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On the Design of Nonlinear Discrete-Time Adaptive Controller for damaged Airplane

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airplane in presence of asymmetric left-wing damaged. Variations of the aerodynamic parameters, mass and moments of inertia, and the center of gravity due to damage are all considered in the nonlinear mathematical modeling. The proposed discrete-time nonlinear MRAC algorithm applies the recursive least square (RLS) algorithm as a parameter estimator as well as the error between the real damaged dynamics and a model of nominal undamaged aircraft to generate the desired control commands. The discrete-time adaptive control algorithm is augmented with a Nonlinear Dynamic Inversion (NDI) control strategy and is implemented on the NASA generic transport model (GTM) airplane while considering the effect of wing damage and un-modeled actuator dynamics. The stability of the proposed nonlinear adaptive controller is demonstrated through Popov's hyperstability theory. Simulation results of the introduced controller are compared with the classical discrete-time adaptive control strategy. The results demonstrate the effective performance of the proposed algorithm in controlling the airplane in presence of abrupt asymmetric damage.

Keywords: Discrete-time, Nonlinear Adaptive Control, Nonlinear Dynamic Inversion, Parameter Uncertainties, Structural Damage, Un-Modeled dynamic.

INTRODUCTION

Flight safety and accurate control are major concern issues in modern aviation. Loss of Control (LoC) remains one of the dominant contributors to fatal aircraft accidents over the past few decades even in the presence of triple redundancy-system designs of today's modern airplanes. Hence, designing an appropriate algorithm to control the airplane for accomplishing the mission in presence of fault, failure, and damage is a real concern for most control engineers. Consequently, various control strategies ranging from classical to advanced approaches have been proposed and applied to the impaired aircraft [1]–[3].Classical control algorithms are not suitable for fault, failure or damage Scenarios since failure or damage can change the airplane's parameters and result in performance degradation [4].

Failure or damage in passenger aircraft can result loss of performance and consequently, loss of control. To stop such situations, flight control designers are trying to improve control algorithms to safely retrieve the damaged aircraft[5]. Advanced control algorithms can help by adapting to the failure or damage or instead using robust algorithms. In Ref.[6], a robust integrated controller based on H-infinity algorithm for nonlinear longitudinal dynamics of a Boeing 747-100/200 aircraft in the presence of fault diagnosis objectives is presented.

A robust integrated fault-tolerant flight control system which accommodates different types of actuator failures and control effector damage is presented in Ref.[7].

It is clear that, the robust algorithm, in general, do not exhibit appropriate performance considering large parameter variations. Safe recovery of an

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airplane in failures or damage conditions is started in NASA[8]. In the following, for different wing damages on a generic transport model (GTM) Wind-tunnel experiments have been conducted[9]. GTM airplane which is a case study investigated in this paper is a 5.5-percent scaled model of a business jet airplane.

Initial researches are focused on design adaptive controller for a left-wing damaged airplane. In Ref.[10]. 6-DOF equations For damage scenario is derived according Center of Gravity (C.G.) variation.

To deal with the structural damage, identification system is often used to handle the model uncertainty. In different researches several online identification algorithms are introduced. Laban [11]proposes a two-step real-time method to identify the aerodynamic models. Lombaerts[12] applies adaptive NDI and real-time identification for damaged airplane. Lombaerts et al.[13]is utilized NDI technique with real-time identification based on Kalman filter for a damaged airplane.

In Ref. [14]nonlinear dynamics model of aircraft with asymmetric damage is linearized. Then, a model reference adaptive controller is designed to deal with the damaged airplane. Performance of several existing adaptive controllers is investigated for GTM Wing damaged in Refs.[15], [16]. Tang[17]presented an adaptive control strategy for an airplane with nonlinear uncertainties and actuator failures.

adaptive control algorithm in joint space is an approach for unknown or un-modeled dynamics which has been presented in Ref.[18]. Adaptive feedback linearization control is utilized to control a system with fast convergence [19]. Optimal adaptive controller using Markov parameters approach is introduced in Ref.[20].

Instabilities of adaptive controllers have been demonstrated in several conditions[21]–[23].To deal with these instabilities, the use of robust-adaptive control algorithms such as L1 adaptive controller [3], [24] is recommended.

Applying projection function along with adaptive algorithms and using the dead zone methodare popular strategies that make the adaptive controllers robust. Additionally, to improve the performance of adaptive control, combined algorithm such as fuzzy-adaptive, neural network adaptive and sliding mode-adaptive algorithms have been proposed. In Ref.[25]adaptive controller with projection function strategy is designedto deal withasymmetricwing damage aircraft.In Ref.[26], a robust adaptive control strategy is presented which is used fuzzy system to approximate the uncertainties. An algorithm to control a damaged airplane using neural network in a hybrid direct–indirect is designed in Ref.[16]. Neural network adaptive control approach to compensate the damage effects of an airplane is presented In Ref.[27]. Particularly in nonlinear dynamic systems, adaptive control and neural control scheme are often combined. Experimental evaluation via flight tests for applying adaptive neural network controller to a flying-wing type unmanned aerial vehicle experiencing partial wing-loss is introduced in Ref.[28].A control algorithm based on neural network for an airplane with control surface lock is introduced in Ref.[29]. In[30] an enhanced automation was presented to recover the control of aircraft under damages.

Nonlinear L1 adaptive control for damage airplane is introduced in Ref.[3].

