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Science Article

Model Reference Adaptive Control Using a New Adaptation Law Based on Power-Series Quadratic Lyapunov Functions for the Single Degree of Freedom Wing Rock Phenomenon

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In this research, new adaptation law for updating parameters of the model reference adaptive control and the model reference adaptive control with feedback integrators for a specific class of nonlinear systems with additive parametric uncertainty are presented. The innovation presented in this paper is the consideration of a new form for Lyapunov functions candidate to prove the stability of the closed-loop system. In general, Lyapunov functions candidate, which is used to prove stability and to derive rules for updating control parameters, include two sets of quadratic expressions. The first quadratic expression contains the trajectory tracking error and the second category includes the error of estimating the controller parameters. In this research, it is proved that by selecting quadratic expressions including the variable of trajectory tracking error in the form of power series, a new adaptation law is obtained that includes quadratic expressions in terms of the variable of tracking error in the form of power series. This type of adaptation law can be considered as an adaptation law derived from quadratic Lyapunov functions, except that the gain adaptation matrix parameters vary with time. It has been shown that by using an adaptive controller with a feedback integrator, the tracking error tends to zero faster and the flying object roll angle tracks the reference trajectory after a shorter time. In order to evaluate the control performance of the designed controllers, the system of one degree of freedom of the Wing Rock phenomenon has been used.

Keywords: *Model reference adaptive control, Model reference adaptive control with feedback integrator, Quadratic Lyapunov functions in the power series, New adaptation law, Wing Rock phenomenon.*

Introduction

The model reference adaptive controller or MRAC is one of the effective algorithms in controlling uncertain dynamical systems. Whitaker first introduced MRAC in the 1950s with the goal of controlling high-performance aircraft [1]. At the same time, several attempts were made to design adaptive systems for fighter aircraft autopilots, which led to the development of these algorithms in the 1960s. The state space model of dynamic systems and the Lyapunov stability criterion were introduced in this decade, which led to the introduction of several adaptive algorithms in the 1970s based on the Lyapunov stability method. The MRAC structure is such that the closed-loop

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system includes the system, reference model, state feedback controller, and adaptation laws to update the controller parameters. The derivation of adaptation laws and proving the stability of the closed-loop system is done by considering the Lyapunov quadratic function. The structure of most quadratic Lyapunov functions used in the design of adaptive controllers consists of two sets of quadratic expressions. The first category is a quadratic expression in terms of state-tracking error variables, while the second category contains quadratic expressions containing the error of estimating the controller parameters. Quadratic functions (in terms of parameters or variables) have a special place in control science due to having some properties. The quadratic functions are convex, which makes it easier to calculate derivatives, which has increased the popularity of this type of cost function in linear and nonlinear control science. The Lyapunov stability method has been extensively studied for designing adaptive systems, however, very little variation has been shown in the choice of these quadratic structures. In some studies, by selecting the quadratic expression, including the error of tracking in the logarithmic form, the rules of adaptation to the normalized form have been obtained [2-3]. This variation of choice in Lyapunov functions is further observed in the analysis of nonlinear systems, which are mentioned in some of these studies. A study on piecewise quadratic Lyapunov functions as a convex optimization problem with respect to linear matrix inequalities is presented in the reference [4]. Composite quadratic Lyapunov functions based on a set of quadratic functions are presented by [5] and also the stability analysis of nonlinear systems with polyhedral Lyapunov functions is reviewed by references [6-7]. In reference [8] with quadratic Lyapunov function, an adaptive controller is designed to examine the status of a spacecraft. What led to the writing of this article is the introduction of the new Lyapunov functions candidate in the form of the power series as well as the introduction of new adaptation rules derived from the proposed Lyapunov functions, and it is shown that numerous different adaptation rules can be obtained from the proposed Lyapunov function. It is also shown that by using an adaptive controller with a feedback integrator, the tracking error converges to zero faster and a faster transient mode performance is achieved than a conventional adaptive controller.

Design of model reference adaptive control with quadratic Lyapunov function - power series

In this part, based on Lyapunov stability method, the model reference adaptive control using Lyapunov function in the power series is designed. System (1) shows the state space of a certain class of nonlinear systems with additive parametric uncertainty [9]:

$$\dot{x}(t) = Ax(t) + B\Lambda\big(u(t) + g(x(t))\big) \tag{1}$$

In equation (1), x is the system state vector, u is the control signal, B is the control matrix and is known, and A and Λ are fixed matrices and are generally unknown. Λ is a diagonal matrix with strictly positive entries and it is assumed that both matrices of A and $B\Lambda$ are controllable. g(x(t)) is the additive uncertainty of the system, which is assumed to be written in the form of a linear combination of N Lipschitz functions:

$$g\big(x(t)\big) = \theta^T(t)\psi\big(x(t)\big) \tag{2}$$

In equation (2), θ is the matrix of unknown parameters and $\psi(x(t))$ is the regressor vector that its components are functions of x:

$$\psi\big(x(t)\big) = \Big(\psi_1\big(x(t)\big),\psi_2\big(x(t)\big),\psi_3\big(x(t)\big), \dots ,\psi_N\big(x(t)\big)\Big)^T \tag{3}$$

The reference model in the form of equation (4) is considered:

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t) \tag{4}$$

In (4) A_m is the Hurwitz matrix and r(t) is the input command bounded signal. According to the definition of tracking error are:

$$\tilde{x}(t) = x(t) - x_m(t) \tag{5}$$

The purpose of designing a model reference adaptive control is so that ensures that the tracking error tends to zero over time. In other words:

$$\lim_{t\to\infty} \|x(t) - x_m(t)\| = 0 \tag{6}$$

Without the presence of parametric uncertainties in the system, which indicates that the matrices A, θ, Λ are fixed and known, the control law with feedback and feedforward gain in the form (7) is considered:

$$u(t) = K_x^T x(t) + K_r^T r(t) - \theta^T \psi\big(x(t)\big) \tag{7}$$

By placing (7) in (1) the result is:

$$\dot{x}(t) = Ax(t) + B\Lambda[K_x^T x(t) + K_r^T r(t)] = \big(A + B\Lambda K_x^T\big)x(t) + B\Lambda K_r^T r(t) \tag{8}$$

Assumption: Considering state matrix A and control matrix B with full rank, matrices K_x and K_r are available such that the equation (9) holds:

$$\begin{cases} A + B\Lambda K_x^T = A_m \\ B\Lambda K_r^T = B_m \end{cases} \quad (9)$$

Now, considering the parametric uncertainty in the system, the input control signal u is considered in the form (10):

$$u(t) = \hat{K}_x^T(t)x(t) + \hat{K}_r^T(t)r(t) - \hat{\theta}^T(t)\psi\big(x(t)\big) \quad (10)$$

In (10), $\hat{K}_x(t)\hat{K}_r(t)$ and $\hat{\theta}(t)$ are estimates of the ideal gain rates k_x , k_r and θ and that must be updated on line. By placing (10) in (1) the result is:

$$\dot{x}(t) = \left(A + B\Lambda\hat{K}_x^T(t)\right)x(t) + B\Lambda(\hat{K}_r^T(t)r(t) - \left(\hat{\theta}(t) - \theta\right)^T\psi\big(x(t)\big) \quad (11)$$

Subtract equation (4) from (11) and dynamics of the tracking error becomes (12):

$$\dot{\tilde{x}} = \left(A + B\Lambda\hat{K}_x^T(t)\right)x(t) + B\Lambda(\hat{K}_r^T(t)r(t) - \left(\hat{\theta}(t) - \theta\right)^T\psi\big(x(t)\big) - A_m x_m(t) - B_m r(t) \quad (12)$$

Using (9) tracking error dynamics is converted to form (13):

$$\dot{\tilde{x}}(t) = A_m\tilde{x}(t) + B\Lambda[\left(\hat{K}_x(t) - K_x\right)^T x(t) + \left(\hat{K}_r(t) - K_r\right)^T r(t) - \left(\hat{\theta}(t) - \theta\right)^T\psi\big(x(t)\big)] \quad (13)$$

By defining parameter estimation matrices to relation form:

$$\begin{cases} \tilde{\theta}(t) = \hat{\theta}(t) - \theta \\ \tilde{K}_x(t) = \hat{K}_x(t) - K_x \\ \tilde{K}_r(t) = \hat{K}_r(t) - K_r \end{cases} \quad (14)$$

By placing relation (14) in (13), the dynamics of the tracking error in the form (15) is obtained:

$$\dot{\tilde{x}}(t) = A_m\tilde{x}(t) + B\Lambda[\tilde{K}_x^T(t)x(t) + \tilde{K}_r^T(t)r(t) - \tilde{\theta}^T(t)\psi\big(x(t)\big)] \quad (15)$$

