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Process Improvement of Experimental Measurements, Using D-optimal Models

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In this paper, the application of D-optimal models as an alternative to response surface models (RS models) for design of experiments (DOE) was examined. Two D-optimal models for tilt-rotors in the wind tunnel experiment, as a form of quadratic functions, were generated based on a chosen optimality criterion. This optimality criterion was used to generate the optimized sampled points in the design space in order to minimize the variance of the coefficients for the quadratic functions. The main advantage of D-optimal modeling process is alleviating the high computational burden of constructing the RS models. Error analysis of the developed models was performed using analysis of variance (ANOVA). The ANOVA of the D-optimal thrust and rolling moment models for tilt-rotors showed that the lateral position of the downwind tilt-rotor relative to the upwind tilt-rotor is the most significant variable affecting the rolling moment and thrust variations. The results also showed that all the models were significant with more than 95% confidence level.

NOMENCLATURE

A	Rotor area
C	Blade chord length
C_T	Thrust coefficient$(T/2\rho A(R\Omega)^2)$
C_{MX}	Rolling moment coefficient $(M_x/2\rho AR(R\Omega)^2)$
D	Rotor diameter
$f(x)$	Quadratic portion of a D-optimal model
$F-value$	Ratio of MSR to MSE
R	Blade radius
S	Wing span
SSE	Sum of squares due to error $(\sum_{i=1}^{n}(y_i-(\hat{y}_i))^2)$
SSR	Sum of squares due to model $(\sum_{i=1}^{n}(\hat{y}_i-(\bar{y}_i))^2)$
SST	Total sum of square $= SSE + SSR$
m	Number of model coefficients
MSR	Mean square due to model $= SSR/(m-1)$
MSE	Mean square due to error $= SSE/(n-m)$
n	Number of response values (observation)
N	Number of rotor blade
$(X/D, Y/S, Z/S)$	Longitudinal, lateral and vertical location of downwind air vehicle w.r.t. upwind air vehicle
y	Actual response
$\bar{y}$	Mean value of actual response
$\hat{y}$	Predicted or model response
ε	Random errors
β	Response coefficients of predicted model
μ	Advance ratio $= V_\infty/R\Omega$
Ω	Rotational speed (RPM)
ρ	Air density

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INTRODUCTION

Generally, the Design of Experiments (DOE) is an attractive tool for planning experiments so that the data obtained can be analyzed to yield valid conclusions.

Planning an experiment starts with determining the objectives of an experiment and proceeds with selecting the variables, DOE, execution of the design, checking the data and data interpretation and analysis. The objectives of an experiment are commonly classified into comparative, screening, response surface and regression modeling. In all cases, the data sampled are used to derive the fitted models, linking the output and input (variables). It has been found that the characteristics of the fitted models strongly depend on the number and the arrangement of sample points [1]. Consequently, DOE is a major part of an experiment before doing that experiment.

Studies have shown that the most popular methods for DOE are Response Surface Methods (RSMs) [1]. Response surface models (RS models) obtained from RSMs are commonly quadratic functions (second-order polynomial) in which they fit to the sampled data using least-squares regression (LSR). In RSMs, the data are sampled using classic methods such as Central Composite Design (CCD), Box-Behnken Design (BBD), etc. Each method has special features for data collection and RS model building. Once the RS model is generated, a maximum, a minimum or an area where the response is stable over a range of factors can be obtained. Recent reviews of RSMs in aerospace and mechanical engineering are available in the literature [2-4] so they have not been repeated here.

In all classic methods, the number of sampled points depends on the number of variables (factors) in the RS model. As the number of factors increases, the number of sample points required for RS models quadratically increases. Moreover, classic methods are not sufficient for non-linear design space. These are, in fact, the major limitations for the CCD and the BBD application in the non-linear DOE. Barton [5] has shown that higher-order RS models (*e.g.* cubic, quarteric...) can be used for modeling a non-linear design space, but they are unstable and they need a broad range of sample points, particularly in high dimensions. Many researchers recommend the use of a sequential RSM with move limits [6], or a trust region method [7], instead of higher-order RS models. In addition, the Hierarchical and Interactive Decision Refinement Methodology (HIDRM) is a sequential RSM that is used to separate the design space into sub-regions. Then, it fits each region with a separate model [8]. Most of the sequential methods have been developed for single-objective optimization problems whereas much of the engineering design is multi-objective. Barton [5] has reported that the design space cannot be separated into sub-regions that are good for all objectives of the multi-objective optimization problems. Moreover, Koch *et.al.* [9] have discussed about the difficulties of RS models for multi-objective designs. Unlike classic RSMs, D-optimal is a computerized design method, which is suitable for modeling non-linear design space. The optimized sample points calculated from the design space can approximate the quadratic coefficients well. The main advantage of D-optimal models is to alleviate the high computational expense of constructing models for high-dimensional problems. For the same example problems with five factors, D-optimal models require at least 21 sampled data to construct the quadratic model whereas RSMs consider 50 observations at different locations with only 8 center points. Comparisons of DOE methods are available elsewhere in the literature [10-14].

