JAST, Vol. 14, No 1, pp 117-127

JAST Journal of Aerospace Science and Technology

Science Article

Design of Multi-Input Multi-Output Controller for an Unmanned Aerial Vehicle by Eigenstructure Assignment Method

Ali Motamedi[1], Abolghasim Naghash[2]
1&2 - Amirkabir University of Technology, Department of Aerospace Engineering
* Postal Code; 15875-4413 ,Hafez Street, Tehran
Email: * a_motamedi@aut.ac.ir

The purpose of this paper is to present a Multi-Input Multi-Output (MIMO) linear controller based on the eigenstructure assignment method for a fixed-wing Unmanned Aerial Vehicle (UAV) in longitudinal and lateral-directional channels. To this end, a six degree-of-freedom model of the aerial vehicle is considered, where dynamic modes of the system in each channel are analyzed, and the effect of each dynamic mode on state and output variables of the system is investigated. Then, the eigenvalue and eigenvector parameters of the designed controller are appropriately assigned for the system dynamic modes in each channel. In addition, the system requirements of each dynamic mode are satisfied with the proposed controller, and the adverse interaction between the system state variables is minimized. The capability and effectiveness of the designed controller in a desired maneuver are demonstrated with a nonlinear model simulation of a fixed-wing UAV. In this regard, the results in longitudinal and lateral-directional channels are presented.

***Keywords**: Eigenstructure Assignment, Multi-Input - Multi-Output Controller, Unmanned Aerial Vehicle*

Introduction

One of the simplest methods in designing control systems is designing the controller based on the linear model of the system. The use of linear model-based controllers has long been common for many systems, so that even now, with the advent of modern and complex control methods, the investigation of the system behavior with a linear system based-controller is the basis for designing these controllers. In designing a flight control system, one of the first steps is to use linear models of aircraft to produce an initial controller structure. An aircraft or any flying object has a relatively complex nonlinear dynamics so that extensive flight range, alteration of the aerial vehicle structure (in some configurations), system nonlinearizedities, and model uncertainties in the system cause the designed controller to not have the desired performance of the designer in real flight conditions in most of the cases. Therefore, based on the conventional method in designing controllers, it is necessary to make repeated adjustments after designing the controller to bring the controller performance closer to the desired conditions by the designer. As is clear, this step will involve time-consuming and costly non-analytical processes in the controller design process. Therefore, if more attention is paid to the design of the controller in the initial analyzes, these settings will be reduced in the next steps of

1 Ph.D. student, Corresponding Author
2 Associate Professor

Received: 29 May 2021
Accepted: 11 July 2021

the controller design process to achieve a high-performance controller.
In order to meet the mentioned tips, modern methods have been developed to implicitly investigate the interaction between different states in a system by fully displaying a multi-input multi-output controller in a linear system. Among modern control methods, the eigenstructure assignment method is one of the methods that has been used to address these challenges [1]. In a linear time-invariant system without control input, the system response is generated based on system initial conditions and eigenvectors. Therefore, in the case of setting the transient response of a system using feedback, the adjustment of eigenvectors will be as important as the placement of eigenvalues [2]. The design parameters in the eigenstructure assignment method are the eigenvalues and some special elements of eigenvectors so that by specifying them, the feedback gain matrix will be uniquely obtained. Therefore, assigning the appropriate values for these parameters makes it possible to achieve a controller with the desired performance at an optimal time and cost.
The eigenstructure assignment method has been used to control various systems [3-5]. In [6], this method is used to help adjust an energy-based nonlinear regulator for the positioning of a ship on the horizontal plane. In [2, 7 ,8], this method has been used very appropriately and significantly in the design of an aerial vehicle controller. In [9], a robust controller based on an eigenstructure assignment technique has been designed for the F-16 aircraft. Also, in order to achieve good performance characteristics in the F-16 high angle of attack maneuver, this control structure has been used in [10]. Mortazavi and Naghash in [11] presented a step-by-step algorithm with acceptable accuracy for calculating state and control weighting matrices, using the inherent relationship between the controller adjustable parameters and the desired eigenstructure. Mehrabian and Roshanian in [12] designed a Skid-to-turn missile autopilot using scheduled eigenstructure assignment method. In [13], the linear control of the ducted fan aerial vehicle has been performed by modeling of a three-degrees of freedom for a specific maneuver using an eigenstructure assignment method. Also, in [14] this control structure has been used to control an unmanned aerial vehicle.
In the present paper, in order to take advantage of the eigenstructure assignment method for a specific fixed-wing unmanned aerial vehicle, the controller based on the eigenstructure assignment method is designed based on the dynamics of a six-degrees of freedom of aerial vehicle and the expected performance of the aerial vehicle for the two longitudinal and lateral-directional channels.

Eigenstructure assignment method

In general, the state space model for a linear system is introduced as follows.