Lyapunov's function is now considered a candidate in the quadratic form - power series (16):

$$V\big(\tilde{x}(t), \tilde{K}_x(t), \tilde{K}_r(t), \tilde{\theta}(t)\big) = \sum_{i=1}^{n}\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^i + \mathrm{tr}\big([\tilde{K}_x^T(t)\Gamma_x^{-1}\tilde{K}_x(t) + \tilde{K}_r^T(t)\Gamma_r^{-1}\tilde{K}_r(t) + \tilde{\theta}^T(t)\Gamma_\theta^{-1}\tilde{\theta}(t)]\Lambda\big) \quad . \quad \alpha_i \in R^+ \quad (16)$$

In relation (16), $\Gamma_x = \Gamma_x^T > 0, \Gamma_r = \Gamma_r^T > 0, \Gamma_\theta = \Gamma_\theta^T > 0$ are adaptation gain and $P = P^T > 0$, the Lyapunov algebraic equation answer is:

$$A_m P + P{A_m}^T = -Q \quad (17)$$

The time derivatives of the Lyapunov function candidate are:

$$\dot{V} = \left(\dot{\tilde{x}}^T(t)P\tilde{x}(t) + \tilde{x}^T(t)P\dot{\tilde{x}}(t)\right)\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} + 2\mathrm{tr}(\left[\tilde{K}_x^T(t)\Gamma_x^{-1}\dot{\tilde{K}}_x(t) + \tilde{K}_r^T(t)\Gamma_r^{-1}\dot{\tilde{K}}_r(t) + \tilde{\theta}^T(t)\Gamma_\theta^{-1}\dot{\tilde{\theta}}(t)\right]\Lambda) \quad (18)$$

The time derivative of the above Lyapunov function is evaluated along the dynamics of the tracking error (15):

$$\dot{V} = -\left(\tilde{x}^T(t)Q\tilde{x}(t)\right)\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} + 2\tilde{x}^T(t)PB\Lambda\tilde{K}_x^T(t)x(t)\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} + 2\tilde{x}^T(t)PB\Lambda\tilde{K}_r^T(t)r\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} - 2\tilde{x}^T(t)PB\Lambda\tilde{\theta}^T(t)\psi(x(t))\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} + 2\mathrm{tr}(\left[\tilde{K}_x^T(t)\Gamma_x^{-1}\dot{\tilde{K}}_x(t) + \tilde{K}_r^T(t)\Gamma_r^{-1}\dot{\tilde{K}}_r(t) + \tilde{\theta}^T(t)\Gamma_\theta^{-1}\dot{\tilde{\theta}}(t)\right]\Lambda) \quad (19)$$

Using the identity that exists between the effect of a matrix and the constituent vectors of that matrix, the relation (20) is used:

$$\begin{aligned} &\tilde{x}^T(t)PB\Lambda\tilde{K}_x^T(t)x(t)\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} = \mathrm{tr}(\tilde{K}_x^T(t)x(t)\tilde{x}^T(t)PB\Lambda\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1}) \\ &\tilde{x}^T(t)PB\Lambda\tilde{K}_r^T(t)r(t)\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} = \mathrm{tr}(\tilde{K}_r^T(t)r(t)\tilde{x}^T(t)PB\Lambda\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1}) \\ &\tilde{x}^T(t)PB\Lambda\tilde{\theta}^T(t)\psi\big(x(t)\big)\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} = \mathrm{tr}\big(\tilde{\theta}^T(t)\psi\big(x(t)\big)\tilde{x}^T(t)PB\Lambda\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1}\big) \end{aligned} \quad (20)$$

Using (20), relation (19) turns to the form of relation (21):

$$\dot{V} = -\tilde{x}^T(t)Q\tilde{x}(t)\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} + 2\mathrm{tr}\left(\tilde{K}_x^T(t)x(t)\tilde{x}^T(t)PB\Lambda\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} + \tilde{K}_x^T(t)\Gamma_x^{-1}\dot{\tilde{K}}_x(t)\Lambda\right) + 2\mathrm{tr}\left(\tilde{K}_r^T(t)r(t)\tilde{x}^T(t)PB\Lambda\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} + \tilde{K}_r^T(t)\Gamma_r^{-1}\dot{\tilde{K}}_r(t)\Lambda\right) + 2\mathrm{tr}\left(\tilde{\theta}^T(t)\psi\big(x(t)\big)\tilde{x}^T(t)PB\Lambda\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} - \tilde{\theta}^T(t)\Gamma_\theta^{-1}\dot{\tilde{\theta}}(t)\Lambda\right) \quad (21)$$

Set the expressions inside the effect matrix of relation (21) to zero:

$$\tilde{K}_x^T(t)\left(x(t)\tilde{x}^T(t)PB\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} + \Gamma_x^{-1}\dot{\tilde{K}}_x(t)\right)\Lambda = 0$$
$$\tilde{K}_r^T(t)\left(r(t)\tilde{x}^T(t)PB\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} + \Gamma_r^{-1}\dot{\tilde{K}}_r(t)\right)\Lambda = 0$$
$$\tilde{\theta}^T(t)\left(\psi\big(x(t)\big)\tilde{x}^T(t)PB\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} + \Gamma_\theta^{-1}\dot{\tilde{\theta}}(t)\right)\Lambda = 0 \quad (22)$$

Finally, the rules of adaptation to form (23) are obtained:

$$\dot{\tilde{K}}_x(t) = \dot{\hat{K}}_x(t) - \dot{K}_x = \dot{\hat{K}}_x(t) = -\Gamma_x x(t)\tilde{x}^T(t)PB\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1}$$
$$\dot{\tilde{K}}_r(t) = \dot{\hat{K}}_r(t) - \dot{K}_r = \dot{\hat{K}}_r(t) = -\Gamma_r r(t)\tilde{x}^T(t)PB\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1}$$
$$\dot{\tilde{\theta}}(t) = \dot{\hat{\theta}}(t) - \dot{\theta} = \dot{\hat{\theta}}(t) = \Gamma_\theta\psi\big(x(t)\big)\tilde{x}^T(t)PB\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} \quad (23)$$

The time derivative of the Lyapunov function is simplified as follows:

$$\dot{V} = -\tilde{x}^T(t)Q\tilde{x}(t)\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} \le 0 \quad (24)$$

Therefore, it can be concluded that $\tilde{x}(t), \tilde{K}_x(t), \tilde{K}_r(t), \tilde{\theta}(t)$ are uniformly bounded. Since the signal r(t) is bounded and the matrix A_m is stable, it can be concluded that $x_m(t)$ is a bounded signal, so the system state vector x is also bounded $x(t) = \tilde{x}(t) + x_m(t)$. We also know that θ is a fixed vector and $\hat{\theta}(t) = \tilde{\theta}(t) + \theta$ is therefore uniformly bounded. As a result, ψ(x(t)) is also bounded and from the dynamics of the system (1), it can be concluded that $\dot{x}(t)$ is also bounded. Now we calculate the second derivative of the Lyapunov function:

$$\ddot{V} = -\big[\tilde{x}^T(t)\big(A_m^TQ + QA_m\big)\tilde{x}(t) + 2\tilde{x}^T(t)PB\Lambda\tilde{K}_x^T(t)x(t) + 2\tilde{x}^T(t)PB\Lambda\tilde{K}_r^T(t)r(t) - 2\tilde{x}^T(t)PB\Lambda\tilde{\theta}^T(t)\psi\big(x(t)\big)\big]\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} - \tilde{x}^T(t)Q\tilde{x}(t)[\tilde{x}^T(t)\big(A_m^TP + PA_m\big)\tilde{x}(t) + 2\tilde{x}^T(t)PB\Lambda\tilde{K}_x^T(t)x(t) + 2\tilde{x}^T(t)PB\Lambda\tilde{K}_r^T(t)r(t) - 2\tilde{x}^T(t)PB\Lambda\tilde{\theta}^T(t)\psi(x(t))]\sum_{i=1}^{n} i(i-1)\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-2} \quad (25)$$

Equation (25) is bounded because all its variables and parameters are bounded, so the second derivative of Lyapunov's function is also bounded:

$$\ddot{V} < \infty \quad (26)$$

Therefore, it can be concluded that $\dot{V}$ is continuously uniform, and since V is bounded from below and $\dot{V} \le 0$, therefore, Barbalat's lemma [10] can be used and it concludes:

$$\lim_{t\to\infty}\dot{V} = 0 \quad (27)$$

Since Lyapunov function candidate is radially unbounded, it can be concluded that the error generally and radially and asymptotically converges to zero:

$$\lim_{t\to\infty}\|x(t) - x_m(t)\| = 0 \quad (28)$$

Note: System (1) is also controlled by the adaptation law (29) by considering the control law (10) [9]:

$$\dot{\hat{K}}_x(t) = -\Gamma_x x(t)\tilde{x}^T(t)PB$$
$$\dot{\hat{K}}_r(t) = -\Gamma_r r(t)\tilde{x}^T(t)PB$$
$$\dot{\hat{\theta}}(t) = \Gamma_\theta\psi\big(x(t)\big)\tilde{x}^T(t)PB \quad (29)$$

Table 1 presents some examples of the adaptation laws derived from the Lyapunov function in the power series.

