

Scientific-Research Article

Fast Terminal Sliding Mode Controller for a Quadrotor Unmanned Aerial Vehicle

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This paper presents robust nonlinear control law for a quadrotor UAV using fast terminal sliding mode control. Fast terminal sliding mode idea is used for introducing a nonlinear sliding variable that guarantees the finite time convergence in sliding phase. Then in reaching phase for removing chattering and producing smooth control signal, continuous approximation idea is used. Simulation results show that the proposed algorithm is robust against parameter uncertainty and has better performance than conventional sliding mode for controlling a quadrotor UAV.

Keywords: quadrotor UAV, fast terminal sliding mode - finite time convergence.

Introduction

Control under heavy uncertain conditions is one of the central topics of modern control theory. Sliding mode control (SMC) is one of the most robust and effective tools to cope with heavy uncertain conditions. Sliding mode control works on the concept of designing a control to drive the system to the desired sliding manifold [1-3]. These systems have the advantage of that under the sliding manifold the close loop system behavior is robust to external disturbances. In reaching phase the parameters of sliding mode can be adjusted such that the convergence time of sliding variable to sliding surface be finite, however the system states in the sliding phase cannot converge to zero in finite time. So that the equilibrium point corresponding to the system is asymptotically stable. Here we have the first problem in conventional sliding mode control is that its finite time in reaching phase only [4-7].

In conventional SMC, the most common used sliding variable is the linear sliding surface which is based on linear combination of the system states by using an appropriate time-invariant coefficient [8-10]. This can be said that, an arbitrary linear manifold is considered as a sliding variable, which can guarantee the asymptotic stability [11-13]. Accomplishing finite-time error convergence is more desirable in practice. Instead of using a linear sliding

surface, terminal sliding mode control (TSMC) with a nonlinear sliding variable is presented [10]. The terminal sliding mode was developed by adding the nonlinear fractional-power item into the sliding mode to offer some superior properties, such as finite time convergence, faster and better tracking precision. This is accomplished by using a nonlinear sliding line which results in finite time convergence in sliding phase. Also nonlinear switching hyper planes in TSMC can improve the transient performance substantially [8-13], [14].

Traditional SMC and TSMC have another problem, such as discontinuous control that often causes chattering in control input [1]. One approach to eliminate chattering in control signal is to use a continuous approximation of the discontinuous sliding mode controller. Using approximation, lead to tracking to within a guaranteed precision rather than perfect tracking in general. So as such here terminal sliding mode with boundary precision is used to control the nonlinear system.

In this paper the both ideas of conventional sliding mode and terminal sliding mode with continues approximation in boundary layer are used. The paper is organized as follows. In section II Modeling of the Quadrotor is presented. Then in section III conventional SMC and in section IV the proposed method is given. In section V the proposed method is simulated to control a small Quadrotor UAV. The conclusion is given in section VI.

Modeling of UAV Quadrotor

Unmanned Aerial Vehicles (UAV) have shown a growing interest thanks to recent technological projections, especially those related to instrumentation. [15]

Despite the progress, researchers must deal with serious difficulties, related to the control of such systems, particularly in the presence of atmospheric turbulences. In addition, the navigation problem is complex and requires the perception of an often constrained and evolutionary environment.

In figure 1 a type of these UAV Quadrotor can be seen.



Fig. 1 Typical Quadrotor

The quadrotor have four propellers in cross configuration. The two pairs of propellers (1,3) and (2,4) as described in Fig. 2, turn in opposite directions. By varying the rotor speed, one can change the lift force and create motion. Thus, increasing or decreasing the four propeller's speeds

together generates vertical motion. Changing the 2 and 4 propeller's speed conversely produces roll rotation coupled with lateral motion. Pitch rotation and the corresponding lateral motion; result from 1 and 3 propeller's speed conversely modified. Yaw rotation is more subtle, as it results from the difference in the counter-torque between each pair of propellers.

