# A Comprehensive Approach to Develop a Continuous Fuzzy Guidance Law for Maneuvering Targets

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*Based on the idea of Continuous Fuzzy Guidance Law (CFGL), a "three-phase fuzzy guidance" (TFG) law is proposed for the class of surface to air homing missiles. The current approach enables the guidance law to track a maneuvering target from the beginning of the launch phase up to the terminal one while it dynamically attempts to keep miss-distance, flight time and control effort at a minimum acceptable level. The guidance law developed here depends on four factors: line-of-sight (LOS) angle, LOS rate, LOS angular acceleration, and relative distance to the target. To show the relative superiority of the approach, the performance of the new guidance law has been compared with that of proportional navigation guidance. The results confirm the validity of the idea; as for a TFG, we get quite comparable results. The current approach also shows a relatively good robustness for a wide variety of flight conditions.*
***Keywords***: *Guidance, Fuzzy,Maneuver, Target*

## Nomenclature

| | | | |
|---|---|---|---|
| $N$ | Effective Navigation Ratio | $a_{12}, a_{21}, a_{22}$ | Aerodynamic Coefficients |
| $V_C$ | Missile-Target Closing Velocity [m/sec] | $b_{11}, b_{12}$ | Aerodynamic Coefficients |
| $V_M$ | Missile Velocity [m/sec] | $T_a$ | Aerodynamic Time Constant [sec] |
| $V_T$ | Target Velocity [m/sec] | $K_a, K_Q, K_A$ | Autopilot Loop Gains |
| $L$ | Missile Lead Angle [rad] | $K_1, K_2, K_3$ | Aerodynamic Coefficients |
| $HE$ | Missile Heading Error [rad] | $\lambda$ | LOS Angle [rad] |
| $A_C$ | Missile Acceleration Command [m/sec$^2$] | $\dot{\lambda}$ | LOS Rate [rad/sec] |
| $A_T$ | Target Acceleration [m/sec$^2$] | $\ddot{\lambda}$ | LOS Angular Acceleration [rad/sec2] |
| h1, h2 | Height [m] | $\gamma_T$ | Target Flight Path Angle [rad] |
| $W_I$ | Stabilizing Loop Gain [rad/sec] | $\gamma_M$ | Missile Flight Path Angle [rad] |
| $W_T$ | Target Aerodynamic Constant [rad/sec] | $\omega_{ns}$ | Un-damped Natural Frequency [rad/sec] |
| | | $\omega_T$ | Angular Target Acceleration [rad/sec$^2$] |
| | | $\xi_s$ | Damping Constant |

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## 1 Introduction

During the last two decades, development of appropriate guidance laws, suitable for intercepting

missiles in a wide variety of flight conditions, has attracted considerable attention [1, 2]. This is mainly true about highly maneuverable aircraft, for which conventional approaches may not be sufficient to obtain both tracking and interception, unless there is perfect knowledge about the system dynamics and also extensive computational capabilities are available. The conventional approaches to this subject include: (1) Exact feedback linearization [3, 4]; (2) Sliding mode control [5, 6]; (3) Adaptive control [7, 8]; and, the last but not the least, (4) LQ-based control system [9, 10]. It is, therefore, appropriate to investigate other advanced control theories to improve existing performance capabilities. In this line of thought, Fuzzy logic controllers have shown to exhibit suitable properties which help eliminating measurement deficiencies, or other difficulties such as changing climatic conditions [11].

In fact, Fuzzy controllers have been used in many different fields wherever the system under consideration is not well-defined, uncertain or model free. There are, however, a limited number of published works available that discuss the subject of Fuzzy guidance for aerial vehicles in a systematic manner. Most references introduce a Fuzzy guidance law which achieves good performance in a limited number of flight conditions. Some are limited to only one phase of flight such as terminal phase [12] or midcourse one [13]. There are also few references that discuss the so-called "Integrated Fuzzy logic guidance" law by considering a group of four discrete controllers [14]; three of the integrated controllers are utilized for three phases, whilst the fourth one is used as a switch. From a practical point of view, using four controllers separately leads to onboard size increment and further cost. In addition, there will be a necessity to insure a proper hardware matching and accurate synchronization among the controllers, whereas software integration by considering single controller will eliminate such difficulties and decrease the implementation complexities.

In this work, we introduce a flexible TFG Law with extensive design freedom based on Fuzzy logic. The developed TFG is based on the fact that a smart control system does not need to be sensitive to the target maneuvering at all times. In fact, while the target is far away, the defending missile does not need any extraordinary effort to stir toward the target. However, as the target gets closer, the missile demands considerable control effort to manage both miss-distance and flight-time. It is obvious that, ideally, we desire to manage the whole process in a continuous manner without the necessity to distinguish different flight phases or using

any switching techniques. Such an idea is quite interesting in the sense that the control system is smart enough to distinguish the missile flight phases and, therefore, the designer is not forced to adjust the guidance system for some pre-selected flight phases prior to the actual flight.

