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# Noise-Induced Miss Distance Analysis of First-Order Explicit Guidance Law

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## ABSTRACT

**Keywords:** *Explicit Guidance, Optimal Guidance, Miss Distance Analysis, Seeker Noise*

*This study presents the miss distance analysis of the first-order explicit guidance law due to seeker noise using the adjoint method. For this purpose, linearized equations are utilized and the adjoint model is developed. Then the first-order equations are obtained and converted into nondimensional ones. The analysis is carried out for different values of the power of the alpha function, defined as the time decrease rate of the zero-effort miss distance to unit control input. The unity power gives the first-order optimal guidance strategy, minimizing the integral of the square of the commanded acceleration during the total flight time. The seeker and control system is assumed as a fifth-order binomial transfer function. Due to computational error and stability consideration, the effective navigation ratio is kept constant for very small time-to-go until intercept, and its effect on the miss distance is also investigated. Finally, approximate formulas are obtained using curve fitting method for rms miss distance due to seeker noise.*

## Nomenclature

| | |
|---|---|
| $K_L$ | **Time-varying coefficient of guidance law** |
| $m$ | **Power of alpha function** |
| $n$ | **Order of the guidance and control system** |
| $n_L, n_T$ | **Interceptor and target accelerations** |
| $N$ | **Effective navigation ratio** |
| $N_\mathrm{m}$ | **Effective navigation ratio associated with the power of alpha function** |
| $R_A$ | **Reference range** |
| $s$ | **Laplace domain variable** |
| $t$ | **Time** |
| $t_{go}$ | **Time-to-go until intercept** |
| $t_f$ | **Total flight time of the engagement** |
| $T$ | **Equivalent time constant** |
| $T_j$ | **Control system time constants** |
| $u_N$ | **Line-of-sight angle measurement noise** |
| $u_\mathrm{GL}$ | **Glint noise input** |
| $u_\mathrm{FN}$ | **Range independent noise input** |
| $u_\mathrm{RN}$ | **Active range dependent noise input** |
| $u_\mathrm{RNA}$ | **Semiactive range dependent noise input** |
| $v_c$ | **Missile-target closing velocity** |
| $x_j$ | **State variables of seeker and control system** |
| $y$ | **Missile-target separation perpendicular to initial line-of-sight** |
| $\alpha$ | **Alpha function** |
| $\lambda$ | **Line-of-sight angle** |
| $\tau = t/T$ | **Normalized time** |
| $\Phi$ | **Power spectral density for noise sources** |

**Superscript:**

| | |
|---|---|
| $\dot{(\ )}$ | **Time derivative** |
| $\ddot{(\ )}$ | **Second derivative with respect to time** |
| $(\ )'$ | **Derivative with respect to normalized time** |
| $\hat{(\ )}$ | **Normalized variable** |

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# Introduction

The single-lag optimal guidance law (OGL) is utilized as an alternative to proportional navigation (PN) for the final stage of minimum phase interceptors. In order to guide the non-minimum phase interceptors, higher-order guidance laws may be used [1-3].

The first-order OGL can be improved in several aspects. One of these aspects is the modification of the equivalent navigation ratio in this guidance law [4,5]. Considering that the first-order OGL, similar to PN, is obtained by minimizing the integral of squared commanded acceleration, modification of its effective navigation ratio is suggested. The optimal effective navigation ratio in PN is 3 when minimizing the integral of the squared commanded acceleration; however, the value of 3 is not necessarily used in the guidance law. The effective navigation ratio of PN is usually chosen between 3 and 5 [7].

In the explicit guidance law (EGL) presented in Ref. [5], the closed-loop guidance equation was obtained using a chosen profile of the acceleration command. If the selected profile is chosen proportional to the alpha function (the time decrease rate of the zero-effort miss to unit control input), the OGL is obtained based on the least integral square control effort; although, the selected profile can be chosen proportional to the alpha function to a power greater than zero (usually greater than 1) in EGLs.

Improvements in widely used guidance laws, such as PN and single-lag optimal guidance in a way that does not change its structure, can be of great interest to the industry. This causes most designers' analyses of the guidance and control parameters are applicable under these guidance laws. One of these is proposing a variable navigation coefficient [8-12]. If the effective navigation ratio of PN is taken as a function of some current state variables (such as line-of-sight rate), the guidance gain is somehow like a closed-loop design. If the effective navigation ratio as the guidance gain is chosen/obtained as a function of time (an open-loop design), the guidance performance depends on initial conditions, disturbances, and target maneuver.

Adjoint method is a well-known technique to analyze the performance of linear time-varying guidance and control systems. It is a powerful tool due to its simplicity, accuracy, and relatively low computational burden, especially in miss distance analysis in the presence of noise [13-17]. In addition, normalizing the governing equations and their numerical solution can be used for all values of the nondimensional parameters by producing the results all at once. It has equal importance to the problem's analytical solution and will be significant from the practical point of view.

The noise-induced miss distance analytical solution is available in literature only for a first-order control system and special cases [7,18]. In Ref. [7], miss distance formulas of PN due to glint noise, range independent noise, active/semiactive range dependent noise were obtained analytically for the first-order control system, only for integer effective navigation ratios. In the solution, a formula has been provided for each integer navigation ratio. The solution to the problem is analytically obtained in Ref. [18] using series solution for non-integer effective navigation ratios, whereas the number of series terms is finite for integer effective navigation ratios. In Ref. [19], approximate formulas of miss distance of a modified PN strategy with lateral acceleration feedback was presented for the second-order guidance and control system due to seeker noise sources and radome effect.

For practical application, the order of the system should be increased, which is considered the fifth order in most literature. This issue has been investigated in Ref. [20] for the guidance and control system up to the 30th order. Also, using curve fitting, Ref. [21] has presented the approximate formulas of miss distance due to time delay for the guidance and control system up to the 30th order in the worst case in the presence of initial heading error, target maneuver, and seeker noises.

