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Optimization of Composite Stiffened Cylindrical Shell using PSO Algorithm

M. Fakoor[1], P. Mohammadzade[2*] and E. Jafari[3]

1, 2, 3. Faculty of New Science and Technologies, University of Tehran

Postal code: 3456746, Tehran, Iran

mfakoor@ut.ac.ir

Composite stiffened cylindrical shells are widely used as primary elements in aerospace structures. In the recent years, there has been a growing research interest in optimum design of composite stiffened cylindrical shell structures for stability under buckling load. This paper focuses upon the development of an efficient optimization of ring-stringer stiffened cylindrical shell. The optimization problem used in this study involves weight minimization of ring-stringer stiffened composite cylindrical shell with buckling load and stress, which are considered as design constraints. The proposed methodology is based on Particle Swarm Optimization (PSO) algorithm. The material of shell is composite, but the material of stiffeners is considered to be isotropic. The approach adopted in modeling utilizes the Rayleigh-Ritz energy method and the stiffeners are treated as discrete members. In addition, a 3-D Finite Element (FEM) model of the ring-stringer stiffened cylindrical shell is developed that takes into consideration the exact geometric configuration. The results obtained using the Rayleigh-Ritz energy method are compared with those using 3-D FE model. The proposed methodology is implemented on the ring-stringer stiffened cylindrical shell using the PSO algorithm. The obtained results show a 13% reduction in the weight of the ring-stringer stiffened cylindrical shell whilst all the design constraints are satisfied. In addition, the results show that the proposed methodology provides an effective way of solving composite stiffened cylindrical shell design problems.

Keywords: Optimization, PSO algorithm, Cylindrical shell, Composites

Introduction

Cylindrical shells are widely used in a variety of industries, including aerospace, marine, automotive, oil and gas. One of the main advantages of composite structures over the conventional structures is that the stiffness and strength of composite structures can be tailored by selecting orientation of fibers. In addition, significant weight reduction offered by composite materials makes composites a more attractive option in the design of cylindrical shells. In designingcylindrical shells with a minimum weight, the load bearing aspects of these structures will be generally constrained with a loss of stability under axial compression. Therefore, the buckling load and the weight of structures are two main design considerations in their design [1]. In addition, the design flexibility offered by composites introduces additional design variables (e.g., fiber orientation, number and layer thickness) and the complex mechanical behavior of such materials make the design of composite structures a more difficult and time consuming process. These features of composites have motivated the use of optimization algorithms in the design of such structures, which has been the focus of many research programmers during the past two decades [2-5].

A survey paper by Ganguli [6] has provided a historical review of optimal design of composite structures and a study of composite structures with respect to industrial application has also been presented in [7]. Bruyneel [8] has presented a review of problems, solution procedures and applications of optimization of laminated

1. Associate Professor
2. Assistant Professor (Corresponding Authour)
3. M. Sc.

composite structures. In the recent years, there has been significant progress in the application of optimization on composite cylindrical shells [9-12].

Many researchers attempted to employ Genetic Algorithm (GA) in the optimization of composite structures [13-16]. Park et al. [17] have presented optimization of composite laminate stacking sequence by considering the uniform tensile axial loads to enhance strength using the theory of first-order transverse shear deformation based on Genetic Algorithm (GA). Gharib and Shakeri [18] have presented optimization of the stacking sequence of a laminated cylindrical shell with natural frequency and buckling load as the objective functions using GA and artificial neural networks (NN). Sadiqfar et al. [19] introduced GA in multi-objective optimization of stiffened cylindrical shells to achieve the minimum weight and maximum axial buckling load. Talebi et al. [20] proposed a novel tank design algorithm using GA for enhancing roll stability of fuel tank shape considering the multi-objective optimization of the problem. The main drawbacks of the use of GA in optimization ofcomposite structures is a high number of evaluations required and the corresponding high computational time.

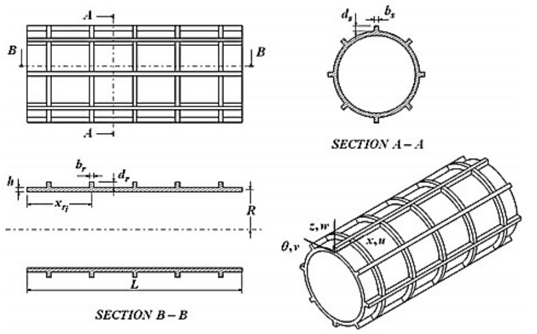

Fig. 1 Geometry of stiffened cylindrical shell

Particle swarm optimization (PSO) is a meta-heuristic optimization algorithm that is computationally more efficient than GA. This algorithm is an evolutionary algorithm introduced by Kennedy and Eberhart [21] and is inspired by the social behavior of animals such as fish and birds that live together in small and large groups.The fast convergence rate, the ease of implementation and the number of parameters set are other advantages of PSO [22-23]. Suresh et al. [24] has employed PSO as a multi-agent search method to maximize elastic coupling of a composite helicopter rotor blade. Chang et al. [25] introduced a method based on PSO for stacking sequence design. Some researchers used PSO algorithm in the optimization problem of composite cylindrical shells.