In this research, two numerical and one categorical factors in 3-level (*i.e.*, low, medium and high) were used to construct the quadratic D-optimal models. All the 12 unknown coefficients of the models were estimated on the basis of optimized data points. A total of 22 optimized points were calculated from 51 initial candidate points using the steepest descent technique. The candidate points were vertices, center points, centers of edges, triple blends, interior points, etc. all of which are located all through the design space. In this direction, D-optimal models were developed for the tilt-rotor aerodynamic interaction problem. The error analysis of the models showed that the thrust and rolling moment variations had been well designed with more than 95% confidence level.

RSM: RS MODELS

Using LSR, RSMs develop RS models by fitting the sample data. The actual response can be written as [15]:

$$y = f(x) + \varepsilon \tag{1}$$

where $f(x)$ is an unknown response function and ε is the random error. The actual response, Eq. 1, can be written in terms of a series of $n-th$ observations as follows:

$$y_i = \beta_0 + \sum_{j=1}^{k} \beta_j x_{ij} + \varepsilon_i \qquad i = 1\ldots, n \tag{2}$$

where x_{ij} denotes the $i-th$ observation of variable x_j. The $\beta's$ in Eq. 2 can be estimated using the method of LSR as:

$$L = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{i=1}^{k} \beta_j x_{ij} \right)^2 \tag{3}$$

The function L will be minimized when $(\partial L/\partial \beta)$ is set to zero. Eq. 2 may be written in matrix notation as:

$$y = X\beta + \varepsilon \tag{4}$$

where y is an $(n \times 1)$ vector of observations, X is an $(n \times p)$ matrix of the levels of independent variables (design matrix), β is a $(p \times 1)$ vector of the regression coefficients, and ε is an $(n \times 1)$ vector of random errors. The vector of least square estimators $\hat{\beta}$ is determined in a way that it minimizes:

$$L = \sum_{i=1}^{n} \varepsilon_i^2 = (y - X\beta)^T (y - X\beta) \tag{5}$$

This condition is simplified as:

$$X^T X \hat{\beta} = X^T y \tag{6}$$

Thus:

$$\hat{\beta} = \left(X^T X\right)^{-1} X^T y \tag{7}$$

The fitted regression model, therefore, is corresponded to:

$$\hat{y} = X\hat{\beta} \tag{8}$$

The reader is referred to [15] for more details on the development of RSMs. The process of modeling repeats similarly when $f(x)$ in Eq. 1 is considered as the quadratic RS model:

$$\beta_0+\sum_{j=1}^{k} \beta_j x_j+\sum_{j=1}^{k} \beta_{jj} x_j^2+\sum_{i<j}^{k} \beta_{ij} x_i x_j+\varepsilon_i \qquad i = 1, \ldots, n \tag{9}$$

in which there are $(k+1)(k+2)/2$ unknown coefficients to be estimated and k is the number of factors. When constructing a quadratic model, the design variables need to be evaluated at least at three locations (3-level) of the design space to estimate the coefficients in the model. This leads to a (3^k) factorial design of the experiments that requires (3^k) data samples. However, the CCD [15] has become a popular alternative for the second-order RS models. CCDs are (2^k) factorial designs augmented by the $2k$ star (axial) points as well as the central points to allow for the estimation of the second-order coefficients. For two-factor cases, the CCD considers at least 9 observations at different design points with only a center point. Additional details on LSR and RSMs can be found in many documents, including [1, 16-18].

D-OPTIMAL MODELS

In general, there are conditions where some type of computer-generated design may be appropriate: 1) an irregular design experimental region, 2) a non-standard model (*i.e.* quarteric), 3) unusual sample size (*e.g.* categorical factors), and 4) the need to reduce the number of runs required by a standard RSM. The usual approach is to specify a model, determine the region of interest, select the number of observations, specify the optimality criterion, and then choose the design points from a set of candidate points. Thus, with the choice of the second-order RS model, Eq. 9, associated with the (D-optimality) criterion the D-optimal model can be obtained as:

$$y = \beta_0 + \sum_{j=1}^{k} \beta_j x_j + \sum_{j=1}^{k} \beta_{jj} x_j^2 + \sum_{i<j}^{k} \beta_{ij} x_i x_j + \varepsilon_i$$

$$Maximize \left|X^T X\right| \tag{10}$$

where $\left|X^T X\right|$ is determinant of the information matrix $X^T X$ in Eq. 7. Thus, finding a design matrix X from a set of candidate points that maximize the determinant of information matrix means finding a design region where the factor effects are maximally independent of each other (determinant of the correlation matrix is non-zero). Using Eq. 10, the expected prediction error for the factors will also be minimized. The optimal region can be obtained using the Steepest Ascent (Descent) Method (SAM) in two phases. The first phase is composed of a sequence of line searches in the direction of maximum improvement. Each search in the sequence is continued until there is evidence that the direction chosen dose not result in further improvements. The sequence of line searches is performed as long as there is no evidence of lack of fit for a simple first-order model of the form given in Eq. 2. The second step is performed when there is lack of linear fit in the first step, and, instead, a quadratic function, Eq. 9, is, therefore, fitted. The SAM, finally, resulted in the minimized variance coefficients given in Eq. 6. Figure 1 shows a flow diagram of the two phases of the optimization process.

