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Optimum Design of Filament-Wound Laminated Conical Shells for Buckling Using the Penalty Function

M. Z. Kabir[1], A. Rojhani Shirazi[2]

Optimum laminate configuration for minimum weight of filament–wound laminated conical shells subject to buckling load constraint is investigated. In the case of a laminated conical shell, the thickness and the ply orientation (the design variables) are functions of the shell coordinates, influencing both the buckling load and its weight. These effects complicate the optimization problem considerably. The first level of complexity is attributed to the correlation between the volume and the buckling load and their dependence on the fiber configuration. The second level of complexity is associated with the high computational cost involved in calculation of the buckling load. Thus, the main objective of this study is to solve the optimization problem as well as to reduce the computational cost associated with it. Based on the characteristic buckling behavior of laminated conical shells, the usual penalty function method is used.

NOMENCLATURE

W	Weight of the shell.
$[A],[B],[D]$	Stiffness matrices.
P^*	Specified buckling load.
ρ	Mass density.
V	Volume of the shell.
$\pm\theta_1^{(i)}$	Fiber orientations.
S	Longitudinal coordinate in the cone surface.
S_1	Longitudinal coordinate at the small end of the cone.
t_1^i	Thickness of the layer at the small end.
α	Cone semi-vertex angles.
$\{N\}^T$	Membrane force vectors.
$\{M\}^T$	Bending moment vectors.
$[\bar{Q}]$	Laminated transformed reduced stiffness matrix.
$\{\Delta\}$	Displacement vector.
$[L]$	Coefficients of the linear operator ($L_{ij} = L_{ji}$).
$\widehat{N}_1$	Axial force.
$\widehat{N}_2$	Lateral force.
β_n	$(\frac{n\pi}{L})$.
β_m	$\frac{2m}{(R_1+R_2)}\cos\alpha$.
u_1, v_1, w_1	Amplitudes to be determined for each mode (m,n).
$(a_{ij} = a_{ji})$	Submatrices identified from the eigenvalue problem.
$\phi(x,r)$	Penalty function.
$S.F$	Safety factor.

1. *Associate Professor, Dept. of Civil & Environment Eng., AmirKabir Univ. of Tech., Tehran, Iran.*
2. *M.Sc. Graduate, Dept. of Civil & Environment Eng., AmirKabir Univ. of Tech., Tehran, Iran.*

INTRODUCTION

Multi–layered angle–ply composites are important structural materials in a number of areas of engineering, namely in aerospace, civil and mechanical engineering. Laminated conical shells are usually used as a connection between two cylindrical shells of different diameters. The main goal is then minimization of the weight of the structure under the constraint of carrying the applied load. The design variables are usually the fiber orientations and the thickness.

In the case of a shell of laminated composite material, unlike an isotropic conical shell, the thickness and the material properties vary with the shell coordinates, which ultimately results in coordinate dependence on stiffness matrices (A, B, D).

The calculation of the "exact" buckling load of a laminated conical shell, taking into account the variation of the stiffness coefficients, requires an analytic representation of these functions, depending on many factors such as the manufacturing process, variation of the ply angle, and the change in the thickness of the shell (see Baruch, Arbocz & Zhang [1,2]). In addition, calculation of the buckling load by solving the system of non linear governing partial differential equations with variable coefficients complicates the problem considerably. As far as the authors know, the only work that has been done to calculate the exact buckling load with consideration of variation of the stiffness coefficients is that of Goldfeld and Arbocz [3]. The buckling loads were calculated by means of the computer code STAGS- A [4] by adding a user written subroutine WALL.

Unlike a laminated cylindrical shell, in the case of a laminated conical shell, due to the variation of thickness and ply orientation with the shell coordinates, the volume of the shell is also dependent on the ply configuration. Thus, for optimization, both the buckling load (usually, the constraint) and the weight of the laminated conical shell (usually, the objective function) are strong functions of the ply configuration (the design variables). Also the fiber orientation and the thickness of the conical shell are strong functions of the shell coordinates, and cannot be taken as nominal values. This complicates the optimization procedure significantly.

Furthermore, since the structural analysis is based on variable stiffness coefficients, high computational cost is involved in calculating the buckling load.

The only work that deals with the optimization of a laminated conical shell is that of Brown and Nachals [5]. The design objective was to tailor the stiffness and thermal expansion characteristics of the composite to achieve maximum strength. The design variables were the ply orientations. Dependence of the material properties on the shell coordinates was approximated by taking them piecewise constant along the axial direction. That led to no consideration of the dependence of the weight of the shell on the chosen ply orientation.