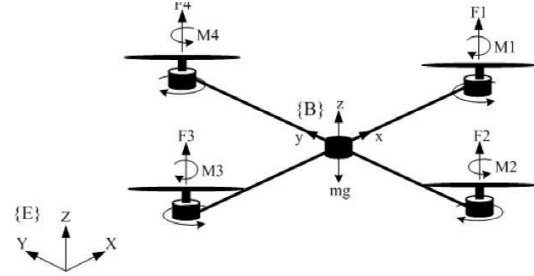


Fig. 2 Quadrotor configuration

Here we will make the following assumptions:

- The quadrotor structure is rigid and symmetrical.
- The center of mass and o' coincides.
- The propellers are rigid.
- Thrust and drag are proportional to the square of the propellers speed.

Under these assumptions we will the following dynamical equation (1):

$$\begin{aligned}
 \dot{x}_1 &= x_2 \\
 \dot{x}_2 &= a_1 x_4 x_6 + a_2 x_2^2 + a_3 \bar{\Omega} x_4 + b_1 U_2 \\
 \dot{x}_3 &= x_4 \\
 \dot{x}_4 &= a_4 x_2 x_6 + a_5 x_4^2 + a_6 \bar{\Omega} x_2 + b_2 U_3 \\
 \dot{x}_5 &= x_6 \\
 \dot{x}_6 &= a_7 x_4 x_2 + a_8 x_6^2 + b_3 U_4 \\
 \dot{x}_7 &= x_8 \\
 \dot{x}_8 &= a_9 x_8 + U_x \frac{U_1}{m} \\
 \dot{x}_9 &= x_{10} \\
 \dot{x}_{10} &= a_{10} x_{10} + U_y \frac{U_1}{m} \\
 \dot{x}_{11} &= x_{12} \\
 \dot{x}_{12} &= a_{11} x_{12} + \frac{\cos x_1 \cos x_3}{m} U_1 - g
 \end{aligned} \tag{1}$$

And the integers are defied as below(2):

$$\begin{aligned}
a_1 &= \left(\frac{I_y - I_z}{I_x} \right), a_2 = \frac{-K_{f\alpha x}}{I_x}, a_3 = \frac{-J_r}{I_x} \\
a_4 &= \left(\frac{I_z - I_x}{I_y} \right), a_5 = \frac{-K_{f\alpha y}}{I_y}, a_6 = \frac{J_r}{I_y} \\
a_7 &= \left(\frac{I_x - I_y}{I_z} \right), a_8 = \frac{-K_{f\alpha z}}{I_z}, a_9 = \frac{-K_{f\alpha x}}{m} \quad (2) \\
a_{10} &= \frac{-K_{f\alpha y}}{m}, a_{11} = \frac{-K_{f\alpha z}}{m} \\
b_1 &= \frac{d}{I_x}, b_2 = \frac{d}{I_y}, b_3 = \frac{1}{I_z}
\end{aligned}$$

Further calculations and explanations of this method can be found in [15-18].

Control design using Conventional Sliding Mode

Consider a nonlinear system

$$\dot{X}^{(n)} = f(X) + u + w, |w| \leq \varepsilon \quad (3)$$

Where $f(x)$ is a known nonlinear part, is a bounded uncertainty $X = [x \ \dot{x} \ \ddot{x} \ \dots \ x^{(n-1)}]^T$ is system state and u is the control input. Then sliding variable is

$$S = \left(\frac{d}{dt} + \lambda \right)^{n-1} \tilde{X} \quad (4)$$

Where λ is a strictly positive constant $\tilde{x} = x_d - X$, and x_d is the desired state. Tracking $X = x_d$ of system is equivalent to . Conventional SMC makes S equal to $S = 0$ zero in finite time and then maintain that condition. This controller consists of a reaching mode and a sliding mode, then we have:

$$u = u_{eq} + u_{reach}, u_{reach} = -k \tanh(S) \quad (5)$$

In this controller u_{eq} cancels the known terms of the sliding dynamics, and if the uncertainties exists use the Lyapunov stability theorem to get the necessary conditions, as follow:

$$\dot{V} = \frac{1}{2} \dot{S}^2 \quad (6)$$

$$\dot{V} = S\dot{S} \leq -\eta|S| \quad (7)$$

Where η is a positive constant, which implies that [1-3]:

$$t_{reach} \leq \frac{|S(0)|}{\eta} \quad (8)$$

This algorithm has a problem. By introducing linear sliding variable as shown in (4), in sliding phase we have asymptotic stability. In next section we use both terminal sliding mode idea to cope with this problems.