MODELING ERROR ESTIMATION

Each approximated model was constructed based on the results obtained at n sample points. The accuracy of D-optimal models was estimated using the R-squared and adjusted R-squared as follows:

$$R - squared = SSR/SST = 1 - (SSE/SST) \tag{11}$$

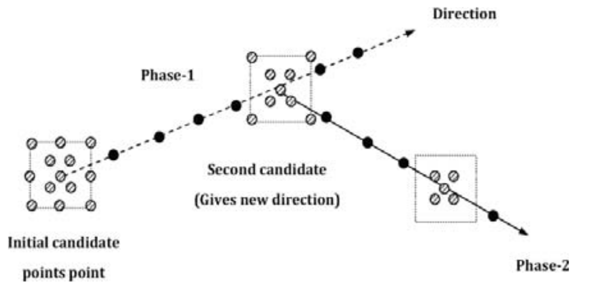

Figure 1. The sequence of line searches for a 2-factor optimization problem.

$$adjusted\,R - squared = (SSR(n-1))\,/\,(SST(n-m)) =1-(SSE(n-1))/(SST(n-m)) \quad (12)$$

where n and m are the number of sample points and the number of model coefficients, respectively. The sum of square about the mean of the actual data (SST) is defined as:

$$SST = SSE + SSR \quad (13)$$

where the sum of square error SSE is defined as the difference between the actual data y_i and the predicted value from the D-optimal model $\hat{y}_i$ and sum of square of regression SSR is defined as the difference between the predicted values from the D-optimal model, $\hat{y}_i$ and the mean of the actual data $\bar{y}$.

$$SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \,, SSR = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2 \quad (14)$$

TEST PROBLEM: TILT-ROTORS AERODYNAMIC INTERACTIONS

The design problem in question involves the examination of the aerodynamic coefficients (thrust and rolling moment variations) of two tilt-rotors in tandem flight model [19]. The relative positions of the tilt- rotors were specified with a total of three factors. Figure 2 illustrates the relative position of two tilt-rotors. As seen in Figure 2, the flow pattern passing over the downwind tilt-rotor strongly changes with the position of the upwind tilt-rotor.

The relative position was parameterized with a total of three factors (A, B and C), chosen to represent the longitudinal, lateral and vertical positions of the downwind tilt-rotor at three different stations, respectively. Since there were no sufficient data available, the first factor X/D was categorized into three levels (*i.e.*, low, medium and high) whereas the other two factors were considered as numeric a factors. Table 1 shows the relevant test plan of the tilt-rotors. As seen in Table 1, the categorical factor was segmented into 2.5, 5 and 10 that were considered for the actual test plan. More details of the measurement setup and scaling are reported in [19].

The aerodynamic coefficients (C_T/σ and C_{MX}/σ) were considered as responses y_1 and y_2. In conventional set up case, the total data were at least 380 [19].

Table 1. Tilt- rotor test plan.

Experiment plan			
Number of points (Observations)	>380		
X/D	Low=2.5	Medium=5	High=10
μ	0.1	0.1	0.1
Tilt-rotor (1)			
RPM	6356	6356	6356
Reference Conditions			
C_T/σ	0.121	0.121	0.121
C_{MX}/σ	0.0085	0.0085	0.0085
Tilt-rotor (2)			
RPM	6319	6319	6319
Trim Condition			
C_T/σ	0.121	0.121	0.121

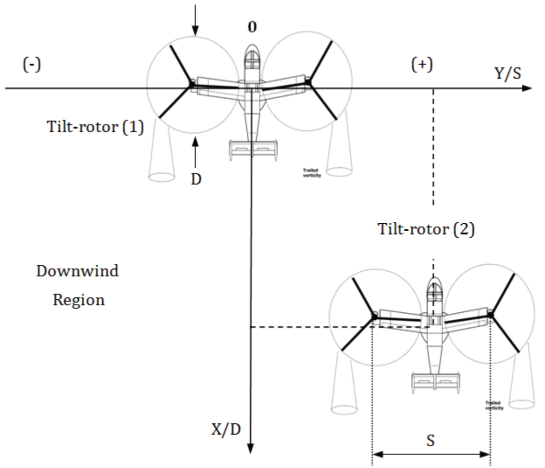

Figure 2. The relative positions of the two tilt-rotors in tandem flight.

Overall, 22 sample points were calculated from a set of 51 candidate points using the steepest decent method. Consequently, the experimental space was converted into what is shown in Table 2. The range of each factor, [a, b], was reduced to a common scale, [-1, +1], regardless of its relative magnitude. The coded

D-OPTIMAL MODEL BUILDING

Initially, 51 candidate points including 4 vertices, 4 center of edges, 4 axial check points, 4 interior points and one overall centroid were considered for the 3-D design of the experiment space. Figure 3 illustrates the total number of candidate points.