In this study, the approximate formulas of the normalized steady-state miss distance coefficients due to seeker noise sources have been obtained under the first-order EGL in terms of the power of the alpha function for a fifth-order binomial guidance and control system. This work has been utilized the adjoint method, plotting the results and fitting a polynomial function. In addition, the effective navigation ratio is kept constant for a very small time-to-go until intercept and its effect on noise-induced miss distance under the first-order EGL is investigated.

### First-order EGL

The optimal guidance strategy for a first-order control system is given by [7]:

$$n_c = N(v_c\dot{\lambda} - K_L n_L) \quad (1)$$

where $n_c$ is the acceleration command, $v_c$ is the missile-target closing velocity, $\dot{\lambda}$ is the line-of-sight (LOS) rate, $n_L$ is the interceptor acceleration, $N$ is the effective navigation ratio, and $K_L$ is a time-varying coefficient. Here, the optimal effective navigation ratio is denoted by $N_1$ and its relation is [1]:

$$N_1 = \frac{6x^2(e^{-x} + x - 1)}{2x^3+3 + 6x - 6x^2-12xe^{-x}-3e^{-2x}} \quad (2)$$

Also, coefficient $K_L$ is given by [1]:

$$K_L = \frac{e^{-x}+x-1}{x^2} \ , x = \frac{t_{go}}{T} > 0 \quad (3)$$

where $t_{go}$ is the time-to-go until intercept. In explicit guidance law for a first-order control system, the alpha function (the time decrease rate of the zero-effort miss distance to unit control input) is given by [22]:

$$\alpha = T(e^{-x} + x - 1) \quad (4)$$

If the profile of the commanded acceleration is chosen proportional to $\alpha^m$, the original equation of the EGL can be explicitly integrated only for integer values of m, and its equations for $m$=1,2,3 were obtained in Ref. [22]. If the power of m is not an integer, the EGL guidance gain may be computed approximately. The diagram of the equivalent navigation ratio ($N_m$) corresponding to the powers of the alpha function from 1 to 3 is shown in Fig. 1 versus the normalized time to go until intercept. Here, the approximate equation of Ref. [23] is used for computing the guidance gain for integer and non-integer values of $m$.

$$N_m = mN_1 - \frac{2(m-1)}{1-2K_L} \ , \quad x > 0.005 \quad (5)$$

Approximate Eq. 5 agrees with the formulas of EGL guidance gain for integer values of $m$=1,2,3, as seen in Fig. 1. Since Eqs. (2) and (3) are singular at $x$=0, the computing of $K_L$ and $N_1$ for very small values of $x$ has a huge numerical error, as discussed in Appendix A.

It should be noted that the alpha function for the perfect control system is equal to the time-to-go until intercept. Therefore, there is a simple relationship between the power of the alpha function and the effective navigation ratio in PN, that is, $m$=$N$-2. This implies that the values of $m$=1,2,3 are in accordance to the values of $N$=3,4,5.

In the following, the approximate formulas are obtained for computing the normalized steady-state miss distance coefficients due to seeker noise sources using the adjoint method and plotting the normalized steady-state miss distance coefficients versus $m$.

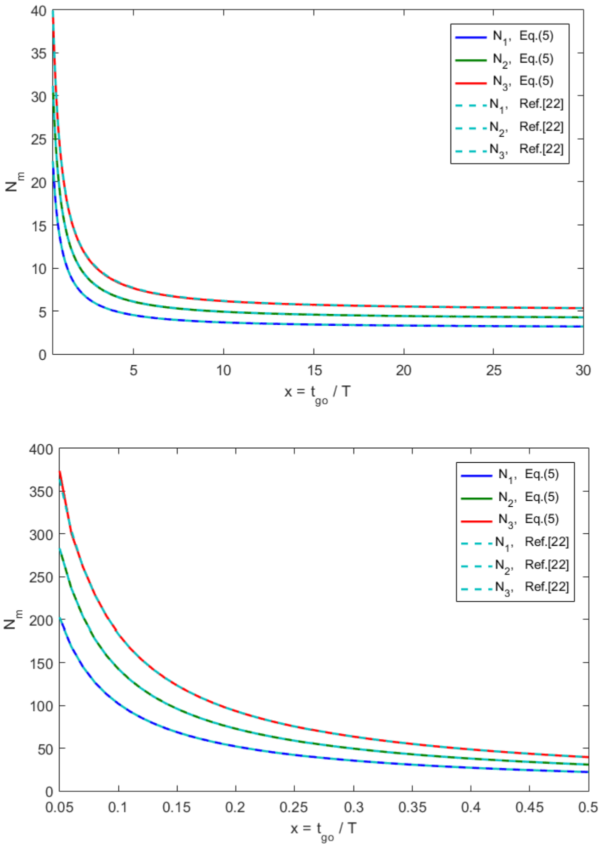


**Figure 1.** The accuracy of the approximate Eq. (5) for the equivalent navigation ratio

### Linearized equations of guidance problem

The block diagram of the linearized equations of the first-order EGL is shown in Fig. 2. This block diagram simplifies for the first-order OGL with $m = 1$ ($N_m = N_1$ for $m$=1). In the figure, s is the Laplace domain variable. In linearization, the interceptor acceleration ($n_L$) and the target acceleration ($n_T$) are assumed to be in the direction perpendicular to the initial LOS. Also, the LOS angle ($\lambda$) is approximated by $\lambda$=$y/v_c t_{go}$ using small angle approximation where $y$ is the separation between the interceptor and its target along the direction perpendicular to the initial LOS ($\ddot{y} = n_T - n_L$). The closing velocity is also assumed to be constant. The LOS angle measurement noise is denoted by $u_N$ and is modeled according to Ref. [7]:

$$u_N = \frac{u_{GL}}{v_c t_{go}} + u_{FN} + \frac{v_c t_{go}}{R_A} u_{RN} + \left(\frac{v_c t_{go}}{R_A}\right)^2 u_{RNA} \qquad (6)$$

where $u_{GL}$ is the glint noise input, $u_{FN}$ is the range independent noise input, $u_{RN}$ is the active range dependent noise input and $u_{RNA}$ is the semiactive range dependent noise input.