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

In equation (1), the parameters x, u, y are considered as state variables, system inputs and system outputs, respectively. Based on the dimensions of matrix A, in a n-order open-loop system, n eigenvalues and n eigenvectors can be considered as follows.

$$\Lambda = [\lambda_1 \quad \cdots \quad \lambda_i \quad \cdots \quad \lambda_n] \tag{2}$$

$$V = [v_1 \quad \cdots \quad v_i \quad \cdots \quad v_n] \tag{3}$$

So that

$$AV = V\Lambda \tag{4}$$

By defining the left eigenvector as follows:

$$W^T = [w_1 \quad \cdots \quad w_i \quad \cdots \quad w_n] \tag{5}$$

For system (1), equation (6) can always be considered:

$$WA = \Lambda W \tag{6}$$

Solving equation (1) gives the following equation that determines the direct relationship between eigenvalues and eigenvectors of the system and their behaviors [1].

$$\begin{aligned} y(t) &= \sum_{i=1}^{n} C v_i w_i^T e^{\lambda_i t} x_0 \\ &+ \sum_{i=1}^{n} C v_i w_i^T \int_0^t e^{\lambda[t-\tau]} B u(\tau) d\tau \end{aligned} \tag{7}$$

In equation (7), the first term is known as homogeneous component and the second term is known as forced component. Based on this relationship, it is clear that the dynamic behavior of the aircraft at any given moment is influenced by four factors: system eigenvalues λ_i, system eigenvectors $v_i$and w_i , system initial conditions x_0 and system inputs u.
For this control method, the control input is designed as a state feedback controller so that it can be written:

$$u = -Kx \tag{8}$$

For a closed loop system, the following equation can be considered for the designed controller based on the eigenvalues and eigenvectors of the system:

$$(A - BK)V = \Lambda V \quad (9)$$

The equation (9) can be rewritten as follows:

$$(A - BK)v_i = \lambda_i v_i \quad (10)$$

$$[A - \lambda_i I \quad B]\begin{bmatrix} v_i \\ K v_i \end{bmatrix} = 0 \quad (11)$$

For a non-trivial solution of the equation (11) we have $[\,v_i \quad K v_i]^T$ that is a member of the null space $[A - \lambda_i I \quad B]$.

The null space of the matrix $[A - \lambda_i I \quad B]$ contains a set of orthogonal vectors. Each combination of these vectors results in a vector that, when used as an eigenvector, produces a closed loop system with the desired eigenvalue λ_i. Therefore, the desired vector v_i^d can be projected on this null space to obtain an achievable eigenvector v_i^a, so that it would maintain the best possible closed-loop eigenvalue and optimal coupling. This is shown geometrically for a mode of a three-dimensional system in Figure 1. Therefore, if a three-dimensional vector is desirable, that vector, as shown in Figure 1, is pictured on the null space, and the result is the closest achievable vector that can provide the required eigenvalues [1].

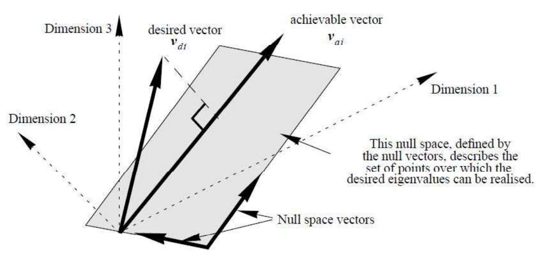

Figure 1 - Diagrammatic representation of a two-dimensional achievable space in a three-dimensional state-space [1]

Based on the equation (10) and considering the vector $m_i = -K v_i^a$ which is actually the columns of the matrix $M = -KV$, the vector v_i^a can also be represented as follows:

$$v_i^a = (\lambda_i I - A)^{-1} B m_i \quad (12)$$

Therefore, based on the information presented in the above, vector v_i^d may not be available in any subspace. Therefore, although not every v_i^d can be reached, but the distance between v_i^d and v_i^a can be reduced as much as possible by defining a cost function as follows:

$$J_i = \frac{1}{2}\left(v_i^a - v_i^d\right)^* P_i\left(v_i^a - v_i^d\right) \quad (13)$$

By solving equation (9) using the Lagrange multiplier method and defining the cost function as (14), the matrix equation (15) can be obtained:

$$\bar{J}_i = \frac{1}{2}\left(v_i^a - v_i^d\right)^* P_i\left(v_i^a - v_i^d\right) + \mu_i^*[(\lambda_i I - A)v_i^a - B m_i] \quad (14)$$

$$\begin{bmatrix} (\lambda_i I - A) & -B & 0 \\ 0 & 0 & -B^T \\ P_i & 0 & (\lambda_i I - A)^* \end{bmatrix}\begin{bmatrix} v_i^a \\ m_i \\ \mu_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ P_i v_i^d \end{bmatrix} \quad (15)$$

Accordingly, by solving equations (15) and specifying the values of m_i and v_i^a and forming matrices M and V using the values of m_i and v_i^a, the value of the feedback gain matrix K can be calculated as follows:

$$K = -MV^{-1} \quad (16)$$

Required Characteristics to Determine Eigenstructures

Eigenstructures include eigenvalues and eigenvectors, and an exclusive controller with a different performance is obtained with each selection of these structures. The selection of eigenvalues is simpler than the selection of eigenvectors, so the selection of eigenvectors is of considerable importance; these values will be determined based on the requirements and desired characteristics of the system.

Decoupling requirements

In general, decoupling of the system modes and the elimination of the interaction of system modes with each other are achieved by correctly adjusting the system eigenvectors. This requirement can be described based on the concept shown in Figure 2. In Figure 2, in addition to the discussion of the vectors v_i^d, v_i^a and the desired eigenvalues, the decoupling goal in assigning the optimal eigenvector is also shown. This figure shows that in addition to having the desired closed loop eigenvalue, this system mode also needs to be separated from another dimension of system. Therefore, as it is clear in the last figure for this system, the only achievable eigenvector can be obtained from the intersection between the achievable vector space and the plane formed from dimensions 1 and 3 [1].

