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# A Comparison Study on Various Finite Element Models of Riveted Lap JointUsingDynamic Model Updating

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*Various models have been proposed in literature so far to simulate the behavior of riveted structures. In order to find the most accurate numerical method in modeling the dynamic behavior of riveted structures, a comparison study is performed on several of these models in this research. For this purpose, experimental modal analysis tests are conducted on a riveted plate to verify the efficacy of the numerical models. Moreover, finite element model updating is used to reduce the difference between analytical and experimental results. First, the material properties of plates are optimized using the experimental results obtained from modal tests of a simple plate. Next, optimization is performed for the physical properties of the rivet. In the end, it is concluded the fastener model proposed by Rutman can bring about the most accurate results when it is used in combination with solid plates and gap elements in the contact region.*

**Keywords:** Rivet, Riveted plates, Modal parameters, Finite Element Model Updating

## Introduction

Engineering structures are usually are comprised of several subsets which are connected by various types of joints like weld, bolts and rivets.The riveted joint has turned out to be one of the popular joint types in jointing the aerospace structures because it can be easily disassembled, maintained and inspected.

Identification of the characteristics of such joints in assembling structures is of particular interest and many studies are conducted to evaluate and simulatethe dynamiceffectsof such joints [1-5]. Moreover, Finite Element (FE) model updating has been widely used for precise simulation of jointed structures [6-10].

Especially the subparts of many aerospace structures, such as wings and fuselage, consist of many riveted joints which can influence the dynamic behavior of the whole structure dramatically. Consequently, identifying an accurate model to simulate the behavior of riveted joints has become a necessary area of inquiry which has also broadened the use of FE model updating (Douradoand Meireles [11]). Experimental and numerical analysis and also model updating of the dissimilar plate with rivet joint was done by Rahman et al. [12].

So far many FE models have been proposed for analytical simulation of rivets and riveted joints. One of such models is the use of rigid elements with infinite stiffness as rivets [13-16].Moreover, the use of beam and spring elements is a common way of rivet modeling (Naarayan et al. [15] and Brewer[16]). 3D solid modeling of rivets along with the connected structures is also one of the most common methods, particularly in the case of static or stress analyses (Atre and Johnson [17]).The dynamic

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characteristic of joined riveted assembly structure can be significantly affected by these joints due to local effects in the mating areas of the riveted joints such as surface contact, clamping force and slips (Yones et al.[18]).A hierarchical model updating strategy based on Bayesian inference applied to identify the unknown parameters in the typical bolt-jointed structure models is investigated by Zhan et al.[19]

In some models, the upper and lower caps of rivets, as well as the rivet shank which is modeled by a spring or beam elements, are includedin the modeling [20,21]. Rutmanet al.[22]proposed a particular methodfor simulating the behavior of fasteners which can be utilized for riveted joints as well.The model has been successfully implemented as a utility in Nastran software. Dourado and Meireles [11] suggested a simple method in which the rivet is replaced by eight combination elements. Shokrollahi and Adel introduced the new concept of thebolted joint affected region to simulate the dynamical behavior of bolted lap joints [20].

With such a variety in the proposed models, a comparison study between the characteristic, especially the dynamic characteristics, predicted by each model seems necessary. The results of such study canhelp engineers to find an accurate yet simple model for simulating the dynamic behavior of riveted lap joints.Thus, in this research, the dynamic characteristics predicted by each of the proposed FE modelsare evaluated and compared with experimental tests.

For this purpose, modal tests have been conducted on a plate connected by rivets to two other plates. The modal parameters resulted from FE analysis of each rivet model are compared with the experimental test. FE model updating was used in order to reduce the difference between the FE results and experimental tests. Therefore, the results obtained using the test of simple plate were used to optimize the material properties of the plates. Furthermore, updating the rivet model was also performed for various cases. To sum up, finding the most accurate method in modeling the riveted joint is pursued in this research.

## Experimental Tests

Two impact hammer modal tests are conducted separately. The first experiment is performed on a simple plate while in the other experiment,two riveted platesare tested. Both experiments are conducted in a free-free boundary condition and each structureis suspended by two rubber bands. OnePCB accelerometer sensor and a five channel B&K analyzer are usedfor data acquisition. Pulse Lab Shopsoftware is selected as the user interface for defining the test setupand also calculating the Frequency Response Functions (FRFs) resulted from the tests. The extracted FRFs are imported into STAR software for modal identification purposes.

### Test of Simple Plate

The experiment is conducted on a 200mm by 500mm rectangular aluminum plate with a thickness of 2mm. 91 points with equal spacing are marked on the plate as the location of hammer impacts. The accelerometer sensor is fixed at a corner of the plate during the test. The structure is suspended by rubber bands.Averaging is also performed to reduce the noise effect and each impact is repeated 5 times.