An adaptive reconfigurable scheme composed of on-line observers for damage detection as well as an adaptive controller was applied on a tailless fighter aircraft with wing damage in Ref.[31]. A Linear Parameter-Varying (LPV) model-based estimation with an indirect adaptive control was presented in Ref.[32]to recover the wing-damage airplane.

Simulating the effect of wing tip loss and controlling the damage airplane were also examined in Refs.[33], [34] in which an adaptive Proportional Integral Derivative (PID) [33]and an adaptive MRAC algorithm with nonlinear dynamic inversion[34] were applied to the dynamics of the wing-damaged airplane. MRAC scheme was also applied on a wing-damaged airplane in Ref.[35]where a virtual-command was introduced to the standard MRAC to maintain the tracking error within a small range and provide more robustness.

Obviously, the implementation of adaptive controllers on electrical boards necessitates the control law to be designed in discrete-time. Although there are several researches regarding the control of the asymmetric damaged airplane as pointed out above, discrete-time controller design applied to damaged airplane is rare according to the literature [36]. The classical method of discrete-time model reference adaptive control strategy for linear systems was introduced by [37]–[39].

This paper introduces a novel discrete-time nonlinear adaptive control algorithm and implements the introduced algorithm to the left-wing damaged airplane. In the proposed architecture the RLS algorithm as a state estimator along with the errors between sensors outputs and a nominal (un-damaged) mathematical model outputs are applied in the adaptive control loop. The discrete-time adaptive algorithm is augmented with a Nonlinear Dynamic Inversion (NDI) controller as the baseline controller. Comparing the results of applying the proposed controller on the inner-dynamics of the GTM model with 30% of left-wing damage with the results of classical discrete-time adaptive algorithm demonstrates the performance superiority of our proposed control strategy.

The remainder of the paper includes the following sections. The damaged airplane dynamic model is briefly introduced in section 2. In Section 3, first, the nonlinear dynamic inversion as a baseline controller is designed and then the classical discrete-time adaptive algorithm is modified and augmented with the NDI strategy. Section 4 illustrates the numerical results of applying the controller to the GTM model in several failure Scenarios. Conclusions are presented in Section 5.

Mathematical Modelling

Here, mathematical model of damaged airplane is briefly presented. More detailed can be found in Refs.[4], [30], [40], [41].

Damaged Airplane model

The airplane Centre of Gravity (C.G.) position is changed due to structural damage. If "$\Delta x, \Delta y, \Delta z$" are the damaged airplane C.G shift location with respect to the reference C.G. location in body coordinates and with assumption a rigid body airplane, the equations of motion are derived as[40]:

$$\begin{aligned} X = m[&-(r^2+q^2)\Delta x + (-\dot{r}+pq)\Delta y \\ &+(pr+\dot{q})\Delta z + \dot{u} - rv + qw + g\sin\theta] \end{aligned} \quad (1)$$

$$\begin{aligned} Y = m[&(pq+\dot{r})\Delta x - (r^2+p^2)\Delta y + (-\dot{p}+qr)\Delta z \\ &+\dot{v} - pw + ru - g\sin\varphi\cos\theta] \end{aligned} \quad (2)$$

$$\begin{aligned} Z = m[&(-\dot{q}-pr)\Delta x + (\dot{p}+qr)\Delta y - (q^2+p^2)\Delta z \\ &+\dot{w} - qu + pv - g\cos\varphi\cos\theta] \end{aligned} \quad (3)$$

$$\begin{aligned} L = &m\Delta y(\dot{w} - qu + pv - g\cos\varphi\cos\theta) \\ &- m\Delta z(\dot{v} - pw + ru - g\sin\varphi\cos\theta) \\ &+ I_{xx}\dot{p} - I_{xz}\dot{r} - I_{xy}\dot{q} - I_{xz}pq + I_{xy}pr \\ &+ (I_{zz} - I_{yy})qr + I_{yz}(r^2 - q^2). \end{aligned} \quad (4)$$

$$\begin{aligned} M = &-m\Delta x(\dot{w} - qu + pv - g\cos\varphi\cos\theta) \\ &+ m\Delta z(\dot{u} + qw - rv + g\sin\theta) \\ &+ \dot{q}I_{yy} - \dot{p}I_{xy} + pqI_{yz} - qrI_{xy} \\ &- \dot{r}I_{yz} + (I_{xx} - I_{zz})pr + I_{xz}(p^2 - r^2). \end{aligned} \quad (5)$$

$$\begin{aligned} N = &m\Delta x(\dot{v} - pw + ru - g\sin\varphi\cos\theta) \\ &- m\Delta y(\dot{u} + qw - rv + g\sin\theta) \\ &+ \dot{r}I_{zz} - \dot{p}I_{xz} - \dot{q}I_{yz} - prI_{yz} + qrI_{xz} \\ &+ (I_{yy} - I_{xx})pq + I_{xy}(q^2 - p^2). \end{aligned} \quad (6)$$

Based on equations (4) to (6), due to the variation of C.G. location, linear and angular acceleration is coupled and additional moment terms are generated. In the equations (1) to (6) m and $I_{(..)}$ are the damaged airplane mass and components of the inertia tensor.

Aerodynamic Model

The stability derivatives of NASA GTM model are extracted using a modified vortex-lattice code [41].Rolling moments and pitching moment coefficients vs. angle of attack are illustrated In Figures 1, 2 [5]. Abrupt lift reduction and its asymmetric distribution are the main effects of the wing tip loss. Therefore, the vertical force and pitching moment coefficients slope ($C_{Z_\alpha}, C_{m_\alpha}$) are reduced and several new aerodynamic coefficients such as C_{l_α}, C_{l_q} , C_{n_q} , and cross coupling between longitudinal and lateral states are generated.