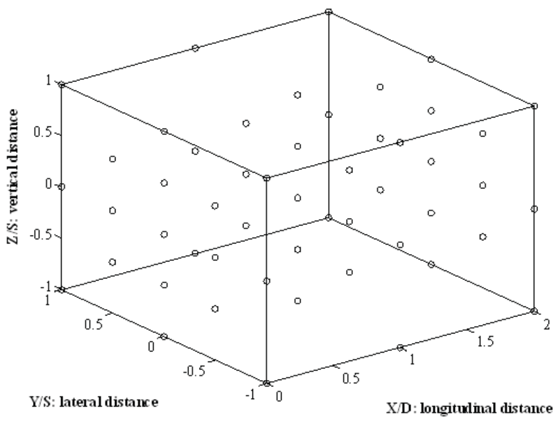

Figure 3. Distribution of the candidate points in the design space.

factor is defined as:

$$X_{coded} = \frac{(x_{actual} - \bar{x})}{(b-a)/2}, \quad \bar{x} = (a+b)/2 \qquad (15)$$

The mentioned 22 sample points were sufficient for determination of the D-optimal quadratic models as:

$$C_T/\sigma = 0.014 + 0.003A - 0.002B - 0.0001C[1] - 0.002C[2] + 0.003AB - 0.004AC[1] + 0.0003AC[2] + 0.002BC[1] - 0.002BC[2] + 0.008A^2 - 0.005B^2 \qquad (16)$$

and the rolling moment coefficient:

$$C_{MX}/\sigma = 0.006 + 0.029A - 0.002B - 0.011C[1] - 0.003C[2] + 0.006AB - 0.009AC[1] + 0.006AC[2] - 0.002BC[1] + 0.002BC[2] - 0.0007A^2 - 0.005B^2 \qquad (17)$$

where, A, B and C are the coded factors. C[1] and C[2] are the differences of levels 1 and 2 of C factor from the overall average response, respectively.

RESULTS

The accuracy of D-optimal models, Eqs. 16 and 17, was estimated using R-squared relations in Eqs. 11-14. In the case of $n = 22$ and $m = 12$, the R-squared values of the thrust and rolling moment were 0.9605 and 0.9543, respectively. Consequently, each model predicts the actual response with good accuracy. Further examination is shown in Figures 4 and 5. As shown, the predicted values are close to the actual values of thrust and rolling moment so the D-optimal models can well approximate the actual data.

The residual values (random errors) of each model are shown in Figures 6 and 7. The reference lines at (-3) and (+3) emphasize that the residuals are bounded for a better understanding. As seen in the figures, there are no systematic trends apparent for the residuals. Since the residuals have no systematic trend, the models have well fitted to the data. Moreover, the residual plots illustrate the constant standard deviation in the data so the assumption of constant standard deviation for random errors is sufficiently satisfied.

Table 2. D-optimal test plan (matrix).

Coded Factor				Response	
Sample	A	B	C	C_T/σ	C_{MX}/σ
1	-1	0	Medium	-0.005	-0.04
2	-1	+1	Low	-0.0123	-0.021
3	-1	-1	High	-0.011	-0.032
4	-1	-1	High	-0.0134	-0.046
⋮	⋮	⋮	⋮	⋮	⋮
21	+1	0	Medium	0.001	0.043
22	+1	-1	Low	-0.026	0.021

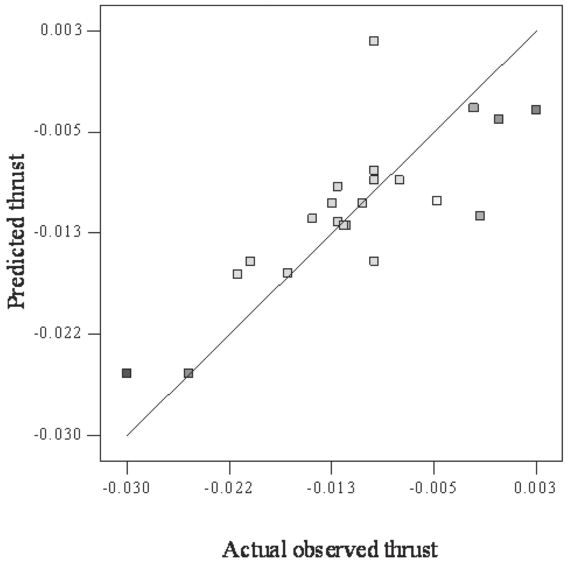

Figure 4. Actual response vs. predicted thrust.