The modal parameters are identified by the automatic curve fitting option of the STAR software. The extracted modal parameters can be seen in Table 1.

### Test of Riveted Plates

The second experiment is conducted on two 250 by 200mm rectangular aluminum plates which are connected to each other by four aluminum rivets. The thickness of the plates is 2mm and the diameter of each rivet is 5mm. Fig.1 shows a drawing of the tested structure (dimensions are in millimeter).

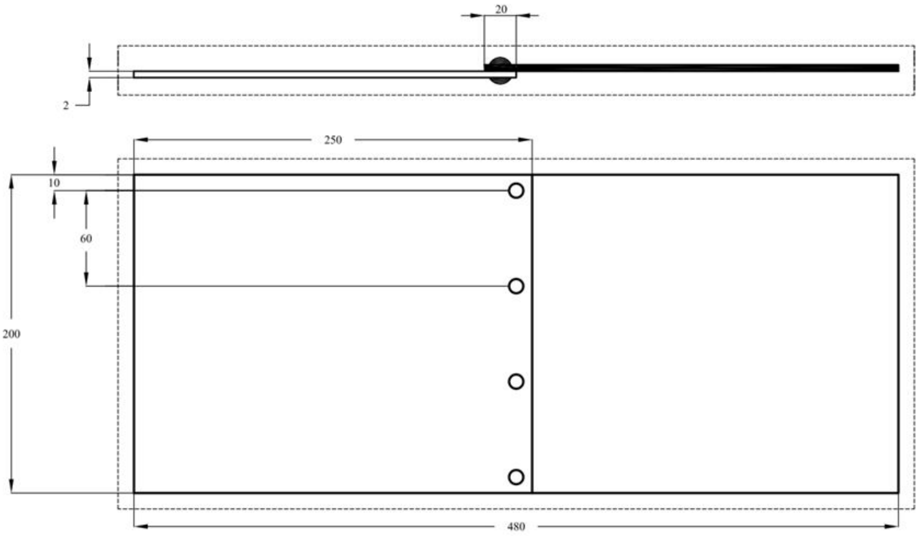

**Fig. 1**The geometry of the riveted lap joint 172×110mm (96 x 96 DPI)

**Fig. 2**Setup of experimental modal test conducted on the riveted lap joint 165x151mm (220 x 220 DPI)

In this section, the test setup and settings are like before, except for the fact that 87 points are marked on the structure as the locations through which hammer impacts act (Fig. 2).

## Finite Element Model of Simple Plate (Comparison and Updating)

In this section, the modal parameters of the tested aluminum plate, described in section 2.1, are calculated using MSC Nastran FE software. The plate is modeled by 5mm isotropic QUAD shell elements. The material properties considered for Aluminum are as follows

$$E = 70\ Gpa,\ \upsilon = 0.33, \qquad \rho = 2700\frac{kg}{m^3}$$

The numerical natural frequencies, shown in Table 1,are obtained using normal modesolution of Nastran.

The difference between the experimental results and the analytical results obtained form the simple plate can be decreased with the help of

FE model updating.For this purpose, FEMtools software is utilized. The material properties of Aluminum are selected as the updating parameters. The optimized properties of the materialare calculated as follows

$$E = 69.998\,Gpa,\ \upsilon = 0.3925, \quad \rho = 2932.28\frac{kg}{m^3}$$

Table 1presents the effect of the executed updating on the natural frequencies calculated through FE analysis.

**Table 1.** Results of FE model updating for the simple plate

| **Mode Number** | **Experimental Freq. (Hz)** | **Numerical Freq. (Hz) (before updating)** | **Difference (before updating)** | **Numerical Freq. (Hz) (after updating)** | **Difference (after updating)** |
|---|---|---|---|---|---|
| 1 | 41.42 | 42.12 | 1.67 | 40.50 | 2.21 |
| 2 | 60.24 | 63.86 | 6.02 | 60.00 | 0.39 |
| 3 | 114.64 | 117.19 | 2.22 | 113.11 | 1.36 |
| 4 | 129.70 | 137.36 | 5.91 | 129.69 | 0.01 |
| 5 | 213.83 | 230.04 | 7.60 | 218.54 | 2.20 |
| 6 | 269.87 | 277.64 | 2.88 | 273.43 | 1.32 |
| 7 | 293.21 | 288.68 | 2.22 | 289.89 | 1.93 |
| 8 | 331.00 | 350.31 | 5.87 | 334.73 | 1.13 |
| 9 | 369.08 | 384.45 | 4.22 | 371.25 | 0.59 |

It can be deduced that updating has decreased the difference between the analytical and experimental natural frequencies calculated for the simple plate. The conducted material updating, as the first step in model updating, can come into use in the FE analysis of riveted plates. The conducted material updating can also reduce the difference between the analytical and experimental models of riveted lap joint. Therefore, it can be assumed that the remaining difference originates from errors in rivet modeling, which is going to be removed in the second step of updating and will be discussed later.