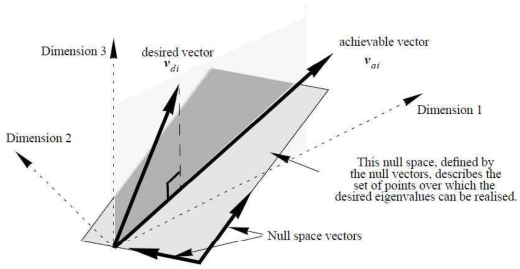

Figure 2 - Diagrammatic representation of decoupling in a three-dimensional state space [1]

Performance Requirements

Performance Requirements can be divided into two categories, which are:
1. The controller is designed in such a way that the input commands are followed by a provided rise time and a specific maximum overshoot.
This characteristic can be achieved by selecting the desired optimum values that have the lowest time constant (for first-order modes) or a minimum damping and frequency (for second-order modes) in every closed loop mode. It is tried to select the eigenvalues as close as possible to the open loop eigenvalues of the system.
2. Characteristics for acceptable perturbation in one variable resulting from command inputs or changes in other variables.
This feature is performed by decoupling operations on the desired eigenvector, so that the effects of the interactions of the variables on each other are reduced.

Unmanned Aerial Vehicle Model

Most of the models presented in the references consider the aircraft to be rigid, and its elastic effects are most often ignored. Accordingly, like any other rigid body, the aerial vehicle will have a six-degrees of freedom, three-degrees related to the transfer motion and three-degrees related to the rotational motion of the aerial vehicle. The basis of an aerial vehicle's performance is based on the nonlinear equation of motion, but in some references, in order to use linear control methods, the linear model of the aerial vehicle is extracted by linearizing the nonlinear equations of motion in specific conditions. In [15], a linear dynamic model for a fixed wing aerial vehicle is extracted based on dimensional stability-control derivatives in two longitudinal and lateral-directional channels. (Such as equations (17) and (18)).

$$\begin{bmatrix}\dot{w}\\ \dot{q}\\ \dot{\theta}\\ \dot{u}\end{bmatrix}=\begin{bmatrix}Z_w & Z_q+V_T & -g\sin\gamma & Z_u\\ \bar{M}_w & \bar{M}_q & \bar{M}_\theta & \bar{M}_u\\ 0 & 1 & 0 & 0\\ X_w & X_q & -g\cos\gamma & X_u\end{bmatrix}\begin{bmatrix}w\\ q\\ \theta\\ u\end{bmatrix}+\begin{bmatrix}0 & Z_{\delta e}\\ 0 & \bar{M}_{\delta e}\\ 0 & 0\\ X_{\delta t} & X_{\delta e}\end{bmatrix}\begin{bmatrix}\delta t\\ \delta e\end{bmatrix} \tag{17}$$

$$\begin{bmatrix}\dot{r}\\ \dot{\beta}\\ \dot{p}\\ \dot{\phi}\end{bmatrix}=\begin{bmatrix}N_r^{'} & N_\beta^{'} & N_p^{'} & 0\\ -1 & Y_v & 0 & \frac{-g}{U_0}cos\gamma_0\\ L_r^{'} & L_\beta^{'} & L_p^{'} & 0\\ \frac{sin\gamma_0}{cos\theta_0} & 0 & \frac{cos\gamma_0}{cos\theta_0} & 0\end{bmatrix}\begin{bmatrix}r\\ \beta\\ p\\ \phi\end{bmatrix}+\begin{bmatrix}N_{\delta a}^{'} & N_{\delta r}^{'}\\ 0 & Y_{\delta r}^{*}\\ L_{\delta a}^{'} & L_{\delta r}^{'}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta a\\ \delta r\end{bmatrix} \tag{18}$$

Equation (17) is used for the longitudinal channel and equation (18) is used for the lateral-directional channel of the aerial vehicle. As can be seen, the models shown are multi-input models, each of which includes two control inputs and four mode variables.
Here, in order to design and simulate the behavior of the controller, an unmanned aerial vehicle weighing 13 kg is considered. Based on the linear models presented in the equations (17) and (18) and by substituting the values related to each parameter, the longitudinal and lateral-directional dynamic model of the aerial vehicle is shown below.

$$\begin{bmatrix}\dot{w}\\ \dot{q}\\ \dot{\theta}\\ \dot{u}\end{bmatrix}=\begin{bmatrix}-4.24 & 29.43 & 0 & -0.65\\ -4.64 & -6.14 & 0 & -0.45\\ 0 & 1 & 0 & 0\\ 0.22 & 0 & -9.81 & -0.07\end{bmatrix}\begin{bmatrix}w\\ q\\ \theta\\ u\end{bmatrix}+\begin{bmatrix}0 & -2.92\\ 0 & -40.35\\ 0 & 0\\ 0.39 & -0.62\end{bmatrix}\begin{bmatrix}\delta t\\ \delta e\end{bmatrix} \tag{19}$$

$$\begin{bmatrix}\dot{r}\\ \dot{\beta}\\ \dot{p}\\ \dot{\phi}\end{bmatrix}=\begin{bmatrix}-0.39 & 35.95 & -28.09 & 0\\ -1 & -0.62 & 0 & -0.32\\ 12.79 & -131.61 & -27.63 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}r\\ \beta\\ p\\ \phi\end{bmatrix}+\begin{bmatrix}-12.98 & -24.23\\ 0 & 0.14\\ -150.22 & 0.28\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta a\\ \delta r\end{bmatrix} \tag{20}$$

Longitudinal Linear Model Analysis

The longitudinal linear model presented by equation (19) consists of four state variables w, q, θ, u , which are the vertical velocity component, the rate of change of the pitch angle, the pitch angle and the horizontal component of the aerial vehicle velocity along the x axis of the aerial vehicle's body coordinate system, respectively. Also, the two control inputs include the throttle angle and the elevator deflection angle. The aircraft's open-loop longitudinal linear model typically includes short-period and phugoid dynamic modes. In general, phugoid mode is a mode with low frequency and low damping that occurs approximately at a constant angle of attack. While the short-period mode is a high frequency mode with high damping that can be seen at almost constant speed. The open-loop eigenvalues and the corresponding eigenvectors for system (19) are expressed in Table 1.