This paper focuses on the optimization of composite stiffened cylindrical shell. The methodology presented in this paper involves the minimization of the weight of the stiffened cylindrical shell subjected to buckling and stress constraints as well as the side constraints of the design variables. The main motivations and contributions of this paper are: (I) development of an efficient optimization algorithm using PSO for optimum design of composite stiffened cylindrical shell, (II) comparative study of computational performance of GA and PSO on the composite stiffened cylindrical shell, (III) study ofthe effect of number of ring and stringer on buckling load, (IV) study of effect of L/R on buckling load.

$$\begin{aligned} u &= u_0(x,\theta) + z\varphi_x(x,\theta) \\ v &= v_0(x,\theta) + z\varphi_\theta(x,\theta) \\ w &= w_0(x,\theta) \end{aligned} \quad (1)$$

Composite stiffened cylindrical shell equations

Consider a homogeneous circular cylindrical shell with thickness h, length L, mean radius R, mass density ρ, modulus of elasticity E, Poisson's ratio v and shear modulus $G = E/2(1+v)$. The shell is reinforced by Nrrings of equal or unequal spacing and Ns stringers of equal spacing. A coordinate system (x,y,z) is fixed on the middle surface of the shell at one of its two ends,height of the stringer and the ring are shown with $\mathrm{d_s}$and $\mathrm{d_r}$, respectively, and the widths correspond to $\mathrm{b_s}$ and $\mathrm{b_r}$. The intervals of the mid-shell to the geometric center of the stringer and the ring are marked with the $\mathrm{z_s}$and $\mathrm{z_r}$symbols, respectively.As shown in Fig. 1., the displacements of the different points of the shell are related to the displacements of mid surfaces by the Eq. 1. Where(u,v,w) are the orthogonal components of displacement of an arbitrary point (x,θ,z) in the shell along the coordinates(x,θ,z), respectively, and (u_0,v_0,w_0) are theinitial displacements of the shell mid-surface at point(x,θ,z). Shell strain energy could be stated as follows:

$$U_\varepsilon = \frac{1}{2}\int_0^l \int_0^{2\pi} \varepsilon^T [S] \varepsilon R \, d\theta \, dx \quad (2)$$

Where[S] is the stiffness matrixand$R$is mean radius.The strain vector, ε can be written as:

$$\varepsilon^T = \{ \varepsilon_x^0 \varepsilon_\theta^0 \gamma_{x\theta}^0 k_x k_\theta k_{x\theta} \} \quad (3)$$

$$\begin{aligned} &\varepsilon_x^0 = u_{,x} \;\; ; \;\; \varepsilon_\theta^0 = \frac{1}{R}\left(v_{,\theta} + w\right) \;\; ; \\ &\gamma_{x\theta_m} = \left(\frac{1}{R}u_{,\theta} + v_{,x}\right) \\ &k_x = -w_{,xx} \; ; \;\; k_\theta = \frac{1}{R^2}\left(v_{,\theta} - w_{,\theta\theta}\right) \\ &k_{x\theta} = \frac{1}{R}\left(v_{,x} - 2w_{,x\theta}\right) \end{aligned} \quad (4)$$

Where ε_x^0 ،ε_θ^0 ،$\gamma_{x\theta}^0$ are the mean surface strains which are defined with the curvature of the middle surface k_x،k_θ ،$k_{x\theta}$ in Equation 4.

According to the Love's first approximation:

To obtain the stiffened equations, the displacement according to the Euler-Bernoulli theory in the directions x, θ, z are defined as follows:

$$u_0(x,\theta) = A\cos(\lambda x)\cos(n\theta)$$
$$v_0(x,\theta) = B\sin(\lambda x)\sin(n\theta) \quad \lambda = \frac{m\pi}{L} \qquad (5)$$
$$w_0(x,\theta) = C\sin(\lambda x)\cos(n\theta)$$

$$\varepsilon_{ss} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2} \qquad (6)$$

The strain in axial direction is stated as:

The strain in circumferential direction is stated as:

$$\varepsilon_{rs} = \frac{1}{R}\left(\frac{\partial v_0}{\partial \theta} - \frac{z}{R}\frac{\partial^2 w_0}{\partial \theta^2} + w_0\right) \qquad (7)$$

And the strain energy of the rings is [26-27]:

$$U_r = \sum_{K=1}^{N_r}\left[\frac{1}{2}E_{rk}\int_0^L\int_{A_{rk}}\varepsilon_{rs}^2 A_{rk}\,dx\right] + \sum_{K=1}^{N_r}\left[\frac{1}{2}G_{rk}J_{rk}\int_0^{2\pi}\frac{1}{R^2}\left(\frac{\partial^2 w_0}{\partial x\,\partial\theta}\right)^2 R\,dx\right] \qquad (8)$$

G_{rk} ,J_{rk}،A_{rk}،E_{rk}،$N_r$are torsional rigidity,polar moment of inertia, cross-section, elastic modulus and the number of rings, respectively.

The strain energy of the stringers is [28]:

$$U_s = \sum_{K=1}^{N_s}\left[\frac{1}{2}E_{sk}\int_0^L\int_{A_{sk}}\varepsilon_{ss}^2 A_{sk}\,dx\right] + \sum_{K=1}^{N_s}\left[\frac{1}{2}G_{sk}J_{sk}\int_0^L\frac{1}{R^2}\left(\frac{\partial^2 w_0}{\partial x\,\partial\theta}\right)^2 dx\right] \qquad (9)$$

G_{sk} ,J_{sk}،A_{sk}،E_{sk}،$N_s$are torsional rigidity, polar moment of inertia, cross-section, elastic modulus, and the number of stringer, respectively.

Potential energy due to external forces including internal and external axial pressure are stated as follows:

Axial load [29]:

$$V_{N_a} = -\frac{N_a}{2}\int_0^L\int_0^{2\pi}\left[\left(\frac{\partial u_0}{\partial x}\right)^2 + \left(\frac{\partial v_0}{\partial x}\right)^2 + \left(\frac{\partial w_0}{\partial x}\right)^2\right]R^2\,dx\,d\theta \qquad (10)$$

$N_a$is axial loading.

Potential energy due toinner pressure [29]:

$$V_P = -\int_0^L\int_0^{2\pi}\frac{P}{2}\left\{\left[\frac{\partial^2 w_0}{\partial x^2} + w_0\right]w_0\right\}dx\,d\theta \qquad (11)$$

There fore, potential energy function could be written as follows:

$$\Pi = U_\varepsilon + U_r + U_s + V_{Na} + V_P \qquad (12)$$

Where U_ε is shell strain energy, U_r is strain energy of the rings, $U_s$is strain energy of the stringers, V_{Na} is potential energy due to external forces and $V_P$is potential energy due toinner pressure.

Using the principle of minimum potential energy and Ritz method (minimizing the energy function relative to the Ritz function's coefficients), equilibrium equations can be obtainedas follows:

$$\delta\Pi = 0 \quad \Rightarrow \begin{cases} \frac{\partial \Pi}{\partial A} = 0 \\ \frac{\partial \Pi}{\partial B} = 0 \\ \frac{\partial \Pi}{\partial C} = 0 \end{cases} \qquad (13)$$

To determine buckling load of the shell, Eq.13 can be rewritten as follows:

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix}\begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = 0 \qquad (14)$$

For Eq. 14 to have an unequivocal answer, the determinant of equation 15 must be zeroed:

$$\begin{vmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{vmatrix} = 0 \qquad (15)$$

The elements of the determinant are stated in Appendix (5).To calculate yield stresses, at first, the values of A, B and C are obtained by Eq. 14,then, by replacing the revalues into Eq.s1 and 5, the displacement field will be achieved. After obtaining the displacements, by using Eq. 1, strains and curvature will be obtained. Employing Hooke's relation for orthotropic materials, the stresses are obtained for each laminate as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_\theta \\ \tau_{x\theta} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix}\left\{\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_{x\theta}^0 \\ \gamma_{x\theta}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x \\ k_\theta \\ k_{x\theta} \end{Bmatrix}\right\} \qquad (16)$$

Eq.16 can be used for any point of the shell. Strain equations are employed to calculate stress in rings and stringers as follows:

$$\sigma_r = \frac{E_r}{R}\left(\frac{\partial v_0}{\partial\theta} - \frac{z}{R}\frac{\partial^2 w_0}{\partial\theta^2} + w_0\right) \qquad (17)$$

$$\sigma_s = E_s\left(\frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2}\right) \qquad (18)$$

Analysis of Composite Cylindrical Shell Using ANSYS

The modeling of composite cylindrical shell using ANSYS software is performed for validation of the the theoretical analysis. SHELL99 Linear Layered Structural Shell has been chosen. The boundary condition of simply supported is also selected as follows:

$$N_x = M_x = w = v = 0 \qquad at \quad x = 0 \;\; or \;\; L \qquad (19)$$

The shell structure is considered as a composite material with the following properties (Table1).