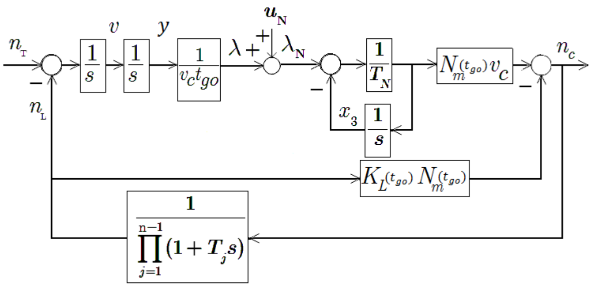


**Figure 2.** Block diagram of the linearized guidance problem

These inputs are assumed as white-noise error sources, and the spectral density is denoted by $\Phi$ with the same subscript as its corresponding input. The power spectral density of active/semi-active range dependent noise is defined at a reference range $R_A$ [7].
The seeker and the noise filter are jointly modeled by a first-order transfer function with time constant $T_N$. The control system is also modeled by a ($n$-1) order transfer function as follows:

$$\frac{n_L}{n_c}(s) = \frac{1}{\prod_{j=1}^{n-1}(1 + T_j s)} \qquad (7)$$

where $T_j$ represents the time constants of the control system and $n$ is the order of the guidance and control system. Assuming that all time constants are equal, the equivalent time constant is given by:

$$T = (n-1)T_j + T_N \ , \ (T_j = T/n) \qquad (8)$$

Using the block diagram of Fig. 2 and assuming that all the time constants of the guidance and control system are equal ($T_j = T_N$), the state equations are obtained for $n$>2 as follows [20]:

$$\dot{y} = v \qquad (9)$$

$$\dot{v} = n_T - n_L \qquad (10)$$

$$\dot{x}_3 = n(\lambda_N - x_3)/T \qquad (11)$$

$$\dot{x}_4 = n(n_c - x_4)/T \qquad (12)$$

$$\begin{cases} \text{for} & j = 5:1:n+2 \\ & \dot{x}_j = n(x_{j-1} - x_j)/T \\ \text{end} & \end{cases} \qquad (13)$$

where $x_4$ , $x_5$ ,…, $x_{n+1}$ are the state variables of the control system transfer function that is modeled as the product of ($n$-$1$) first-order transfer function and

$$n_L = x_{n+2} \qquad (14)$$

$$n_c = \frac{nN_m v_c(\lambda_N - x_3)}{T} - K_L N_m n_L \qquad (15)$$

$$\lambda_N = u_N + y/v_c t_{go} \qquad (16)$$

## Adjoint model for noise miss distance calculation

By applying the rules of the adjoint method from Ref. [7], the adjoint model of the block diagram of Fig. 2 is obtained as shown in Fig. 3. The state equations of the adjoint block diagram are obtained only in the presence of noise as follows ($n$>2):

$$\dot{x}_2 = x_3 \qquad (17)$$

$$\dot{x}_3 = y_1/v_c\, t \qquad (18)$$

$$\dot{x}_4 = y_1 \quad (19)$$

$$\dot{x}_5 = n(z_1 - x_5)/T \quad (20)$$

$$\begin{cases} \text{for} \quad j = 6:1:n+3 \\ \quad \dot{x}_j = n(x_{j-1} - x_j)/T \\ \text{end} \end{cases} \quad (21)$$

$$\dot{x}_{\text{FN}} = y_1^2 \quad (22)$$

$$\dot{x}_{\text{RN}} = (y_1 v_c t/R_A)^2 \quad (23)$$

$$\dot{x}_{\text{RNA}} = y_1^2\,(v_c t/R_A)^4 \quad (24)$$

$$\dot{x}_{\text{GL}} = (y_1/v_c t)^2 \quad (25)$$

where

$$y_1 = (N_m v_c x_{n+3} - x_4)/T_N \quad (26)$$

$$z_1 = -x_2 - K_L N_m x_{n+3} \quad (27)$$

In the adjoint model of Fig. 3, $x_5, x_6, \ldots, x_{n+3}$ are the state variables of the control system, which are modeled as the product of ($n$-1) first-order transfer function, and the state variables, $x_{\text{FN}}$, $x_{\text{RN}}$, $x_{\text{RNA}}$ and $x_{\text{GL}}$ are related to the seeker noise sources. For numerical solution, the initial values of the adjoint state variables are set to zero, except for $x_3(0) = 1$. According to Fig. 3, the standard deviation of the miss due to noise is calculated as follows:

$$\sigma_j = \sqrt{\Phi_j x_j(t_f)} \quad j\text{=GL, FN, RA, RNA} \quad (28)$$

Using the following change of variables:

$$\hat{x}_2 = x_2/T\ ,\ \hat{x}_3 = x_3\ ,\ \hat{x}_4 = x_4/Tv_c \quad (29)$$

$$\hat{z}_1 = \frac{z_1}{T}\ ,\ \hat{x}_j = \frac{x_j}{T}\ ,\ j\text{=5, 6, … } n\text{+3} \quad (30)$$

$$\hat{x}_{\text{GL}} = Tx_{\text{GL}}\ ,\ \hat{x}_{\text{FN}} = \frac{x_{\text{FN}}}{Tv_c^2} \quad (31)$$