So now we consider conventional sliding mode, then we have the following equations for both sliding surfaces and also the controller as follow (9,10):

$$\begin{aligned}
S_\phi &= x_2 + \lambda_1 x_1 \\
S_\theta &= x_4 + \lambda_2 x_3 \\
S_\psi &= x_6 + \lambda_3 x_5 \\
S_x &= x_8 + \lambda_4 x_7 \\
S_y &= x_{10} + \lambda_5 x_9 \\
S_z &= x_{12} + \lambda_6 x_{11}
\end{aligned} \quad (9)$$

And the controllers:

$$\begin{aligned}
U_2 &= \frac{1}{b_1} [-q_1 \tanh(S_\phi) - a_1 x_4 x_6 - a_2 x_2^2 - a_3 \bar{\Omega} x_4 - \lambda_1 x_2] \\
U_5 &= \frac{1}{b_2} [-q_2 \tanh(S_\theta) - a_4 x_2 x_6 - a_5 x_4^2 - a_6 \bar{\Omega} x_2 - \lambda_2 x_4] \\
U_4 &= \frac{1}{b_3} [-q_3 \tanh(S_\psi) - a_7 x_4 x_2 - a_8 x_6^2 - \lambda_3 x_6] \\
U_x &= \frac{m}{U_1} [-q_4 \tanh(S_x) - a_9 x_8 - \lambda_4 x_8] \\
U_y &= \frac{m}{U_1} [-q_5 \tanh(S_y) - a_{10} x_{10} - \lambda_5 x_{10}] \\
U_1 &= \frac{m}{\cos \phi \cos \theta} [-q_6 \tanh(S_\psi) - a_{11} x_{12} - \lambda_6 x_{12}]
\end{aligned} \quad (10)$$

Control design using Fast Terminal Sliding Mode

Consider a nonlinear system with relative degree 2 and matched uncertainty as follow:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f(x_1, x_2) + g(x_1, x_2)u + w, |w| \leq \alpha \quad (11)
\end{aligned}$$

For stabilization with conventional SMC, the linear sliding variable is introduced as:

$$S = x_2 + \lambda x_1 \quad (12)$$

Now assume that the sliding mode controller is able to reaching to sliding surface in finite reaching time. Therefore we yields

$$S = 0 \Rightarrow x_2 = -\lambda x_1 \quad (13)$$

$$\dot{x}_1 = x_2 \Rightarrow \dot{x}_1 = -\lambda x_1 \quad (14)$$

It means that the state variable x_1 is asymptotically stable. For stabilizing the state variables, in this paper terminal sliding mode control is designed with introducing nonlinear sliding variable as follow[14,19]:

$$S = x_2 + \alpha x_1 + \beta x_1^{\frac{q}{p}} \quad (15)$$

where $\beta > 0$ is a design constant, both p and q are positive odd integers and satisfy the following condition:

$$1 < \frac{p}{q} < 2 \quad (16)$$

When the system reaches the sliding surface $S = 0$, the motion of the system can be described by the following nonlinear differential equation:

$$\dot{x}_2 = -\beta x_1^{q/p} \Rightarrow \dot{x}_1 = -\alpha x_1 - \beta x_1^{q/p} \quad (17)$$

Sliding time can be calculated as follows:

$$t_s = \frac{p}{\alpha(p-q)} \ln \frac{\alpha |x_0|^{1-\frac{q}{p}} + \beta}{\beta} \quad (18)$$

when t_r is the time of reaching sliding variable to sliding surface from an initial condition.