Overall, the idea is to develop a guidance system with variable sensitivity to target maneuvers. This will be achieved by, first, adopting only LOS angle to create the guidance commands when the target is relatively far and only we need low sensitivity to its maneuvers. As the target gets closer, the derivative of the LOS angle will be used. This scheme insures an increased sensitivity to target maneuvers. As the third stage, and as the target gets closer and closer, we bring into account both the first and second order derivatives of LOS angle to provide the highest guidance sensitivity. In this approach, we have three sub-fuzzy guidance laws each of which has its own characteristics; nonetheless, we use relative target-missile distance $R_{TM}$ to secure smooth transmission among the sub-guidance laws. To further highlight the achieved improvement, a comparative study has been conducted to compare the developed TFG law and the Proportional Navigation Guidance (PNG). The comparison clearly shows the relative superiority of the developed TFG law.

The paper is organized as follows: first, we give an overview on proportional navigation; in Section 3, we describe the basis of the proposed TFG law. The effectiveness of the TFG is demonstrated through a suitable case in Section 4; and, finally, in Section 5, we discuss the achievements and possible future enhancements.

## 2 Overview of PNG

The first missile which was successfully tested in 1950, known as "Lark", was the first one in its kind that used proportional navigation guidance [15]. The idea behind its guidance was simple. Theoretically, it issues acceleration commands perpendicular to the instantaneous LOS for which the magnitudes are proportional to the LOS angle rate and the closing velocity . In mathematical terms, this can be stated as:

$$A_c = NV_c \dot{\lambda} \qquad\qquad (1)$$

Equation (1), basically, implies that the PNG law seeks to null the LOS rate by making the missile turn rate remain directly proportional to the LOS rate. Neglecting gravitational force and drag effects for simplicity, such a guidance law seems quite logical and effective, especially, for a 2D point mass missile target engagement

geometry as shown in Fig. 1. Nonetheless, the actual real-world scenarios raise different complexities which make PNG not as effective as one might expect. Therefore, different studies attempt to develop new ideas which are comparable to PNG and, at the same time, are more flexible to handle actual real-world scenarios. The TFG described in this work is an attempt in this line of work.
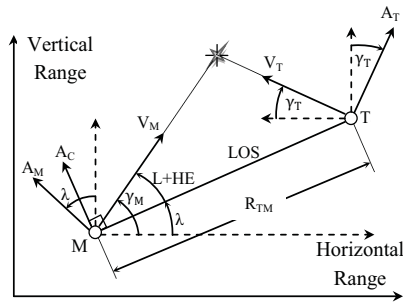


**Figure 1. 2D missile-target engagement geometry.**

## 3   A New Approach to Design Based on Fuzzy Guidance Law

In a general sense, missile guidance needs to be investigated in three distinguished phases: initial, midcourse, and terminal, as shown in Fig. 2. During the first phase, which is also known as launch or boosting phase, the launcher aims the missile in an appropriate direction, based on the perceived target aircraft. This aiming establishes the initial LOS that the missile uses during the initial phase. At the end of the launch phase, the missile is expected to reach some predetermined altitude where the second phase starts. The second or midcourse phase of the flight is relatively the longest phase in both distance and time. During midcourse phase, the missile makes necessary corrections to stay on the desired course. The objective of the midcourse phase is, therefore, to guide the missile toward a position which is the nearest possible position that allows the missile to enter its final phase. The last few seconds of the engagement, which is very critical, constitutes the terminal guidance phase. The success during this phase is obviously very crucial as it determines the success or failure of the entire mission. The objective of the final phase is to bring the missile into contact or, otherwise, the closest proximity with the target. This phase, obviously, requires a very fast response with a high degree of accuracy.