Different damage models of the GTM model such as wing and tail damage is presented in Ref.[42]. In this paper, it is assumed that the percentage of wing tip loss is as a damage[5], [30].

In this paper, aerodynamic coefficients are defined by Taylor series expansion [13]. For example, equation (7) presents the rolling moment coefficient in the body coordinate system.

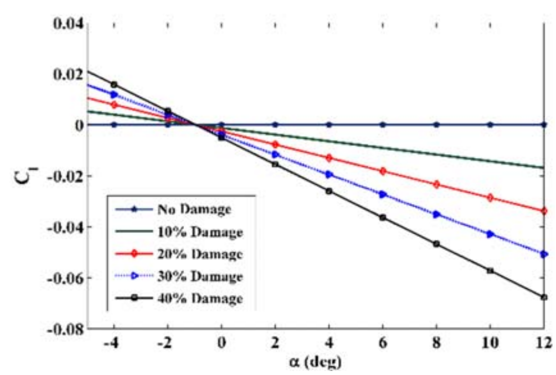

Figure 1. Rolling moment coefficient versus α [5].

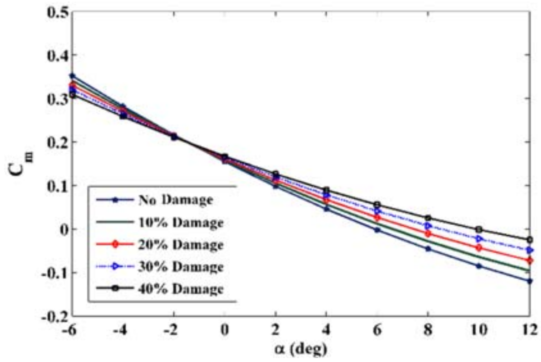

Figure 2. Pitching moment coefficient versus α [5]

Different damage models of the GTM model such as wing and tail damage is presented in Ref.[42]. In this paper, it is assumed that the percentage of wing tip loss is as a damage[5], [30].

In this paper, aerodynamic coefficients are defined by Taylor series expansion [13]. For example, equation (7) presents the rolling moment coefficient in the body coordinate system.

$$C_l = C_{l_0} + C_{l_\alpha}\alpha + C_{l_{\alpha^2}}\frac{\alpha^2}{2} + C_{l_\beta}\beta + C_{l_{\beta^2}}\frac{\beta^2}{2} + C_{l_{\hat{p}}}\hat{p} + C_{l_{\hat{p}^2}}\frac{\hat{p}^2}{2} + C_{l_{\hat{q}}}\hat{q} + C_{l_{\hat{q}^2}}\frac{\hat{q}^2}{2} + C_{l_{\hat{r}}}\hat{r} + C_{l_{\hat{r}^2}}\frac{\hat{r}^2}{2} + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_e}}\delta_e + C_{l_{\delta_r}}\delta_r . \quad (7)$$

As was mentioned before, loss of the wing tip as a damage leads to sudden change in the C.G. location and inertia. Thus, several new aerodynamic coefficients and cross dynamic coupling are generated. Additionally, it leads to abrupt change in aerodynamic coefficients. Therefore, the airplane controller must able to recover and control the damaged airplane in the presence of un-un-modeled dynamics and parameter uncertainties.

Control Architecture

Inthissection,anovelcontrolarchitecturebasedonthe airplanesensorsoutputsandanonlinear mathematical model is introduced. Nonlinear Dynamic Inversion (NDI) is used as baseline controller and is augmented with the discrete-time adaptive algorithm. The proposed controller enhances the performance of the NDI controller in the presence of airplane damage. The error signal (the difference between airplane sensors outputs and the undamaged mathematical model outputs) is used to develop the discrete-time adaptive controller. The adaptive controller guarantees that the closed-loop system tracks the input angular rates, considering uncertainties and un-modeled actuators dynamics due to airplane damage. In fact, the adaptive controller is used to compensate for the NDI control algorithm properties due to the structural damage. More details concerning the design process of this framework are described as follows.

Nonlinear Dynamic Inversion

The nonlinear dynamic inversion (NDI) is commonly used to control nonlinear systems, especially in aerospace applications. The rotational and translational dynamics of the airplane are separated by multiple time scales approach with the assumption that the rotational dynamics is much faster than the translational. The airplane nonlinear dynamics is considered as below:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} . \quad (8)$$

where the dimension of $\mathbf{f}(\mathbf{x})$, $\mathbf{g}(\mathbf{x})$ which are the non-linear functions equals to n, $\mathbf{x} = [p \;\; q \;\; r]^T$ is the state vector which is n-dimensional and q is the pitch rate, p is the roll rate, r is the yaw rate, and $\mathbf{u}$ is n-dimensional input. Considering $\dot{x} = \frac{x((k+1)T_s) - x(kT_s)}{T_s}$, and using the shorthand notation of indices k instead of the more precise one *kTs*; where *Ts* is the sampling period; the discrete form of equation (8) is derived as blew:

$$\mathbf{x}(k+1) = \mathbf{f}_1(\mathbf{x}(k)) + \mathbf{g}_1(\mathbf{x}(k))\mathbf{u}(k) \quad (9)$$

where, $\mathbf{f}_1(\mathbf{x}(k)) = Ts\,\mathbf{f}(\mathbf{x}(k)) + \mathbf{x}(k)$ and $\mathbf{g}_1(\mathbf{x}(k)) = Ts\,\mathbf{g}(\mathbf{x}(k))$ are discrete-time nonlinear functions. If $\mathbf{g}_1(\mathbf{x}(k))^{-1}$ exists, the input $\mathbf{u}(k)$ which is $(\delta_e, \delta_a, \delta_r)$ can be obtained from equation (10), where the pseudo input U(k) can be generated by the desired dynamics[43]:

$$\mathbf{u}(k) = \mathbf{g}_1(\mathbf{x}(k))^{-1}\left[\mathbf{U}(k) - \mathbf{f}_1(\mathbf{x}(k))\right] \quad (10)$$