Table 1. Mechanical properties of cylindrical shell

Input parameters	*Value*
L	144 [mm]
R	82.5 [mm]
E_1	$146e9$
E_2	$10.8e9$
G_{12}	$5.78e9$
ϑ_{12}	0.29
h_i	0.04,0.06,0.14,0.26 [mm]
θ	42,40,86,25 $[deg]$

The distances of the rings and stringers were considered equal. The ring and stringer are made of aluminum. The specification of rings and stringers

Table 2. Mechanical properties of stiffeners

Input parameters	*Value*
N_r	2
N_s	4
$b_r = b_s$	$1\ mm$
$d_r = d_s$	$2.5\ mm$

Comparison of Analytical and Numerical Results

Table 3. Comparative results for verification of analytical results (no stiffened)

	n	m	P_{cri} $[N/m]$	σ_Y $[Mpa]$
Numerical	5	1	100031	185.2
Analytical	5	1	107235	196.1

Table 3 contains the comparison of analytical and numerical results.

As shown in Table 3, the numerical results extracted from ANSYS software approximately coincide with the analytical results with only 5 to 6 percent error for the non-stiffened cylindrical shell. Table4 includes the same results as Table 3 for stiffened structure.

Table 4. Comparative results for Verification of analytical results (Stiffened)

	n	m	P_{cri} $[N/m]$	σ_Y $[Mpa]$
Numerical	13	5	135041	159.6
Analytical	13	5	143523	171.7

As is listed in Table 4, the results of ANSYS software matches with an error of 5.9 to 7 percent in the analytical results of the stiffened cylinder.
In the presented method, the order of the reported values is negligible considering the engineering works. In the case of stiffened shells, the above approach could be applicable.
In order to analyze the shell stress and obtain the amount of yield stress, instead of the single pressure, a desired axial force in circumferential direction on the shell structure was applied, after obtaining the buckling load, the amount of yield stress for calculated buckling load can be determined

The Effect of Some Parameters on Buckling Load

With MATLAB software, the effect of the number of rings and stringers on buckling load were examined, the corresponding diagram is shown in Fig.2.

From Fig. 2 it can be concluded that at small circumferential wave number (i.e n = 1,2) the influences of number of rings and stringers are insignificant. Also, at large number of circumferential waves,the buckling load increases with number of rings and stringers. The unstiffened shell has the lowest critical load in n=6 and m=1, and in ouranalytical procedure, buckling occurred in the same wave. Therefore, as the number of stringers increases, buckling will occur in a higher load.

By passing from the half wave of 1 to 10, at first,the buckling loads are decreases and then,theywill increase. As shown in Appendix A, in the buckling equation, sinus is also involved, due to the periodic state of sinus function, the same situation has happened for buckling loads. The increase in the number of rings has a greater effect on buckling load in comparison with the increase in the number of stringers.
The effects of Length to cylinder radius ratio arealso shown in Fig. 3.

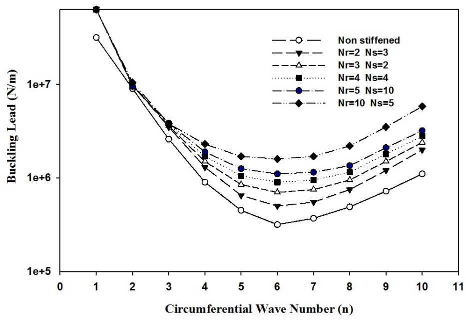

Figure 2. Effect of number of ring and stringer on buckling load.

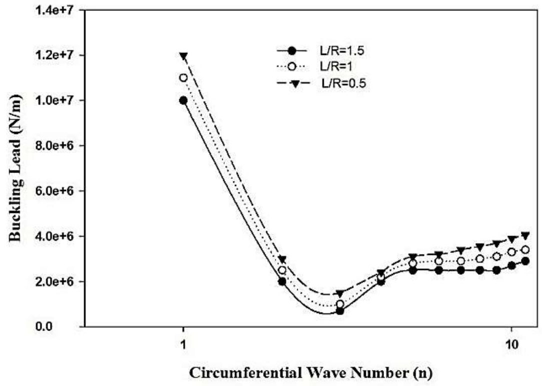

Figure 3. Effect of L/R on buckling load.

