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Investigating the Effects of Noise and Parameter Changing on Modeling of Jet Transport Aircraft

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This paper presents a novel approach to modeling of jet transport aircraft. Initially, basic mathematical models of jet transport are derived. Afterwards by focusing on the bank angle system of the jet transport, considering the aileron as the input, adverse methods of identification are utilized to estimate parameters of the system in an online manner. Then, effects of different types of noise on identification process are analyzed. Eventually, effects of time varying parameters are discussed. Recursive least squares method and its extended version, covariance resetting and forgetting factor methods were the fundamental tools in the system identification process of jet transport. Comprehensive simulations are presented and cast some light on effectiveness and disadvantages of different approaches

Keywords: *"Jet Transport Aircraft", "Bank Angle", "aileron", "System Identification", "Recursive-least-squares", "Noise", "Varying Parameters".*

Introduction

Ailerons can be used to generate a rolling motion for an aircraft. Ailerons are small hinged sections on the outboard portion of a wing. Ailerons usually work in opposition: as the right aileron is deflected upward, the left is deflected downward, and vice versa.

The ailerons are used to bank the aircraft; to cause one wing tip to move up and the other wing tip to move down. The banking creates an unbalanced side force component of the large wing lift force which causes the aircraft's flight path to curve. Airplanes turn because of banking created by the ailerons, not because of a rudder input.

The ailerons work by changing the effective shape of the airfoil of the outer portion of the wing. Changing the angle of deflection at the rear of an airfoil will change the amount of lift generated by the foil. With greater downward deflection, the lift will increase in the upward direction. The aileron on the right wing is deflected up. Therefore, the lift on the left wing is increased, while the lift on the right wing is decreased. For both wings, the lift force of the wing section through the aileron is applied at the aerodynamic center of the section which is some distance (L) from the aircraft center of gravity. This creates a torque

$$T = F \times L \tag{1}$$

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about the center of gravity. If the forces (and distances) are equal there is no net torque on the aircraft. But if the forces are unequal, there is a net torque and the aircraft rotates about its center of gravity. The resulting motion will roll the aircraft to the right (clockwise). If the pilot reverses the aileron deflections (right aileron down, left aileron up) the aircraft will roll in the opposite direction. We have chosen to name the left wing and right wing based on a view from the back of the aircraft towards the nose, because that is the direction in which the pilot is looking.

In this paper we are going to derive mathematical models of a jet transport aircraft. Afterwards we will focus on the bank angle system with aileron as the input. Then, we use classic auto-regressive with exogenous input (ARX) and auto-regressive with exogenous input with moving average noise (ARMAX) structures to model bank angle system of the jet transport aircraft and investigate the effect of noise and parameter changing on identification of this model.

Mathematical Formulation

The state space jet model during cruise flight at MACH=0.8 and H=40,000 ft is described by the following equations:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t) \tag{2}$$

In which

$$A = \begin{bmatrix} -0.0558 & -0.9968 & 0.0802 & 0.0415 \\ 0.5980 & -0.1150 & -0.0318 & 0 \\ -3.0500 & 0.3889 & -0.4650 & 0 \\ 0 & 0.0805 & 1.0000 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0073 & 0 \\ -0.4750 & 0.0077 \\ 0.1530 & 0.1430 \\ 0 & 0 \end{bmatrix} \tag{3}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = 0$$

The system is multi-input multi-output, therefore the following transfer functions could be derived from the preceding model:

$$\frac{yaw\ rate}{rudder} = \frac{-0.475s^3 - 0.2479s^2 - 0.1187s - 0.05633}{s^4 + 0.635s^3 + 0.938s^2 + 0.511s + 0.0036} \tag{4}$$

$$\frac{bank\ angle}{rudder} = \frac{0.114s^2 - 0.2s - 1.373}{s^4 + 0.635s^3 + 0.938s^2 + 0.511s + 0.0036} \tag{5}$$

$$\frac{yaw\ rate}{aileron} = \frac{0.007s^3 - 0.0005s^2 + 0.008s + 0.004}{s^4 + 0.635s^3 + 0.938s^2 + 0.511s + 0.0036} \tag{6}$$

$$\frac{bank\ angle}{aileron} = \frac{0.1436s^2 + 0.0273s + 0.11}{s^4 + 0.635s^3 + 0.938s^2 + 0.511s + 0.0036} \tag{7}$$