# Finite Element Modeling of Riveted Lab Joint and Updating

In this section, various FE models for rivet are investigated. For each model, the calculated natural frequencies are studied for initial and updated modeling.Updating is conducted in two steps. The first step is based on the results obtained from material updating which is performed in the previous section. The second step of updating is performed for the parameters that depend on the rivet modeling.

## MPC

Using MPC is one of the easiest ways in simulating rivet behavior [13-16]. Therefore, an FE model is generated in MSC Patran to make use of this modeling. For this purpose, plates are modeled again with the use of 5 by 5mm QUAD shell elementsand four RBE2 are used at the lab joint to play the role of rivets. The natural frequencies obtained from this modeling are presented in Table2.

In this modeling, there is no parameter, related to the rivet model, to be updated. Therefore, updating approach is only limited to the material properties, which is conducted once before.

**Table 2.** Natural frequencies of riveted lap joint obtained by MPC modeling

| Mode Number | Experimental Freq. (Hz) | Numerical Freq. (Hz) (before updating) | Difference (before updating) | Numerical Freq. (Hz) (after material updating) | Difference (%) (after material updating) |
|---|---|---|---|---|---|
| 1 | 43.38 | 38.44 | 11.39 | 37.08 | 14.53 |
| 2 | 63.74 | 68.40 | 7.32 | 64.29 | 0.86 |
| 3 | 122.23 | 127.20 | 4.07 | 122.88 | 0.53 |
| 4 | 129.61 | 136.43 | 5.26 | 128.75 | 0.66 |
| 5 | 226.27 | 245.58 | 8.54 | 233.62 | 3.25 |
| 6 | 282.13 | 291.14 | 3.20 | 285.88 | 1.33 |
| 7 | 300.38 | 304.40 | 1.34 | 294.40 | 1.99 |
| 8 | 337.31 | 345.2 | 2.34 | 329.53 | 2.31 |
| 9 | 406.14 | 403.89 | 0.55 | 386.86 | 4.75 |
| 10 | 415.45 | 421.31 | 1.41 | 411.78 | 0.88 |

### Beam

As stated before, using a beam element instead of each rivet is a common way of modeling this type of connection. Thus, in a similar manner as described in the previous section (MPC) the FE model is generated, except for the fact that rivets are modeled by beam elements. Therefore, each rivet is modeled by a beam with a circular cross section with 5mmdiameter.The results achieved by this type of modeling are described in Table 3.

**Table 3.** Natural frequencies of riveted lap joint obtained by beam modeling

| Mode Number | Experimental Freq. (Hz) | Numerical Freq. (Hz) (before updating) | Difference (before updating) | Numerical Freq. (Hz) (after material updating) | Difference (%) (after material updating) | Numerical Freq. (Hz) (after rivet updating) | Difference (%) (after rivet updating) |
|---|---|---|---|---|---|---|---|
| 1 | 43.38 | 40.56 | 6.52 | 39.99 | 7.81 | 39.38 | 9.22 |
| 2 | 63.74 | 68.40 | 7.32 | 64.27 | 0.84 | 64.28 | 0.86 |
| 3 | 122.23 | 127.22 | 4.08 | 122.93 | 0.57 | 122.91 | 0.56 |
| 4 | 129.61 | 136.36 | 5.21 | 129.01 | 0.64 | 128.82 | 0.61 |
| 5 | 226.27 | 245.85 | 8.66 | 233.29 | 3.10 | 233.62 | 3.25 |
| 6 | 282.13 | 292.03 | 3.51 | 294.57 | 4.41 | 287.98 | 2.08 |
| 7 | 300.38 | 304.55 | 1.39 | 295.53 | 1.62 | 294.67 | 1.90 |
| 8 | 337.31 | 345.81 | 2.52 | 331.70 | 1.66 | 330.35 | 2.06 |
| 9 | 406.14 | 408.00 | 0.46 | 426.10 | 4.91 | 395.96 | 2.51 |
| 10 | 415.45 | 421.42 | 1.44 | 411.94 | 0.84 | 412.05 | 0.82 |

Furthermore, for this type of modeling, the material properties of beam elements can be good candidates for updating. Moreover, since the rivet caps are not included in the modeling, selecting the moment of inertia of the beam cross section as the updating parameter can somehow compensate for the overlooked bending stiffness.The updated parameters for beam model are shown below.

$$\text{Density} = 2808.38\ \frac{kg}{m^3}\ ,\ \text{Poisson Ratio} = 0.40,$$
$$\text{Young Modulus } 75874.2\ \text{GPa}, \text{Moment of Inertia} = 47.80\ mm^4$$