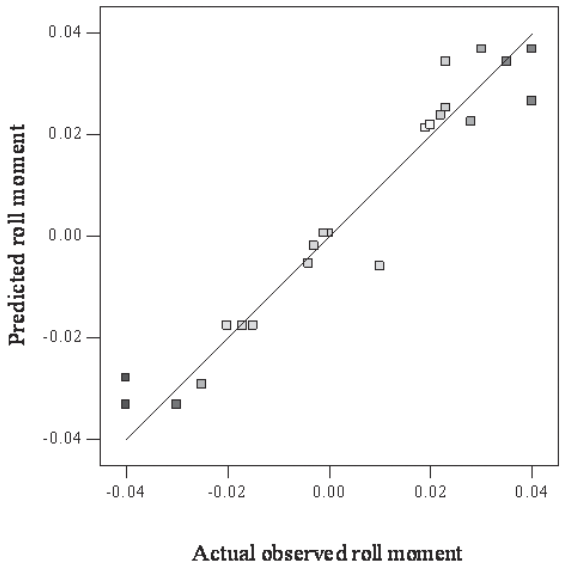

Figure 5. Actual response vs. predicted rolling moment.

The plots of factor Y/S versus residuals are shown in Figures 8 and 9. A residual distribution, as shown in Figures 8 and 9, shows a trend to lower residuals as the value of the response increases. It indicates that we should not transform the responses because the data are fitted well.

Table 3 summarizes the results of the analysis of variance for two D-optimal models. As seen, F- value of the rolling moment is 24.4. The F-value is defined as the ratio of model mean square (MSR) to residual mean square (MSE). It implies that the rolling moment model is significant. There is only a 0.01% chance for an error that affects the rolling response. The p-

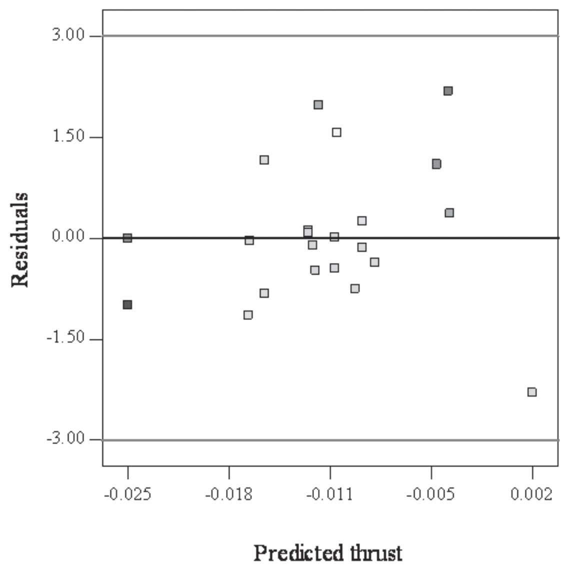

Figure 6. Residual vs. predicted thrust response.

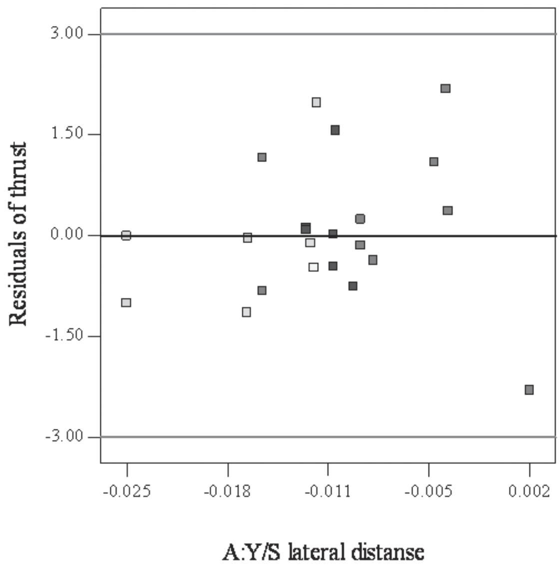

Figure 8. Residual vs. predicted lateral distance variable.

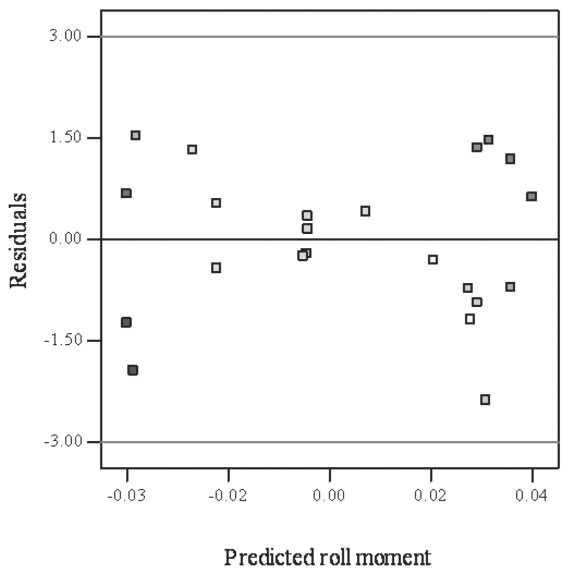

Figure 7. Residual vs. predicted rolling moment response.