Using the control input described in Eq. (10), the closed loop system dynamics would be linear ($\mathbf{x}(k+1) = \bar{\mathbf{I}}\,\mathbf{x}(k)$, $\bar{\mathbf{I}}$: identity matrix).

The fastest dynamic is used as the inner loop in the multiple time scale separation architecture therefore, the body angular rates are concerned as the inner loop. In this research, the airplane body axes rotational rates are used for NDI loop to control the damage airplane.

By assuming I_{xy} and I_{yz} to be zero and defining $\boldsymbol{\omega} = [p \quad q \quad r]^T$, the moment equations of motion are written as:

$$\dot{\boldsymbol{\omega}} = \mathbf{I}^{-1}\begin{bmatrix} L \\ M \\ N \end{bmatrix} - \mathbf{I}^{-1}\boldsymbol{\omega}\times(\mathbf{I}\boldsymbol{\omega}) \quad (11)$$

$$\boldsymbol{\omega}(k+1) = \left\{ \mathbf{I}^{-1}\begin{bmatrix} L(k) \\ M(k) \\ N(k) \end{bmatrix} - \mathbf{I}^{-1}\boldsymbol{\omega}(k)\times(\mathbf{I}\,\boldsymbol{\omega}(k)) \right\} Ts \quad (12)$$
$$+\boldsymbol{\omega}(k)$$

Based on nominal model of airplane and using Eq. (12) and Eq. (13), the control surfaces deflection commands can be derived according to Eq. (14)[3]:

$$\begin{bmatrix} C_l \\ C_m \\ C_n \end{bmatrix} = \begin{bmatrix} C_{l_0} + C_{l_\beta}\beta + C_{l_p}\dfrac{pb}{2V} + C_{l_r}\dfrac{rb}{2V} \\ C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\dot\alpha}}\dfrac{\dot\alpha\bar{c}}{2V} + C_{m_q}\dfrac{q\bar{c}}{2V} \\ C_{n_0} + C_{n_\beta}\beta + C_{n_p}\dfrac{pb}{2V} + C_{n_r}\dfrac{rb}{2V} \end{bmatrix}$$
$$+\begin{bmatrix} C_{l_{\delta a}} & 0 & C_{l_{\delta r}} \\ 0 & C_{m_{\delta e}} & 0 \\ C_{n_{\delta a}} & 0 & C_{n_{\delta r}} \end{bmatrix}\begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \end{bmatrix} \quad (13)$$
$$= \begin{bmatrix} C_{l_{states}} \\ C_{m_{states}} \\ C_{n_{states}} \end{bmatrix} + \begin{bmatrix} C_{l_{\delta a}} & 0 & C_{l_{\delta r}} \\ 0 & C_{m_{\delta e}} & 0 \\ C_{n_{\delta a}} & 0 & C_{n_{\delta r}} \end{bmatrix}\begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \end{bmatrix}.$$

$$\mathbf{u}(k) = F^{-1}\times\left\{ \frac{1}{\bar{q}S}\left(T_s^{-1}\mathbf{I}\,(\boldsymbol{\omega}_d(k+1) - \boldsymbol{\omega}_d(k)) + (\boldsymbol{\omega}_d(k)\times\mathbf{I}\boldsymbol{\omega}_d(k)) - \begin{bmatrix} L_T \\ M_T \\ N_T \end{bmatrix} \right) - \begin{bmatrix} bC_{l_{states}} \\ \bar{c}C_{m_{states}} \\ bC_{n_{states}} \end{bmatrix} \right\} \quad (14)$$

where:

$$\mathbf{u}(k) = \begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \end{bmatrix}, F = \begin{bmatrix} bC_{l_{\delta a}} & 0 & bC_{l_{\delta r}} \\ 0 & \bar{c}C_{m_{\delta e}} & 0 \\ bC_{n_{\delta a}} & 0 & bC_{n_{\delta r}} \end{bmatrix} \quad (15)$$

In equation (13), $\bar{c}$, S, b, are the mean aerodynamic chord, wing area and span, the subscripts T, d represent the thrust related parameters and desired input command, respectively. Aerodynamic coefficients variations, mass and inertia changes due to the airplane damage are considered as uncertainties. In fact, lack of robustness of the controller is leads to unknown plant dynamics. Hence, the closed loop dynamics is given in Eq.

$$\mathbf{x}(k+1) = \bar{\mathbf{I}}\,\mathbf{U}(k) + \Delta\mathbf{f}_1(\mathbf{x}(k), \mathbf{u}(k)). \quad (16)$$

where the inversion dynamic error is $\Delta\mathbf{f}_1(\mathbf{x}(k), \mathbf{u}(k))$.