In this paper we focus on the last transfer function, that is, bank angle as the output and aileron a s the input. Fig. 1 shows the schematic dynamic of the jet transport aircraft.
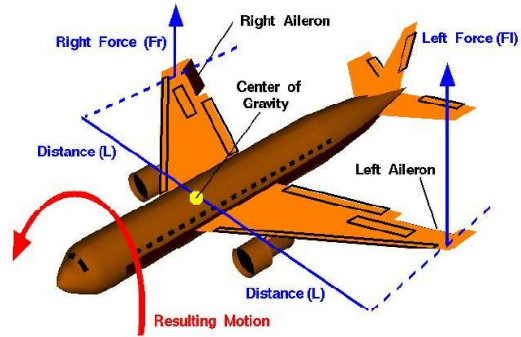

Fig. 1: Jet transport schematic

Using zero order hold method and sampling time of 0.01 we discretize this model.

$$G_d(z^{-1}) = \frac{b_1 z^3 + b_2 z^2 + b_3 z}{z^4 + a_1 z^3 + a_2 z^2 + a_3 z + a_4} \tag{8}$$

Where actual a_i and b_i are :

$$
\begin{aligned}
a_1 &= -2.01 \\
a_2 &= 1.705 \\
a_3 &= -0.7771 \\
a_4 &= 0.2101 \\
b_1 &= 0.06364 \\
b_2 &= 0.2369 \\
b_3 &= 0.4441 \\
b_3 &= -0.009138
\end{aligned} \tag{9}
$$

Afterwards, assuming that we don't know a_i and b_i we try to estimate these parameters under diverse conditions.

Parameter Estimation of Jet Transport Aircraft

Considering the derived model in the previous section, bank angle as the output and aileron as the

input, we utilize RLS method to estimate parameters of this system.

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\varepsilon(t)$$

$$\varepsilon(t) = y(t) - \varphi^T(t)\hat{\theta}(t-1)$$

$$\text{(10)}$$

$$K(t) = \frac{P(t-1)\varphi(t)}{1 + \varphi^T(t)P(t-1)\varphi(t)}$$

$$P(t) = P(t-1) - \frac{P(t-1)\varphi(t)\varphi^T(t)P(t-1)}{1 + \varphi^T(t)P(t-1)\varphi(t)}$$

In which $\theta = [a_1 a_2 a_3 a_4 b_1 b_2 b_3 b_4]$ is the parameters vector K, is the correcting gain, φ is the vector of regressors, P is somehow the covariance matrix and is the one-step ahead error.

To solve the recursive equations (10), for initialization we consider $P(0) = \rho I$ where $\rho = 10^5$ and $\theta(0) = 0$.

Figures 2, 3 and 4 show the RLS estimation results for a single sinusoid, sum of three sinusoid signals with different frequencies and a white noise as inputs.
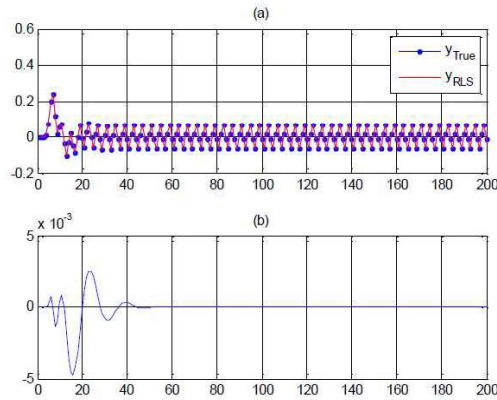

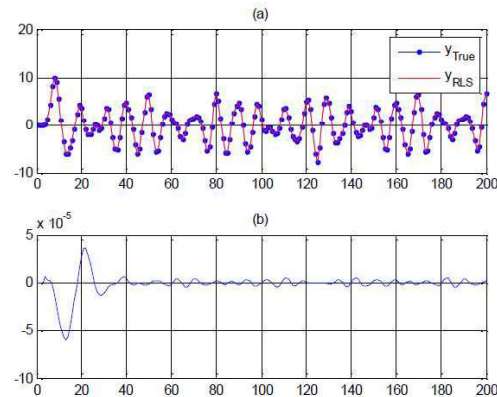**Fig. 2:**(a) RLS estimation, one sinusoid. (b) RLS Residue

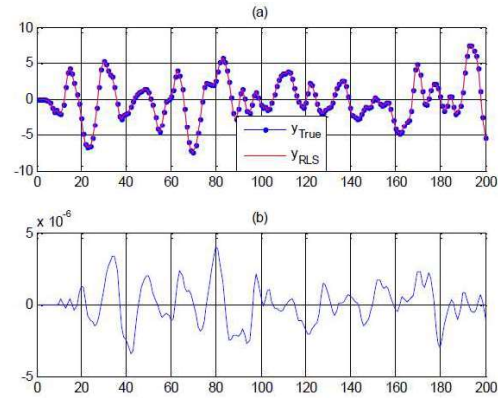**Fig. 3:**(a)RLS estimation, sum of four sinusoids. (b) RLS Residue.