In this work, a preliminary investigation was made into the characteristic buckling behavior of laminated conical shell; it was found that unlike the case of buckling of plates and cylindrical shells, the buckling load curve (load versus an appropriate shell parameter) of laminated conical shells with cone semi–vertex angles $\alpha > 10°$ are smooth and convex. There is no discontinuity in the slope of the buckling load curve due to change in the circumferential wave number. This behavior is typical of the buckling of conical shells (see Goldfeld, Sheinman and Baruch [6] for buckling curves of isotropic conical shells). Thus, the penalty function technique can be an effective procedure in solving the optimization problem of a laminated conical shell for buckling.

To summarize, the objective of this study is to develop an adaptive penalty function to obtain the minimum weight of filament–wound laminated conical shells subjected to buckling load constraint by optimizing the laminate stacking sequence (ply orientation angles and thickness).

PROBLEM DESCRIPTION

Laminated conical shells are usually used as a connection between two cylindrical shells of different diameters. Thus, the shell geometry is assumed to be given data, as well as the applied load, and the optimization problem will be formulated as the minimization of the weight W of the shell under the constraint of specified buckling load P^* (given as the applied load including an appropriate knock–down factor):

$$\begin{cases} \text{Minimize:} \\ W = \rho.V(\pm\theta_1^{(1)}, ..., \pm\theta_1^{(n)}, \pm t_1^{(1)}, ..., t_1^{(n)}) \\ \text{Subject to:} \\ P_{cr}(\pm\theta_1^{(1)}, ..., \pm\theta_1^{(n)}, \pm t_1^{(1)}, ..., t_1^{(n)}) \geq P^* \end{cases} \quad (1)$$

Where ρ is the mass density and V is the volume of the shell. The design variables are the fiber orientations $\pm\theta_1^{(i)}$, and the thickness $t_1^{(i)}$ at the small end of the shell for each layer (i).

Design Variables

In the present work, a laminated conical shell made by means of a horizontal helical filament winding machine and a conical shell mandrel is considered. Using a geodesic path (the shortest distance between two points on a surface) to represent the fiber orientation leads to the following effects (see Baruch, Arbocz and Zhang [1,2] and Goldfeld and Arbocz [3]):

- Variable ply angle along the axial coordinate (constant in the circumfernatial direction), the ply angle being larger at the small end:

$$\theta^{(i)}(S) = \arcsin\left(\frac{S_1}{S}\sin\theta_1^{(i)}\right) \quad (2)$$

- Where $\theta_1^{(i)}$ is the fiber orientation of the (i)th ply at the small end of the shell ($\theta_1^{(i)}$=0 is defined as the fiber in longitudinal direction), S is the longitudinal coordinate in the cone surface, and S_1 is the longitudinal coordinate at the small end of the cone, see Figure 1. In this work, the investigation focuses on ply orientations in the range of $\pm\theta_1^{(i)} = 0°$ to $80°$. In the case of a fiber orientation of 90°, a different procedure must be used, and the assumption of a geodesic path is no longer valid. The fiber orientation and the thickness then remain constant along the axial direction. In that case, therefore, the buckling load can be calculated on the basis of constant stiffness coefficients. That case is not treated here.

- Variable thickness $t^{(i)}$ for each layer along the axial coordinate (constant in the circumferential direction) is:

$$t^{(i)}(s) = t_1^{(i)} \frac{s_1 \cos\theta_1^{(i)}}{s \cos\theta^{(i)}} \tag{3}$$

where $t_1^{(i)}$ is the thickness of the layer at the small end. Since the volume of fiber across each specific cross–section along the axial direction must remain constant, the thickness of each layer at the small end is larger than that at the large end.

Objective Function – Volume of a Conical Shell
The variation of thickness in the axial direction implies that the volume of the shell is also a function of the fiber configuration. The volume V of a conical shell can be expressed as:

$$V = \int_{s1}^{s2} 2\pi . r(S).t(S)ds \tag{4}$$

where

$$r(S) = S.\sin\alpha \tag{5}$$

Substituting Eq. (2) into Eq. (3) and putting the result with Eq. (5) into Eq. (4) one obtains, after some