$$\hat{x}_{\text{RN}} = \frac{R_A^2}{T^3 v_c^4} x_{\text{RN}}\ ,\ \hat{x}_{\text{RNA}} = \frac{R_A^4}{T^5 v_c^6} x_{\text{RNA}} \quad (32)$$

the adjoint equations are normalized as follows:

$$\hat{x}_2' = \hat{x}_3 \quad (33)$$

$$\hat{x}_3' = \hat{y}_1/\tau \quad (34)$$

$$\hat{x}_4' = \hat{y}_1 \quad (35)$$

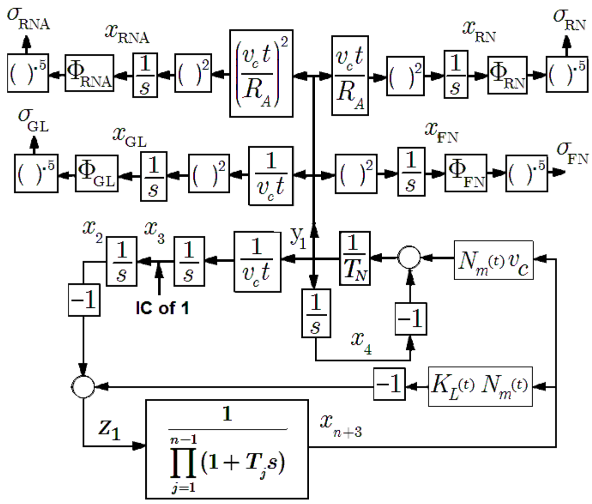


**Figure 3.** Block diagram of adjoint model of linearized EGL with seeker noise sources

$$\hat{x}_5' = n(\hat{z}_1 - \hat{x}_5) \quad (36)$$

$$\begin{cases} \text{for} \quad j = 6:1:n+3 \\ \quad \hat{x}_j' = n(\hat{x}_{j-1} - \hat{x}_j) \qquad n > 2 \\ \text{end} \end{cases} \quad (37)$$

$$\hat{x}'_{\mathrm{FN}} = \hat{y}_1^2 \tag{38}$$

$$\hat{x}'_{\mathrm{RN}} = \hat{y}_1^2 \tau^2 \tag{39}$$

$$\hat{x}'_{\mathrm{RNA}} = \hat{y}_1^2 \tau^4 \tag{40}$$

$$\hat{x}'_{\mathrm{GL}} = \hat{y}_1^2 / \tau^2 \tag{41}$$

where

$$\hat{y}_1 = n(N_m \hat{x}_{n+3} - \hat{x}_4) \tag{42}$$

$$\hat{z}_1 = -\hat{x}_2 - K_L N_m \hat{x}_{n+3} \tag{43}$$

where $(\ )'$ is the derivative with respect to normalized time $\tau = t/T$. By numerically solving the adjoint equations, the normalized miss distance coefficients due to seeker noise sources as in reference [20] are obtained by substitution Eqs. (31) and (32) into Eq. (28), that is,

$$K_{\mathrm{GL}}(\tau_f) = \frac{\sigma_{\mathrm{GL}}}{\sqrt{\Phi_{\mathrm{GL}}/T}} = \sqrt{\hat{x}_{\mathrm{GL}}(\tau_f)} \tag{44}$$

$$K_{\mathrm{FN}}(\tau_f) = \frac{\sigma_{\mathrm{FN}}}{v_c\sqrt{T\Phi_{\mathrm{FN}}}} = \sqrt{\hat{x}_{\mathrm{FN}}(\tau_f)} \tag{45}$$

$$K_{\mathrm{RN}}(\tau_f) = \frac{R_{\mathrm{A}}\sigma_{\mathrm{RN}}}{\Phi_{\mathrm{RN}}^{0.5} T^{1.5} v_c^2} = \sqrt{\hat{x}_{\mathrm{RN}}(\tau_f)} \tag{46}$$

$$K_{\mathrm{RNA}}(\tau_f) = \frac{R_{\mathrm{A}}^2\sigma_{\mathrm{RNA}}}{\Phi_{\mathrm{RNA}}^{0.5} T^{2.5} v_c^3} = \sqrt{\hat{x}_{\mathrm{RNA}}(\tau_f)} \tag{47}$$

in which $\tau_f = t_f/T$. The mentioned normalized coefficients are namely the normalized miss distance coefficient due to glint noise ($K_{\mathrm{GL}}$), the normalized miss distance coefficient due to range-independent noise ($K_{\mathrm{FN}}$), the normalized miss distance coefficient due to semi-active range dependent noise ($K_{\mathrm{RN}}$), and the normalized miss distance coefficient due to active range dependent noise ($K_{\mathrm{RNA}}$).

## Results and Discussion

By using the adjoint equations of the previous section, the normalized miss distance coefficients due to seeker noise sources under the first-order EGL are obtained for different values of the power of alpha function, *m*, for a fifth-order guidance and control system. The numerical solutions of the normalized miss distance coefficients due to seeker noise sources are plotted in Fig. 4 versus the normalized final time under the first-order EGL for *m* = 1, 2 and 3. As seen in the figures, the normalized miss distance coefficients almost reach steady-state values at least after 5 to 6 times of the equivalent time constant of the guidance and control system; however, the values of the normalized steady-state miss distance coefficients are, here, picked out at $t_f/T = 10$.