### Spring

As it is suggested in the literature, spring elements are good candidates for modeling rivets. Moreover, it is suggested to use the following equation for calculating spring stiffness (Naarayan, 2009).

$$K_s = \frac{Ed}{B+C\left(\frac{d}{t_1}+\frac{d}{t_2}\right)}$$

In which E represents the Young modulus of plate material, d rivet diameter, and $t_1$ and $t_2$ the thickness of the upper and lower plates, respectively. For aluminum rivet, B is 0.5 and C equals 0.8. Since the spring elements do not exhibit any bending stiffness on their own, rotational stiffness values need to be defined for the springs. Therefore,the two plates are again modeled by the use of shell elements and are connected to each other at the rivet locationsby fourCBUSH elements which are a combination of axial and rotational springs. For this element, type axial and rotational stiffness values can be defined along different axes.Table 4 shows the natural frequencies obtained through CBUSH modeling.

For this modeling approach, the rotational and axial stiffness of the CBUSH elements can be chosen for updating. The updated values for these parameters are exhibited below.

$$\text{Axial Stiffness} = \ 6893920.0\frac{KN}{m},$$
$$\text{Bending Stiffness} = 76272.2\ \frac{KN.m}{rad}$$

**Table 4**. Natural frequencies of riveted lap joint obtained by spring modeling

| Mode Number | Experimental Freq. (Hz) | Numerical Freq. (Hz) (before updating) | Difference (before updating) | Numerical Freq. (Hz) (after material updating) | Difference (%) (after material updating) | Numerical Freq. (Hz) (after rivet updating) | Difference (%) (after rivet updating) |
|---|---|---|---|---|---|---|---|
| 1 | 43.38 | 40.79 | 5.97 | 39.31 | 9.39 | 39.92 | 7.99 |
| 2 | 63.74 | 68.40 | 7.32 | 64.29 | 0.86 | 64.29 | 0.86 |
| 3 | 122.23 | 127.15 | 4.03 | 122.79 | 0.46 | 122.81 | 0.48 |
| 4 | 129.61 | 136.78 | 5.53 | 129.13 | 0.37 | 129.48 | 0.10 |
| 5 | 226.27 | 246.04 | 10.03 | 233.80 | 3.30 | 233.91 | 3.38 |
| 6 | 282.13 | 277.58 | 1.61 | 273.32 | 3.12 | 273.58 | 3.03 |
| 7 | 300.38 | 303.65 | 1.09 | 293.78 | 2.20 | 294.02 | 2.12 |
| 8 | 337.31 | 346.85 | 2.83 | 331.26 | 1.79 | 333.32 | 1.18 |
| 9 | 406.14 | 378.77 | 6.74 | 363.41 | 10.52 | 364.75 | 10.19 |
| 10 | 415.45 | 421.55 | 1.47 | 411.80 | 0.88 | 412.04 | 0.82 |

## Fastener

Two utilities are available in Patran for modeling fasteners which can be used for simulating rivet behavior as well. One of these utilities, which is developed by Rutman, will be discussed in the next section. In order to use the other utility, plates need to be modeled with the use of shell elements. Thus, the previous modeling utilized for plates and rivets is replaced by the fastener model in the utility. Table 5 presents the natural frequencies obtained using this modeling.

**Table 5.** Natural frequencies of riveted lap joint obtained by fastener modeling

| Mode Number | Experimental Freq. (Hz) | Numerical Freq. (Hz) (before updating) | Difference (before updating) | Numerical Freq. (Hz) (after material updating) | Difference (%) (after material updating) | Numerical Freq. (Hz) (after rivet updating) | Difference (%) (after rivet updating) |
|---|---|---|---|---|---|---|---|
| 1 | 43.38 | 40.92 | 5.67 | 40.38 | 6.93 | 40.34 | 7.02 |
| 2 | 63.74 | 68.40 | 7.32 | 64.27 | 0.84 | 64.27 | 0.84 |
| 3 | 122.23 | 127.20 | 4.07 | 122.89 | 0.54 | 122.85 | 0.51 |
| 4 | 129.61 | 136.43 | 5.26 | 129.11 | 0.39 | 129.24 | 0.29 |
| 5 | 226.27 | 245.85 | 8.66 | 233.29 | 3.10 | 233.29 | 3.10 |
| 6 | 282.13 | 289.01 | 2.44 | 292.94 | 3.83 | 290.81 | 3.08 |
| 7 | 300.38 | 304.30 | 1.30 | 295.27 | 1.70 | 295.00 | 1.79 |
| 8 | 337.31 | 346.19 | 2.63 | 332.65 | 1.38 | 332.94 | 1.30 |
| 9 | 406.14 | 399.56 | 1.62 | 411.68 | 1.36 | 406.22 | 0.02 |
| 10 | 415.45 | 421.29 | 1.41 | 416.23 | 0.18 | 411.42 | 0.97 |

In this model, some rotational and axial springs, as well as beam elements, are used for rivet simulation. Therefore, springs stiffness values and material properties of beam are selected for updating.