Table 1– Some examples of the adaptation laws corresponding to Lyapunov functions in quadratic - power series form

Lyapunov function candidate	Corresponding adaptation laws
$V = \tilde{x}^T(t)P\tilde{x}(t) + tr([\tilde{K}_x^T(t)\Gamma_x^{-1}\tilde{K}_x(t) + \tilde{K}_r^T(t)\Gamma_r^{-1}\tilde{K}_r(t) + \tilde{\theta}^T(t)\Gamma_\theta^{-1}\tilde{\theta}(t)]\Lambda)$	$\dot{\hat{\mathbf{K}}}_x(t) = -\mathbf{\Gamma}_x\mathbf{x}(t)\tilde{\mathbf{x}}^T(t)\mathbf{PB}$ $\dot{\hat{\mathbf{K}}}_r(t) = -\mathbf{\Gamma}_r\mathbf{r}(t)\tilde{\mathbf{x}}^T(t)\mathbf{PB}$ $\dot{\hat{\boldsymbol{\theta}}}(t) = \mathbf{\Gamma}_\theta\boldsymbol{\psi}\big(\mathbf{x}(t)\big)\tilde{\mathbf{x}}^T(t)\mathbf{PB}$
$V = \tilde{x}^T(t)P\tilde{x}(t) + \big(\tilde{x}^T(t)P\tilde{x}(t)\big)^2 + tr([\tilde{K}_x^T(t)\Gamma_x^{-1}\tilde{K}_x(t) + \tilde{K}_r^T(t)\Gamma_r^{-1}\tilde{K}_r(t) + \tilde{\theta}^T(t)\Gamma_\theta^{-1}\tilde{\theta}(t)]\Lambda)$	$\dot{\hat{\mathbf{K}}}_x(t) = -\mathbf{\Gamma}_x\mathbf{x}(t)\tilde{\mathbf{x}}^T(t)\mathbf{PB}(\mathbf{1} + \mathbf{2}\tilde{\mathbf{x}}^T(t)\mathbf{P}\tilde{\mathbf{x}}(t))$ $\dot{\hat{\mathbf{K}}}_r(t) = -\mathbf{\Gamma}_r\mathbf{r}(t)\tilde{\mathbf{x}}^T(t)\mathbf{PB}\big(\mathbf{1} + \mathbf{2}\tilde{\mathbf{x}}^T(t)\mathbf{P}\tilde{\mathbf{x}}(t)\big)$ $\dot{\hat{\boldsymbol{\theta}}}(t) = \mathbf{\Gamma}_\theta\boldsymbol{\psi}\big(\mathbf{x}(t)\big)\tilde{\mathbf{x}}^T(t)\mathbf{PB}(\mathbf{1} + \mathbf{2}\tilde{\mathbf{x}}^T(t)\mathbf{P}\tilde{\mathbf{x}}(t))$
$V = \tilde{x}^T(t)P\tilde{x}(t) + (\tilde{x}^T(t)P\tilde{x}(t))^2 + (\tilde{x}^T(t)P\tilde{x}(t))^3 + tr([\tilde{K}_x^T(t)\Gamma_x^{-1}\tilde{K}_x(t) + \tilde{K}_r^T(t)\Gamma_r^{-1}\tilde{K}_r(t) + \tilde{\theta}^T(t)\Gamma_\theta^{-1}\tilde{\theta}(t)]\Lambda)$	$\dot{\hat{\mathbf{K}}}_x(t) = -\mathbf{\Gamma}_x\mathbf{x}(t)\tilde{\mathbf{x}}^T(t)\mathbf{PB}\Big(\mathbf{1} + \mathbf{2}\tilde{\mathbf{x}}^T(t)\mathbf{P}\tilde{\mathbf{x}}(t) + \mathbf{3}\big(\tilde{\mathbf{x}}^T(t)\mathbf{P}\tilde{\mathbf{x}}(t)\big)^2\Big)$ $\dot{\hat{\mathbf{K}}}_r(t) = -\mathbf{\Gamma}_r\mathbf{r}(t)\tilde{\mathbf{x}}^T(t)\mathbf{PB}(\mathbf{1} + \mathbf{2}\tilde{\mathbf{x}}^T(t)\mathbf{P}\tilde{\mathbf{x}}(t) + \mathbf{3}(\tilde{\mathbf{x}}^T(t)\mathbf{P}\tilde{\mathbf{x}}(t))^2)$ $\dot{\hat{\boldsymbol{\theta}}}(t) = \mathbf{\Gamma}_\theta\boldsymbol{\psi}\big(\mathbf{x}(t)\big)\tilde{\mathbf{x}}^T(t)\mathbf{PB}(\mathbf{1} + \mathbf{2}\tilde{\mathbf{x}}^T(t)\mathbf{P}\tilde{\mathbf{x}}(t) + \mathbf{3}(\tilde{\mathbf{x}}^T(t)\mathbf{P}\tilde{\mathbf{x}}(t))^2)$

Design of model reference adaptive control with feedback integrator with Lyapunov function in the power series

In this part, the purpose of designing a model reference adaptive control is to track the input signal of the reference model with less tracking error than simple MRAC controller. In this section, a class of unknown nonlinear systems in form (30) is considered [9]:

$$\dot{x}_p(t) = A_p x_p(t) + B_p\Lambda\left(u(t) + \theta^T(t)\psi\left(x_p(t)\right)\right) \quad (30)$$

In equation (30) $x_p(t) \in R^{n_p}$ is system state vector, $u(t) \in R^m$ is system control signal, $B_p \in R^{n\times m}$ is system control matrix and is considered as known, and $A_p \in R^{n\times n}$ and $\Lambda \in R^{m\times m}$ are considered to be unknown fixed matrices. Λ is a diagonal matrix with strictly positive elements and the matrices (A_p, $B_p\Lambda$) are assumed to be controllable, $\theta(t) \in R^{N\times m}$ is a matrix of unknown parameters and $\psi(x_p(t)) \in R^N$ is a regressor vector and known that its entries are functions of x_p. The purpose is to design u in such a way that the output of the system (31) follows the command signal $r(t) \in R^m$.

$$y(t) = C_p x_p(t) \in R^m \quad (32)$$

In equation (31) the C_p matrix is known and constant. By definition:

$$\tilde{x}_y(t) = y(t) - r(t) \quad (33)$$

In equation (32) $\tilde{x}_y(t)$ is the system output tracking error. Integration of (32) results in:
By synergizing equations (30) and (33), the developed open-loop dynamics are obtained:

$$\dot{x}(t) = Ax(t) + B\Lambda\left(u(t) + \theta^T(t)\psi\left(x_p(t)\right)\right) + B_m r(t)$$
$$y(t) = [0_{m\times m} C_p]x(t) = Cx(t) \quad (34)$$

In equation (34) $x(t) = (\tilde{x}_{yI}^T(t) \;\; x_p^T(t))^T \in R^n$ is the system vector of the developed system and $n = n_p + m$. The developed system matrices are:

$$A = \begin{bmatrix} 0_{m\times m} & C_p \\ 0_{n_p\times m} & A_p \end{bmatrix} \quad . \; B = \begin{bmatrix} 0_{m\times m} \\ B_p \end{bmatrix}$$

$$B_m = \begin{bmatrix} -I_{m\times m} \\ 0_{n_p\times m} \end{bmatrix} \quad . \; C = [0_{m\times m} C_p] \quad (35)$$

The purpose is to design a state feedback controller for the developed dynamic system (36).

$$\dot{x}(t) = Ax(t) + B\Lambda\left(u(t) + \theta^T(t)\psi\left(x_p(t)\right)\right) + B_m r(t)$$
$$y(t) = Cx(t) \quad (36)$$

Assumption: There exists a fixed matrix $K_x \in R^{n\times m}$ such that:

$$A_m = A + B\Lambda K_x^T \quad (37)$$

Using Equation (37), Equation (36) turns to (38):

$$\dot{x}(t) = A_m x(t) + B\Lambda(u(t) - K_x^T x(t) + \theta^T(t)\psi\left(x_p(t)\right) + B_m r(t) \quad (38)$$

The control law in the form (39) is considered:

$$u(t) = \hat{K}_x^T(t)x(t) - \hat{\theta}^T(t)\psi\left(x_p(t)\right) \quad (39)$$