It is noted that air density ($A_d$) and temperature ($A_t$) vary with the altitude. Here, we ignore the effect of such parameters in the overall missile performance. However, for a more precise modeling, one might use a look-up table to consider the effects of such parameters based on changing altitudes ($\Delta h = h_2 - h_1$). Obviously, the effect of air density in higher altitudes is less pronounced.
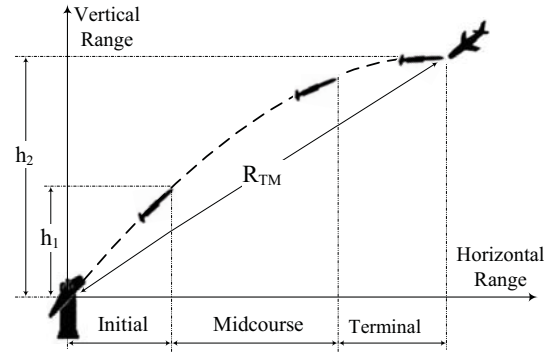


**Figure 2. Guidance phases of flight [16].**

Considering these simplifications, we now need the necessary linguistic tools to develop the Fuzzy guidance system. In the proposed algorithm, the linguistic variables, $\lambda$, $\dot{\lambda}$, $\ddot{\lambda}$, $A_C$ , and $R_{TM}$ are used to express the linguistic sets necessary as listed in Tables 1 and 2.

**Table 1. Linguistic sets of the Fuzzy guidance law.**

| SN | MN | LN |
|---|---|---|
| Small negative | Medium negative | Large negative |
| SP | MP | LP |
| Small positive | Medium positive | Large positive |
| S | ZE | L |
| Small | Zero | Large |

Possible values that could be accepted by antecedent part $\lambda$, $\dot{\lambda}$, $\ddot{\lambda}$, $A_C$ and $R_{TM}$ are introduced in Table 2.

**Table 2. Term sets for each individual variable.**

| $T_\lambda$ | = | LN | MN | SN | ZE | SP | MP | LP |
|---|---|---|---|---|---|---|---|---|
| $T_{\dot{\lambda}}$ | = | LN | MN | SN | ZE | SP | MP | LP |
| $T_{\ddot{\lambda}}$ | = | LN | MN | SN | ZE | SP | MP | LP |
| $T_{AC}$ | = | LN | MN | SN | ZE | SP | MP | LP |
| $T_{R_{TM}}$ | = | L | M | S | | | | |

For a uniform calculation, all of the variables $\lambda$, $\dot{\lambda}$, $\ddot{\lambda}$, $A_c$ and $R_{TM}$ are normalized with the following scheme:

$$V_{norm} = V/V_{max} \tag{2}$$

where, V is the variable that needs to be normalized and  is the maximum value that the variable can accept during the flight. In this approach, all inputs ( $\lambda$, $\dot{\lambda}$, $\ddot{\lambda}$ and $R_{TM}$) have to be divided respectively on their max

values before being fed into the mathematical tool box. On the other hand, the output variable ($A_C$) has to be multiplied by its max value to have the actual output. Further, to be able to manage the missile-target relative distance, we partition the linguistic variable $R_{TM}$ into three segments denoted by L, M, and S as shown in Fig. 3. During the launch phase, we have $R_{TM}$= L which means the missile is very far from its assigned target; during the midcourse phase, $R_{TM}$ = M, and, finally, during the terminal phase, when the missile gets very close to the target, $R_{TM}$= S. Different investigations by the authors suggest that Gaussian distribution for $R_{TM}$ membership functions ensures a smooth transition between any two neighboring phases of the flight.
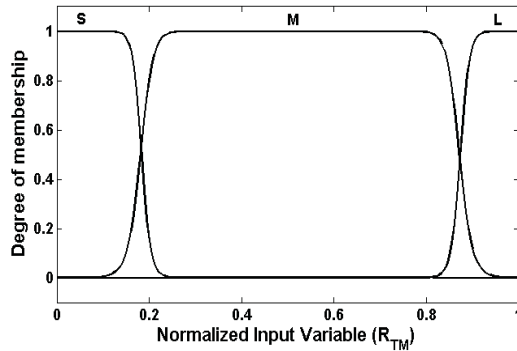


**Figure 3. Membership functions of $R_{TM}$.**

During the development of the Fuzzy guidance law, we need to have a systematic approach as it is not rigorously clear where the initial phase actually ends and the midcourse begins. The following subsections explain the process in a systematic manner.

### 3.1 Launch Phase ($R_{TM}$ =L)

For a surface to air missile, the missile, undoubtedly, has to have an initial command to climb to a certain altitude. Any suitable altitude can be attained by maintaining a positive lead angle toward a suspected target throughout the launch phase. Here, we desire to investigate the guidance commands that help minimize the control effort while attempting to maintain its proper orientation toward the target. On the other hand, keeping the control effort at a minimum might contradict precise target tracking [17]. Nevertheless, minimizing control effort, to some degree, can also be achieved by monitoring LOS angle as a base to derive the necessary guidance law. Therefore, we developed an asymmetric guidance rule base depending only on. The output commands as a lateral acceleration would be as in Eq. 3 where  is the input-output mapping of the Fuzzy logic

system:

$$A_C = f_l(\lambda) \qquad\qquad (3)$$

Figure 4 shows the membership functions of . It is interesting that further investigation proved that using other more complex forms of membership functions cannot necessarily have any meaningful advantages, compared to those of triangular ones[18]. However, the density of the membership functions is chosen closer to zero to ensure insensitivity to any target maneuvering.
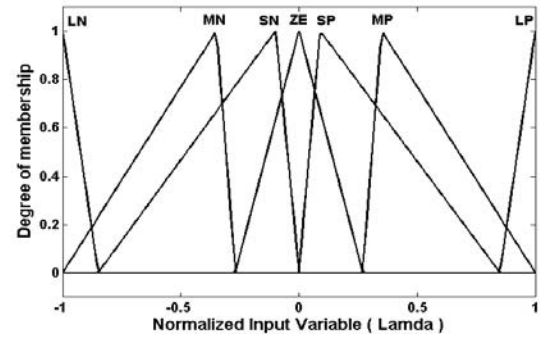


**Figure 4. Membership functions of $\lambda$.**

For this phase, the guidance rules are listed in Table 3. It is noted that the linguistic variables associated with are symmetric, while  ones are not. Moreover, the combination of  when  respectively enables the missile to fly with some positive lead angle.

**Table 3. Fuzzy guidance rules for launch phase.**

| $R_{TM}$  is L | $\lambda$ | LN | MN | SN | ZE | SP | MP | LP |
|---|---|---|---|---|---|---|---|---|
|  | $A_C$ | MN | SN | ZE | ZE | SP | MP | LP |

The first three shaded columns associated with  are intended to allow the missile to climb while the next four columns do not.

### 3.2  Midcourse Phase ($R_{TM}$ =M)

The midcourse phase of the flight could simply be defined as the period that follows the launch phase and continues until the terminal one. Nonetheless, its starting and terminating points could not be defined rigorously and we might use the power of Fuzzy logic to alleviate the problems associated with this vague definition. The main purpose of the midcourse guidance is, however, to guide the missile toward the nearest possible point while the target is maneuvering. In this phase, we use LOS angle rate ($\dot{\lambda}$ ) to construct the necessary rules. Such membership functions are shown in Fig. 5.
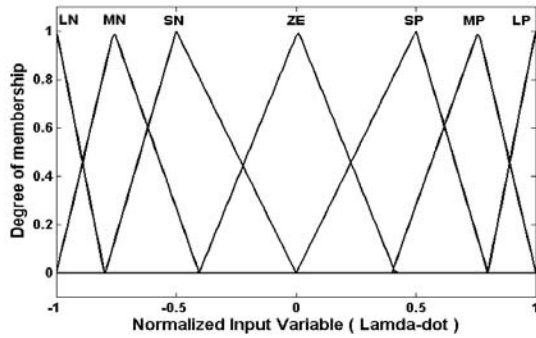
**Figure 5. Membership functions of $\dot{\lambda}$.**

The Fuzzy midcourse guidance law takes the following form:

$$A_C = f_m(\dot{\lambda}). \qquad (4)$$

The Fuzzy midcourse guidance seeks to track the target and allows moderate sensitivity with respect to the target maneuvers. This can be achieved by restricting values to the Fuzzy sets MN and MP as shown in Table 4.

**Table 4. Fuzzy guidance rule for medium phase.**

| $R_{TM}$ is M | $\dot{\lambda}$ | LN | MN | SN | ZE | SP | MP | LP |
|---|---|---|---|---|---|---|---|---|
| | $A_C$ | MN | MN | MN | ZE | MP | MP | MP |

Different case studies conducted by the authors show that lowering sensitivity with respect to the target maneuvering would affect control efforts in a favorable manner and help avoiding large changes to the flight path angle and trajectory; the latter helps a relatively smoother transition into the terminal phase.

### 3.3 Terminal Phase ($R_{TM} = S$)

With the completion of the midcourse phase, a terminal phase starts which is traditionally referred to as the key to any successful interception. However, in an integrated approach to guidance and with the help of proposed TFG, we expect to modify such a level of expectation from terminal phase. During this phase, in addition to , we bring an estimated into account to make the guidance law as sensitive as possible and to correctly predict any target movement as seen by the guidance system. The Fuzzy terminal course guidance law for this phase will then be a function of the following form:

$$A_C = f_t(\dot{\lambda}, \ddot{\lambda}). \qquad (5)$$

However, thorough investigations by the authors reveal that during the terminal stage, $\dot{\lambda}$ acts as the dominating parameter, while $\ddot{\lambda}$ effectively helps fine-tuning the outcome. Furthermore, there exists a design-dependent ratio between $\dot{\lambda}$ and $\ddot{\lambda}$. For the case-studies presented in this work, we take this ratio as 2. This simply means that if $dA_C/d\dot{\lambda}$ (that is, change in $A_C$ due to the change in $\dot{\lambda}$ only) varies in the range of [-6k, 6k], then $dA_C/d\ddot{\lambda}$ (change in $A_C$ due to the change in $\ddot{\lambda}$ only) varies in the range of [-3k, 3k]. Table 5 gives the numerated guidance rule with respect to ($\dot{\lambda}$, $\ddot{\lambda}$) separately.