Table 1 - Dynamic modes and eigenstructures of the longitudinal open-loop dynamic of airplane

	Mode 1	**Mode 2**
Eigenvalues	$-5.193 \pm 11.65i$	$-0.032 \pm 0.255i$
w	0.9290	0.1069
q	0.3689	0.0068
θ	0.0289	0.0266
u	0.0091	0.9939

As it is known, mode 1 is more related to state variable w and mode 2 is more related to state variable u. Based on the definitions made for short-period mode and phugoid mode, it can be concluded from the presented results that mode 1 represents the dynamic mode of short period and mode 2 represents the dynamic phugoid mode of aerial vehicle.

In order to investigate the coupling of each mode with the input and output of the system based on the definitions made for the eigenvectors on the right and left of a model, vectors Cv_i and $w_i^T B$ determine the coupling of each mode with the output and input variables, respectively. If the outputs of the longitudinal model of the aerial vehicle are considered here as equal to the aerial vehicle mode vector, it can be concluded that the output w will have the greatest effect from the short period mode and the output u will have the greatest effect from the phugoid mode.

Longitudinal Controller Design

In order to determine the feedback gain matrix related to the aerial vehicle longitudinal channel controller, system eigenstructures need to be assigned appropriately. Therefore, due to the existence of four outputs and two inputs in this model, we will be able to assign four closed-loop eigenvalues and corresponding eigenvectors. Here, in order to achieve the desired performance and to satisfy the stability requirements based on [15] as well as the existing experience for similar UAVs, the damping ratio and the natural frequency for the short-period and phugoid mode are considered $\zeta_{sp} = 0.7$, $\omega_{n_{sp}} = 11\, rad/s$, and $\zeta_{ph} = 0.28$, $\omega_{n_{ph}} = 9\, rad/s$ respectively. Accordingly, for the short-period mode and the phugoid mode, the eigenvalues $-7.70 \pm 7.68i$ and $-2.43 \pm 8.67i$ are assigned to this aerial vehicle, respectively.

For assigning eigenvectors corresponding to short period and phugoid modes, we can say, as a general rule in the longitudinal dynamics of each aerial vehicle, in the short period mode while the changes in forward speed are zero, the vertical speed component of the aerial vehicle with the pitch rate term has coupling. Also, in phugoid mode, while the vertical speed changes of the aerial vehicle are zero, the pitch angle and the forward speed are coupled together. Therefore, if the vectors v_1 and v_2 represent the eigenvectors corresponding to the short-period mode and the vectors v_3 and v_4 represent the eigenvectors corresponding to the phugoid mode, in order to provide the conditions for the decoupling of these dynamic modes from each other, these vectors are considered as follows [2].

$$\begin{bmatrix} w \\ q \\ \theta \\ u \end{bmatrix} \quad \underset{v_1}{\begin{bmatrix} 1 \\ x \\ y \\ 0 \end{bmatrix}} \quad \underset{v_2}{\begin{bmatrix} x \\ 1 \\ y \\ 0 \end{bmatrix}} \quad \underset{v_3}{\begin{bmatrix} 0 \\ z \\ 1 \\ k \end{bmatrix}} \quad \underset{v_4}{\begin{bmatrix} 0 \\ z \\ k \\ 1 \end{bmatrix}} \tag{21}$$

Based on the design, the optimum eigenvectors for the longitudinal channel of the aerial vehicle are considered as shown in Table 2. It should be noted that the values x, y, z and k assigned here are not unique values and can be changed depending on the designer's opinion. But in the meantime, the expertise of the designer in assigning appropriate values based on the performance requirements that are required for the aerial vehicle is of considerable importance. Therefore, in the case of the selected aerial vehicle in this article, according to the approach observed in the references mentioned and also the experience of the authors, the values mentioned in Table 2 have been selected. By reviewing the results obtained from the simulator and meeting the performance requirements of the aerial vehicle, after adjusting these values several times, the selected values have been approved and used.

Table 2 - Desired Eigenvalues and eigenvectors of the longitudinal closed-loop system

	Short Period		Phugoid	
$\begin{bmatrix} w \\ q \\ \theta \\ u \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0.045+0.18i \\ -0.21-0.01i \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.045+0.18i \\ 1 \\ -0.21-0.01i \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1.3+0.01i \\ 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1.3+0.01i \\ 1 \\ 1 \end{bmatrix}$

Based on the assigned eigenstructure and also considering the Identity matrix for matrix P in equation (15), finally the feedback gain matrix for the longitudinal channel of the UAV is calculated as follows.

$$K_{EA_{long}} = \begin{bmatrix} -33.7 & 0.0554 & 1.6036 & 0.0036 \\ 0.0001 & -0.0001 & -0.0009 & 0.0000 \end{bmatrix} \tag{22}$$

Lateral-Directional Linear Model Analysis

The lateral-directional linear model presented by equation (20) includes the state variables r, β, p, ϕ which are the yaw rate, sideslip angle, roll rate and aerial vehicle bank angle in the body system, respectively. In this model, two control inputs include the angle of the control surface of aileron and rudder of the aerial vehicle. This open-loop lateral-directional linear model includes dutch roll, roll and spiral dynamic modes, so that the dutch roll mode is an oscillating mode and the roll mode is a first-order and fast mode. Spiral mode is also a first-order mode and usually with very poor stability. The open-loop eigenvalues of the system (20) and the corresponding eigenvectors corresponding to each are shown in Table 3.