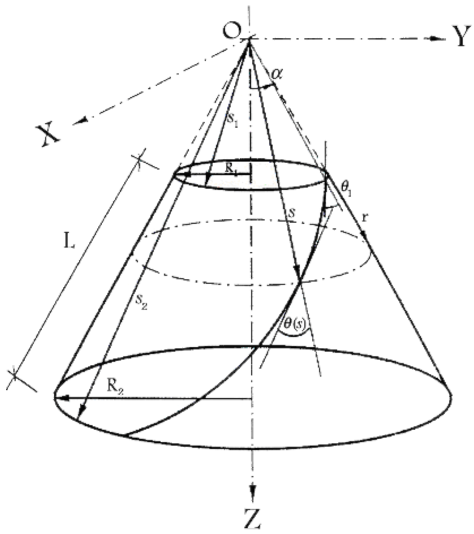

Figure 1. Geometry and sign convention for coordinates and the position of a fiber on a conical shell.

trigonometric manipulation:

$$V = 2\pi . R_1 \sum_{i=1}^{n} t_1^{(i)} \cos\theta_1^{(i)} \left[\sqrt{s_2^2 - s_1^2 . \sin 2\theta_1^{(i)}} - s_1 \cos\theta_1^{(i)} \right] \tag{6}$$

Constraint – Buckling Load
Variation of thickness and material properties with the shell coordinates ultimately results in coordinate dependence of the stiffness matrices (A, B, D). Under the classical laminate theory, Jones [8] and Whitney [9], the constitutive equation reads:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \epsilon \\ X \end{bmatrix} \tag{7}$$

Where $\{N\}^T = \{N_{ss}, N_{\theta\theta}, N_{s\theta}\}$ & $\{M\}^T = \{M_{ss}, M_{\theta\theta}, M_{s\theta}\}$ are the membrane force and bending moment vectors. The coefficients of the elastic stiffness matrix are given by:

$$\begin{aligned} A &= \sum_{k=1}^{N} [\bar{Q}]_k (t_k - t_{k-1}) \\ B &= \tfrac{1}{2} \sum_{k=1}^{N} [\bar{Q}]_k (t_k^2 - t_{k-1}^2) \\ D &= \tfrac{1}{3} \sum_{k=1}^{N} [\bar{Q}]_k (t_k^3 - t_{k-1}^3) \end{aligned} \tag{8}$$

[A], [B] and [D] are, respectively, the membrane, coupling and flexural rigidities, and $[\bar{Q}]_k$ the laminate transformed reduced stiffness matrix of the k-th layer. The winding angle θ of a conical shell depends on the winding process and is a function of the longitudinal coordinate S, see Eq. (2), which ultimately influences the $[\bar{Q}]_k$ matrix. Furthermore, the thickness of a lamina also changes in the longitudinal direction, see Eq. (3). Therefore, the $[A]$, $[B]$ and $[c]$ matrices are strong functions of S. This leads to calculating the buckling load by solving a system of non – linear governing partial differential equations with variable coefficients. In this work, the buckling loads are calculated through the classical shell theory.

Classical shell theory (C.S.T):

$$\{\Delta\} = \{u_0, v_0, w_0\}^T$$

$$L_{11} = A_{11} \frac{\partial^2}{\partial x_1^2} + A_{66} \frac{\partial^2}{\partial x_2^2}$$

$$L_{12} = (A_{12} + A_{66}) \frac{\partial_2}{\partial x_1 \partial x_2} = L_{21}$$

$$L_{13} = -B_{11} \frac{\partial^3}{\partial x_1^3} (B_{12} + 2B_{66}) \frac{\partial^3}{\partial x_1 \partial x_2^2} + \frac{A_{12}}{\rho} \frac{\partial}{\partial x_1} = L_{31}$$

$$L_{22} = A_{66} \frac{\partial^2}{\partial x_1^2} + A_{22} \frac{\partial^2}{\partial x_2^2}$$

$$L_{23}=-(B_{12}+2B_{66})\frac{\partial^3}{\partial x_1^2\partial x_2}-B_{22}\frac{\partial^3}{\partial x_2^3}+\frac{A_{22}}{\rho}\frac{\partial}{\partial x_2}=L_{32}$$

$$L_{33}=D_{11}\frac{\partial^4}{\partial x_1^4}+2(D_{12}+2D_{66})\frac{\partial^4}{\partial x_1^2\partial x_2^2}+D_{22}\frac{\partial^4}{\partial x_2^4}+\left(\widehat{N}-2\frac{B_{12}}{\rho}\right)\frac{\partial^2}{\partial x_1^2}+\frac{A_{22}}{\rho^2}-2\frac{B_{22}}{\rho}\frac{\partial^2}{\partial x_2^2} \tag{9}$$

$$\begin{bmatrix} L_{11} & L_{12} & L_{31}\\ L_{21} & L_{22} & L_{23}\\ L_{31} & L_{32} & L_{33}\end{bmatrix}\begin{bmatrix}u_0\\ v_0\\ w_0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \tag{10}$$