It can be found that bythe increase in length to the mean radiusratio, the critical load of the stiffened cylindrical shell decreases; this trend is proportional to the ratio of thickness to radius, as the thickness of shell increases, the buckling occurs in higher loads.

Particle Swarm Optimization (PSO)

PSO algorithm is inspired by the behavior of a group of birds, fish or insects in search of food or a certain way. Each particle expresses its position and speed using its memories and other particles, the basic steps in the PSO algorithm are as follows:

1. Generate the population of primary particles with random position values and initial random velocity.
2. Determine the new velocity vector for each particle of the population using the best previous position of each particle up to that moment, the best position experienced in the total particle population, and also the velocity vector of each particle before each other.
3. Change and improve the current position of each particle using the velocity vector of the previous position of each particle.
4. Repeat from step 2 until the desired stop criteria.

The vector of the velocity and position of each particle can be expressed by the following Equations:

$$V_k^i = wV_{k-1}^i + C_1 r_1 \left(Pbest^{\,i} - X_{k-1}^i\right) + C_2 r_2 \left(Gbest_{k-1}^g - X_{k-1}^i\right) \quad (20)$$

$$X_k^i = X_{k-1}^i + V_k^i \quad (21)$$

In these equations, i represents a particle, and the index k represents the number of repetitions, V represents the velocity, and X represents the position, and $r_1$and $r_2$are random numbers with uniform distribution in the interval $\{0,\ 1\}$. The coefficients C_1 and C_2 are particle acceleration constants. W is the coefficient of inertia.$Pbest^{\,i}$is the best experienced by the particle up to that moment, $Gbest_{k-1}^g$ is the best experienced situation in the whole population that is repeated.

Formulation of optimization problem

This section describes formulation of the optimization of the ring-stringer composite stiffened cylindrical shell. The optimization problem involves minimization of the weight of the ring-stringer composite stiffened cylindrical shell. The constraints of the problem are buckling load and stress as well as the side constraints of the design variables. The design variables and constraints are shown in Tables 5 and 6. Formulation of the optimization problem can be written as:

Minimize an objective function $f(x) = W_t$
Subject to constraint:
$g_j(x) \leq g_0(x) \qquad j = 1, \dots, m$

$x_L \leq x_i \leq x_U \quad i = 1, \dots, n$

x is variable parameter and x_L is lower limitation and x_U is upper limitation as defined in Table 5.

$g_1(x) = -F_{critical} \leq F_{ultimate}$

Where $F_{critical}$ is buckling loadthat is derived by equation 20 and is a function of variable parameters.

$g_2(x) = \sigma_{yield} \leq \sigma_{ultimate}$

Where σ_{yield} is maximum yield stress of structure that is derived by equation 22 and is a function of variable parameters.The most straight-forward technique to take constraints into account is the incorporation of penalty functions to the objective function. To get faster results and better control of the optimization process, some scale factors of the function have to be introduced to minimize (f). The objective function would be as follows:

$$f(x) = f(x) + \alpha(Max\left\{\frac{g_j(x)}{g_0(x)} - 1{,}0\right\}$$

Where$\alpha > 0$.

Table 5. Design Variable Parameter

Notation	Design variable	Upper limitation	Lower limitation	Unit
x_1, x_2, x_3	$h_i(i = 1,2,3)$	1.2	0.5	$[mm]$
x_4	N_r	10	1	$[mm]$
x_5	b_r	8	2	$[mm]$
x_6	d_r	8	2	$[mm]$
x_7	N_s	10	1	$[mm]$
x_8	b_s	8	2	$[mm]$
x_9	d_s	8	2	$[mm]$
x_{10}	ρ_{Stiff}	6500	2000	$[Kg/m^3]$
x_{11}	ϑ_{12}	0.35	0.15	
x_{12}, x_{13}, x_{14}	$\theta_i(i = 1,2,3)$	90	0	$[Deg]$
x_{15}		15	2	$[GPa]$
x_{16}	E_{11}	450	80	$[GPa]$
x_{17}	E_{Stiff}	200	50	$[GPa]$
x_{18}	ϑ_{Stiff}	0.3	0.25	$[GPa]$
x_{19}	E_{22}	40	10	$[GPa]$

Objective function

As indicated in the previous section, the objective function is the minimization of the total weight of the ring-stringer composite stiffened cylindrical shell (W_t), which can be expressed as follows:

The objective function of this research is the mass, or the weight of the cylindrical shell. Total mass is a function of the mass of the shell, plus the mass of the mounted stiffeners.

$$W_t = W_{sh} + W_r + W_s \tag{22}$$

W_t Is the total weight and W_r and$W_s$are the weights of the rings and stringers, respectively.