Now we use idea of boundary terminal sliding mode control in reaching phase. The sliding mode control (5) contains the continuous nonlinear function. This nonlinearity can solve chattering problem. $\tanh(\cdot)$

Now we consider the following sliding surfaces and controllers so we have (19,20):

$$\begin{aligned} S_\phi &= x_2 + \lambda_1 x_1 + \beta_1 x_1^{\alpha_1} \\ S_\theta &= x_4 + \lambda_2 x_3 + \beta_2 x_3^{\alpha_2} \\ S_\psi &= x_6 + \lambda_3 x_5 + \beta_3 x_5^{\alpha_3} \\ S_x &= x_8 + \lambda_4 x_7 + \beta_4 x_7^{\alpha_4} \\ S_y &= x_{10} + \lambda_5 x_9 + \beta_5 x_9^{\alpha_5} \\ S_z &= x_{12} + \lambda_6 x_{11} + \beta_6 x_{11}^{\alpha_6} \end{aligned} \quad (19)$$

And the controllers:

$$\begin{aligned} U_2 &= \frac{1}{b_1} \left[-q_1 \tanh(S_\phi) - a_1 x_4 x_6 - a_2 x_2^2 - a_3 \bar{\Omega} x_4 - \lambda_1 x_2 - \alpha_1 \beta_1 x_1^{\alpha_1-1} \right] \\ U_3 &= \frac{1}{b_2} \left[q_2 \tanh(S_\theta) - a_4 x_2 x_6 - a_5 x_4^2 - a_6 \bar{\Omega} x_2 - \lambda_2 x_4 - \alpha_2 \beta_2 x_3^{\alpha_2-1} \right] \\ U_4 &= \frac{1}{b_3} \left[-q_3 \tanh(S_\psi) - a_7 x_4 x_2 - a_8 x_6^2 - \lambda_3 x_6 - \alpha_3 \beta_3 x_5^{\alpha_3-1} \right] \\ U_x &= \frac{m}{U_1} \left[-q_4 \tanh(S_x) - a_9 x_8 - \lambda_4 x_8 - \alpha_4 \beta_4 x_7^{\alpha_4-1} \right] \\ U_y &= \frac{m}{U_1} \left[-q_5 \tanh(S_y) - a_{10} x_{10} - \lambda_5 x_{10} - \alpha_5 \beta_5 x_9^{\alpha_5-1} \right] \\ U_1 &= \frac{m}{\cos \phi \cos \theta} \left[-q_6 \tanh(S_z) - a_{11} x_{12} - \lambda_6 x_{12} - \alpha_6 \beta_6 x_{11}^{\alpha_6-1} \right] \end{aligned} \quad (20)$$

Simulation

Now for the simulation we consider the bellow real parameters to simulate the two methods:

$$\begin{aligned} K_p &= 2.9842 \times 10^{-5} \text{ N.m / rad / s} \\ K_d &= 3.2320 \times 10^{-7} \text{ N.m / rad / s} \\ m &= 486 \text{ g} \\ d &= 25 \text{ cm} \\ I_x &= 3.8278 \times 10^{-3} \\ I_y &= 3.8288 \times 10^{-3} \\ I_z &= 7.6566 \times 10^{-3} \\ K_{f_{ax}} &= 5.5670 \times 10^{-4} \text{ N / rad / s} \\ K_{f_{ay}} &= 5.5670 \times 10^{-4} \text{ N / rad / s} \\ K_{f_{az}} &= 6.3540 \times 10^{-4} \text{ N / rad / s} \\ K_{f_{bx}} &= 5.5670 \times 10^{-4} \text{ N / m / s} \\ K_{f_{by}} &= 5.5670 \times 10^{-4} \text{ N / m / s} \\ K_{f_{bz}} &= 6.3540 \times 10^{-4} \text{ N / m / s} \end{aligned}$$

Now we will see the difference of these methods in the simulation. In this simulation the desired states are given as below:

$$\begin{aligned} \phi &= 60^\circ (\pi/3) \\ \theta &= 0 \\ \psi &= 36^\circ (\pi/5) \\ x &= 5 \\ y &= 2 \\ z &= 20 \end{aligned}$$

The tracking position with two different methods is shown in figure 3. As can be seen the proposed method is much faster and gets to the desired position with a smooth action.