At the end times of the flight, or in other words, for very small values of $t_{go}$, the value of x tends to zero. The value of $K_L$ tends to 0.5 when $x \to 0$ according to Eq. (3), and according to Eqs. (2) and (5), the value of the effective navigation ratio, which is a function of *x*, is singular at *x*=0. This problem is investigated in Appendix A. In order to avoid the problem and stability considerations, the effective navigation ratio in the final moments is limited to a certain value as shown in Eq. (48). According to this equation if $t_{go}/T$ becomes less than a designed value for $x_{Nc}$, the effective navigation ratio is limited to the value of $N_m(x_{Nc}T)$. For this case, the value of $N_m$ in Eq. (49) is replaced for adjoint Eqs. (26), (27), (42) and (43).

$$N_m = \begin{cases} N_m(t_{go}) & \text{for} \quad t_{go} \geq x_{Nc}T \\ N_m(x_{Nc}T) & \text{for} \quad t_{go} < x_{Nc}T \end{cases} \tag{48}$$

$$N_m = \begin{cases} N_m(t) & \text{for} \quad t \geq x_{Nc}T \\ N_m(x_{Nc}T) & \text{for} \quad t < x_{Nc}T \end{cases} \tag{49}$$

It should be noted that the graphs in Fig. 4 are also drawn for $x_{Nc} = 0.02$.

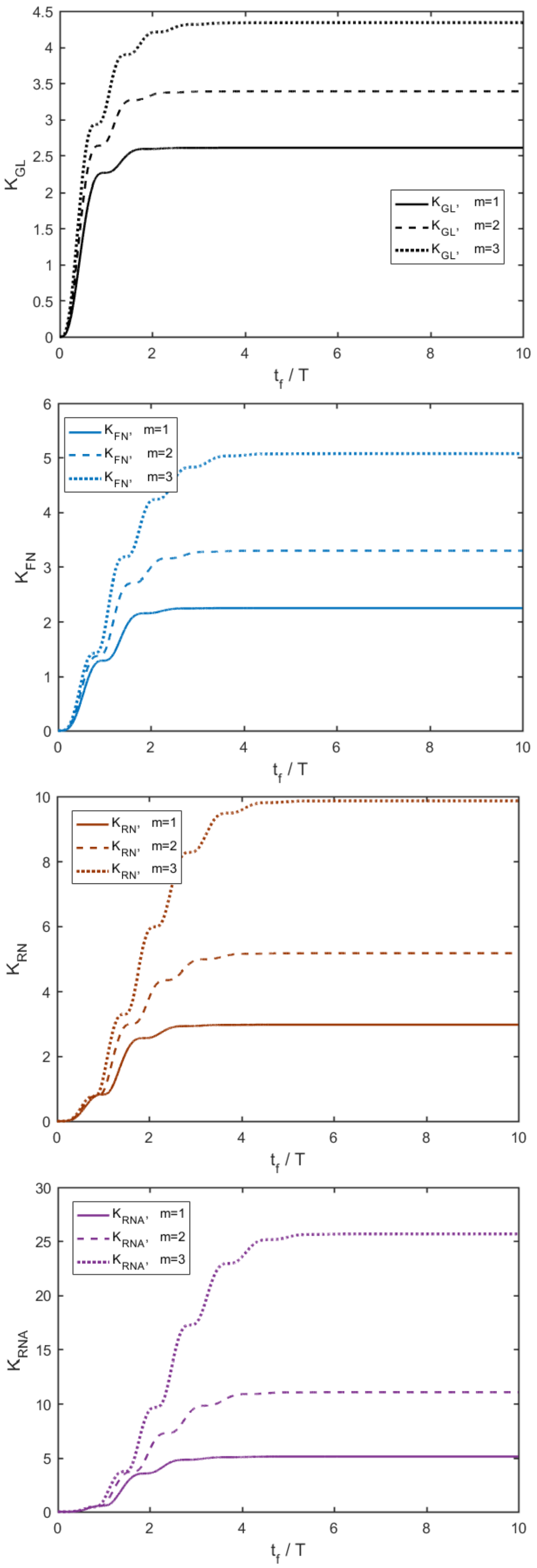


**Figure 4.** Normalized miss distance coefficients due to seeker noise sources versus normalized final time

Figure 5 shows the normalized steady-state miss distance coefficients due to seeker noise sources under the EGL versus the power of the alpha function. As seen in the figure, increasing the power of the alpha function from *m* = 0.2 to 4, the steady-state miss coefficients increases for $x_{Nc}$= 0.02 and 1 for a fifth-order system. The steady-state miss coefficients for $x_{Nc} = 1$ are greater than those for $x_{Nc} = 0.02$. In order to investigate the matter more closely, the steady-state miss coefficients are plotted in Fig. 6 versus $x_{Nc}$. As an important point, the steady-state miss coefficient has a maximum at $x_{Nc} = 1 \pm 0.25$ for *m* = 1,2,3 as seen in Fig. 6.

From a practical point of view, an optimum value of $x_{Nc}$ is determined by using the graphs of the miss distance due to initial heading error, target maneuver and seeker noise sources (similar to Fig. 6).

In the following, the Monte Carlo simulation is used to verify the results of the numerical solution of the adjoint model. Figure 7 shows the convergence of the results of the normalized steady-state miss coefficient due to glint using Euler method with an integration time step of 0.001 s. Therefore, in the Monte Carlo simulation, 2000 runs are made to calculate the steady-state standard deviation of the miss distance from the equations of the direct method, i.e., Eqs. (9-13). The results of the Monte Carlo simulation can be seen with asterisk or circle marks in Figs. 6 and 8, which show the agreement of the results with the adjoint solutions. It should be noted that the results of the present study have been obtained without acceleration limit and with the noise model according to Eq. (1). Applying acceleration saturation causes nonlinearity and does not permit using the adjoint model.