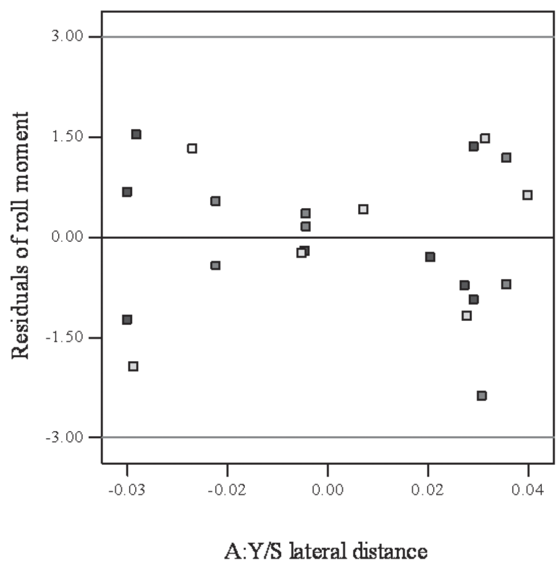

Figure 9. Residual vs. predicted lateral distance variable.

value (see [1]) less than 0.05 shows that factor (A) has significant influence on the rolling moment response. Furthermore, lack of fit F-value of 1.78 shows that lack of fit is not significant relative to the error. It means that there is a 27.11% chance for an error that affects lack of fit. As seen in Table 3, the F-value of the thrust is 16.2. It implies that the thrust model is, therefore, significant. In this case, there is only a 0.01% chance for an error to affect the thrust. The p-value less than 0.05 shows that the model terms are significant. In addition, lack of fit F-value of 6.12 shows that lack of fit is not significant relative to the error. Thus, there is a 3.43% chance for an error to affect the lack of fit.

The main effects (A, B and C) and factor interactions (AB, AC, BC...) are shown in Figures. 10, 11 and 12. As seen in Figure 11a, the rolling moment has got higher effects in the range of factors relative to the thrust. Both Figure 10 and 11 show that the A ($= Y/S$) is a significant factor relative to (B) and (C). The main reason is the occurrence of the interactions between the factors, which are presented in Figure 12.

Table 3. Analysis of variance test (ANOVA).

Model	Sum of squares	Mean squares	F-value	p-value
C_T/σ	8.6E-04	7.77E-05	16.2	< 0.0001
Residual	4.8E-04	4.80E-05		
Lack of fit	4.1E-04	8.25E-05	6.12	0.0343
C_{MX}/σ	1.4E-02	1.26E-03	24.4	< 0.0001
Residual	5.1E-04	5.14E-05		
Lack of fit	3.3E-04	6.59E-05	1.78	0.2711

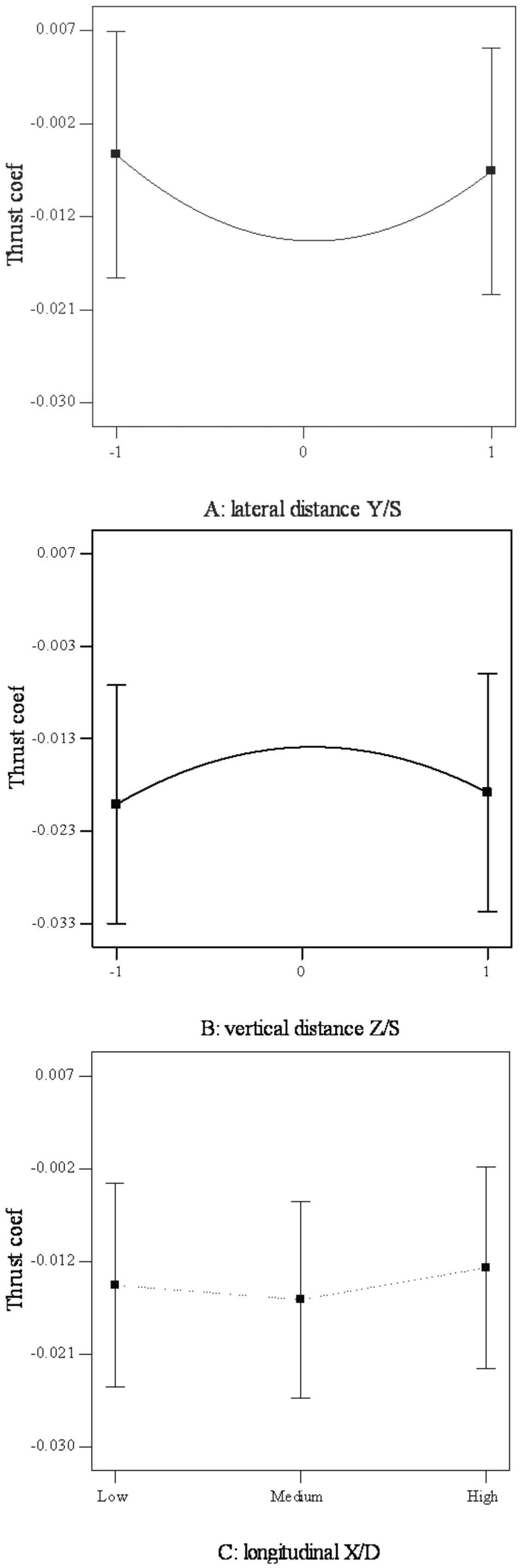

Figure 10. Influence of the main effects on the predicted thrust response.