Table 3 - Dynamic modes and eigenstructures of the lateral-directional open-loop dynamic of airplane

	Mode 1	Mode 2	Mode 3
Eigenvalues	-19.69	$-4.46 \pm 15.07i$	-0.0281
r	0.7975	0.8974	0.2980
β	0.0413	0.0571	0.0235
p	0.6011	0.4366	0.0262
ϕ	0.0305	0.0278	0.9539

From the results presented in Table 3, it is clear that mode 1 with state variable p, mode 2 with state variable r and mode 3 with state variable ϕ have a strong coupling. Based on the specifications of each of these modes, it can be said that the mode 1 shows roll mode, mode 2 shows dutch roll mode and mode 3 shows the aerial vehicle spiral mode.

Here again, if the outputs of the lateral-directional model of the aerial vehicle are considered equal to the aerial vehicle state vector, so that the matrix C is defined as an Identity matrix, the size of vector Cv_i will exactly result in the values given in Table 3 as the output vector of the lateral-directional channel of the aerial vehicle. Here, too, it can be concluded that output p has the greatest effect from the roll mode, output r has the greatest effect from the dutch roll mode, and output ϕ has the greatest effect from the spiral mode.

Lateral-Directional Controller Design

For the lateral-directional channel of the aerial vehicle, as in the design of the controller of the longitudinal channel, the feedback gain matrix is calculated by assigning the corresponding eigenstructures. Based on the desired performance characteristics as well as the stability requirements mentioned in [15] and the experiences in the controller design for similar UAVs, the values of $\zeta = 0.7$ and $\omega_n = 7\ rad/s$ can be considered for the dutch roll dynamic mode as an oscillation mode. Based on this, eigenvalues $-4.90 \pm 4.99i$ will be obtained for this mode. Also, for roll and spiral dynamic modes, in order to achieve the desired level of stability, the values 11.0 and -0.9 have been assigned to these modes, respectively.
For assigning eigenvectors corresponding to roll, dutch roll and spiral modes in order to decouple these modes from each other, for the roll mode, the component corresponding to p (due to the importance of this state variable in roll mode) is considered to be one. For the spiral mode, due to the importance of the bank angle, the

corresponding component of this state variable is considered to be one. Due to the coupling between the variables of roll rate and bank angle of the aerial vehicle in roll and spiral modes, these components are considered corresponding to each other and in order to decouple these modes from the dutch roll mode, the other components of these vectors are considered to be zero. For the Dutch roll mode, the data shows that the yaw rate is of particular importance. Also in this mode, the yaw rate and the sideslip angle are coupled to each other. Therefore, while the values of these components correspond to each other in the dutch roll eigenvectors, in this mode, in order for decoupling from other modes, the roll rate and the bank angle are considered zero and the values corresponding to these state variables in the desired vector roll eigenvectors are considered zero. If the vectors v_1 and v_2 represent the eigenvectors corresponding to the aerial vehicle dutch roll mode and the v_3 and v_4 vectors represent the eigenvectors corresponding to the aerial vehicle roll and spiral mode, respectively, these vectors can be considered as follows [2].

$$\begin{array}{ccccc} \begin{bmatrix} r \\ \beta \\ p \\ \phi \end{bmatrix} & \begin{bmatrix} 1 \\ x \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} x \\ 1 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \\ y \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ y \\ 1 \end{bmatrix} \\ & v_1 & v_2 & v_3 & v_4 \end{array} \tag{23}$$

The values x and y in equation (23) are nonunique uncertain components for which, according to the designer, it is possible to assign different values. Here, based on the design based on the requirements of stability and control as well as the experiences of the authors, the optimum eigenvectors for the lateral-directional channel of the aerial vehicle are considered as shown in Table 4.

In the lateral-directional channel, it is also assigned based on eigenstructures, and also considering the Identity matrix for the matrix P in equation (15), feedback gain matrix based on equation (16) as the values shown in (24) has been concluded.

Table 4 - Desired Eigenvalues and eigenvectors of the lateral -directional closed-loop system

	Roll	Dutch roll		Spiral
$\begin{bmatrix} r \\ \beta \\ p \\ \phi \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ 1 \\ -0.972-0.1i \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0.211+1i \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.211+1i \\ 1 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ -0.972-0.1i \\ 1 \end{bmatrix}$

$$K_{EA_{lat}} = \begin{bmatrix} -0.0716 & 0.7335 & 0.0886 & -0.0494 \\ -0.2134 & -0.1898 & 0.7401 & -0.1499 \end{bmatrix} \tag{24}$$

Simulation of UAV behavior in the presence of a designed controller

In order to prove the performance of the designed controller based on the eigenstructure assignment method, and to study the behavior of the UAV with the specifications presented in equations (19) and (20), a linear model simulation of a six-degrees of freedom has been performed in MATLAB environment. The simulated system is implemented for the longitudinal channel for the initial condition $X_0 = [0\ 0\ 5\ 0]$ and for the lateral-directional channel for the initial condition $X_0 = [0\ 0\ 0\ 5]$ with the implemented controller. The results obtained for the outputs u, θ, q, w in the longitudinal channel are shown in Figure 3 and the outputs r, β, p, ϕ for the lateral-directional channel of the aerial vehicle are shown in Figure 4.