Fig. 4:(a)RLS estimation, white noise. (b) RLS Residue

As can be seen, in all the cases estimations show promising results. Furthermore, Table1 presents the estimated parameters, their actual values and the corresponding cost functions.

Table 1: Parameters estimation under different input signals

	Actual values	One sinusoid	Sum of four sinusoid	White noise
	θ_0	$\hat{\theta}_{LS}$	$\hat{\theta}_{LS}$	$\hat{\theta}_{LS}$
a_1	-2.010	-1.903	-1.918	-2.010
a_2	1.705	1.505	1.704	1.7049
a_3	-0.777	-0.633	-0.777	-0.777
a_4	0.210	0.170	0.210	0.210
b_1	0.063	0.063	0.063	0.063
b_2	0.236	0.243	0.236	0.236
b_3	0.444	0.466	0.444	0.444
b_4	-0.009	0.048	-0.009	-0.009
$V(\hat{\theta}_{LS})$		6.16×10^{-5}	4.64×10^{-8}	1.93×10^{-8}

By analyzing the table 1, It can be inferred that for the single sinusoid signal which is PE of the order of 2, parameters have not converged to their actual values and for the summation of four sinusoid signal which is PE of the order of 8, the accuracy of convergence became relatively better, and eventually the best convergence happened for the white noise signal.

Effects of Noise on Jet Transport modeling

In this section we are going to investigate the effects of different kinds of noise on the identification process. To accomplish this, we try to estimate parameters of the jet transport aircraft system with white noise colored noise and without noise.

We assume that the system in unknown for us and structure of the model - number of poles and zeros-

is the only priori knowledge we have about the system. Fig. 5 shows this estimation.
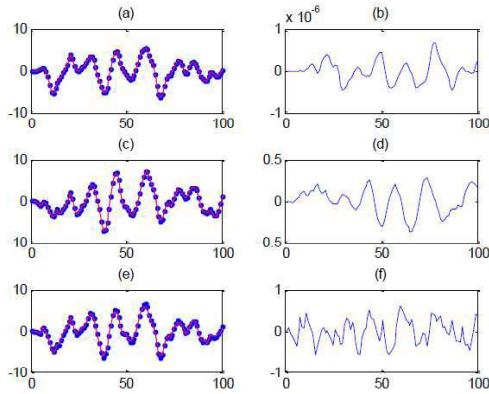

Fig. 5: (a) True output and RLS estimated output without noise. (b) Residue of RLS estimation without noise. (c) True output and RLS estimated output with white noise. (d) Residue of RLS estimation with white noise. (e) True output and RLS estimated output with colored noise. (f) Residue of RLS estimation with colored noise

As can be seen, in the two cases of without noise and with white noise the output of estimation tracks the actual output of the system but colored noise caused a considerable error. Figures 6 and 7 show the parameters convergence in absence and presence of noise.
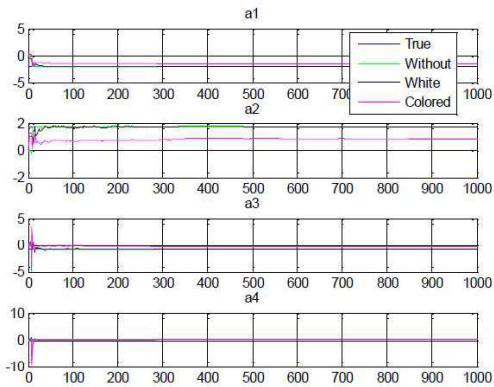

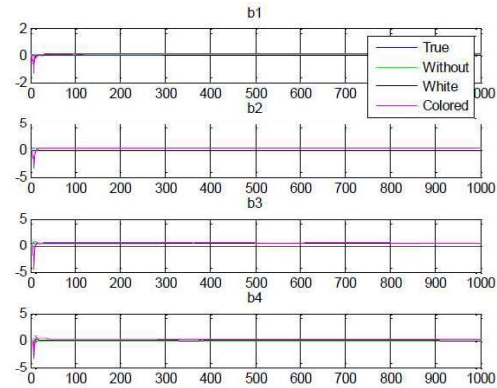**Fig. 6**: Effects of noise on denominator parameters estimation

Fig.7: Effects of noise on denominator parameters estimation

Table 1 summarizes the results of this simulation.

