1(\beta r)$ — type functions are appropriate Bessel functions of the first kind expressed in accordance with the theory of cylindrical functions and wronskian of Bessel functions as follows:

$$J_1(\beta r) = ber_1(\beta_1 r) - bei_1(\beta_1 r)$$

$$Y_1(\beta r) = \frac{J_1(\beta r) Y_0(\beta r) - 2 / \beta \pi z}{J_0(\beta z)}$$

Functions of the type $H(S; \sigma; R)$ are defined as :

$$H(S; \sigma; R) = e^{-\sigma R} \left\{ \sum_{i=0}^{\infty} \left(\frac{\sigma^k}{2(k+1)} \left[S_{k+2, k+1}(R) R + \frac{1}{2} S_{k+3, k+2}(R) \right] + \sum_{j=0}^{\infty} \frac{\sigma^{j+k+1}}{2(j+2)} S_{k+j+4, k+j+3}(R) \right) \right\}$$

The expression (9) is found by using, during integration, the properties of the Bessel and Lommel functions [15]. In this representation, the flowrate is a complex magnitude of the view:

$$Q_{(E)} = Re Q_{(E)} + i Im Q_{(E)}$$

For calculating the actual flowrate it is necessary to take module of $Q_{(E)}$

$$Q_{(E)} = \sqrt{Re^2 Q_{(E)} + Im^2 Q_{(E)}}$$

Now, it is worthwhile to compare $Q_{(E)}$ and Q_0 , that is well-known liquid flowrate between two concentric pipes without any external action :

$$Q_0 = \frac{\pi}{8\eta} \frac{\partial}{\partial z} \left(R_2^2 - R_1^2 + \frac{(R_2^2 - R_1^2)^2}{\ln \left\{ \frac{R_2}{R_1} \right\}} \right)$$

Comparing the flowrates and performing non-complex calculations, one can obtain the following ratio for viscosities

$$\frac{\eta_{(E)}}{\eta_0} = \alpha_1 + \frac{\alpha_2}{\partial P / \partial z}, \quad (10)$$

α_1 and α_2 are the coefficients, which can be found both experimentally and theoretically for any concrete liquid motion and the e.f. applied. Due to the capacitor specifications [12], the parameter α_2 is proportional to the intensity A_0 of the e.f., which allows us to conclude that the bigger the e.f. applied, the greater the viscosity changes.

If the applied e.f. is direct, then one should look for the function $w(r, t)$ in the solution (5) without time aspect,

$$w(r) = u(r) e^{-\sigma r},$$

as the $w(r)$ does not already depend on time, the scheme of further solution will be the same.

THE EXPERIMENT

The experimental scheme for the practical realization of an electric field's effect on the liquid moving between concentric pipes in this study is in fact the same as that developed in [1]. The only difference is the need to adjust the electric cascades for pipe-in-pipe variant of assembling.

The minimal charge surface density at plates σ_{\min} necessary for compensating the pressure drop ΔP along the section L is theoretically established

$$\sigma_{\min} = \frac{\beta \left(1 + \frac{1}{k^2} \right)}{q} \cdot \frac{\Delta P}{L}, \quad (11)$$

where, β is a constant coefficient, q is the total electric charge in liquid volume unit, k is plates curvature for

above construction defined as $k = 1/(D - d)$, where D and d are radii of the external and internal pipes respectively.

To observe the e.f. effect on liquid motion in the capacitor case, the condition

$$\sigma_{pl} \geq \sigma_{\min} \quad (12)$$

should take place ; σ_{pl} is the real value of charge density created by us at plates, σ_{\min} is the threshold value of charge density by the relation (11). Below σ_{\min} , the effect of e.f. on liquids is not observed as each of the plate configuration and liquid type has its corresponding value of σ_{\min} ; the condition (12) however, is always valid. Theoretical estimations show that σ_{pl} is defined by the following formula :

$$\sigma_{pl} = \frac{\epsilon U}{4\pi(D - d)};$$

ϵ -dielectric permeability of liquid involved, U -voltage between the plates.