In general, any designed controller needs to be implemented on a real system and provide the desired performance for it. As it is known, an aircraft has nonlinear dynamics and therefore the study of its behavior requires simulation of a nonlinear six-degrees of freedom of the aircraft. Therefore, based on the observations of the results presented in Figure 3 and Figure 4 and with ensuring the optimal performance of the designed controller, by performing a simulation of a six-degrees of freedom of the UAV in the Simulink/MATLAB environment, the designed controller is applied to the nonlinear system. In order to evaluate the performance of the controller in executing directional commands, a Dubins path including arc-straight line-arc [16 and 17] at a fixed height of 100 m is considered. The simulation result of a nonlinear six-degrees of freedom of aerial vehicle movement in Figure 5 show the ability of a designed controller based on eigenstructures to apply to a nonlinear system. From the displayed results, it is clear that the designed controller was able to control the UAV well in both longitudinal and lateral-directional channels, so that the aerial vehicle in the presence of the controller with optimal operating conditions has been able to achieve stable conditions, and follow the default path.

In order to investigate the effect of noise on the controller performance and its stability, white noise has been added as a measurement noise to the output of the sensors in the developed nonlinear simulator. The standard deviation for white noise applied to each of the measured variables based on the values proposed in [18] is shown in Table 5.

Table 5 - Standard deviation of measurement noise

Measurement	Standard Deviation	Unit
u	0.01	m/sec
w	0.01	m/sec
p	0.13	deg/sec
q	0.13	deg/sec
r	0.13	deg/sec
ϕ	0.30	deg

In the a nonlinear six-degrees of freedom simulation presented to test the controller's stability to random behaviors and atmospheric disturbances, the Dryden wind model [19 and 20] is applied to the UAV as a random wind model. Here, in order to implement the wind model, the relevant block has been used in the Simulink environment of MATLAB software, and based on the profile shown in Figure 6, a random wind has been applied to the UAV as a disturbance. How the aerial vehicle responds to noise and disturbance in the nonlinear simulator, while moving the aerial vehicle on the designed Dubins path, is shown in figure 7 for the UAV control variables and for some of the state variables is shown in Figure 8, Figure 9 and Figure 10. It is noteworthy that in the simulation, the assumption is made on the explicit and direct application of control commands (assuming the existence of an actuator with a transfer function equal to one) to the UAV.

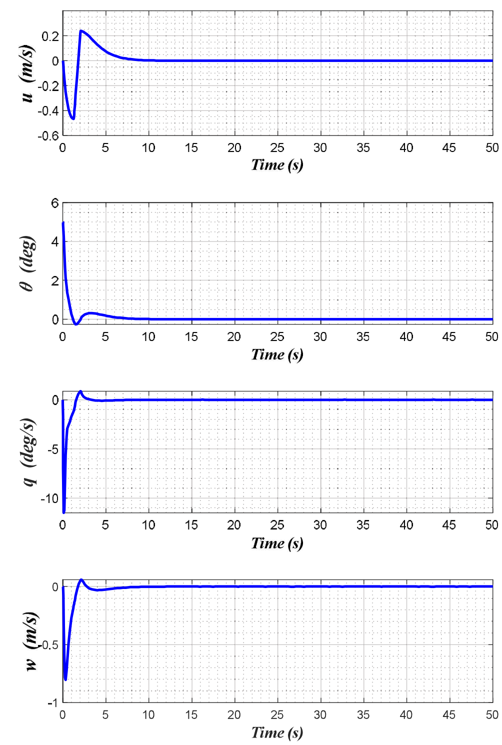

Figure 3 - Behavior of the UAV longitudinal channel output variables in linear system control

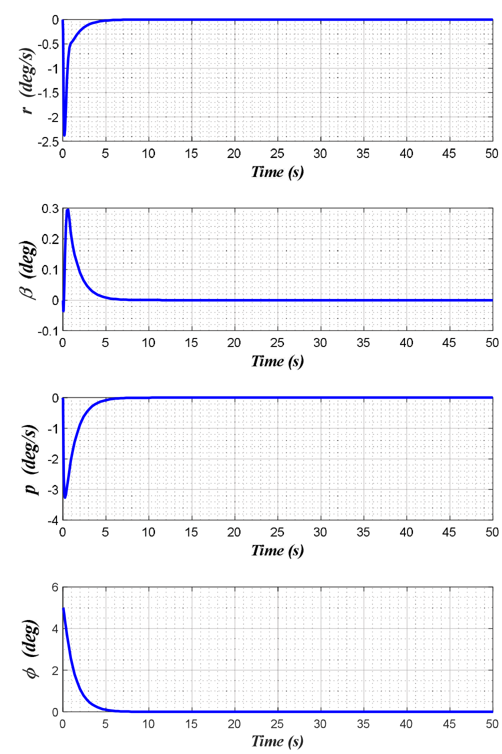

Figure 4 - Behavior of the UAV lateral-directional channel output variables in linear system control

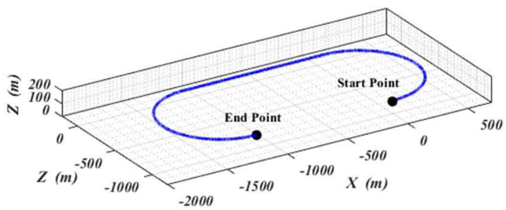

Figure 5 - Simulation of a nonlinear six-degrees of freedom of the UAV movement to track a predetermined Dubins flight path