**Table 5. Numerated guidance rules of ($\dot{\lambda}$, $\ddot{\lambda}$) separately.**

| $\dot{\lambda}, \ddot{\lambda}$ | LN | MN | SN | ZE | SP | MP | LP |
|---|---|---|---|---|---|---|---|
| respect to $\dot{\lambda}$ only | -6k | -4k | -2k | 0k | 2k | 4k | 6k |
| respect to $\ddot{\lambda}$ only | -3k | -2k | -1k | 0k | 1k | 2k | 3k |

By a simple calculation, the numerated guidance rules can be obtained as illustrated in Table 6.

**Table 6. Numerated guidance rule base**

| $A_C$ | | $\ddot{\lambda}$ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | LN | MN | SN | ZE | SP | MP | LP |
| $\dot{\lambda}$ | LN | -9k | -8k | -7k | -6k | -5k | -4k | -3k |
| | MN | -7k | -6k | -5k | -4k | -3k | -2k | -1k |
| | SN | -5k | -4k | -3k | -2k | -1k | 0k | 1k |
| | ZE | -3k | -2k | -1k | 0k | 1k | 2k | 3k |
| | SP | -1k | 0k | 1k | 2k | 3k | 4k | 5k |
| | MP | 1k | 2k | 3k | 4k | 5k | 6k | 7k |
| | LP | 3k | 4k | 5k | 6k | 7k | 8k | 9k |

As it was shown previously, when considering both values ($\dot{\lambda}, \ddot{\lambda}$), the values of will change in the range [-9k, 9k]. Dividing this range into seven parts enables us to express each of them by a linguistic value; the whole linguistic values of $A_c$ are listed in Table 7.

**Table 7. Fuzzy guidance rules for terminal phase.**

| $R_{TM}$ is S | $A_C$ | $\dot{\lambda}$ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | LN | MN | SN | ZE | SP | MP | LP |
| | $\ddot{\lambda}$ | | | | | | | |
| | LN | LN | LN | LN | LN | LN | MN | MN |
| | MN | LN | LN | MN | MN | MN | SN | SN |
| | SN | MN | MN | SN | SN | SN | ZE | ZE |
| | ZE | SN | SN | ZE | ZE | SP | SP | SP |
| | SP | ZE | SP | SP | SP | MP | MP | MP |
| | MP | SP | MP | MP | MP | LP | LP | LP |
| | LP | MP | LP | LP | LP | LP | LP | LP |

The membership functions of are shown in Fig. 6 and the Fuzzy sets described for are shown in Fig. 7. The density of the membership functions associated with is chosen closer to zero; again, such an approach ensures low $A_C$ commands during launch and midcourse phases. On the other hand, decreasing the sensitivity is eliminated in the terminal phase by adopting both $\dot{\lambda}$ and $\ddot{\lambda}$, where the resulting acceleration command is due to both and measurements.
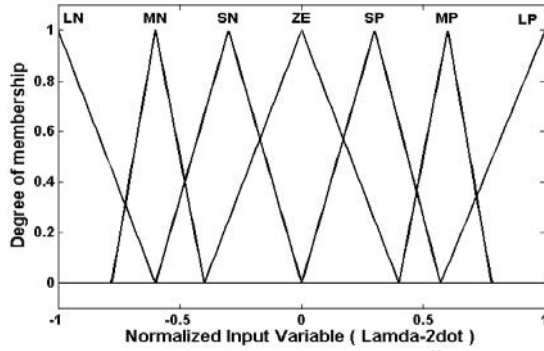


Figure 6. Membership functions of $\ddot{\lambda}$.
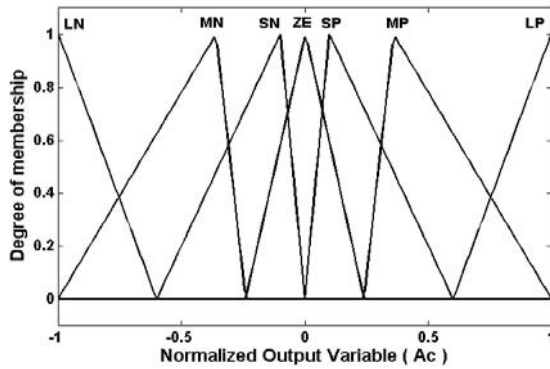


Figure 7. Membership functions of $A_C$.