In equation (39), $\hat{K}_x$ and $\hat{\theta}$ matrices are estimates of feedback gains and regressor vector parameters. placing (39) in (38) results in:

$$\dot{x} = A_m x + B\Lambda\left(\tilde{K}_x^T x + \tilde{\theta}^T\psi(x_p)\right) + B_m r(t) \quad (40)$$

The model reference is considered to be form (41):

$$\dot{x}_m = A_m x_m + B_m r(t)$$
$$y_m = Cx_m \quad (41)$$

According to the definition of state tracking error:

$$e(t) = x(t) - x_{ref}(t) \quad (42)$$

Subtract the equation (41) from (40) yields:

$$\dot{e}(t) = A_m e(t) + B\Lambda\left(\tilde{K}_x^T(t)x(t) - \tilde{\theta}^T(t)\psi\left(x_p(t)\right)\right) \quad (43)$$

Lyapunov function is now considered to be the quadratic- power series form:

$$V\left(\tilde{x}(t).\tilde{K}_x(t).\tilde{\theta}(t)\right) = \sum_{i=1}^{n} \alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^i + tr\left(\tilde{K}_x^T(t)\Gamma_x^{-1}\tilde{K}_x(t)\Lambda\right) + tr\left(\tilde{\theta}^T(t)\Gamma_\theta^{-1}\tilde{\theta}(t)\Lambda\right).\alpha_i \in R^+ \quad (44)$$

In equation (44) $\Gamma_\theta = \Gamma_\theta^T > 0$, $\Gamma_x = \Gamma_x^T > 0$ is the adaptation gain and $P = P^T > 0$ is the unique answer of algebraic Lyapunov equation with $Q = Q^T > 0$:

$$PA_m + A_m^T P = -Q \quad (45)$$

Evaluate the time derivative of the Lyapunov function along the dynamics of the tracking error:

$$\dot{V}\left(\tilde{x}(t).\tilde{K}_x(t).\tilde{\theta}(t)\right) = \Big[-\tilde{x}^T(t)Q\tilde{x}(t) + 2\tilde{x}^T(t)PB\Lambda\left(\tilde{K}_x^T(t)x(t) -\right.$$

$$\tilde{\theta}^T(t)\psi\big(x_p(t)\big)\Big)\Big]\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} + 2\mathrm{tr}\left(\tilde{K}_x^T(t)\Gamma_x^{-1}\dot{\hat{K}}_x(t)\Lambda\right) + 2\mathrm{tr}\left(\tilde{\theta}^T(t)\Gamma_\theta^{-1}\dot{\hat{\theta}}(t)\Lambda\right) \quad (46)$$

Use the identity between the vectors and the matrix effect provided in Form (47):

$$u^T v = \mathrm{tr}(v u^T) \quad (47)$$

Equation (46) turns to (48):

$$\dot{V}\big(\tilde{x}(t), \tilde{K}_x(t), \tilde{\theta}(t)\big) = -\tilde{x}^T(t)Q\tilde{x}(t)\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} + 2\mathrm{tr}\left(\tilde{K}_x^T(t)\{\Gamma_x^{-1}\dot{\hat{K}}_x(t) - x(t)\tilde{x}^T(t)PB\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1}\}\Lambda\right) + 2\mathrm{tr}\left(\tilde{\theta}^T(t)\{\Gamma_\theta^{-1}\dot{\hat{\theta}}(t) - \psi\left(x_p(t)\right)\tilde{x}^T(t)PB\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1}\}\Lambda\right) \quad (48)$$

By selecting the adaptation laws in the form of (49):

$$\dot{\hat{K}}_x(t) = -\Gamma_x x(t)\tilde{x}^T(t)PB\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1}$$
$$\dot{\hat{\theta}}(t) = \Gamma_\theta\psi\left(x_p(t)\right)\tilde{x}^T(t)PB\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} \quad (49)$$

The first time derivative of the Lyapunov function candidate turns to form (50):

$$\dot{V}\big(\tilde{x}(t), \tilde{K}_x(t), \tilde{\theta}(t)\big) = -\tilde{x}^T(t)Q\tilde{x}(t)\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} \le 0 \quad (50)$$

Equation (50) guarantees uniform boundedness of $(\tilde{x}(t), \tilde{K}_x(t), \tilde{\theta}(t))$:

$$V(\tilde{x}(t), \tilde{K}_x(t), \tilde{\theta}(t)) \le V(\tilde{x}(0), \tilde{K}_x(0), \tilde{\theta}(0)) \quad (51)$$

Use equation (44):

$$\tilde{x}^T(t)P\tilde{x}(t) = \frac{1}{\alpha_1}V\left(\tilde{x}(t), \tilde{K}_x(t), \tilde{\theta}(t)\right) - \sum_{i=1}^{n}\frac{\alpha_i}{\alpha_1}(\tilde{x}^T(t)P\tilde{x}(t))^{i} - \frac{1}{\alpha_1}\mathrm{tr}\big(\tilde{K}_x^T(t)\Gamma_x^{-1}\tilde{K}_x(t)\Lambda\big) - \frac{1}{\alpha_1}\mathrm{tr}\big(\tilde{\theta}^T(t)\Gamma_\theta^{-1}\tilde{\theta}(t)\Lambda\big) \quad (52)$$

From equation (52) it can be concluded:

$$\tilde{x}^T(t)P\tilde{x}(t) \le \frac{1}{\alpha_1}V\left(\tilde{x}(t), \tilde{K}_x(t), \tilde{\theta}(t)\right) \quad (53)$$

Inequality of eigenvalues of quadratic expressions used:

$$\lambda_{min}(P)\|\tilde{x}(t)\|^2 \le \tilde{x}^T(t)P\tilde{x}(t) \le \lambda_{max}(P)\|\tilde{x}(t)\|^2 \quad (54)$$

Using (53) and (54), inequality (55) results in:

$$\lambda_{min}(P)\|\tilde{x}(t)\|^2 \le \frac{1}{\alpha_1}V\left(\tilde{x}(t), \tilde{K}_x(t), \tilde{\theta}(t)\right) \le \frac{1}{\alpha_1}V\left(\tilde{x}(0), \tilde{K}_x(0), \tilde{\theta}(0)\right) \quad (55)$$

Inequality (55) results in:

$$\|\tilde{x}(t)\| \le \sqrt{\frac{V\big(\tilde{x}(0), \tilde{K}_x(0), \tilde{\theta}(0)\big)}{\alpha_1\lambda_{min}(P)}} \quad (56)$$

Inequality (56) confirms that the tracking error norm is bounded:

$$\|\tilde{x}(t)\| \in L_\infty \quad (57)$$

From the time derivative of Lyapunov function it can be concluded that:

$$-\int_0^t \dot{V}\left(\tilde{x}(t), \tilde{K}_x(t), \tilde{\theta}(t)\right)dt = V\left(\tilde{x}(0), \tilde{K}_x(0), \tilde{\theta}(0)\right) - V(\tilde{x}(t), \tilde{K}_x(t), \tilde{\theta}(t)) \quad (58)$$

From (50), (54) and (58) result in:

$$\alpha_1\lambda_{min}(Q)\int_0^t\|\tilde{x}(t)\|^2\,dt \le \int_0^t \tilde{x}^T(t)Q\tilde{x}(t)dt \le \int_0^t \tilde{x}^T(t)Q\tilde{x}(t)\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1}\,dt \le V\left(\tilde{x}(0), \tilde{K}_x(0), \tilde{\theta}(0)\right) \quad (59)$$

Considering the first and last expression in inequality (59), inequality (60) results:

$$\int_0^t\|\tilde{x}(t)\|^2\,dt \le \frac{V\big(\tilde{x}(0), \tilde{K}_x(0), \tilde{\theta}(0)\big)}{\alpha_1\lambda_{min}(Q)} \quad (60)$$

Relation (76) confirms that:

$$\|\tilde{x}(t)\| \in L_2 \quad (61)$$

Using (57) and (61), relation (62) concludes:

$$\|\tilde{x}(t)\| \in L_2 \cap L_\infty \quad (62)$$

Since V(t) is bounded and ($r(t) \in L_\infty$), it can be concluded from (41) that $x_m(t)$ is bounded and subsequently it results from the definition of the tracking error:

$$\tilde{x}(t) = x(t) - x_m(t) \quad (63)$$

It is clear that $x \in L_\infty$. The ideal feedback gain and regressor parameters are unknown and fixed, and from the definition of parameter estimation error, we have :

$$\tilde{K}_x(t) = K_x - \hat{K}_x(t)$$
$$\tilde{\theta}(t) = \theta - \hat{\theta}(t) \quad (64)$$