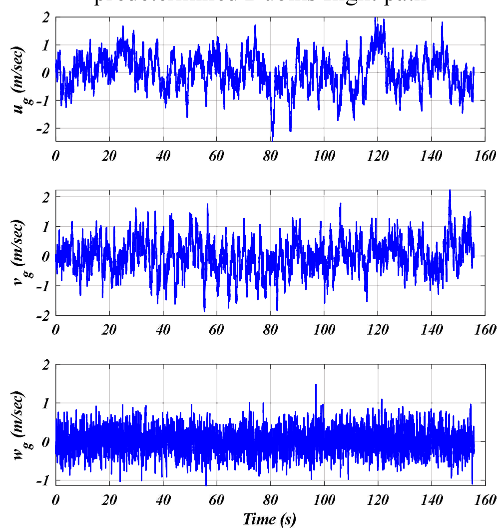

Figure 6 - Display of wind components applied to the UAV, based on the Dryden random model

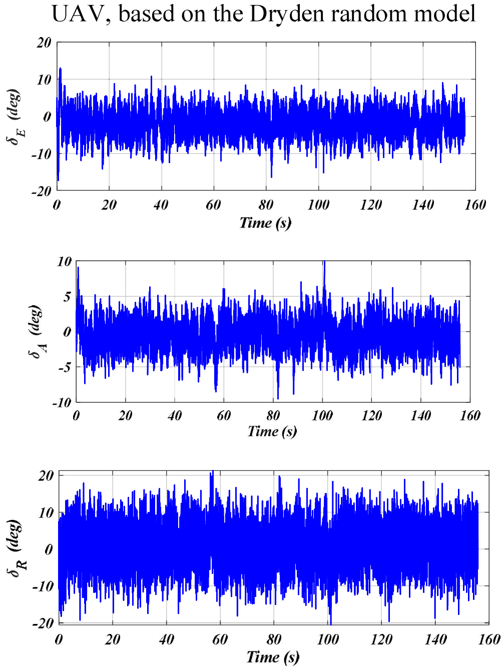

Figure 7 – The UAV control variables for tracking the Dubins path in the presence of atmospheric disturbance and noise measurement

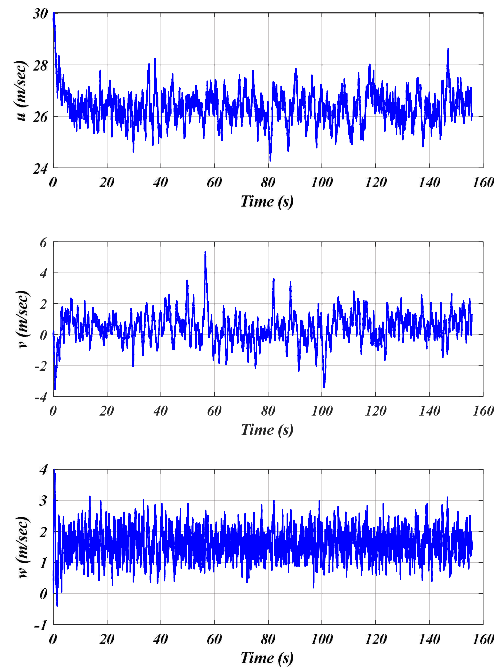

Figure 8 - Changes in speed components u, v, w while tracking the Dubins path in the presence of atmospheric disturbance and noise measurement

The results shown in Figure 7 show that in the absence of actuator dynamics, the frequency of changes in control angles is in the frequency range of disturbance factors on the UAV. Therefore, despite proving the ability of the designed controller, it should be noted that in real implementation conditions, it is necessary to consider the tolerance of the controller in disturbing conditions according to the dynamics of the aircraft and actuators and determine the permissible operating conditions of the UAV.

Conclusion

Based on the observed results, it can be said that the controller designed based on the MIMO eigenstructure assignment method can adequately provide the decoupling conditions between the modes. So that by choosing the right eigenstructures, performance requirements for the controlled system are provided. In general, based on the obtained results, it can be stated that by using the ability to determine the eigenstructure, the nature of the dynamic modes of the system can be evaluated and the effect of each dynamic mode on each of the variables of aircraft state and output can be determined. Also in these conditions, the effect of each input on each aircraft dynamic mode can be evaluated. Therefore, based on this information, the controller design capability is provided by correctly assigning each of the eigenvalues and eigenvectors related to each of the dynamic modes of the system. The performance of the designed controller based on the assigned eigenstructures, in the simulation of sample UAV, shows that this controller achieves the desired performance requirements in an optimal way. The results of nonlinear simulation of a six-degrees of freedom in the presence of uncertain factors including disturbance and measurement noise, and observation of the controller's success in executing commands in these conditions, is a confirmation of the optimal performance of the controller designed based on specified performance requirements.

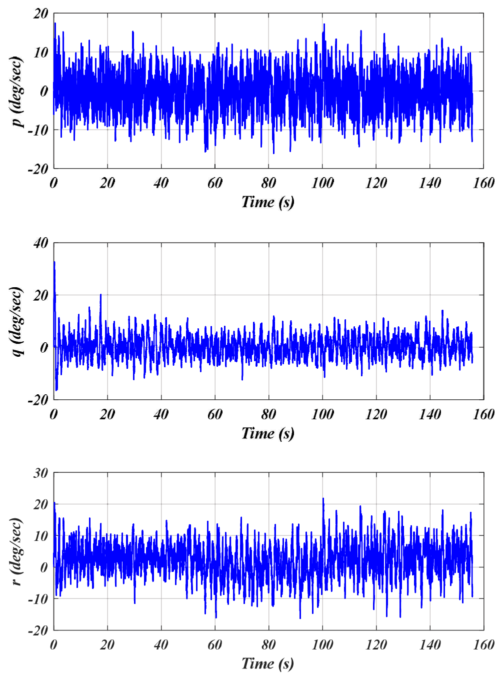

Figure 9 - Changes in angular speed components p, q, r while tracking the Dubins path in the presence of atmospheric disturbance and noise measurement