Several results of e.f. influence on various liquids are given in fig. 1 and table 1. Light circles in figure 1 correspond to the purified and dark ones to the crude oil. Below 350V/m for crude and 400V/m for purified

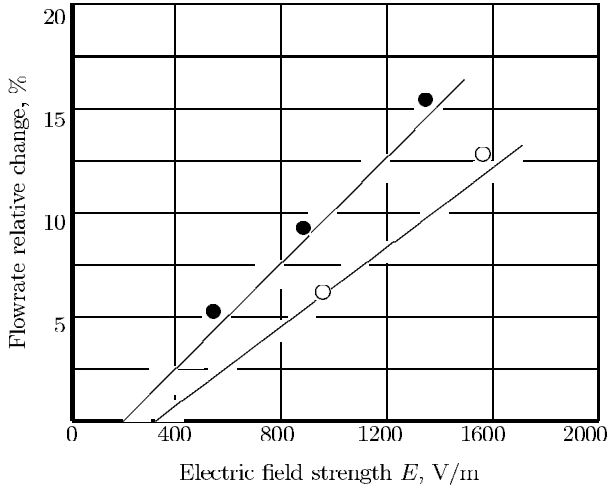


Figure 1. Increase in oil flowrate depending on e.f. strength at the temperature $t = 18^\circ\text{C}$.

oil, the effects could not be observed as not according to the condition (12). In our experiments, we used an alternating e.f., though a direct one may be applied if needed. The study was held at $8 \div 25^\circ\text{C}$ temperature interval. Below 8°C the liquids did not flow by the tubes connecting base elements of the scheme. The table given below shows an increase in flowrate for each liquid. Besides, the flowrate change for purified oil is less than that for the crude one, which may be explained by the small number of electrically active centers as a result of the purification process. Presence of liquid “memory” on e.f.(see the fourth column) allows us to use this effect for practical purposes. The last two columns present the results of calculations for appropriate pressure gain, i.e. , decreasing the pressure drop while increasing the flowrate (the diagram was kindly provided by Roder W. Groombridge, Pipeline Manager - Kvaerner John Brown, Ltd).

CONCLUSIONS

As it follows from our investigations, e.f. is a physical factor, which strongly alters the rheological parameters, especially the viscosity of liquids. Because of space limitations for this paper, the magnetic field effect on liquids (due to alternating e.f.) is not discussed here, but was investigated in our research^[1]. This conclusion gives an opportunity to keep the flow pressure along the pipe length permanent. Indeed, in any z_f point, the pressure is defined by the formula

$$P(z_f) = P_0 - \int_0^{z_f} \frac{\partial P_z}{\partial z} \cdot dz$$

In case of electric cascades, it is possible to vary the $\partial P/\partial z$ value, thereby minimizing the gradient $\partial P/\partial z$, at certain magnitudes of the cascades number, distance between them, applied e.f. , etc.. This allows us to considerably improve all the technological processes connected with the liquids flow. Except for this, the effect of e.f. on fluids and fluid - gas mixture flows could be applied to improve the mixtures escape from the discharge nozzle of the engines and to handle the streamline shape and lift of wing under the Mach

Table 1. Results of measurements under transverse alternating e.f. $E = 1600\text{V/m}$

Type of Liquid	Flowrate change, % $t = 10^\circ\text{C}$	Flowrate change, % $t = 18^\circ\text{C}$	Memory, min	Pressure gain, % $t = 10^\circ\text{C}$	Pressure gain, % $t = 18^\circ\text{C}$
Oil (crude)	$+(20 \div 25)^*$	$+18^*$	$5 \div 6$	$45 \div 50$	≈ 35
Oil (purified)	$+(16 \div 18)$	$+12$	$5 \div 6$	$35 \div 40$	≈ 25
Benzine	$+(15 \div 18)$	$+(10 \div 12)$	$3 \div 4$	$35 \div 40$	$22 \div 25$
Machine oil	$+(18 \div 20)$	$+(10 \div 12)$	$4 \div 5$	$38 \div 42$	$22 \div 25$

*) signs “+” in the second and third columns mean flowrate increasing under e.f.

number $M \leq 1$. The calculations utilized (principally the new method of solving the nonlinear equations for this type of problems given in [16]) show that application of the e.f. leads an increase in the fluids' escape and the wing streamline shape in the required range.

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