In the following, the approximate formulas of the normalized steady-state miss distance coefficients due to seeker noise sources in terms of *m* are obtained for $x_{Nc} = 0.02$ using the curve fitting ($0.5 < m < 4$):

$$K_{\mathrm{GL}} = 0.14m^2 + 0.31m + 2.2 \quad (50)$$

$$K_{\mathrm{FN}} = 0.12m^3 - 0.35m^2 + 1.3m + 1.2 \quad (51)$$

$$K_{\mathrm{RN}} = 0.4m^3 - 1.2m^2 + 3m + 0.74 \quad (52)$$

$$K_{\mathrm{RNA}} = 1.6m^3 - 5.5m^2 + 11m - 2.7 \quad (53)$$

It is worth noting that a lower degree polynomial is preferred in order to obtain an explicit relation for the extremum value of *m* from the total miss distance equation.

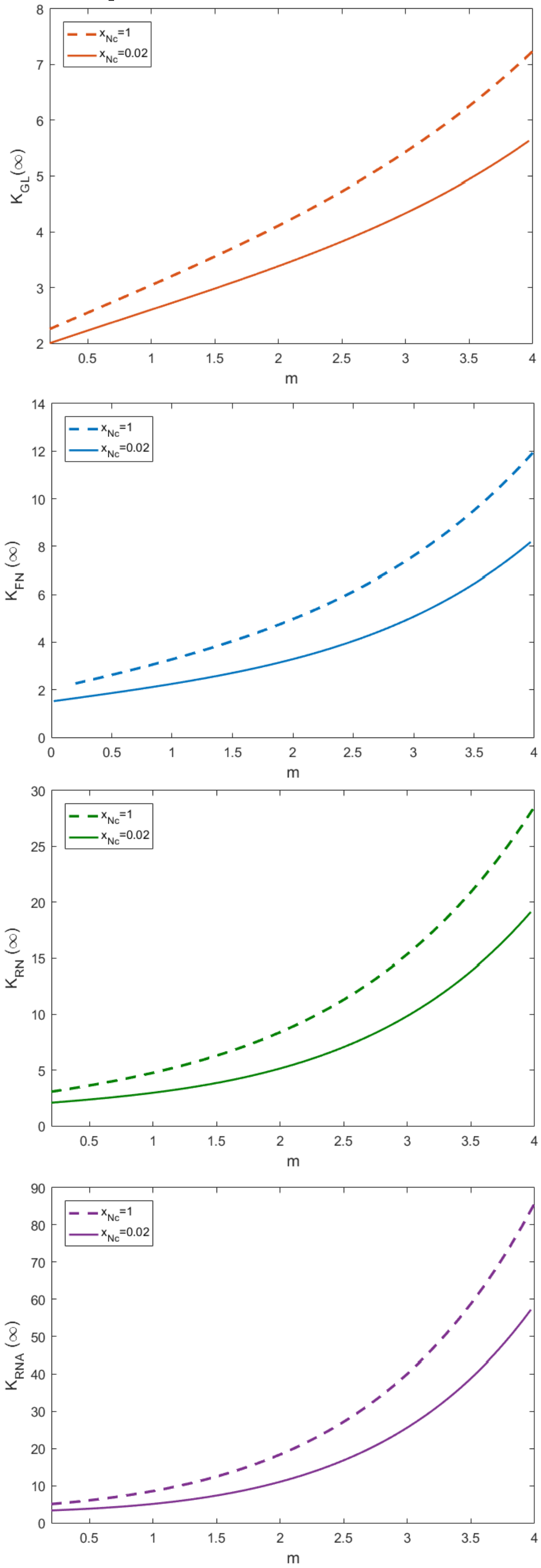


**Figure 5.** Normalized steady-state miss distance coefficients versus *m*

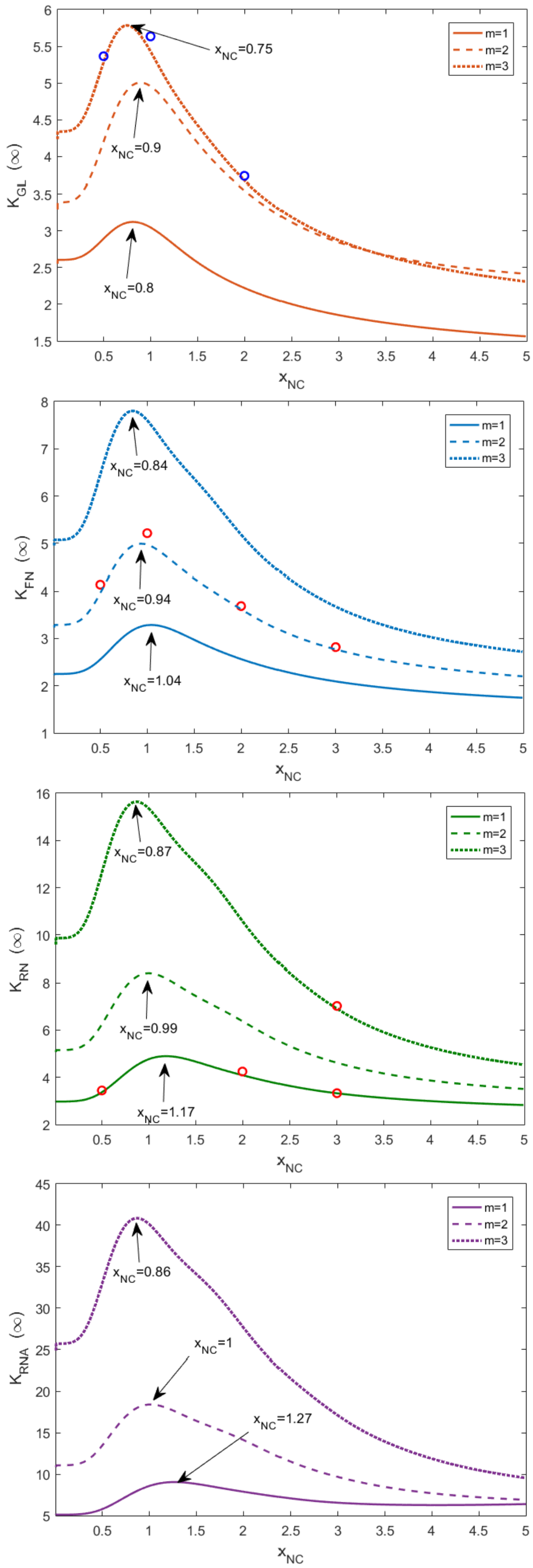


**Figure 6.** Effect of limiting the effective navigation ratio on normalized steady-state miss distance coefficients under EGL ($x_{Nc} \geq 0.01$)