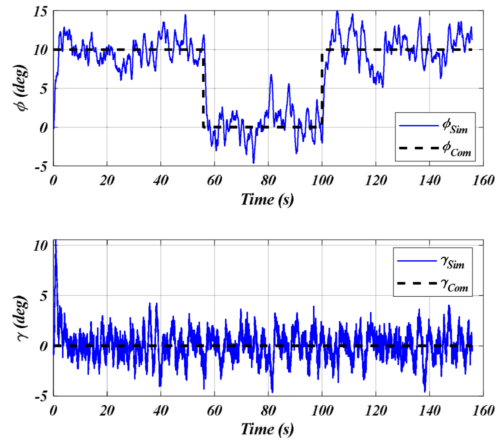

Figure 10 - Changes in the components of the roll angle and flight path of the aerial vehicle, while tracking the Dubins path in the presence of atmospheric disturbance and noise measurement

References

[1] L. Faleiro and R. Pratt, "Eigenstructure assignment applied to the design of an autopilot function for a civil aircraft," Flight control systems (Progress in Astronautics and Aeronautics., vol. 184, pp. 301-347, 2000.
[2] A. Andry, E. Shapiro, and J. Chung, "Eigenstructure assignment for linear systems," IEEE transactions on aerospace and electronic systems, no. 5, pp. 711-729, 1983.
[3] D. Gutiérrez and R. Salazar-Varas, "Using eigenstructure decompositions of time-varying autoregressions in common spatial patterns-based EEG signal classification," Biomedical signal processing and control, vol. 7, no. 6, pp. 622-631, 2012.
[4] M. Serra, F. Resta, and F. Ripamonti, "An active control logic based on modal approach for vibration reduction through the eigenstructure assignement," in 2013 IEEE International Conference on Mechatronics (ICM), 2013: IEEE, pp. 58-62.
[5] Y. Yuan and H. Liu, "An iterative updating method for damped structural systems using symmetric eigenstructure assignment," Journal of Computational and Applied Mathematics, vol. 256, pp. 268-277, 2014.
[6] C. Kazantzidou, T. Perez, and F. Valentinis, "Eigenstructure assignment for the position regulation of a fully-actuated marine craft," IFAC-PapersOnLine, vol. 50, no. 1, pp. 12398- 12403, 2017.
[7] K. M. Sobel and E. Y. Shapiro, "Application of eigenstructure assignment to flight control design: Some extensions," Journal of Guidance, Control, and Dynamics, vol. 10, no. 1, pp. 73-81, 1987.
[8] K. M. Sobel and F. J. Lallman, "Eigenstructure assignment for the control of highly augmented aircraft," Journal of Guidance, Control, and Dynamics, vol. 12, no. 3, pp. 318-324, 1989.
[9] O. Albostan and M. Gökaşan, "Mode decoupling robust eigenstructure assignment applied to the lateral-directional dynamics of the F-16 aircraft," Aerospace Science and Technology, vol. 77, pp. 677-687, 2018.
[10] G. ALBOSTAN and M. Gokasan, "High angle of attack manoeuvring control of f-16 aircraft based on nonlinear dynamic inversion and eigenstructure assignment," in 7th

European Conference for Aeronautics and Space Sciences (EUCASS), 2017.
[11] M. R. Mortazavi and A. Naghash, "Pitch and flight path controller design for F-16 aircraft using combination of LQR and EA techniques," Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, vol. 232, no. 10, pp. 1831-1843, 2018.
[12] A. R. Mehrabian and J. Roshanian, "Skid-to-turn missile autopilot design using scheduled eigenstructure assignment technique," Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, vol. 220, no. 3, pp. 225-239, 2006.
[13] A. Ashrafi, M. Mortazavi, A. gholami, “Linear control for duct fan aerial vehicle by Eigenstructure method,” Twenty-fifth Annual Conference of Mechanical Engineering, Tehran, Tarbiat Modares University, Association of Iranian Mechanical Engineers, ٢٠١٧.
[14] H. Noura and F. Bateman, "Control of an Unmanned Aerial Vehicle," in 7th International Symposium on Mechatronics and its Applications, 2010: IEEE, pp. 1-6.
[15] J. Roskam, Airplane flight dynamics and automatic flight controls. DARcorporation, 1995.
[16] A. Askari, M. Mortazavi, H. Talebi, and A. Motamedi, "A new approach in UAV path planning using Bezier–Dubins continuous curvature path," Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, vol. 230, no. 6, pp. 1103-1113, 2016.
[17] A. Motamedi, Mortazavi, j. Roshani yan, Babaii, “Continuous Curvature Trajectory Planning Based on Dubins Path and Bézier Curve for UAVs,” Scientific Journal of Aeronautical Engineering, vol. 16, no.1, pp. 49-64, 2014.
[18] H. Nobahari and A. Sharifi, "Multiple model extended continuous ant colony filter applied to real-time wind estimation in a fixed-wing UAV," Engineering Applications of Artificial Intelligence, vol. 92, p. 103629, 2020.
[19] R. W. Beard and T. W. McLain, Small unmanned aircraft. Princeton university press, 2012.
[20] U.S. Military Specification, MIL-F-8785C, 1980, http://www.mechanics. iei.liu.se/edu_ug/tmme50/8785c.pdf.