Table 2: Effects of noise on parameters estimation

	Actual Values	Without Noise	White Noise	Colored Noise
	θ_0	$\hat{\theta}_{LS}$	$\hat{\theta}_{LS}$	$\hat{\theta}_{LS}$
a_1	-2.010	-2.010	-1.937	-1.409
a_2	1.705	1.705	1.551	0.668
a_3	-0.777	-0.777	-0.656	-0.141
a_4	0.210	0.210	0.177	0.097
b_1	0.063	0.063	0.078	0.082
b_2	0.236	0.236	0.240	0.271
b_3	0.444	0.444	0.475	0.605
b_4	-0.009	-0.009	0.048	0.318
$V(\hat{\theta}_{LS})$		4.6×10^{-11}	31.432	58.121

Second column of table 2 shows actual values, third column estimations without noise, fourth column estimations with white noise and the last column shows estimation with colored noise. It can be deduced from the table 2 that estimation in presence of white noise has not bias while in the presence of colored noise parameters convergence do not take place perfectly and estimated parameters are biased.

Therefore, to solve this dilemma we apply extended least squares (ERLS) method to identify parameters of the system in the presence of colored noise. ERLS is almost identical with RLS, the only discrepancy is that in ERLS the regressors vector is extended as equation (11) in order to identify noise dynamic.

$$\varphi^T(t) = [-y(t-1) \dots -y(t-n) \\ u(t-1) \dots u(t-n) \, \varepsilon(t-1) \dots \varepsilon(t-n)] \quad (11)$$

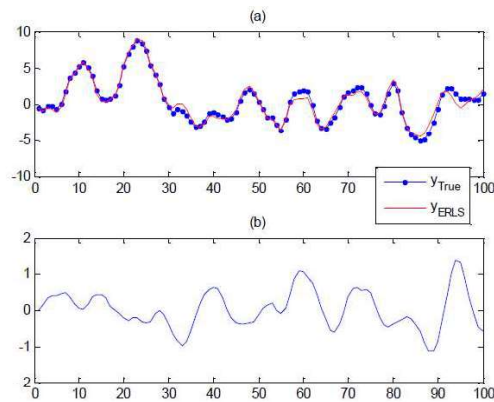Fig 8 shows the result of ERLS method.

Fig. 8: (a)True output and ERLS estimated output with colored noise. (b)Residue of ERLS estimation with colored noise

And figures 9 and 10 show the estimation of parameters of the jet transport aircraft under the influence of colored noise, identified by ERLS method.
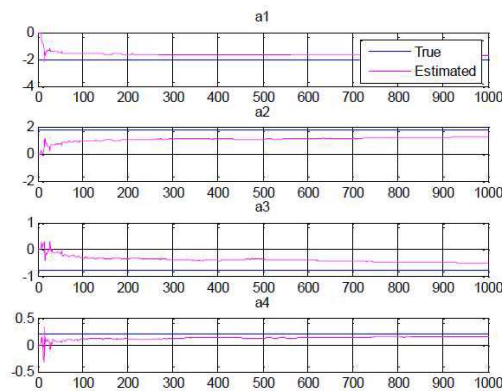

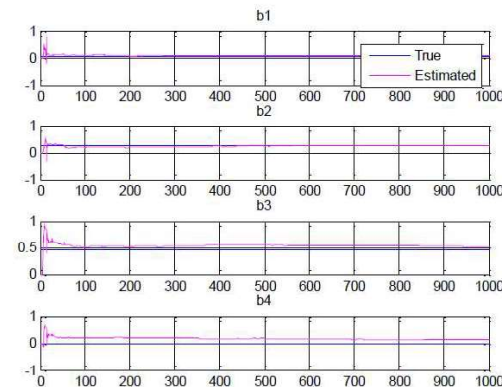**Fig. 9**: Estimation with ERLS method

Fig. 10: Estimation with ERLS method

As can be observed from figure 9 and 10, even by using ERLS method parameters don't converge to their correct values but have a better accuracy compared with solely RLS.