## Rutman Fastener

In this section, the fastener modeling proposed by Rutman et al.[22]is investigated for dynamic studies. In order to use the mentioned simulation, plates are needed to be modeled using solid elements. Thus, HEX8 elements used to mesh the plates and four Rutman fasteners are used in lieu of rivets. Table 6 shows the results obtained through this modeling.

**Table 6.** Natural frequencies of riveted lap joint obtained by Rutman fastener modeling

| Mode Number | Experimental Freq. (Hz) | Numerical Freq. (Hz) (before updating) | Difference (before updating) | Numerical Freq. (Hz) (after material updating) | Difference (%) (after material updating) | Numerical Freq. (Hz) (after rivet updating) | Difference (%) (after rivet updating) |
|---|---|---|---|---|---|---|---|
| 1 | 43.38 | 38.78 | 10.61 | 37.61 | 13.54 | 39.59 | 8.75 |
| 2 | 63.74 | 68.50 | 7.47 | 64.37 | 1.00 | 64.40 | 1.04 |
| 3 | 122.23 | 127.23 | 4.09 | 122.89 | 0.54 | 123.01 | 0.64 |
| 4 | 129.61 | 135.76 | 4.74 | 128.13 | 1.14 | 129.16 | 0.35 |
| 5 | 226.27 | 246.35 | 8.88 | 234.09 | 3.46 | 234.20 | 3.51 |
| 6 | 282.13 | 287.55 | 1.92 | 282.68 | 0.20 | 283.79 | 0.59 |
| 7 | 300.38 | 304.54 | 1.38 | 294.58 | 1.93 | 295.28 | 1.70 |
| 8 | 337.31 | 341.66 | 1.29 | 326.23 | 3.28 | 331.81 | 1.63 |
| 9 | 406.14 | 395.33 | 2.66 | 379.02 | 6.68 | 382.98 | 5.70 |
| 10 | 415.45 | 422.05 | 1.59 | 412.51 | 0.71 | 413.23 | 0.53 |

Again for this model, some rotational and axial springs, as well as beam elements, are used for rivet simulation. Therefore, springs stiffness values and material properties of the beam can be selected for updating.

### Eight Spring Elements

As proposed in Dourado and Meireles [11], using eight spring elements for each rivet can be an efficient method for modeling rivets which are evaluated in this section.For this purpose,shell elements are used for modeling the plate structures. It is also suggested in Dourado and Meireles [11] that the axial stiffness of each spring should be set at about 275 KN/m.

The stiffness values of the springs are chosen as updating parameters and the results are exhibited in Table 7. Moreover, the updated value for spring stiffness is calculated as 386KN/m.

**Table 7.** Natural frequencies of riveted lap joint obtained by eight spring modeling

| Mode Number | Experimental Freq. (Hz) | Numerical Freq. (Hz) (before updating) | Difference (before updating) | Numerical Freq. (Hz) (after material updating) | Difference (%) (after material updating) | Numerical Freq. (Hz) (after rivet updating) | Difference (%) (after rivet updating) |
|---|---|---|---|---|---|---|---|
| 1 | 43.38 | 41.70 | 3.89 | 40.07 | 7.64 | 40.07 | 7.63 |
| 2 | 63.74 | 68.29 | 7.14 | 64.19 | 0.72 | 64.19 | 0.72 |
| 3 | 122.23 | 127.19 | 4.06 | 122.85 | 0.51 | 122.86 | 0.52 |
| 4 | 129.61 | 137.37 | 5.99 | 129.72 | 0.08 | 129.72 | 0.08 |
| 5 | 226.27 | 245.80 | 8.63 | 233.75 | 3.31 | 233.75 | 3.31 |
| 6 | 282.13 | 278.08 | 1.43 | 273.98 | 2.89 | 273.98 | 2.89 |
| 7 | 300.38 | 303.87 | 1.16 | 294.24 | 2.05 | 294.24 | 2.05 |
| 8 | 337.31 | 349.84 | 3.71 | 334.20 | 0.92 | 337.32 | 0.00 |
| 9 | 406.14 | 380.45 | 6.32 | 365.32 | 10.05 | 365.62 | 9.98 |
| 10 | 415.45 | 421.75 | 1.52 | 412.24 | 0.77 | 412.24 | 0.77 |

### Cap and Beam

In the rivet models which are discussed so far, the lower and upper caps were not included in the modeling. It is expected that inserting the caps of rivets into modeling adds to the accuracy. For this purpose, it is suggested to model the rivet caps with the use of two circular shells with the same thickness as the plates (Wronicz and Kaniowski [21]). The rivet shank can again be modeled by beam or spring elements. In this section, beam elements with 5mm diameters, are used as the rivet shanks. The plates are also modeled with the help of shell elements. The modal parameters extracted using this modeling are exhibited in Table 8.