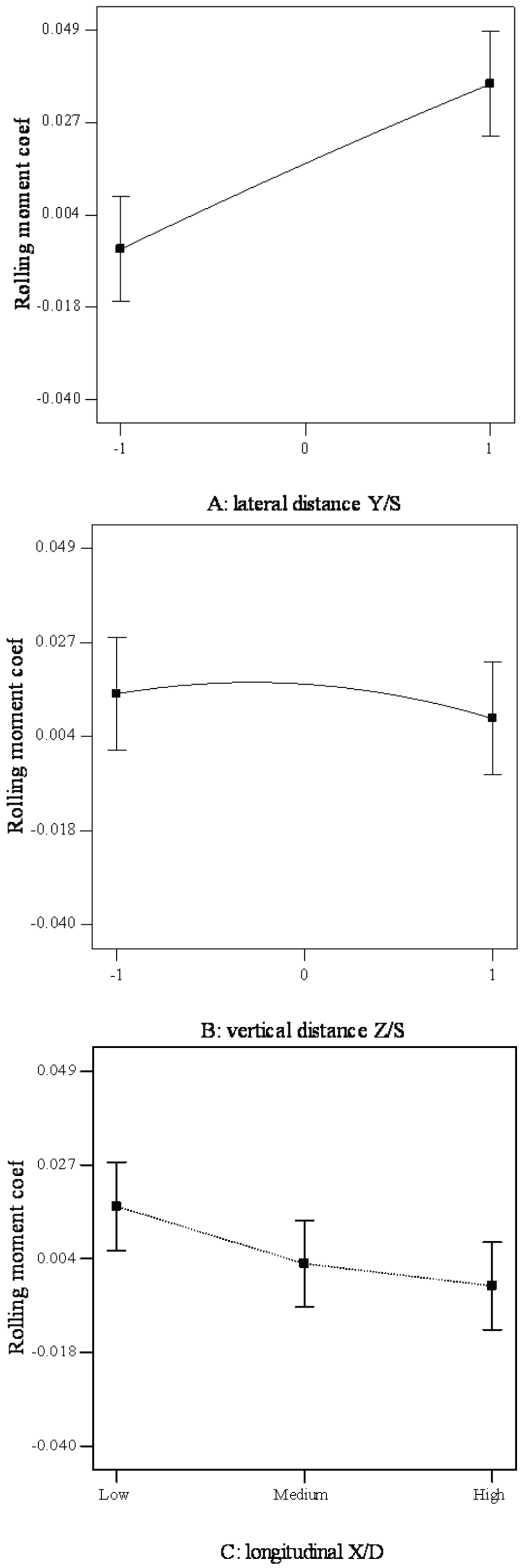

Figure 11. Influence of the main effects on the predicted rolling moment response.

In Figure 12, the interaction term (AB) at the high and low values of (B+) and (B-) is shown. Here, (B+) and (B-) are the highest and the lowest levels of factor (B), respectively. The non-parallel curves in this figure imply that there is a significant interaction between (A) and (B). As seen, the effect of factor (A) depends on the level of factor (B) so the significant interaction is apparent for each of the predicted models. The I- beam range symbols in the graphs are the results of the least significant difference (LSD) calculations. In the case that the points are all outside the range, the differences are caused by the error alone and can be attributed to the factor effects. The I-beam is somewhere overlapped, which means that there is no significant difference between the two points (*i.e.* 95% confidence limit).

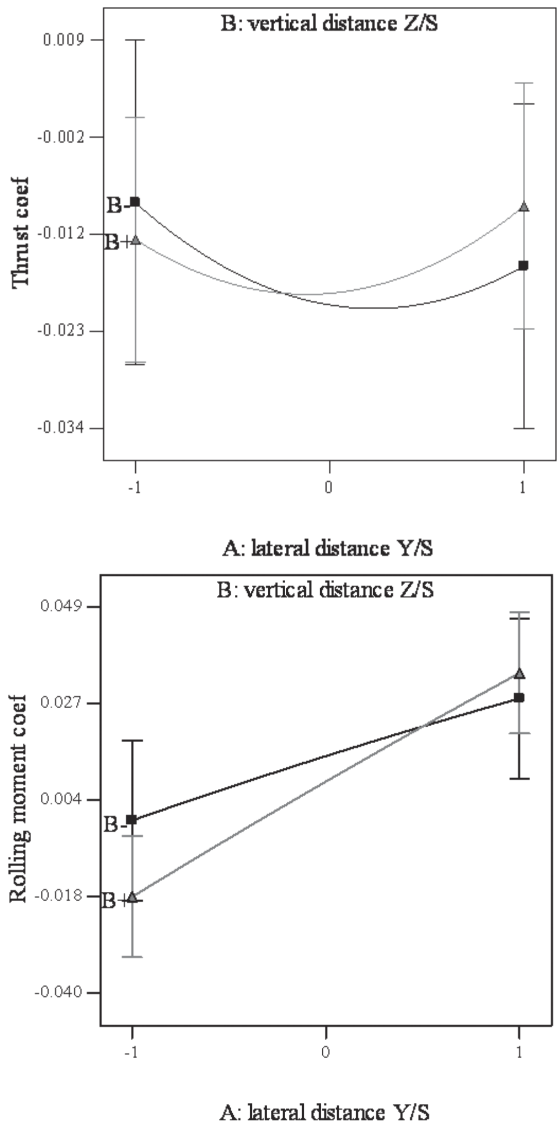

Figure 12. Influence of factor interaction on the predicted response.