Discrete-Time Adaptive Controller

Lack of robustness of NDI controllers against un-modeled dynamic is undeniable. In this section, the design procedure of discrete-time adaptive control strategy which improves the NDI controller performance is described. In the following, the design procedure of the discrete-time adaptive algorithm is just described for an arbitrary state ($\mathbf{x_j}$). According to Eq. (16) which describes the closed loop airplane dynamics with the NDI controller, the j^{th} state of the discrete-time-invariant close loop system (Eq. (16)) is considered as below:

$$x_j(k+1) = u_j(k) + \Delta_j(k). \quad (17)$$

By defining $u_j(k) = -k_m x_j(k) + u_{j_{ad}}(k)$, Eq. (17) is rewritten as Eq.(18) :

$$x_j(k+1) = -k_m x_j(k) + \Delta_j(k) + u_{j_{ad}}(k). \quad (18)$$

The z transform of Eq. (18) is introduced in Eq.(19)

$$x_j(z^{-1})(1 + k_m z^{-1}) = z^{-1}u_{j_{ad}}(z^{-1}) + z^{-1}\Delta_j. \quad (19)$$

In the above equations, k_m is the feedback gain and the design procedure is described as below:

By assuming Δ_j and x_j are bounded, Eq. (20) is obtained from infinity norm of Eq. (19) as below:

$$\|x_j\|(1 + k_m z^{-1}) = z^{-1}\|u_{j_{ad}}\| + z^{-1}\|\Delta_j\|. \quad (20)$$

If $\|\Delta_j\| < \rho_j$, $\|x_j\| < \mathbb{C}_j$ and $\dfrac{\rho_j}{\mathbb{C}_j} < D_j$, then $\|\Delta_j\| < \|x_j\| D_j$ is satisfied and Eq.(20) can be rewritten as following:

$$\|x_j\|(1 + (k_m - D_j)z^{-1}) = z^{-1}\|u_{j\,ad}\|. \quad (21)$$

For stability, k_m is chosen as the root of $(1 + (k_m - D_j)z^{-1})$, which must be smaller than unity.

The nominal j^{th} state of discrete linear time-invariant system x_{nj} is described as below:

$$x_{nj}(k+1) = -k_m\, x_{nj}(k) + u_{jad}(k).$$
$$x_{nj} = \frac{z^{-1}}{1-k_m z^{-1}} u_{jad}. \quad (22)$$

The reference model is defined as:

$$x_{mj}\left(z^{-1}\right) = z^{-1}\frac{\Gamma_j\left(z^{-1}\right)}{C_{1j}\left(z^{-1}\right)} r_j\left(z^{-1}\right). \quad (23)$$

where $C_{1j}(z^{-1})$ and $\Gamma_j(z^{-1})$ are the numerator and denominator of the reference model transfer function. $C_{1j}(z^{-1})$ is asymptotically stable, $r_j(k)$ is a bounded reference sequence. For tracking objective, the following equation should be satisfied[38]

$$C_{1j}\left(z^{-1}\right) x_{nj}\left(z^{-1}\right) = z^{-1}\Gamma_j\left(z^{-1}\right) r_j\left(z^{-1}\right). (24)$$

Moreover, the error equation is defined as:

$$e_j(k) = x_{nj}(k) - x_{mj}(k). \quad (25)$$

It is clear that the control objective is accomplished if the following equation holds[38].

$$\mathrm{C}_{2j}(z^{-1}) e_j(k+1) = 0, \qquad \mathrm{k} > 0. \ (26)$$

where $\mathrm{C}_{2j}(z^{-1})$ is an asymptotic stable polynomial as described in Eq. (27)

$$\mathrm{C}_{2j}(z^{-1}) = 1 + \gamma_{1j} z^{-1} + \gamma_{2j} z^{-2} + \ldots + \gamma_{nj} z^{-Nc_{2j}}. (27)$$

For regulation objective, the control system must reject any disturbance in the initial moment $(x_j(0) \neq 0)$. Therefore, the equation below must be satisfied[38].

$$\mathrm{C}_{2j}(z^{-1})\ x_j(k+1) = 0, \mathrm{k} \geq 0. \quad (28)$$

For a plant with unknown parameters, identification algorithm such as (RLS) is utilized, as it is implemented in this paper. Therefore, The estimated output will be computed as follows[44]:

$$\hat{x}_j(k) = \Phi_j(k)\hat{\Theta}_j(k)$$
$$\hat{x}_j(k) \to x_j(k) \quad (29)$$

where "^" represents the estimation parameters. In Eq. (28), $\hat{x}_j(k)$, $\Phi_j(k)$, and $\hat{\Theta}_j(k)$ represent the estimated outputs, regression vectors and estimated parameters, respectively, which are describe according to Eqs. (29) and (30) as below:

$$\hat{\Theta}_j(k) = \left[\hat{\mathrm{a}}_{1\mathrm{j}}(k), \hat{\mathrm{a}}_{2\mathrm{j}}(k), \ldots, \hat{\mathrm{a}}_{\mathrm{NAj}}(k), \hat{\mathrm{b}}_{0\mathrm{j}}(k), \ldots, \hat{\mathrm{b}}_{\mathrm{NBj}}(k)\right]. \quad (30)$$

$$\Phi_j(k) = \left[-x_{nj}(k-1), \ldots - x_{nj}(k-N_{Aj}), u_{jad}(k-1), \ldots, u_{jad}(k-N_{Bj})\right]. \quad (31)$$