**Table 8.** Natural frequencies of riveted lap joint obtained by beam and cap modeling

| Mode Number | Experimental Freq. (Hz) | Numerical Freq. (Hz) (before updating) | Difference (before updating) | Numerical Freq. (Hz) (after material updating) | Difference (%) (after material updating) | Numerical Freq. (Hz) (after rivet updating) | Difference (%) (after rivet updating) |
|---|---|---|---|---|---|---|---|
| 1 | 43.38 | 39.63 | 8.64 | 38.20 | 11.94 | 38.39 | 11.52 |
| 2 | 63.74 | 68.68 | 7.75 | 64.29 | 0.87 | 64.29 | 0.87 |
| 3 | 122.23 | 127.23 | 4.09 | 122.93 | 0.57 | 122.93 | 0.57 |
| 4 | 129.61 | 135.66 | 4.67 | 127.93 | 1.30 | 128.04 | 1.21 |
| 5 | 226.27 | 246.55 | 8.96 | 233.36 | 3.14 | 233.37 | 3.14 |
| 6 | 282.13 | 284.84 | 0.96 | 292.40 | 3.64 | 292.70 | 3.75 |
| 7 | 300.38 | 304.32 | 1.31 | 295.45 | 1.64 | 295.49 | 1.63 |
| 8 | 337.31 | 342.67 | 1.59 | 326.68 | 3.15 | 327.17 | 3.01 |
| 9 | 406.14 | 388.89 | 4.24 | 416.94 | 2.66 | 416.49 | 2.55 |
| 10 | 415.45 | 421.46 | 1.45 | 411.84 | 0.87 | 411.88 | 0.86 |

For this type of modeling, the material properties of the beam and rivet caps could be selected for optimization.

## Spring and Caps

The modeling approach in this section is similar to what weout lined in the previous section. Except, the rivet shanks are modeled with CBUSH elements which can exhibit some level of bending stiffness. The material properties of the rivet caps and the spring stiffness values are available for updating. Model updating yields only slight changes in rivet material properties and spring stiffness in comparison to the values obtained in spring modeling. The natural frequencies obtained from this modeling approach are depicted in Table 9.

**Table 9.** Natural frequencies of riveted lap joint obtained by spring and cap modeling

| Mode Number | Experimental Freq. (Hz) | Numerical Freq. (Hz) (before updating) | Difference (before updating) | Numerical Freq. (Hz) (after material updating) | Difference (%) (after material updating) | Numerical Freq. (Hz) (after rivet updating) | Difference (%) (after rivet updating) |
|---|---|---|---|---|---|---|---|
| 1 | 43.38 | 39.96 | 7.88 | 38.50 | 11.26 | 38.21 | 11.93 |
| 2 | 63.74 | 68.68 | 7.75 | 64.55 | 1.28 | 64.31 | 0.90 |
| 3 | 122.23 | 127.21 | 4.08 | 122.86 | 0.52 | 122.85 | 0.51 |
| 4 | 129.61 | 136.09 | 5.00 | 128.44 | 0.90 | 128.35 | 0.97 |
| 5 | 226.27 | 246.70 | 0.27 | 234.44 | 3.61 | 233.99 | 3.41 |
| 6 | 282.13 | 277.93 | 1.49 | 273.61 | 3.02 | 273.13 | 3.19 |
| 7 | 300.38 | 304.01 | 1.21 | 294.09 | 2.10 | 294.19 | 2.06 |
| 8 | 337.31 | 344.15 | 2.03 | 328.52 | 2.61 | 327.95 | 2.77 |
| 9 | 406.14 | 377.96 | 6.94 | 362.27 | 10.80 | 361.80 | 10.92 |
| 10 | 415.45 | 421.91 | 1.56 | 412.20 | 0.78 | 412.23 | 0.77 |

### 3D modeling

The most common method, especially for static analyses, in modeling riveted structures is the use of 3D modeling approach for rivets and surfaces. Therefore, the 3D model of the tested structure is generated in commercial CAD software. The model is imported into MSC Patranand is meshed by TET10 elements. The calculated natural frequencies are represented in Table 10.