Therefore, the estimated parameters are bounded or $\hat{K}_x(t), \hat{\theta}(t) \in L_\infty$. Since $x(t) \in L_\infty$ subsequently is $x_p(t) \in L_\infty$, so the regressor vector functions are bounded. From (39) it can be concluded that $u(t) \in L_\infty$ and also from (56) it can be concluded that $\dot{x}(t) \in L_\infty$. The second time derivative of the Lyapunov function candidate becomes

$$\ddot{V}\left(\tilde{x}(t),\tilde{K}_x(t),\tilde{\theta}(t)\right) = -\left(\dot{\tilde{x}}^T(t)Q\tilde{x}(t) + \tilde{x}^T(t)Q\dot{\tilde{x}}(t)\right)\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} - \tilde{x}^T(t)Q\tilde{x}(t)\left(\dot{\tilde{x}}^T(t)P\tilde{x}(t) + \tilde{x}^T(t)P\dot{\tilde{x}}(t)\right)\sum_{i=1}^{n} i(i-1)\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-2} \quad (65)$$

Since $\dot{x}(t) \in L_\infty$ denotes that $\ddot{V} \in L_\infty$, so $\dot{V}$ is a uniform function of time. Lyapunov's function is bounded from the bellow, and $\dot{V} \le 0$ and $\dot{V}$ are continuously uniform, and from Barbalat's lemma it can be concluded that V tends to a fixed limit and the Lyapunov derivative tends to zero.

$$\lim_{t\to\infty} -\tilde{x}^T(t)Q\tilde{x}(t)\sum_{i=1}^{n} i\alpha_i(\tilde{x}^T(t)P\tilde{x}(t))^{i-1} = 0 \quad (66)$$

Or

$$\lim_{t\to\infty} \tilde{x}^T(t)Q\tilde{x}(t) = 0 \quad (67)$$

Which confirms that:

$$\lim_{t\to\infty} \|\tilde{x}(t)\| = 0 \quad \text{or} \quad x(t) \to x_m(t) \quad (68)$$

Since the Lyapunov function candidate is radially unbounded, it is concluded that the closed-loop system is asymptotically and globally stable. so, it was proved that x asymptotically follows $x_m(t)$. Therefore:

$$y(t) = Cx(t) \quad (69)$$

It tracks the output of the model reference asymptotically and globally:

$$y_m(t) = Cx_m(t) \quad (70)$$

From (41) relation we know that $y_m(t)$ tracks the bounded command signal r(t). Therefore y(t) follows r(t).

Wing Rock phenomenon and its governing dynamic equations

Most fighter aircraft perform high-angle attack maneuvers in order to achieve superiority in aerial combat. Flying with a high angle of attack enters the aircraft in a non-linear area, which may be outside the flight envelope designed for the flying object, and this creates dangerous phenomena for the aircraft to fly. Examples of these nonlinear phenomena are jump response, yaw motion deviation, pitch angle oscillations, and wing rock.

Wing rock phenomenon that occurs at high attack angles includes lateral oscillations, the most important characteristic of these oscillations is the oscillation around the longitudinal axis of the flying object with a fixed amplitude and frequency. In the Wing Rock phenomenon, periodic changes in aerodynamic coefficients are the cause of limit cycle oscillations, and this is a clear example of parametric uncertainty in the system. The flow separation from the wings occurs at a high angle of attack. In this case, the adverse pressure gradient continues along the flow, and as a result, the velocity gradient on the surface turns zero. Due to the flow separation as the aircraft moves forward, the compressive drag force increases and the flow separated from the wing surface produces vortices. Due to the asymmetry of the vortices produced in the two wings, nonlinear periodic changes in aerodynamic coefficients occur, which causes adverse oscillations in the roll angle of the aircraft.

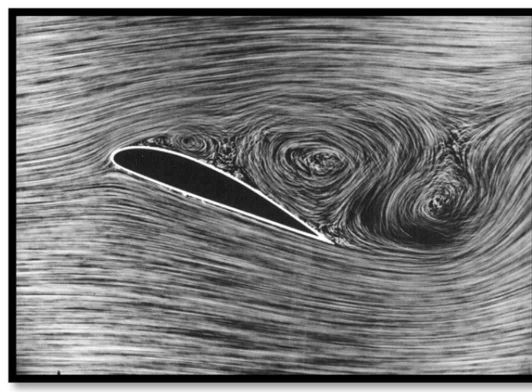

Figure 1 - Fluid flow isolated from the wing surface at a high angle of attack

Delta wing fighters with high sweep back angles are mainly faced with this phenomenon. Experiments have shown that vortices produced at the leading edge of the wing are the most important cause of this phenomenon. Although leading edge vortices are required to generate high lift force in order to create high attack angles, by combining these vortices and wings with low aspect ratio and nonlinear changes in the derivative, the destruction of the roll angle and the effects of the angle of attack and the angle of the lateral glide angle are the main reasons for this phenomenon. Extensive research has been done on this nonlinear phenomenon, some of which are presented in references [11-20].
Various control approaches have been proposed to control this nonlinear phenomenon. In [21], a

forty-nine rule based fuzzy logic controller is presented to control this phenomenon, and the consistency and performance of the transient mode of this controller are analyzed. Two types of nonlinear controllers based on linear feedback and sliding-mode were suggested in [22]. In [23] an adaptive fuzzy controller is designed to control the effect of wing rock in the presence of uncertainty and unknown disturbances. The unknown nonlinear function is detected by a fuzzy approximator and the adaptation laws are derived by Lyapunov stability method. In [24], a neural adaptive controller with trajectory tracking technique L_2 was presented and in [25], with the help of a fuzzy slider-mode controller, it was shown that the system state variables reach the desired state model without overshoot. In the reference [26] with a combination of direct adaptive control methods and uncertainty observer, two robust trajectory tracking controllers are presented, which the researchers concluded that uncertainty observers are possible for real-time applications. In the reference [27] with a combined adaptive controller, the Wing phenomenon was controlled by considering the time delay. It was also shown that by combining CAPC and LQR, a better transient mode performance is obtained than LQR. And in the references [28-32] with the approach of adaptive and robust adaptive controller, the control of this nonlinear phenomenon of the limit cycle has been studied.

In this study, with the aim of evaluating the controllers designed according to the dynamics of this phenomenon with the help of adaptive control algorithms of the reference model with accumulative parametric uncertainty and model reference adaptive control with accumulative uncertainty and feedback integrator, this nonlinear phenomenon is controlled. Both of the above algorithms are direct algorithms that are designed by Lyapunov method.

Although the Wing Rock phenomenon occurs in a space of six-degrees of freedom, the obvious features of this phenomenon, which are in fact the oscillations of the roll angle around the longitudinal axis of the flying object , can also be examined as one degree of freedom. Wind tunnel experiments produce limit cycle oscillations as one degree of freedom to the extent that in this experiment a flying object is on a device in a free rotation around the longitudinal axis. Fitted mathematical models from wind tunnel experiments presented second-order models, which in this study the model presented by the paper [33] is used.

$$\ddot{\phi}(t) = c_1 C_L - c_2 \dot{\phi}(t) \tag{71}$$

In equation (71) ϕ is the roll angle and also:

$$c_1 = \frac{\rho S c L_c^2}{2 I_{xx}} \tag{72}$$

$$c_2 = \frac{\mu_x L_c}{I_{xx} U_c} \tag{73}$$

In equation (72), ρ is air density, S is area of wing, C wing chord, L_c is characteristic length, I_{xx} is moment of inertia of the wing, μ_x is rotational axis damping coefficient and U_c is characteristic speed. The values of these parameters are specified in Table 2. By selecting these numbers, the values c = 0.355 and c = 0.001 are obtained. The moment coefficient of the roll angle is considered in form (74) [33] :

$$C_L\left(\phi(t),\dot{\phi}(t)\right) = a_1\phi(t) + a_2\dot{\phi}(t) + a_3\phi^3(t) + a_4\phi^2(t)\dot{\phi}(t) + a_5\phi(t)\dot{\phi}^2(t) \tag{74}$$

In equation (74) both variables ϕ and $\dot{\phi}$ are assumed to be measurable. The values of a_i, which depend on the angle of attack, are shown in Table 3. Placing (74) in (71) results in:

$$\ddot{\phi}(t) = c_1 a_1 \phi(t) + (c_1 a_2 - c_2)\dot{\phi}(t) + c_1 a_3 \phi^3(t) + c_1 a_4 \phi^2(t)\dot{\phi}(t) + c_1 a_5 \phi(t)\dot{\phi}^2(t) \tag{75}$$

Table 2 - delta wing flying object specifications [33]

$\rho(\frac{Kg}{m^3})$	$S(m^2)$	$C(m)$	$L_c(m)$	$I_{xx}(Kg.m^2)$	$\mu_x \frac{Kg.m^4}{s}$	$U_c(\frac{m}{s})$
1.2	0.0324	0.429	0.10725	2.7×10^{-4}	0.378×10^{-4}	15