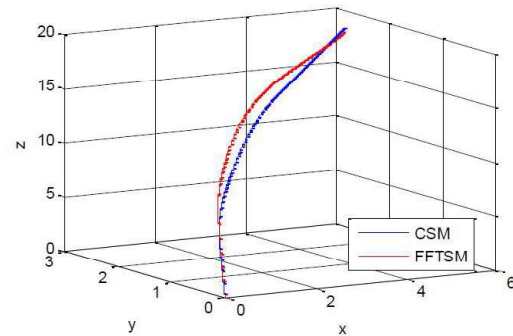


Fig. 3 tracking of position (X,Y,Z)

In figure 4, 5, 6, the angular positioning of the UAV is shown. As can be seen the proposed algorithm is much smoother and faster than the conventional sliding mode.

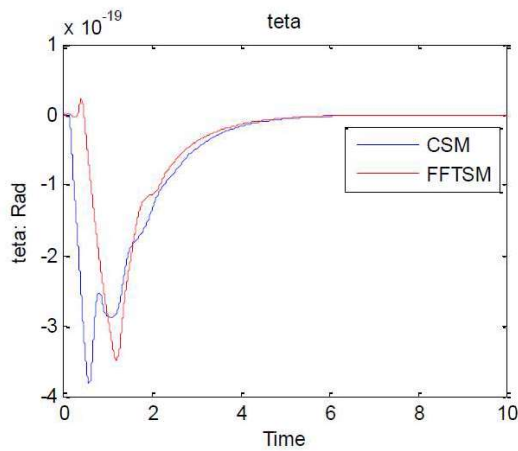


Fig. 4 angular positions of Teta
Phi

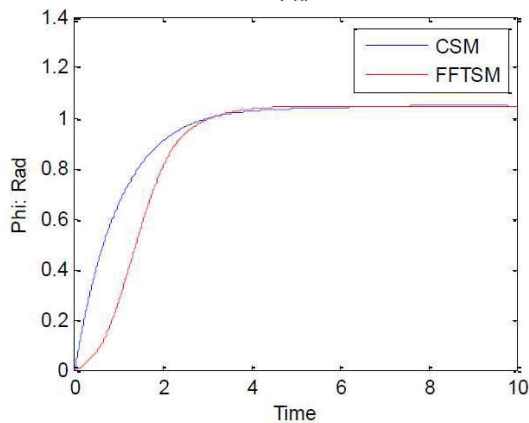


Fig. 5 angular positions of Phi
Yaw

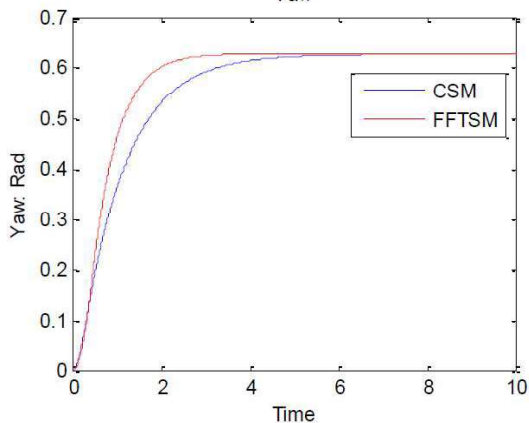


Fig. 6 angular positions of Yaw

Now we will see the controller input that must be given to each state in figure 7. As can be seen the controller input is similar but the proposed method is much faster than the conventional method.

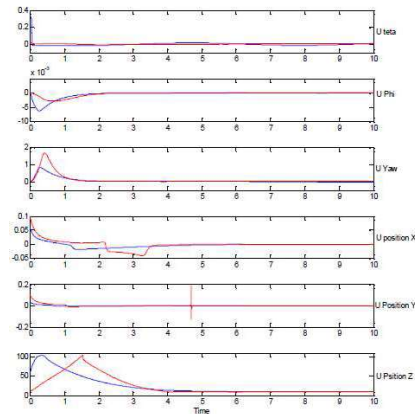


Fig. 7 Controller input

As can be seen the proposed method is sufficient enough to work much faster, now we will see if the value λ changes in the proposed method, how much will it make the system response faster. The changes can be seen in figures 8, 9, 10, 11, 12.