**Table 10.** Natural frequencies of riveted lap joint obtained by 3D modeling

| Mode Number | Experimental Freq. (Hz) | Numerical Freq. (Hz) (before updating) | Difference (before updating) | Numerical Freq. (Hz) (after material updating) | Difference (%) (after material updating) | Numerical Freq. (Hz) (after rivet updating) | Difference (%) (after rivet updating) |
|---|---|---|---|---|---|---|---|
| 1 | 43.38 | 43.97 | 1.35 | 42.59 | 1.83 | 42.58 | 1.84 |
| 2 | 63.74 | 69.20 | 8.57 | 65.04 | 2.05 | 65.04 | 2.05 |
| 3 | 122.23 | 127.66 | 4.44 | 123.40 | 0.96 | 123.40 | 0.96 |
| 4 | 129.61 | 138.78 | 7.07 | 131.08 | 1.13 | 131.07 | 1.13 |
| 5 | 226.27 | 250.06 | 10.52 | 237.68 | 5.04 | 237.68 | 5.04 |
| 6 | 282.13 | 299.00 | 5.98 | 293.50 | 4.03 | 293.48 | 4.02 |
| 7 | 300.38 | 307.12 | 2.24 | 296.85 | 1.18 | 296.85 | 1.18 |
| 8 | 337.31 | 360.76 | 6.95 | 345.14 | 2.32 | 345.11 | 2.31 |
| 9 | 406.14 | 426.24 | 4.94 | 413.79 | 1.88 | 413.74 | 1.87 |
| 10 | 415.45 | 429.76 | 3.44 | 417.59 | 0.52 | 417.59 | 0.52 |

For this modeling, the material properties of the rivets are available for optimization and they changed a little after updating as indicated below.

$$\text{Density} = 2977.02\frac{kg}{m^3}\ ,\ \text{Poisson Ratio} = 0.39,\qquad \text{Young Modulus} = 69512.60\ \text{GPa}$$

## More Discussion

In the previous section, various modeling approaches for riveted lap joints were investigated. For each model, updating was performed in two steps. The first step involved the optimization of material properties based on modal test conducted on the simple plate. The second step of updating involved the rivet modeling. An in-depth study of the natural frequencies obtained using various modeling approaches reveals that the first updating step had a more dramatic effect on calculated modal parameters. While for almost every model, the difference between the experimental results and numerical results had not gone through a drastic change after the second step of model updating.

Moreover, a comparison between the beam and spring models and the equivalent models with cap reveals that considering the rivet caps in the modeling does not add anypalpable benefits to the accuracy of the obtained results.

Furthermore, in the models discussed so far, there are some major problems in the identified mode shapes. In other words, for some mode shapes, the plate elements cross into each other in the contact region. Figs.3 and 4 represent such problem in some of the identified mode shapes in various models. This issue highlights the weakness of the present modeling approach since the modeled joint is not modeled as stiff as the real structure.

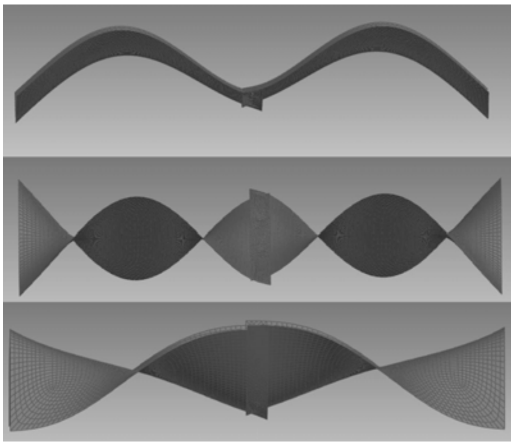

**Figure 3.** Defected mode shapes of the riveted lap joint obtained in models with shell plates 230x201mm (96 x 96 DPI)

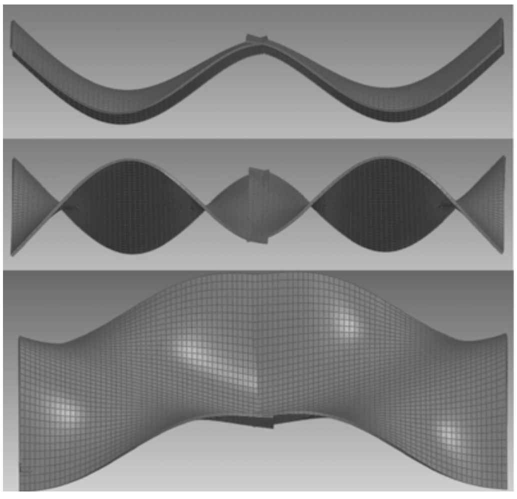

**Figure 4.** Defected mode shapes of the riveted lap joint obtained by Rutman fastener model 240x230mm (96 x 96 DPI)

Moreover, the nonlinear nature of contact phenomena and the accompanying phenomena like friction cannot be included in the linear normal mode solution. Gap elements are available in Patran software and they can be regarded as easy tools to avoid the mentioned problem.Gap elements behave like springs except that they only show stiffness under compression loading. These elements do not show any resistance to tension. Utilizing gap elements in the contact region of the plates can prohibit the entrance of plate elements into each other (as depicted in Figs.3 and 4). It should be noted that for the use of gap elements, plates should be modeled using solid elements because contact happensin this type of modeling only.