Table 3 - The values of the parameters of the equation (74) in terms of angle of attack (degree) [33]

α	a_1	a_2	a_3	a_4	a_5
15	-0.01026	-0.02117	-0.14181	0.99735	-0.83478
21.5	-0.04207	0.01456	0.04714	-0.18583	0.24234
22.5	-0.04681	0.01966	0.05671	-0.22691	0.59065
25	-0.05686	0.03254	0.07334	-0.3597	1.4681

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Phase plane analysis of the Wing Rock phenomenon

By considering $x_1(t) = \phi(t)$ and $x_2(t) = \dot{\phi}(t)$, the system dynamics are converted into state space:

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = c_1a_1x_1(t) + (c_1a_2 - c_2)x_2(t) + c_1a_3x_1(t)^3 + c_1a_4x_1(t)^2x_2(t) + c_1a_5x_1(t)x_2(t)^2 \quad (76)$$

Next, calculate the equilibrium points of the nonlinear system (76):

$$\dot{x}_1(t) = 0$$
$$\dot{x}_2(t) = 0 \quad (77)$$

Which results in:

$$x_2(t) = 0$$

$$a_1x_1(t) + a_3x_1(t)^3 = 0 \quad (78)$$

The answers to Equation (78) are:

$$x_1(t) = 0 \,, x_1(t) = \pm\sqrt{\frac{-a_1}{a_3}} \quad (79)$$

Therefore, 3 equilibrium points were obtained, $A(0,0)$, $B(\sqrt{\frac{-a_1}{a_3}}.0)$, $C(-\sqrt{\frac{-a_1}{a_3}},0)$. Equilibrium points B and C are available when $\frac{a_1}{a_3} < 0$. By referring to Table 1, this condition is met when$\alpha \geq 21.5$.

Table 4 - Equilibrium points related to the attack angle of 25 degrees

Equilibrium point coordinates	Corresponding eigenvalues	Balance point type
A = (0,0)	$\lambda_1 = 0.005 + 0.14i$ $\lambda_1 = 0.005 - 0.14i$	Unstable focus
B= (-0.88,0)	$\lambda_1 = 0.16$ $\lambda_2 = -0.24$	Saddle point
C= (0.88,0)	$\lambda_1 = 0.16$ $\lambda_2 = -0.24$	Saddle point

Next, we specify the type of equilibrium points, which is done using the linearization technique. Considering the general state of the nonlinear system in form (80):

$$\dot{x}_1 = f_1(x_1.x_2)$$
$$\dot{x}_2 = f_2(x_1.x_2) \quad (80)$$

The Jacobin matrices are:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \quad (81)$$

If $(x_{1e}.x_{2e})$ is an equilibrium point of the system (76) then the linearized system corresponding to the above equilibrium point are:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(x_{1e}.x_{2e}) & \frac{\partial f_1}{\partial x_2}(x_{1e}.x_{2e}) \\ \frac{\partial f_2}{\partial x_1}(x_{1e}.x_{2e}) & \frac{\partial f_2}{\partial x_2}(x_{1e}.x_{2e}) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (82)$$

According to the system state space equation:

$$\dot{x}_1(t) = f_1(x_1.x_2) = x_2(t)$$
$$\dot{x}_2(t) = f_2(x_1.x_2) = c_1a_1x_1(t) + (c_1a_2 - c_2)x_2(t) + c_1a_3x_1(t)^3 + c_1a_4x_1(t)^2x_2(t) + c_1a_5x_1(t)x_2(t)^2 \quad (83)$$

The linearized system around the equilibrium point is obtained in the form (84):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
$$A = \begin{bmatrix} 0 & 1 \\ c_1a_1 + 3c_1a_3x_{1e}^2 + 2c_1a_4x_{1e}x_{2e} + c_1a_5x_{2e}^2 & (c_1a_2 - c_2) + c_1a_4x_{1e}^2 + 2c_1a_5x_{1e}x_{2e} \end{bmatrix} \quad (84)$$

Figures 2 to 5 show the nonlinear behavior of the wing rock phenomenon at the angle of attack α = 25 deg with the initial conditions $\dot{\phi}(0) = 0.1$ and $\phi(0) = 0$.

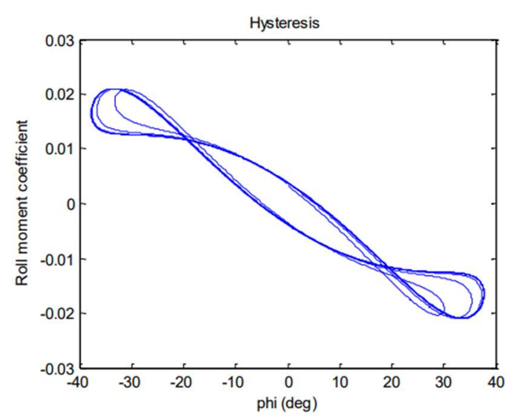

Figure 2 - Roll moment coefficient hysteresis

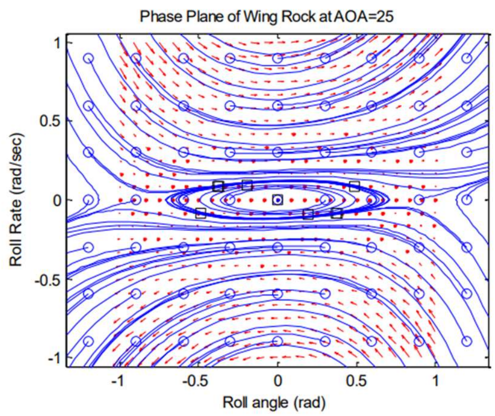

Figure 3 - Phase plane of the Wing Rock phenomenon (2 saddle equilibrium points and an unstable focus)

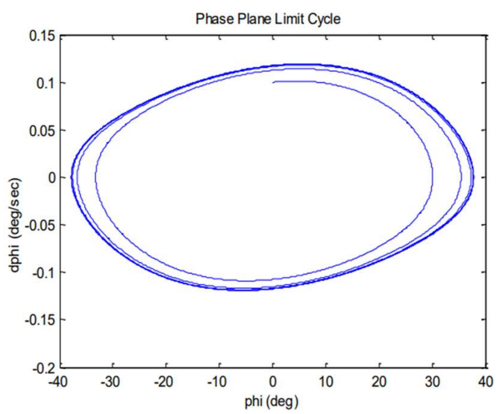

Figure 4 - Wing Rock phenomenon limit cycle

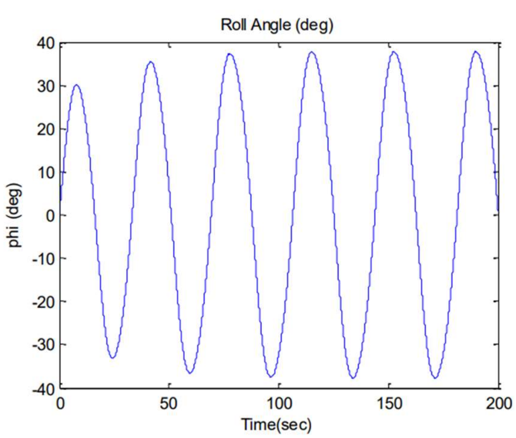

Figure 5 - Rolling angle oscillation response (build up phase)

Wing Rock phenomenon control model

The control model studied in this study is in the form (85):

$$\ddot{\phi}(t) = c_1a_1\phi + (c_1a_2 - c_2)\dot{\phi} + c_1a_3\phi^3 + c_1a_4\phi^2\dot{\phi} + c_1a_5\phi\dot{\phi}^2 + d_0u \tag{85}$$

In (85), u(t) is the input of the control surfaces (aircraft ailerons) and d_0 is the impact factor of the control surfaces. By defining the mode vector $x(t) = (x_1(t), x_2(t))^T = (\phi(t), \dot{\phi}(t))^T$ equation (85) is written in the form of state space (86):

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ c_1a_1 & c_1a_2 - c_2\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix} d_0\left(u + \frac{c_1a_3}{d_0}x_1^3(t) + \frac{c_1a_4}{d_0}x_1^2(t)x_2(t) + \frac{c_1a_5}{d_0}x_1(t)x_2^2(t)\right) \tag{86}$$

Simulation

Evaluation of the performance of the model reference adaptive control designed with the Lyapunov function in the power series

In this part, in order to evaluate the adaptive controller designed in the first part of the system, a degree of freedom of the wing is simulated. Using the data of Table 3 corresponding to the angle of attack of 25 degrees and taking into account the impact factor of the controller:

$$d_0 = 1 \tag{87}$$

The basic conditions for the system are considered as follows:

$$\phi_0 = 5\ \text{deg},\ P_0 = 0 \quad \left(\frac{\text{deg}}{\text{s}}\right) \tag{88}$$

The dynamics of roll angle of the reference model are considered to be in the form of a transfer function (89):

$$\frac{\phi_m}{r} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \tag{89}$$

Equation (89) in the form of state space (90) is considered:

$$\begin{bmatrix}\dot{\phi}_m\\ \dot{P}_m\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\omega_n^2 & -2\xi\omega_n\end{bmatrix}\begin{bmatrix}\phi_m\\ P_m\end{bmatrix} + \begin{bmatrix}0\\ \omega_n^2\end{bmatrix} r(t) \tag{90}$$

The natural frequency, and the damping coefficient and the considered adaptation gain matrices are:

$$\omega_n = 1 \;،\; \xi = 0.7$$
$$\Gamma_r = 100 \;،\Gamma_x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} ،\Gamma_\theta = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 100 \end{bmatrix} \qquad (91)$$

The response form of the closed-loop system is presented with considering the step input with different initial conditions. The maximum allowable changes for aileron are considered (u_{max} = 20 deg) and in the structure of the adaptation law, the adaptation laws corresponding to the second-order Lyapunov function in the power series (Table 1, second row i = 2) are used.