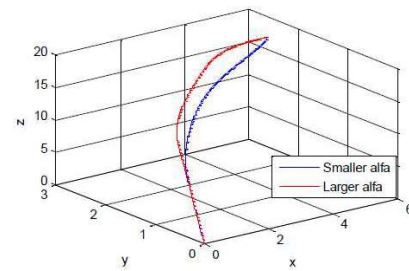


Fig. 8 tracking position of (X,Y,Z)

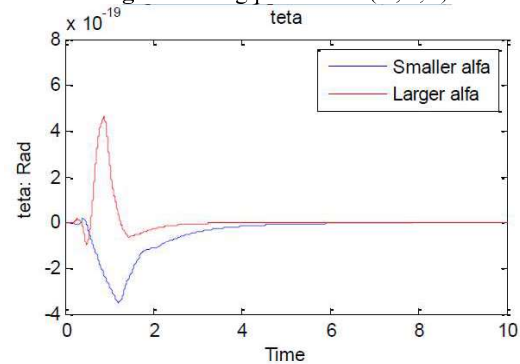


Fig. 9 angular positions of Teta with a larger Landa λ

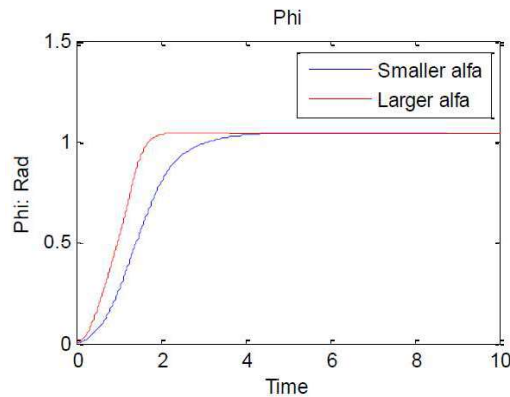


Fig. 10 angular positions of Phi with a larger Landa λ

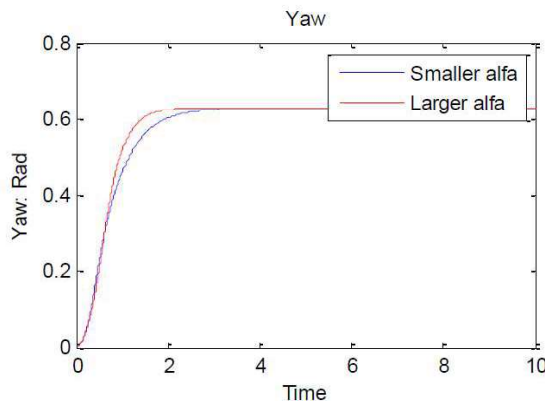


Fig. 11 angular positions of Yaw with a larger Landa λ

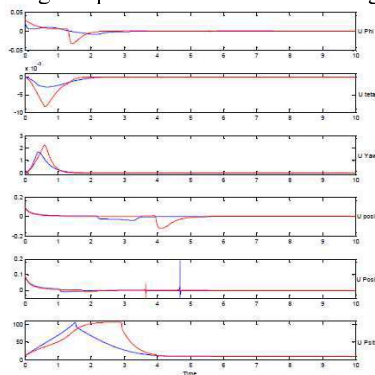


Fig. 12 Controller input with a larger Landa λ

As can be seen by changing the Landa (λ) the system response is much faster and the value of the controller is similar.

Conclusion

As can be seen by the simulation, the Quadrotor UAV is controlled using fast terminal sliding mode. Conventional SMC algorithm can have a good response but it is not finite time. As such the proposed algorithm is finite time and also can be changed as to respond faster. The proposed

algorithm guarantees finite time response with high precision.

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