## FE Model with Gap Elements

### Gap and Rutman Fastener

For this modeling, Rutman fastener model is used along with the plates which are modeled by solid HEX8 elements like before. However, gap elements are used to connect the two plates in their contact region. Unfortunately, FEM tools are not able to import the gap elements as they are. In FEMtools, gap elements are imported as simple springs. Therefore, the properties of the rivet model cannot be updated. The updated material properties are used in the modeling and the obtained natural frequencies are as in Table 11.

### Gap and Beam

As it has been stated before, gap elements can only be used when the connected plates are modeled with solid elements. Therefore, it is expected that the solid modeling yieldsmore precise values for identifying modal parameters. Thus, beam model is again investigated for the case of connected plates modeled with solid gap elements. For this purpose, the beam properties are inserted as in section 4.2, which result from updating. The natural frequencies obtained using this modeling are represented in Table 11.

### Spring and Gap

In this section, the details of modeling are as described in the previous section. Except for the point that spring elements with stiffness values presented in section4.3 are used to model the rivets. The resulted natural frequencies are shown in Table 11.

**Table 11.** Natural frequencies obtained for the riveted lap joint by using gap elements in Rutman fastener, beam and spring models

| Mode Number | Experimental Freq. (Hz) | Numerical Freq. (Rutman fastener) | Difference (%) | Numerical Freq. (Beam) | Difference (%) | Numerical Freq. (Spring) | Difference (%) |
|---|---|---|---|---|---|---|---|
| 1 | 43.38 | 43.83 | 1.02 | 43.54 | 0.37 | 43.58 | 0.46 |
| 2 | 63.74 | 64.76 | 1.60 | 66.20 | 3.86 | 66.20 | 3.86 |
| 3 | 122.23 | 123.06 | 0.68 | 122.22 | 0.01 | 122.22 | 0.01 |
| 4 | 129.61 | 131.56 | 1.50 | 133.72 | 3.17 | 133.91 | 3.32 |
| 5 | 226.27 | 234.46 | 3.62 | 237.45 | 4.94 | 237.46 | 4.95 |
| 6 | 282.13 | 289.12 | 2.48 | 280.38 | 0.62 | 280.60 | 0.54 |
| 7 | 300.38 | 295.52 | 1.62 | 293.33 | 2.35 | 293.33 | 2.34 |
| 8 | 337.31 | 345.51 | 2.43 | 347.21 | 2.93 | 347.66 | 3.07 |
| 9 | 406.14 | 399.12 | 1.73 | 392.70 | 3.31 | 393.49 | 3.12 |
| 10 | 415.45 | 413.64 | 0.44 | 406.00 | 2.27 | 406.08 | 2.56 |

## Conclusions

In this research, the FE modeling and updating approach for riveted lap joints was investigated. As it was stated before, the main objective of this study was to identify an easy and accurate method for modeling the dynamic behavior of riveted structures.

For this purpose, experimental modal tests were conducted on a simple and also a riveted plate structure to verify the accuracy of FE models. First, the modal parameters obtained using various modeling approaches were obtained without any updating. The inordinate difference between the numerical and experimental results revealed the need for updating.

Therefore, FE model updating was performed in two steps. The first step was performed for the material which utilized the results obtained by the modal test of the simple plate. In other words, based the natural frequencies obtained from the simple plate, an equivalent finite element model was updated and the actual material properties of the plate were acquired. The second step of updating involved the rivet modeling, which was conducted for every model that was applicable. This step was performed based on the results obtained from the test of riveted lap joint structure. It was concluded that the first step of modeling was more effective in reducing the difference between the experimental and numerical results.

Moreover, an in-depth investigation of the numerical mode shapes of the riveted lap joint revealed the problem in contact modeling. To overcome the problem, gap elements are added to models which could successfully lower the deviation in experimental and numerical results.It should be noted that gap element is only applicable in cases where the plates are modeled by solid elements because for shell elements there is no contact available in the model. Moreover, including the rivet caps in the modeling did not add a dramatic benefit to the obtained results.

Finally, it is concluded that the fastener model proposed by Rutman can bring about the most accurate results when used in combination with solid plates and gap elements in the contact region. This model can be regarded as an easy, while accurate way of modeling riveted lap joints for dynamic studies. It should be also noted that the results presented in this study can only be generalized to dynamic studies and may not be applicableto static studies or stress analyses.

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