Figures 13 and 14 show a comparison of the D-optimal results with the experimental data, [19]. Each figure has the same value of longitudinal distance (*i.e.*, $X/D = 2.5$) when the advance ratio is 0.1. The top plot has been generated by the current research and the bottom has been taken from [19]. In Figure 13, the contours of the thrust model and actual thrust are fairly similar. As evidenced by the high R-squared we expect the thrust model to approximate the thrust quite well. As seen in Figure 14, the contours of the rolling moment and the actual data are also similar. As expected from the high R-squared, this model also approximates the value of the rolling moment changes quite well.

CONCLUSIONS

In this paper, the application of DOE in experimental planning was examined through a tilt-rotor example. Among the several methods for DOE, the D-optimal model was our candidate due to its optimal performance and capability for modeling non-linear design spaces. The research findings showed that D-optimal models are that are fitted to optimum sample points. The optimized sample points were generated by RSMs. This was the main difference between the D-optimal

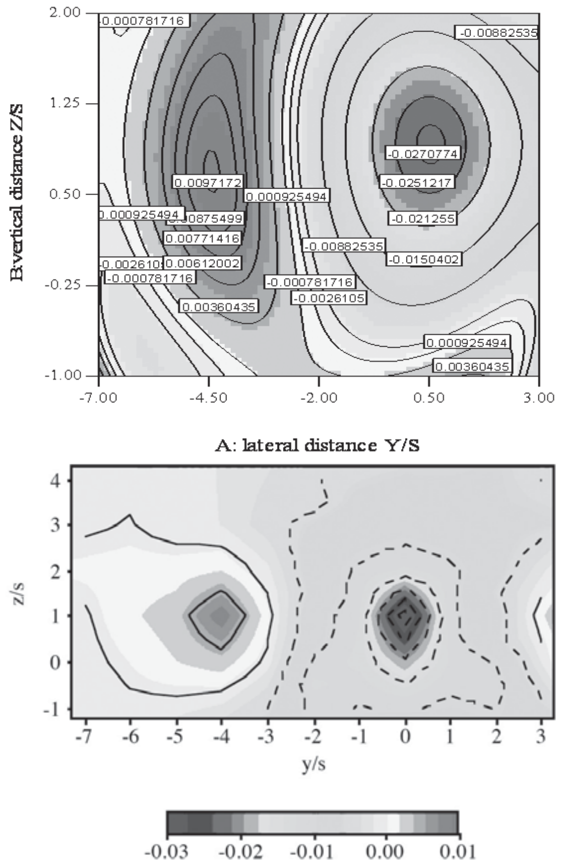

Figure 13. Comparison of the D-optimal thrust prediction with the actual data.

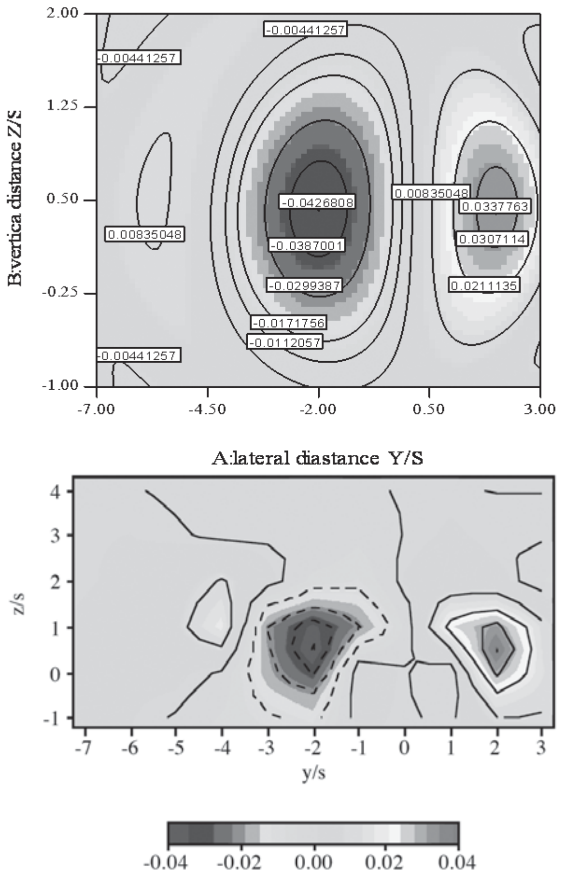

Figure 14. Comparison of the D-optimal rolling moment prediction with the actual data.

models and the classic RSMs. Moreover, the number of optimized sample points (22-point) in comparison to the 380 data points in [19] proved that the D-optimal model can reduce the computational cost with a minimum number of runs. The analysis of variance shows that each model has a high level of accuracy.

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