According to Eq. (29) and using Eq.(25) the control law is obtained as follow:

$$\mathrm{u}_{jad}(k) = \frac{1}{\hat{b}_{0j}}\Big(\mathrm{C}_{2j}(z^{-1})x_{\mathrm{m}j}(k+1) - \mu_1(z^{-1})\hat{x}_j(k) - \mu_2(z^{-1})\mathrm{u}_{jad}(k)\Big). \quad (32)$$

where:

$$C_2(z^{-1}) = A(z^{-1})S(z^{-1}) + z^{-t_d}\mu_1(z^{-1}), Nc_2 = N_A + t_d - 1$$
$$S(z^{-1}) = 1 + s_1 z^{-1} + \ldots + s_{n_S} z^{-N_S}, N_S = t_d - 1$$
$$\mu_1(z^{-1}) = \lambda_0 + \lambda_1 z^{-1} + \ldots + \lambda_{N_{\mu_1}} z^{-N\mu_1}$$
$$N_{\mu_1} = max(\mathrm{N}_A - 1, Nc_2 - t_d)$$
$$\mu_2(z^{-1}) = B(z^{-1})S(z^{-1}) - b_0 \quad (33)$$

The block diagram architecture of the controller is shown below:

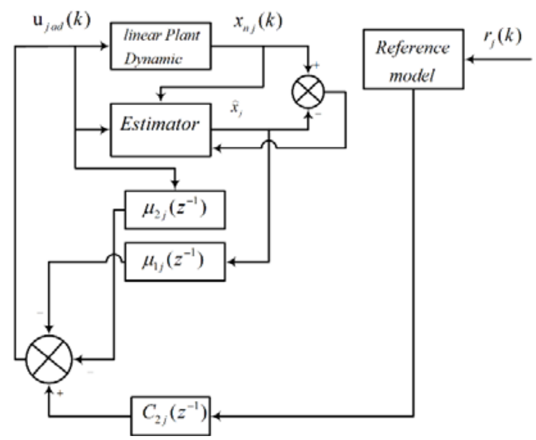

Figure 3. adaptive control architecture

Obviously, there is a deference between Eq. (21) and the actual airplane's output.

$$(34) \qquad \vec{\mathbf{x}}_{\mathbf{N}} - \vec{\mathbf{x}}_n = \vec{\mathbf{\Delta}}$$

where $\vec{\mathbf{x}}_{\mathbf{N}}$ is the output vector obtained from the airplane's output and $\vec{\mathbf{x}}_n$ is the output vector derived from the nominal linear equation of motion Eq. (21). Since the nominal linear model is used in the adaptive controller design, it is necessary to apply the vector $\vec{\mathbf{\Delta}} = [\Delta_1 \quad \Delta_2 \quad \Delta_3]^T$ in the control algorithm. In other words, the nonlinear equation of motion is rewritten as Eq. (35).

$$x_i = \frac{z^{-1}B_i(q^{-1})}{A_i(q^{-1})} u_{ad\,i} + \Delta_i \quad (35)$$
$$x_i(0) \neq 0, \quad \mathrm{i} = 1,2,3$$

According to Eqs. (26), (34) and (35), the input vector is obtained as Eq. (36).

$$
\mathrm{u}_{jad}(k) = \frac{1}{\hat{b}_{0j}}\Big(C_{2j}(z^{-1})x_{mj}(k+1) - \mu_{1j}(z^{-1})\hat{x}_j(k) - \mu_{2j}(z^{-1})\mathrm{u}_{jad}(k) + C_{2j}(z^{-1})\Delta_j\Big). \quad (36)
$$

A block diagram of the proposed control algorithm is shown in Figure 4.

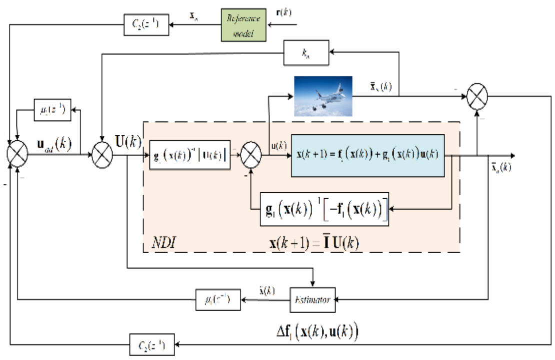

Figure 4. control architecture

Stability proof

The hyperstability concept mainly deals with the stability properties of standard feedback control systems which satisfies the below inequality[45]:

$$
\sum_{k=k_0}^{k_1} u^T(k)y(k) \geq -\gamma_0^2 \quad \text{for all } k_1 \geq k_0. \quad (37)
$$

Considering Popov's hyperstability theory, the necessary and sufficient condition for asymptotic stability of a general transfer function of a linear system $G(z^{-1})$ with a standard feedback controller, is that $G(z^{-1})$ must be strictly positive real. According to Eqs. (21), (25), (29), and (30) the output of the damaged airplane equation of motion is:

$$
\begin{aligned}
x_{pi}(k) &= \hat{b}_{0i}u_i(k) + \hat{b}_{1i}u_i(k-1) \\
&- \hat{a}_{1i}z^{-1}x_{pi}(k-1) - \hat{a}_{2i}z^{-2}x_{pi}(k-2) \\
&+ \varepsilon_i(k) + \Delta_i(k)
\end{aligned} \quad (38)
$$

where:

$$
x_{pi}(k) = \begin{cases} p(k) & if \quad i=1 \\ q(k) & if \quad i=2 \\ r(k) & if \quad i=3 \end{cases}
\qquad
u_i(k) = \begin{cases} T_x(k) & if \quad i=1 \\ T_y(k) & if \quad i=2 \\ T_z(k) & if \quad i=3 \end{cases} \quad (39)
$$