The weight of the cylindrical shell is obtained from Eq.23:

$$W_{Sh} = \sum_{i=1}^{N} w_i = gL\pi\rho\{(r_N^2 - r_1^2)\}$$
$$W_{sh} = gL\pi\rho \quad {}_1(0.165 + x_2 + x_3) + x_2(0.165 - x_1 + x_3) + x_3(0.165 - x_1 - x_2)] \tag{23}$$
$$W_{sh} = 0.165 \,.\, gL\pi\rho(x_1 + x_2 + x_3)$$

Where $r_N$is the radius of n^{th} composite laminate. 3layers of laminates (see Fig. 4) areassumed in the test problem of this paper. Hence, there are:r_1, r_2, r_3, r_4, and ρ is the composite density.

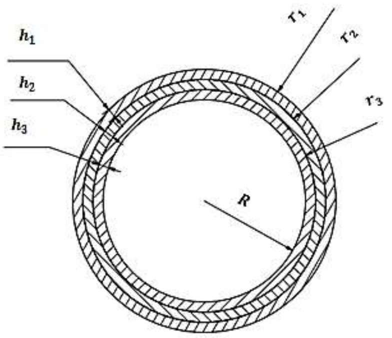

Figure 4. Cylindrical shell (3 laminates).

Rings weight(Fig. 1):

$$W_r = V_r.\rho_r = 2\pi \cdot \rho_r \cdot N_r \cdot \left(R + \frac{h+d_r}{2}\right) \cdot b_r \cdot d_r$$
$$W_r = 2\pi.\rho_r.N_r.\left(\frac{d}{2} + \frac{t+d_r}{2}\right).b_r.d_r \tag{24}$$
$$W_r = 2\pi.x_{10}.x_4.\left(0.0825 + \frac{x_1+x_2+x_3+x_6}{2}\right).x_5.x_6$$

$N_r$is the number of rings, $d_r$is the height of the rings, $b_r$is the width of the rings, $\rho_r$is the density of the rings and t is the thickness of the shell.

Stringer's weight isas follows:

$$W_s = V_s \cdot \rho_s = N_s \cdot \rho_s \cdot b_s \cdot d_s \cdot L$$
$$W_s = 0.144(x_7.x_{10}.x_8.x_9) \tag{25}$$

N_s is the number of stringers, $d_s$is the height of the stringers, b_s is the width, andρ_s is the density of the stringers.

Finally, the objective function is as follows:

$$w_t = 0.165 \,.\, gL\pi\rho.(x_1 + x_2 + x_3) + 2\pi.x_{19}.x_4.\left(0.0825 + \frac{x_1+x_2+x_3+x_6}{2}\right).x_5.x_6 + 0.144(x_7.x_{10}.x_8.x_9) \tag{26}$$

Constant design parameters arepresented in Table6. The design variables includes: properties of the composite material $(E_{11}, E_{22}, G_{12}, \vartheta_{12})$, properties of the stiffener's material $(E_{stiff}, \vartheta_{stiff}, \rho_{stiff})$, number of rings N_r, number of stringer N_s, cross-section of rings $b_r \cdot d_r$, cross-section of stringers $b_s \cdot d_s$. the range of limitation is presented in Table 5.

Table 6. Constant Design Parameter

Input parameters	Value
Length of shell (L)	144 [mm]
Radius of shell	82.5 [mm]
Number of laminates	3
Axial load(Na)	1.5e7 [N/m]
Inner pressure (P)	20 [KPa]
Yield Stress(**σ**)	210 [MPa]

Design Constraints

The design constraints of the test problem are buckling load and yield stress as shown in Table 7.The buckling load is $\frac{F_{critical}}{F_{ultimate}} \geq 1$, and no dimensional constraint buckling means that buckling load of stiffened cylindrical shell ($F_{critical}$) that is obtained with the developed MATLAB code, should be greater than the allowed load $F_{ultimate}$, otherwise, the stiffened cylindrical shell will be buckled.

The Yield stress is $\frac{\sigma_{max}}{\sigma_{ultimate}} \leq 1$, and no dimensional constraint Yield stress means that the maximum stress of stiffened cylindrical shell (σ_{max}) that is obtained with the developed MATLAB code, should be smaller than the allowable stress $\sigma_{ultimate}$, otherwise, the stiffened cylindrical shell will reach to yield point.