To get the actual guidance command output in its crisp form which can be fed as the acceleration command to the missile flight control system, the output Fuzzy is defuzzified using the Center of Area (CoA) method [19], in which the Fuzzy controller first calculates the area under the scaled membership functions and within the range of the output variable. The Fuzzy logic controller, then, uses the following equation to calculate the geometric center of this area:

$$CoA = \frac{\int_{u_{min}}^{u_{max}} u.m_o(u)du}{\int_{u_{min}}^{u_{max}} m_o(u)du}$$

where u is the value of the linguistic variable, and ($u_{min}$, $u_{max}$) represent the range of the linguistic variable, while $m_o$ is the membership function of the output Fuzzy set.

The maximum values of ($\lambda$, $\dot{\lambda}$, $\ddot{\lambda}$ and $A_C$), which are needed for normalization, can be driven from those yielding in the classical guidance laws. In our study, the values used as the maximum ones are presented in Table 8.

Table 8. Maximum values used in normalization.

| $\lambda$ | $\dot{\lambda}$ | $\ddot{\lambda}$ | $A_C$ |
|---|---|---|---|
| [rad] 0.02 | [rad/sec] 0.05 | [rad/sec²] 2.5 | [m/sec²] 200 |

## 4 Case Studies

To demonstrate the potentials of the TFG, we simulate a typical missile-target engagement using the data given in Table 9. With the help of reference [20], a typical guidance system with well-behaved aerodynamics and a perfectly stabilized seeker is constructed as shown in Fig. 8 and the proposed Fuzzy logic-based guidance law is added upon it. The homing loop describing missile-target engagement is also illustrated while the corresponding parameters are listed in Table 10.

Table 9. Data used for simulation.

| | |
|---|---|
| Target position | (5, 10) [km]. |
| Missile position | (0, 0) [km]. |
| Target velocity | $V_T$= 300 [m/sec]. |
| Missile velocity | $V_M$= 1000 [m/sec]. |
| Missile Heading error | HE = 0.52 [rad]. |
| Navigation ratio | N=4. |

Figure 8 illustrates the mathematical representation of the overall system in which the missile's dynamic is considered as a second-order transfer function while the actuator is considered as a constant equal to 1. In the simulation, we assumed that the motors of the missile and the target are able to give the demanded velocities.

The selected time-step for the simulation process is chosen as 0.01 second. This is mainly due to the fact that missile-gyros usually gyrate about 100 cycles per second. Nowadays' chips which are used in the industrial implementation of FLCs, such as Digital Signal Processors (DSPs), Microcontrollers (MCs) and Field Programmable Gate Arrays (FPGAs), readily insure this time-step without any delay which means real time work [21].
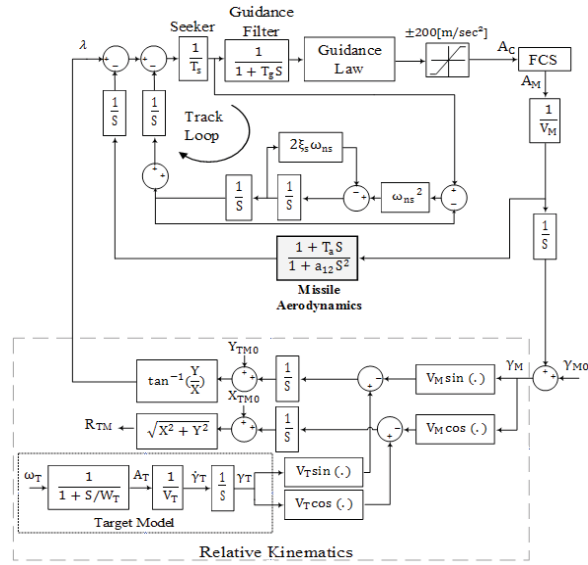
(a)
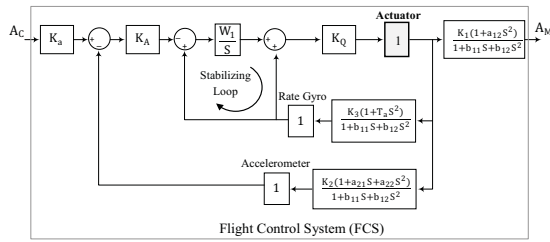


(b)