Effects of Time Varying Parameters

So far we have been considering the parameters of jet transport model to be constant. Whereas in the course of flight and practical situations, parameters may change over time. In this section we make a gallant effort to investigate the effects of these time varying parameters on identification of the system.

To do so, we consider two cases. In one case we change the parameters fast and in the other slow. In simulations this change stars at sampling 1500. We utilize RLS algorithm plus forgetting factor method and covariance resetting to estimate parameters. Needless to mention that covariance resetting and forgetting factor methods are exploited to avoid the matrix gets ill conditioned.

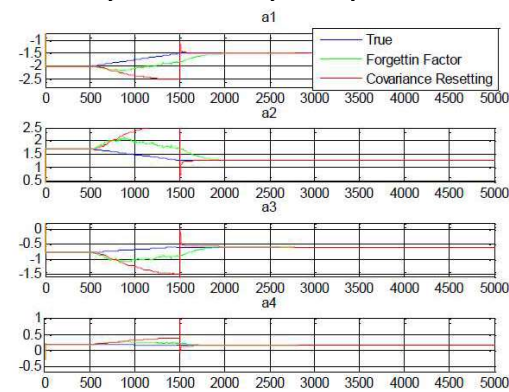Figures 11 and 12 show the parameters estimation while the parameters vary slowly.

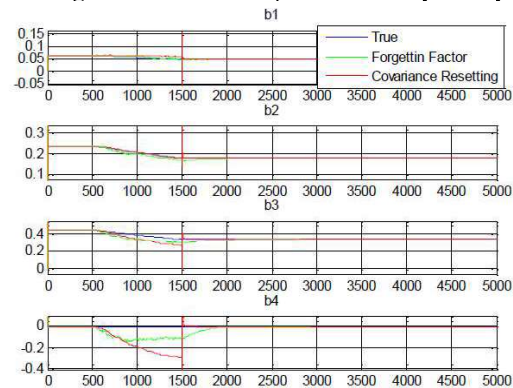**Fig. 11**: Denominator parameters vary slowly

Fig. 12: Denominator parameters vary quickly

As can be seen, when parameters change slowly over time, forgetting factor method shows a better result in comparison with covariance resetting. The reason lies in the fact that in data samples don't jump considerably so giving less importance

to previous samples is enough to estimate parameters properly.

Figures 13 and 14 show the parameters estimation while the parameters vary quickly.
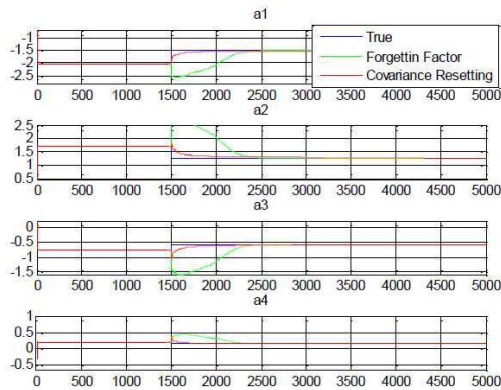

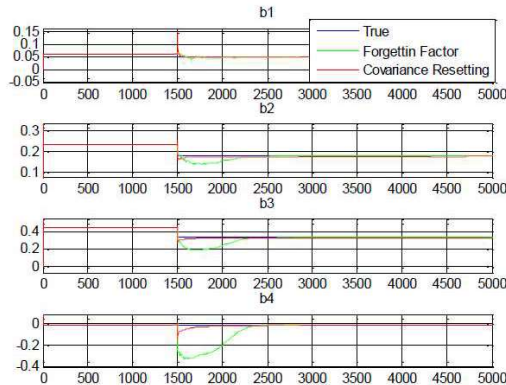**Fig. 13**: Numerator parameters vary quickly

Fig. 14: Numerator parameters vary quickly

On the contrary with former conditions, in this case where parameters vary quickly over a specified time interval, covariance resetting method shows a more desirable result.

Conclusion

Utilizing RLS method and its extended version ERLS, we tried to deal with system identification problem of a jet transport aircraft. Effects of different types of noise and time varying parameters was investigated. It was seen that RLS method estimates parameters without any bias even in presence of white noise, but is impractical when the colored noise shows up. When parameters vary slowly over time forgetting vector shows better results while when parameters change quickly overtime covariance resetting method is more effective.

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