By using Eqs. (13), (31) ,(33), and (34) the error is obtained as below:

$$
\begin{cases}
e_1(k) = \rho_{11}e_1(k-1) + \rho_{12}e_1(k-2) + \Lambda_1(k) \\
e_2(k) = \rho_{21}e_2(k-1) + \rho_{22}e_2(k-2) + \Lambda_2(k) \\
e_3(k) = \rho_{31}e_3(k-1) + \rho_{32}e_3(k-2) + \Lambda_3(k)
\end{cases}. \quad (40)
$$

$$
\begin{cases}
G_1(z^{-1}) = \dfrac{e_1(k)}{\Lambda_1(k)} = \dfrac{1}{1-\rho_{11}z^{-1}-\rho_{12}z^{-2}} \\
G_2(z^{-1}) = \dfrac{e_2(k)}{\Lambda_2(k)} = \dfrac{1}{1-\rho_{21}z^{-1}-\rho_{22}z^{-2}} \\
G_3(z^{-1}) = \dfrac{e_3(k)}{\Lambda_3(k)} = \dfrac{1}{1-\rho_{31}z^{-1}-\rho_{32}z^{-2}}
\end{cases}. \quad (41)
$$

As a result of Popov's hyperstability theory, for asymptotic stability, $G_1(z^{-1})$, $G_2(z^{-1})$ and $G_3(z^{-1})$ must be strictly positive real. Therefore, following conditions must be satisfied: 1- the error transfer functions (Eq. (36)) must be positive real; 2- $\Lambda_1(k)$, $\Lambda_2(k)$ and $\Lambda_3(k)$ must be bounded. The stability of the estimation algorithm has been proved in Ref. [44]. The first condition of stability is achieved by appropriate selections of model reference (Eq.23) and $C_{2j}(z^{-1})$. Since the external disturbances, sensor noise, and the un-modeled dynamic are bounded and therefore, $\Lambda_1(k)$, $\Lambda_2(k)$ and $\Lambda_3(k)$ are consequently bounded.

Simulation

Numerical results show the performance of the designed discrete-time adaptive controller in three damage Scenarios. The properties of the closed-loop control system are compared with a conventional discrete-time MRAC strategy. All simulations are conducted on the nonlinear dynamics of GTM and a linear model of actuators with saturation and rate limit.

Flight Regime and Failure

Elevator, aileron, and rudder are used to control the airplane. The airplane is flying at an altitude of 1000 m and the velocity is *56 m/s*. The dynamics model of control surfaces is assumed to be the first-order transfer functions with rate limit of *50 deg/s*, time constant of 0.05 sec. (*T = 0.05*) and the saturation bound is, ± 25 deg. The actuator models has not been considered in controller design; therefore, the controller must be robust against the un-modeled actuator dynamics and the airplane damage. In severe damage Scenarios, other methods such as engine differential thrust may be used to control the airplane.

In this paper, just the basic control inputs ($[\delta_a\ \delta_e\ \delta_r]$) are used in all Scenarios. All Scenarios considers a 30% of left-wing damage at time instant of *t=10 sec*. Failure scenarios used in the simulations are listed as below:

- Scenario 1, stabilize body angular rates and maintain them at zero.
- Scenario 2, maintaining roll and yaw rates at zero and tracking the pitch rate input.
- Scenario 3, tracking the yaw and pitch rate commands and maintaining the roll rate at zero.
- In Scenario 1, and Scenario 2 and Scenario 3 there is a 30% damage on the left wing.

In this paper, the proposed adaptive algorithm is compared with a conventional discrete-time adaptive control algorithm which is indexed as MRAC2 and MRAC1, respectively.

Results

Figures 5-7 present the properties of the designed controller in Scenario 1 where the control system stabilizes the angular rates at zero in presence of structural damage. Figure 5 is the body angular rate responses. In Figure 6, airplane angles $[\theta\ \phi\ \beta]$ are illustrated versus time. Figure 7 depicts the control effort due to 30% wing tip lost where the roll performance is the mostly affected respecting others at *t=10 sec*. The performance of the designed strategy is far desirable than conventional MRAC results. Figures 8-10 represent airplane body rate responses, airplane body angular rates, angles, and control deflections during Scenario 2. Figure 8 illustrates that tracking the pitch rate command is successfully accomplished. The performance of the proposed algorithm in Scenario 3 is shown in figures 11-13. The body angular rates are shown in Figure 11. Figure 12 illustrates airplane angles and Figure 13 shows the control effort versus time. The simulation and numerical results illustrate the desirable performance of the introduced algorithm in comparison with the classical adaptive controller.

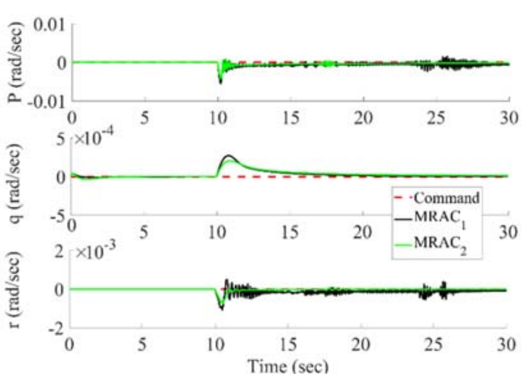

Figure 5. Body angular rates (Scenario 1).