Table 7. Design constraints

Buckling Constraint	$\frac{-F_{max}}{F_{ultimate}} - 1 \leq 0$	$F_{max}$is calculated by code in matlab	$F_{ultimate} = 1.5e7$
Yield Stress Constraint	$\frac{\sigma_{max}}{\sigma_{ultimate}} - 1 \leq 0$	σ_{max} is calculated by code in matl	$\sigma_{ultimate} = 210e6$

Results

Fig. 5 shows the convergence diagram of weighting function to the number of generation based on genetic algorithms.

Due to the large difference between the scale of variable parameters (as mentioned some of them are on the scale of millimeters and some on the Giga scale), the range of all variables was set between {1, 11} and then, after obtaining the final answer of 17.776, the final values of all variable parameters that are between 1 and 11, were returned by Eq.26 to the initial scale.

For example, if the range of variables are between L and U and the obtained value from the optimization procedure is S, the final value of the parameter can be obtained as follows:

$$x = \frac{(U-L)}{10} * (S-1) + L \qquad (27)$$

After making this scale conversion for all parameters, the final and optimized values of the variables are obtained.By obtaining variables, the amount of shell weight that is the objectivefunction in its original range can be calculated; after this stage, the objective function 4.132 is obtained. The optimum value of the particle swarm algorithm (see Fig. 6) is also performed with the same scale variation as was mentioned before. Finally, the final weighting function was achieved as 4.1280,which weighed 13.2% relative to the initial shell weighing 4.78.

After optimizing both the genetic algorithm and the particle swarm, the optimized weight decreased by 13.1% compared to the initial weight, while it has not reached critical loads and has not reached the yield stress (Table 8).

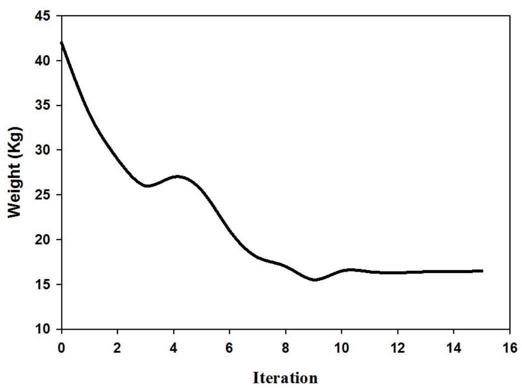

Figure 5. Convergence of weight to iteration (Genetic Algorithm)

Table 8. Result of optimization

Algorithim	Results [kg]	Error	Critical Load [N/m]	Yield Stress [Mpa]
PSO	3.128		13054403	206
GA	3.132	0.12	14067323	207.3

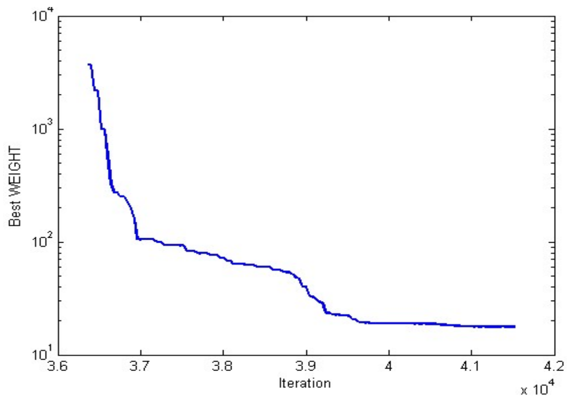

Figure 6. Convergence of weight to iteration (PSO Algorithm**)**

Final values of parameters arelistedinTable 9.

Table 9. Final value of variable parameters

Notation	Design variable		Unit
x_1, x_2, x_3	$h_i(i = 1,2,3)$	0.69,0.53,0.78	*[mm]*
x_4	N_r	3	
x_5	b_r	4.1	*[mm]*
x_6	d_r	5.8	*[mm]*
x_7	N_s	4	
x_8	b_s	2.1	*[mm]*
x_9	d_s	4.7	*[mm]*
x_{10}	ρ_{Stiff}	2604	$[Kg/m^3]$
x_{11}	ϑ_{12}	0.29	
x_{12}, x_{13}, x_{14}	$\theta_i(i = 1,2,3)$	35,42,38	*[Deg]*
x_{15}		5.9	$[GPa]$
x_{16}		432	$[GPa]$
x_{17}	E_{Stiff}	182	$[GPa]$
x_{18}	ϑ_{Stiff}	0.29	
x_{19}	E_{22}	15	$[GPa]$