**Figure 8. (a) Simplified homing loop, (b) flight control system.**

**Table 10. Selected nominal parameters.**

| $T_a$ | $T_a$ | $T_s$ | $K_a$ | $K_A$ | $K_Q$ |
|-------|-------|-------|-------|-------|-------|
| 0.32 | 0.12 | 0.1 | 2.94 | 0.38 | 0.35 |
| $K_1$ | $K_2$ | $K_3$ | $a_{12}$ | $a_{21}$ | $a_{22}$ |
| -1.57 | -1.57 | -1.16 | -0.00053 | 0.000645 | -0.00032 |
| $b_{11}$ | $b_{12}$ | $W_1$ | $W_T$ | $\omega_{ns}$ | $\xi_s$ |
| 0.01 | 0.003 | 18.4 | 2 | 100 | 1 |

Engagement accuracy of the prior guidance laws against 16 different scenarios of a maneuvering target helps us to fully examine the performance of the control system. In these case studies, the target accelerates in the range of [-40, 40] m/sec² and we desire to achieve minimum miss-distance. The 16 scenarios of the target maneuvering are shown in Table 11.

**Table 11. Different scenarios of target maneuvering.**

| $A_T$ | Up | 40 | 35 | 30 | 25 | 20 | 15 | 10 | 5 |
|-------|------|----|----|----|----|----|----|----|----|
| [m/sec²] | Down | -40 | -35 | -30 | -25 | -20 | -15 | -10 | -5 |

Table 12 lists the results: miss distance ($M_D$), flight time ($F_T$), and control effort ($C_{Eff}$) for a few of the prior scenarios, while the root-mean-square (RMS) values for the 16 scenarios are also added. An example which illustrates how the missile attacks the target is mapped in Fig. 10. Figure 10-a outlines the missile-target trajectories for scenario 1 in case of the upward target maneuvering, while Fig. 10-b plots the trajectories for scenario 5 in case of the downward maneuvering. Figure 11 maps the time histories of the missile lateral acceleration for the mentioned two scenarios respectively, where the control effort is calculated as follows:

$$C_{Eff} = \int_0^{FT} A_C^2 \, dt \tag{7}$$

Furthermore, Fig. 12 was included to show the time history of the control surface angle for both scenarios.

**Table 12. Performance of (PNG) and (TFG) for different target accelerations.**

| Target Acceleration [m/sec²] | Guidance Law | $M_D$ [m] | $F_T$ [sec] | $C_{Eff} \times 10^4$ [m²/sec³] |
|---|---|---|---|---|
| Scenario 1 $A_T = 40$ | PNG | 7.81 | 10.17 | 4.63 |
| | TFG | 8.20 | 10.18 | 2.64 |
| Scenario 2 $A_T = 20$ | PNG | 13.3 | 9.36 | 4.36 |
| | TFG | 13.6 | 9.38 | 1.92 |
| Scenario 3 $A_T = 0$ | PNG | 18.1 | 8.94 | 3.38 |
| | TFG | 13.7 | 9.95 | 0.86 |
| Scenario 4 $A_T = -20$ | PNG | 20.3 | 8.77 | 8.56 |
| | TFG | 12.7 | 8.76 | 3.47 |
| Scenario 5 $A_T = -40$ | PNG | 15.5 | 8.88 | 14.6 |
| | TFG | 8.50 | 8.79 | 8.95 |
| RMS of 16 scenarios | PNG | 15.1 | 9.25 | 6.13 |
| | TFG | 11.6 | 9.27 | 3.62 |