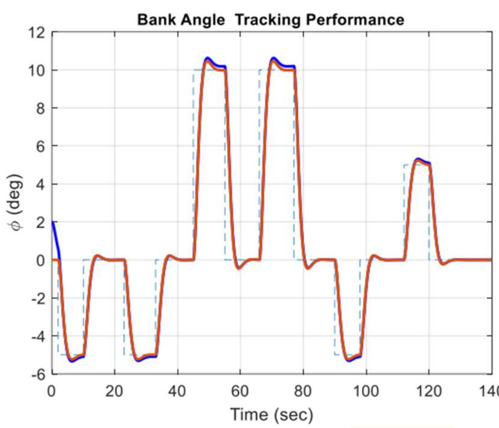

Figure 6 - Tracking the reference trajectory with the second-order adaptive controller in the power series

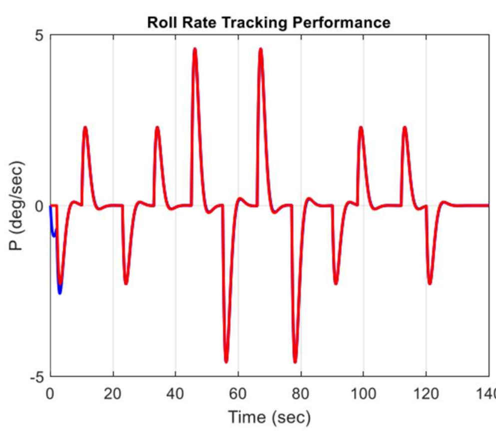

Figure 7: Trajectory tracking of the roll angle rate reference

As shown in Figures 6 and 7, the system mode variables (roll angle and roll angle rate) track the input command trajectory after a period of time and it is clear that these angles are out of hysteresis mode and the wing rock phenomenon is well controlled by the designed controller.

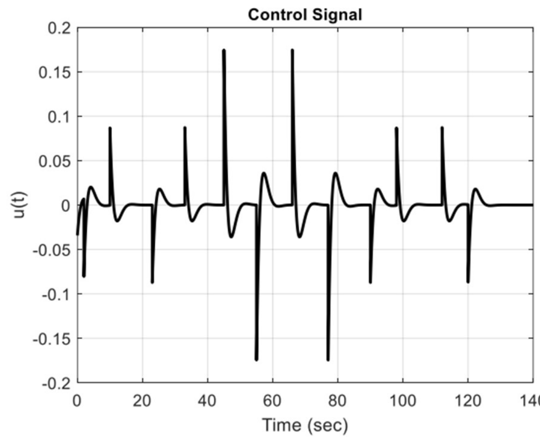

Figure 8 - Control signal (in the allowable range of 20-and 20 + degrees)

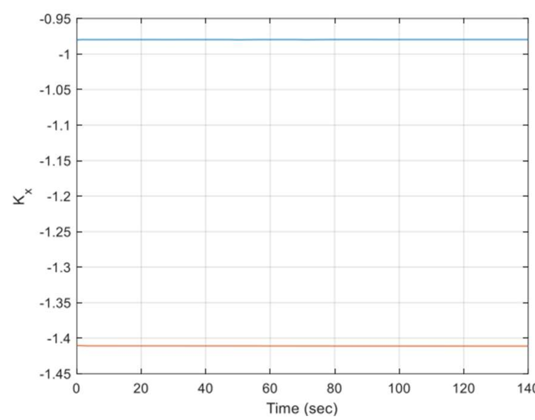

Figure 9 – boundedness of controller feedback parameters estimation

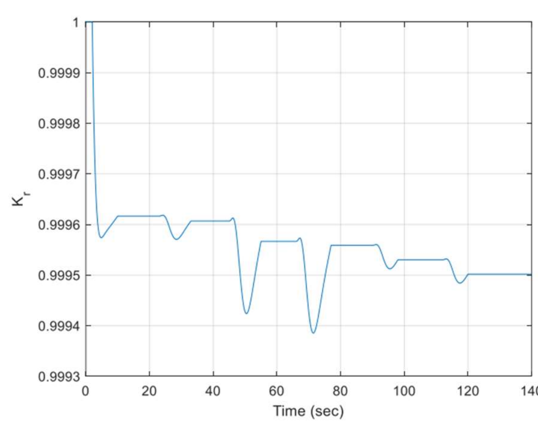

Figure 10 - boundedness of controller feed forward parameters estimation

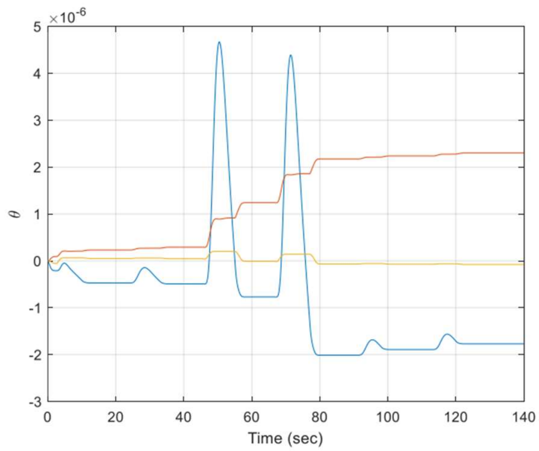

Figure 11 - Boundedness estimation of regressor vector parameters

Since the model reference adaptive control, despite guaranteeing the stability of the closed-loop system, does not provide the convergence of the controller parameters and the regressor vector parameters accumulative uncertainty, but guarantees the boundedness of the estimation of the above parameters. This is well illustrated in Figures 9 to 11.

Comparison of performance of controller designed with Lyapunov function in the power series with controller designed with quadratic Lyapunov function

In this section, to compare the performance of the controller designed with Lyapunov function in the power series with the controller designed with standard Lyapunov function (29), the Lyapunov function in the power series 2 in the form (92) is used:

$$V = 0.1\,(\tilde{x}^T(t)P\tilde{x}(t)) + 0.5\left(\tilde{x}^T(t)P\tilde{x}(t)\right)^2 + \mathrm{tr}([\tilde{K}_x^T(t)\Gamma_x^{-1}\tilde{K}_x(t) + \tilde{K}_r^T(t)\Gamma_r^{-1}\tilde{K}_r(t) + \tilde{\theta}^T(t)\Gamma_\theta^{-1}\tilde{\theta}(t)]\Lambda) \quad (92)$$

The adaptation law derived from Lyapunov function is as follows:

$$\begin{aligned}\dot{\hat{K}}_x(t) &= -\Gamma_x x(t)\tilde{x}^T(t)PB(0.1 + \tilde{x}^T(t)P\tilde{x}(t))\\ \dot{\hat{K}}_r(t) &= -\Gamma_r r(t)\tilde{x}^T(t)PB\left(0.1 + \tilde{x}^T(t)P\tilde{x}(t)\right)\\ \dot{\hat{\theta}}(t) &= \Gamma_\theta \psi\left(x(t)\right)\tilde{x}^T(t)PB(0.1 + \tilde{x}^T(t)P\tilde{x}(t))\end{aligned} \quad (93)$$

The simulation results of the controller with the above adaptation law in comparison with the adaptation controller with the adaptation law (29) are presented in Figures 12 to 15.