Conclusion

In this research,optimization of buckling load forring/ stringer stiffened cylindrical laminated composite shells subjected to axial compression was considered. The maximum strength to weight ratio of these structures was assumed. Evolution algorithms are employeddue to discrete design variables and also the need for global searches.In this study, PSO algorithm has been utilized in addition to the genetic algorithm.The constraints wereassumed as buckling load and the yield stress.Governing equations for composite stiffened cylindrical shell using the principle of minimum energy were written relative to the Ritz coefficients.For kinematic relations of the Love Kirchhoff approximation, stiffeners are considered as separate elements;then, the conjugation relationship between the stiffeners and the shell was applied. Shell loading is assumed to be internal and axial force and the boundary conditions were simply supported. Related constraints wereconsidered in m separatedMATLAB files and are called in each replication and checked by an algorithm in whichthe direction of the objective function in the region was acceptable. After optimizing both the genetic algorithm and the particle swarm, the optimized weight decreased by 13.1% compared to the initial weight, while it didnot reach critical buckling loads and didnot reach the yieldstress.Also, the final values of optimization by each genetic and particle accumulation werevery close, but the particle swarm approachwasmuch faster than the genetic method.

Appendix

$$\alpha_{11} = L\pi^2 \left\{ -N_a R\lambda^2 - \frac{A_{66}n^2}{R} - A_{11}R\lambda^2 + \frac{B_{66}n^2}{R} + \frac{D_{66}n^2}{4R^3} \right\}$$

$$\alpha_{12} = Ln\pi^2\lambda \left\{ A_{12} + A_{66} + \frac{B_{12}}{R} + \frac{B_{66}}{R} - \frac{3D_{66}}{4R^2} \right\}$$

$$\alpha_{13} = L\pi^2\lambda \left\{ A_{12} + B_{11}R\lambda^2 + \frac{B_{12}n^2}{R} + \frac{2B_{66}n^2}{R} - \frac{D_{66}n^2}{R^2} \right\} + \frac{1}{2} N_s L\pi \; {}^3 A_3 z_3 (E_s + N_a)$$

$$\alpha_{21} = \alpha_{12}$$

$$\alpha_{22} = L\pi^2 \left\{ -N_a R\lambda^2 - \frac{D_{22}n^2}{R^3} - \frac{2B_{22}n^2}{R^2} - \frac{A_{22}n^2}{R} - 4B_{66}\lambda^2 - A_{66}R\lambda^2 - \frac{4D_{66}\lambda^2}{R} \right\} - \frac{2n^2\pi^2 A_r E_r}{R} \sum_{k=1}^{N_r} (sin\,\lambda x_k)^2 - \frac{1}{2} N_s L\pi \;{}_s{}^2 A_s N_a$$

$$\alpha_{23} = Ln\pi^2 \left\{ -\frac{A_{22}}{R} - B_{12}\lambda^2 - \frac{B_{22}n^2}{R^2} - 2B_{66}\lambda^2 - \frac{B_{22}}{R^2} - \frac{D_{22}n^2}{R^3} - \frac{D_{12}\lambda^2}{R} - \frac{4D_{66}\lambda^2}{R} \right\} - \frac{2n^2\pi^2 A_r E_r}{R} \sum_{k=1}^{N_r} (sin\,\lambda x_k)^2 - \frac{N_s Ln\pi A_s z_s \lambda^2 N_a}{2R} - \frac{2N^3\pi^2 A_r z_r E_r}{R^2} \sum_{k=1}^{N_r} (sin\,\lambda x_k)^2$$

$$\alpha_{31} = \alpha_{13}$$

$$\alpha_{32} = \alpha_{23}$$

$$\alpha_{33} = L\pi^2 \left\{ -N_a\lambda^2 R - 2B_{12}\lambda^2 - \frac{D_{22}}{R^3} - \frac{2D_{22}n^2}{R^3} - \frac{D_{22}n^4}{R^3} - \frac{2B_{22}}{R^2} - \frac{A_{22}}{R} - \frac{2D_{12}\lambda^2}{R} - \frac{2D_{12}\lambda^2 n^2}{R} - \frac{4D_{66}\lambda^2 n^2}{R} - D_{11}R\lambda^4 \right\}$$
$$- \left\{ \frac{2n^4\pi^2 I_{or}E_r}{R^3} + \frac{2\pi^2 A_r E_r}{R} \right\} \sum_{k=1}^{N_r} (sin\,\lambda \;\; {}_k)^2 - \frac{2n^2\pi^2\lambda^2 G_r J_r}{R} \sum_{k=1}^{N_r} (\cos\lambda x_k)^2 - \frac{1}{2} L N_s \pi\lambda^4 I_{or}(E_s + N_a)$$
$$- \frac{4n^2\pi^2 A_r E_r z_r}{R^2} \sum_{k=1}^{N_r} (sin\,\lambda x_k)^2 - \frac{PL\pi(n^2-1)}{4}$$

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