Overall, the results show that the TFG law gives a relatively better performance in terms of the miss distance and control effort while demonstrating a negligible time increase.
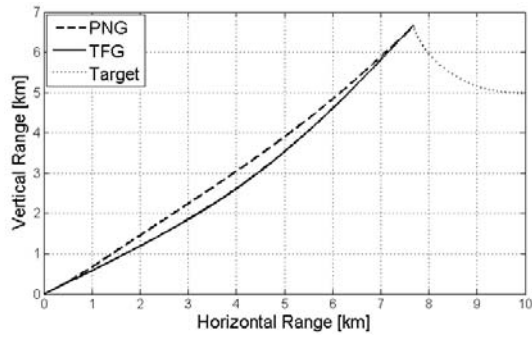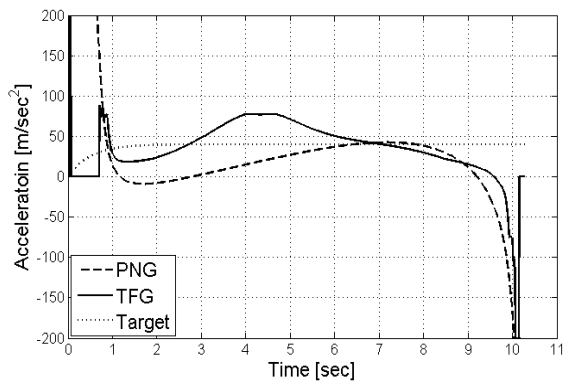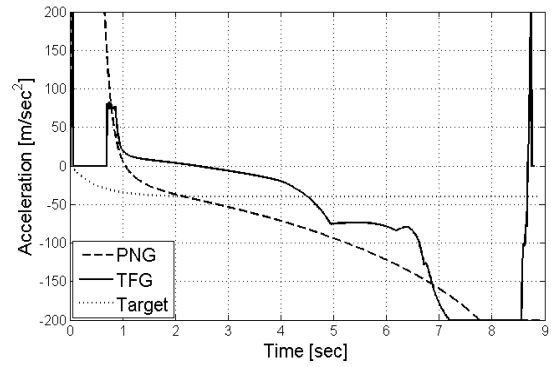
(a)
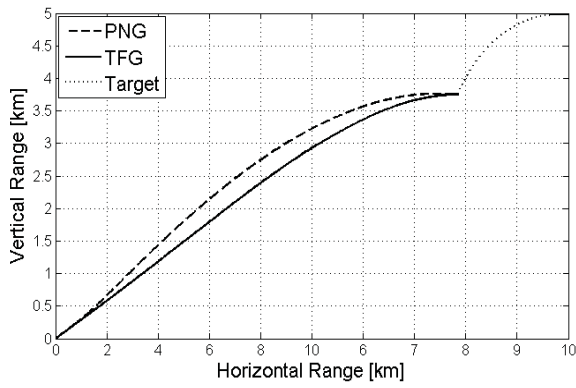


(b)

**Figure 10. Comparison of trajectories,**
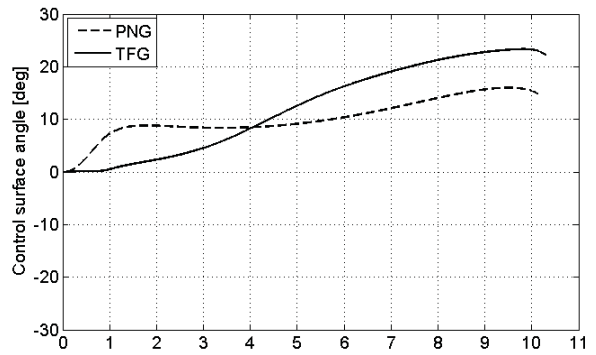**(a) Upward maneuver, (b) Downward maneuver.**
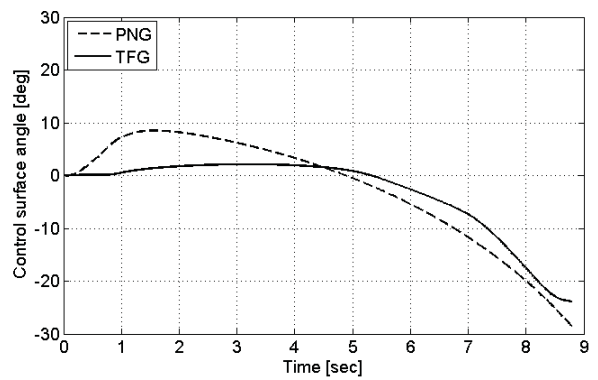


(b)

**Figure 11. Comparison of acceleration,**
**(a)   Up-ward maneuver,   (b) Down-ward maneuver.**



(a)



(a)



(b)

**Figure 12. Time history of control surface angle,**
**Upward maneuver, (b)   Downward maneuver.**

It is also noticeable that, as expected, the designed guidance law gives a straight-line trajectory during the initial phase as shown in Fig. 10. This is because of intentional low sensitivity selected to the target maneuvering during the launch phase, while the trajectory curves toward the target during the other two phases. This could be the source of a slight increase in time to intercept. On the other hand, TFG requires about 60% of the total control effort demanded by PNG to guide the missile toward its target which is quite important.

## 5 Conclusion

In this work, a TFG law for a surface to air homing missile was investigated. The introduced design includes three sub-fuzzy guidance laws which switched together according to the relative distance between the target and the missile. The new guidance law enables the missile to track a maneuvering target from the launch up to the interception. The performance of the TFG law was compared with the classical PNG one, and root-mean-square values of miss distance, control effort and flight time are calculated for various scenarios of the target maneuvering. The results showed relative superiority of the introduced approach in terms of miss distance, control energy expenditure, and inte2frception time. Nevertheless, further investigation might be required to examine the effect of sensor noise and uncertainties in both missile and target dynamics.

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