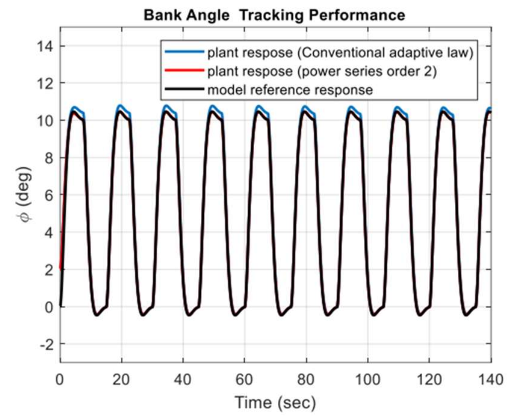

Figure 12 - Roll angle reference model tracking by 2 adaptive controllers (red = second-order power series and blue is common adaptive controller)

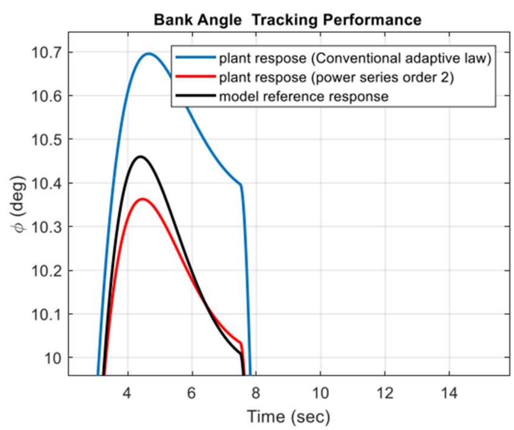

Figure 13 - Magnification of Figure 11

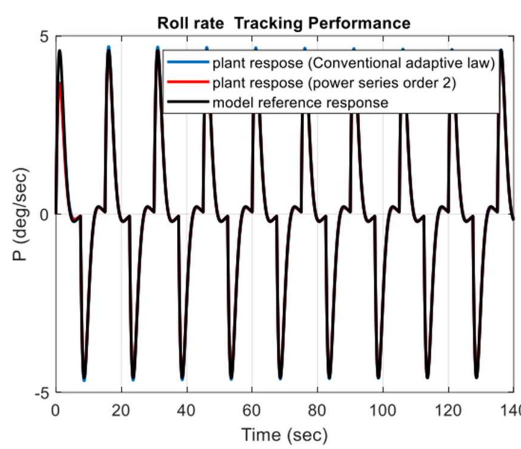

Figure 14: Roll angle rate reference model tracking by 2 adaptive controllers (red = 2nd order power series and blue is conventional adaptive controller)

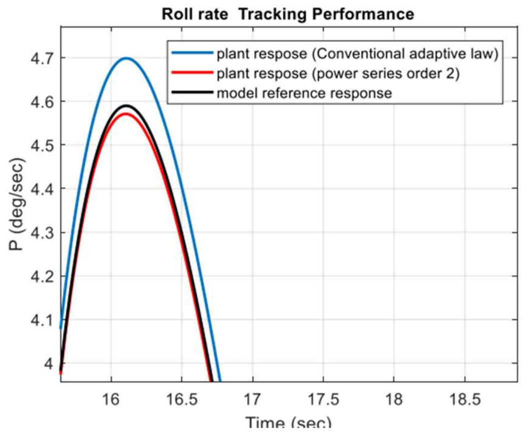

Figure 15 - Magnification of Figure 13

As is well illustrated in the figures above, the controller provides a relatively better response than the conventional adaptive controller with the rules of the second-order power series. The second norm of the tracking error signal and the second norm of the energy signal consumed are presented in Table 5.

Table 5 – Numerical comparison of the second norm of the tracking error and the control signal

$\phi = 5(\text{deg})$	$\lVert\tilde{x}\rVert_2$	$\lVert u(t)\rVert_2$
Controller designed with quadratic Lyapunov function	0.572	4.46
Controller designed with second-order Lyapunov function in the power series	0.362	4.40

Evaluation of the performance of the adaptive controller of the reference model with a feedback integrator designed with the Lyapunov function in the power series

Using the relation (30) and the data in Table 2 for α = 25 deg:

$$A_p = \begin{bmatrix} 0 & 1 \\ -0.0201 & 0.01 \end{bmatrix}, B_p = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (94)$$

Roll angle is considered as adjusted output:

$$y = [1 \quad 0]x_p = \phi \quad (95)$$

Which results in:

$$C_p = [1 \; 0] \quad (96)$$

Developed open loop systems include:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -0.0201 & 0.01 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, C = [0 \; 1 \; 0] \quad (97)$$

In order to stabilize the developed system, LQR controller is used:

$$u_{LQR}(t) = -K_{LQR}^{T} x(t) \quad (98)$$

By selecting values:

$$Q_{LQR} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, R_{LQR} = 1 \quad (99)$$

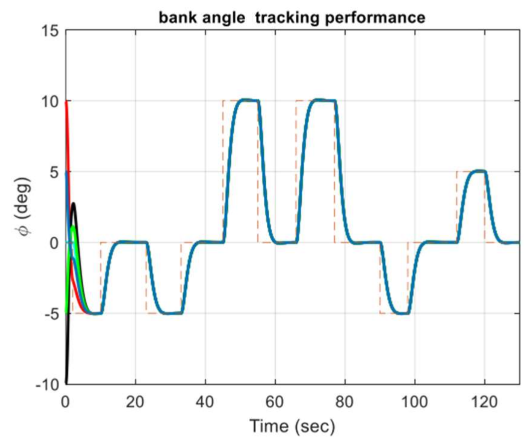

Figure 16 - Trajectory tracking the reference (roll angle)

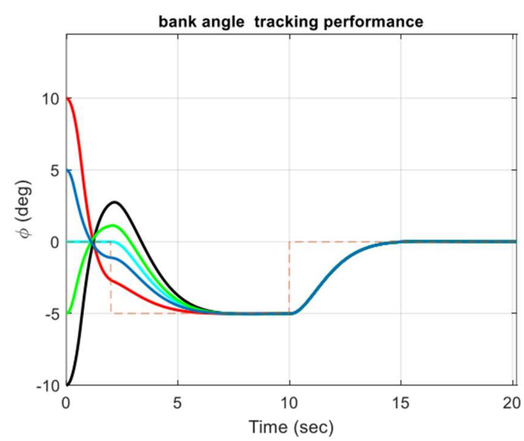

Figure 17 - Magnification of Figure 15

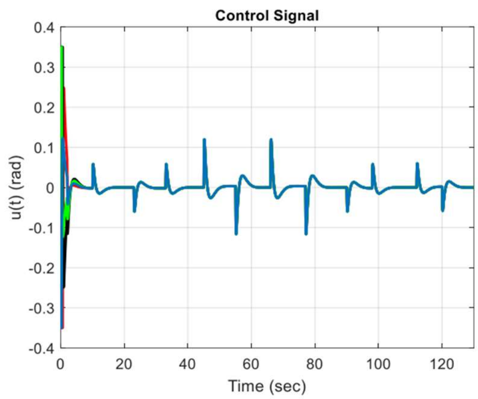

Figure 18 - Control signal (within the allowable range of 20 and +20 degrees)

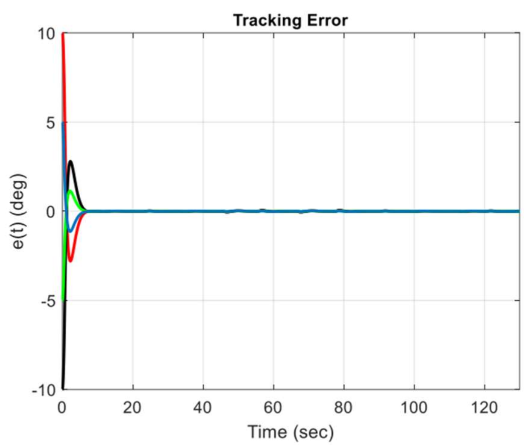

Figure 19 – Roll angle tracking error

The simulation results are presented in Figures 15 to 18. It is clear that after the transient state, the tracking error goes to zero and the control signal remains within the allowable range. As can be seen in Figures 6 and 11, the tracking error decreases over time and by tending the time towards unbounded, the tracking is achieved. However, in the adaptation state with the feedback integrator, the convergence error occurs much faster, which can be seen in Figures 15 and 18.

Conclusion

The innovation presented in this paper is considering Lyapunov function in power series form based on quadratic polynomials tracking error and it was proved that the adaptation rules derived from this Lyapunov function include quadratic expressions in power series form. The adaptation laws derived from quadratic Lyapunov function in the power series can be considered similar to the adaptation law derived from the common quadratic Lyapunov function, except that the benefits of its adaptation vary with time. The simulations show the proper performance of these controllers. In the adaptive mode, the proper transient mode performance and the convergence of the tracking error to zero with tending the time to unboundedness and in the feedback integrator mode, a faster convergence of the tracking error to zero was observed. The tracking error and control signal norm in the second-order adaptive controller in the power series mode were less than the tracking error and control signal norm in the conventional adaptive controller, which are specified in Table 5. Lyapunov function in the form of power series can be used in designing different types of robust adaptive and adaptive controllers designed by Lyapunov method.

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