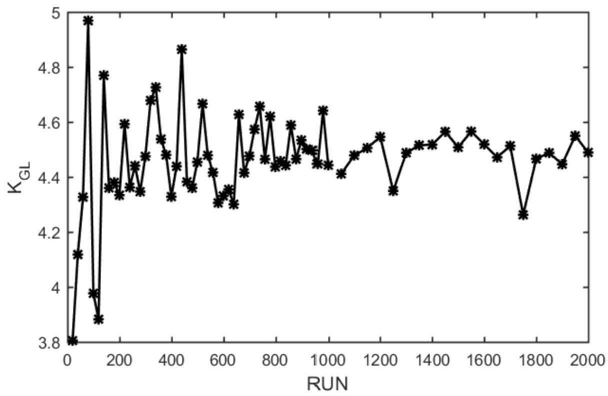


**Figure 7.** Convergence of steady-state miss coefficient in Monte Carlo simulation ($m = 3$)

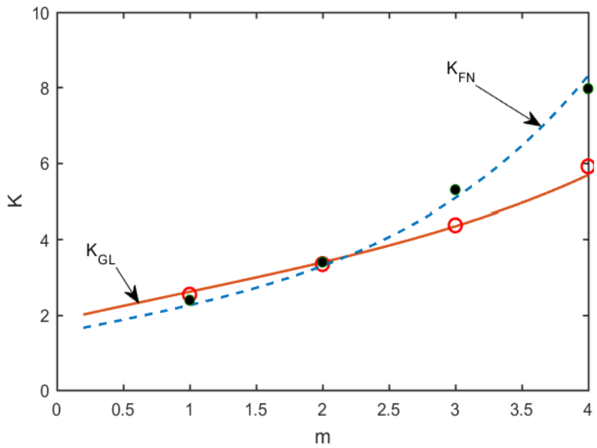


**Figure 8.** Adjoint solutions agree with Monte Carlo simulation

The approximate function of the second degree for $0.8 < m < 2.5$ can be obtained as follows:

$$K_{\mathrm{GL}} = 0.056m^2 + 0.62m + 1.93 \quad (54)$$

$$K_{\mathrm{FN}} = 0.28m^2 + 0.21m + 1.76 \quad (55)$$

$$K_{\mathrm{RN}} = 0.97m^2 - 0.69m + 2.7 \quad (56)$$

$$K_{\mathrm{RNA}} = 3.3m^2 - 3.8m + 5.6 \quad (57)$$

The accuracy of the preceding approximate formulas can be seen in Fig. 9.

## Conclusions

This study presents the miss distance analysis of the first-order EGL due to seeker noise sources, including glint noise, range independent noise, active range dependent noise and semi-active range dependent noise using the linearized equations. The present analysis has been performed

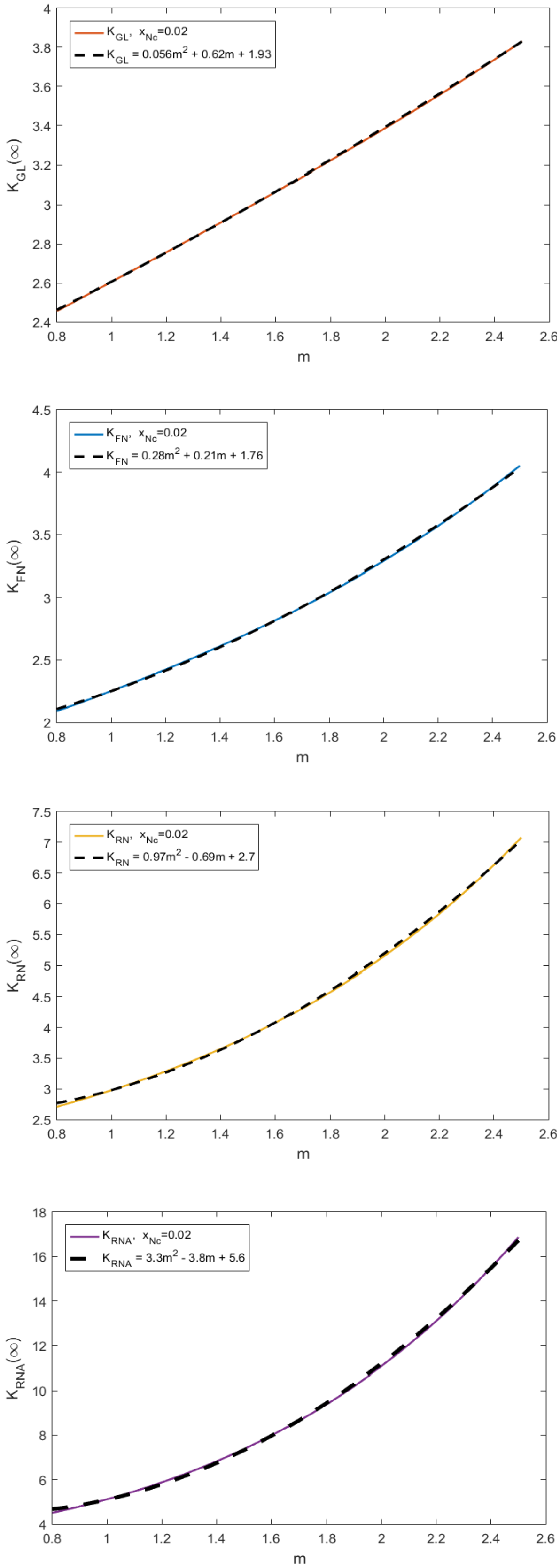


**Figure 9.** Accuracy of the approximate formulas for normalized steady-state miss coefficients due to seeker noise sources

for different values of the power of the alpha function, the time decrease rate of the zero-effort-miss to unit control input for the four mentioned seeker noise sources. It should be noted that the first-order EGL simplifies to the first-order optimal strategy when the power of the alpha function is set to 1.