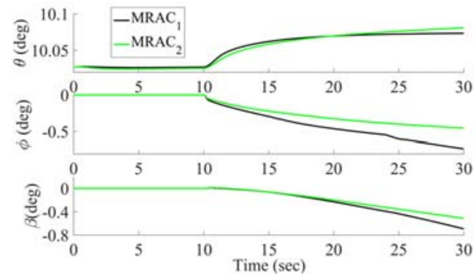

Figure 6. Body attitude angles (Scenario 1).

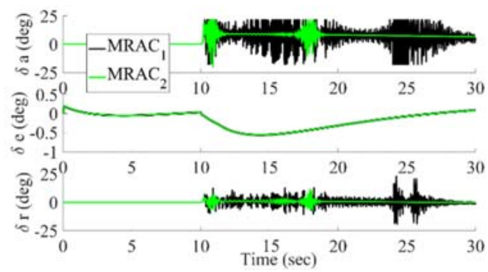

Figure 7. Control surface deflections (Scenario 1).

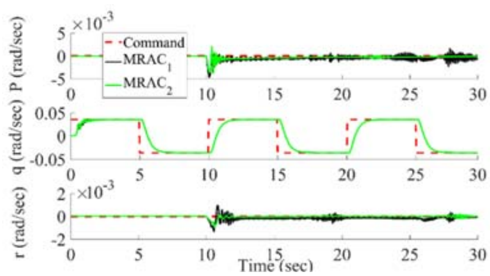

Figure 8. Body angular rates (Scenario 2).

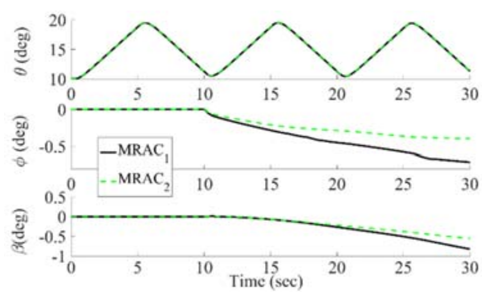

Figure 9. Body attitude angles (Scenario 2).

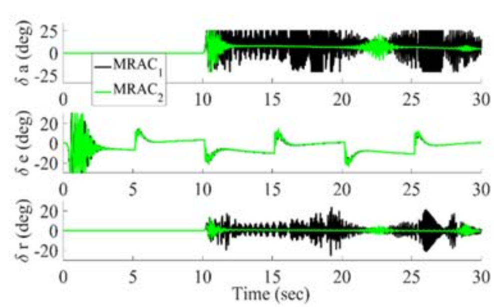

Figure 10. Control surface deflections (Scenario 2).

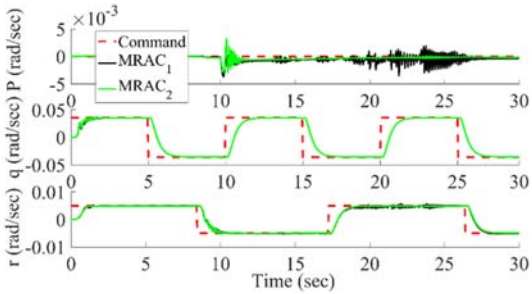

Figure 11. Body angular rates (Scenario 3).

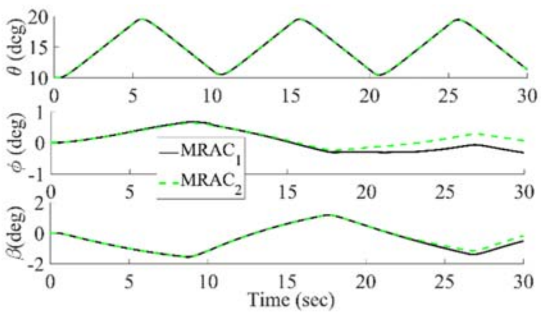

Figure 12. Body attitude angles (Scenario 3).

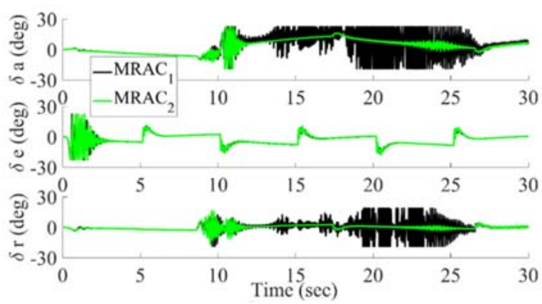

Figure 13. Control surface deflections (Scenario 3).

Conclusion

In this research, a novel discrete-time MRAC adaptive control strategy is introduced and developed to control the airplane in the presence of abrupt asymmetric damage and un-modeled actuators. In the nonlinear dynamics model, the effects of damage including mass and inertial, center of gravity, and aerodynamic coefficient variation have been accounted. The control algorithm is composed of NDI controller as a baseline, which is augmented with discrete-time adaptive control algorithm.

The simulation results demonstrate the effectiveness of the proposed algorithm in both regulating and tracking performance for controlling 30% of left-wing damaged airplane especially when compared with the classical discrete-time adaptive control algorithm. The controller satisfactorily accommodate wing damage disturbances and track the desired pitch and yaw angular rates while stabilizing the airplane roll channel.

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