In our analysis, the seeker and control system is modeled by a fifth-order binomial transfer function. Due to the error of numerical solution and stability considerations, the effective navigation ratio has been kept constant for the final moments of the engagement and its effect on the miss distance has been investigated. The appropriate time interval for applying restrictions to the effective navigation ratio can significantly affect the miss distance when the power of the alpha function increases. In order to verify the simulation results of the adjoint model, the Monte Carlo simulation has been utilized.

Finally, the approximate equations for rms miss distance due to four seeker noise sources have been obtained using curve fitting in terms of the power of the alpha function in the two forms of the quadratic and cubic polynomial functions. The quadratic interpolation has less accuracy but simplifies the relation of miss distance budget to obtain the optimal values of the parameters.

## Appendix A

As mentioned earlier, at the final moments of the engagement, for very small values of $t_{go}$, the value of $x = t_{go}/T$ tends to zero. Therefore, according to Eq. (3), the value of $K_L$ approaches 0.5 when $x \to 0$. Figure 10 shows the numerical error in $K_L$ for very small values of $x$. According to Eqs. (2) and (5), the effective navigation ratio is singular at x=0. In Fig. 11, the values of $N_m$ are plotted according to the equations of Ref. [22] and compared to approximate Eq. (5). The numerical error of the effective navigation ratio becomes very large for very small values of $x$. The sensitivity of the numerical error for very small values of $x$ increases with the increase of the value of $m$ as seen in Fig. 11. Hence, the effective navigation ratio is computed for $x > 0.005$ using Eqs. (2) and (5).

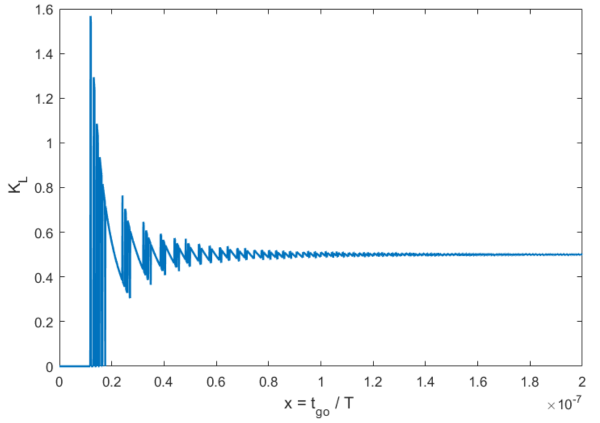


**Figure 10.** Sensitivity of $K_L$ to very small values of x

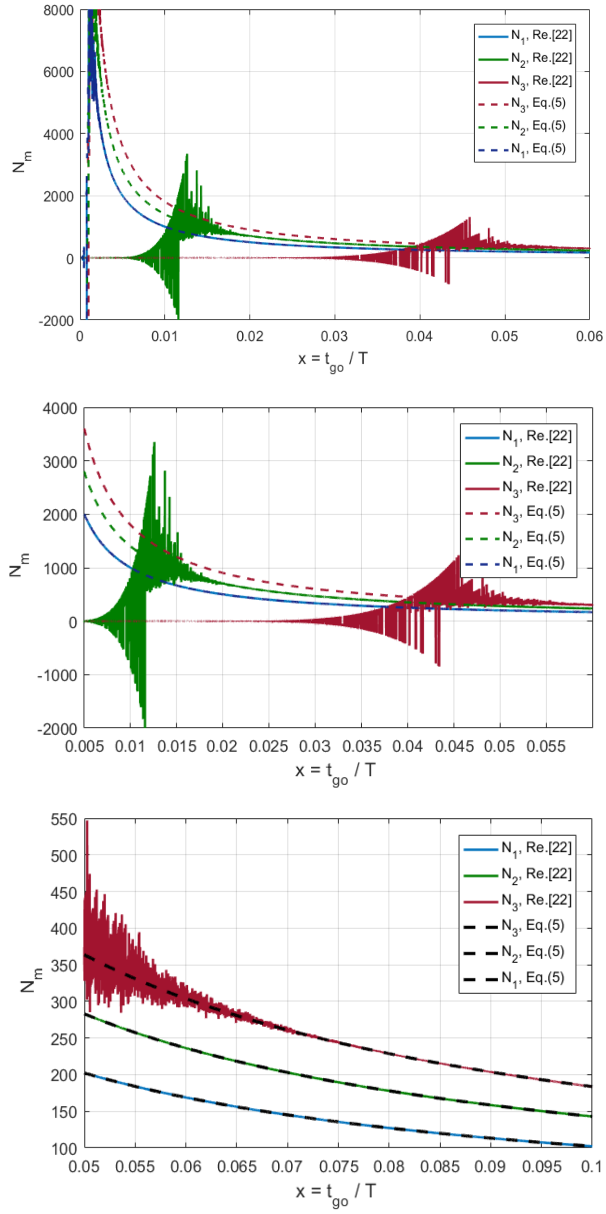


**Figure 11.** Sensitivity of $N_m$ for very small values of x

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