$$[L]\,\{\Delta\}=\{0\} \tag{11}$$

$$\begin{cases} u_0=u_1\cos\beta_n x_1\cos\beta_m x_2\\ v_0=v_1\sin\beta_n x_1\sin\beta_m x_2\\ w_0=w_1\sin\beta_n x_1\cos\beta_m x_2\end{cases} \tag{12}$$

$$\begin{cases}\widehat{N}_1=\lambda\bar{N}_1\\ \widehat{N}_2=\lambda\bar{N}_2\end{cases} \tag{13}$$

$$(\bar{N}_1\beta_n^2+\bar{N}_2\beta_m^2)=\frac{(a_{11}a_{22}a_{33}+a_{12}a_{33}a_{13}+a_{13}a_{12}a_{23})}{+\lambda(a_{11}a_{22}-a_{12}^2)}-\frac{(a_{11}a_{23}^2+a_{22}a_{13}^2+a_{33}a_{12}^2)}{+\lambda(a_{11}a_{22}-a_{12}^2)} \tag{14}$$

$$\begin{cases} a_{11}=-(A_{11}\beta_n^2+A_{66}\beta_m^2)\\ a_{12}=a_{21}=(A_{12}+A_{66})(\beta_m\beta_n)\\ a_{13}=a_{31}=B_{11}\beta_n^3+(B_{12}+2B_{66})(\beta_m^2\beta_n)+\left(\frac{2A_{12}}{\rho_2+\rho_1}\right)\beta_n\\ a_{22}=-(A_{66}\beta_n^2+A_{22}\beta_m^2)\\ a_{23}=a_{32}=-(B_{12}+2B_{66})(\beta_m\beta_n^2)+B_{22}\beta_m^3+\left(\frac{2A_{22}}{\rho_2+\rho_1}\right)\beta_m\\ a_{33}=-\left[D_{11}\beta_n^4+2(D_{12}+2D_{66})(\beta_m^2\beta_n)+D_{22}\beta_m^4+\frac{4A_{22}}{(\rho_2+\rho_1)}+\left(\frac{4B_{12}}{\rho_2+\rho_1}\right)\beta_n^2+\left(\frac{4B_{22}}{\rho_2+\rho_1}\right)\beta_m^2\right]\end{cases} \tag{15}$$

Boundary Conditions

A combination of boundary conditions may be assumed to exist at the edges of the shell. Here we classify these boundary conditions for simply supported (SS1 – SS2 – SS3 – SS4) filament–wound, truncated circular conical shells. The simply supported boundary conditions of the shells considered are:

$u_0=u_0(x_1\,,x_2)\,,\; v_0=v_0(x_1\,,x_2)\,,$

$w_0=w_0(x_1\,,x_2)\,,\; \phi_1=\phi_1(x_1\,,x_2)=-\left(\frac{\partial w_0}{\partial x_1}\right)\,,$

$\phi_2=\phi_2(x_1\,,x_2)=-\left(\frac{\partial w_0}{\partial x_2}\right)\,,$

$N_1=N_1(x_1,x_2)=N_{11},\; N_2=N_2(x_1,x_2)=N_{22}$

$N_6=N_6(x_1,x_2)=N_{12}=N_{21}$

$M_1=M_1(x_1,x_2)=M_{11}$

$M_2=M_2(x_1,x_2)=M_{22}$

$M_6=M_6(x_1,x_2)=M_{12}=M_{21}$

$SS1: w_0=M_1=\phi_2=N_1=N_6=0$

$SS2: w_0=M_1=\phi_2=u_0=N_6=0$

$SS3: w_0=M_1=\phi_2=N_1=v_0=0$

$SS4: w_0=M_1=\phi_2=u_0=v_0=0$

In Figure 2, the axial compressive bucking loads of angle – ply CFRP (carbon fiber reinforced plastic, E_{11}=97.5GPa, E_{22}=8.3GPa, G_{12}=4.1GPa and V_{12}=0.32, Tong [10]) laminated conical shell $\alpha=45°$, R_1=0.2m, L=0.4m, SS3-SS4) are given for different angle ply orientations with a thickness t_1=2 mm , (R_1/t_1)=100 at the small end of the shell. It is seen that in general the larger the ply angle, the lower the buckling load. The reason for this is the rapid reduction in thickness along the longitudinal direction of the shell with increasing ply angle, see Eq. 3. For small ply orientation (up to about $\theta_1=30°$) the buckling load slightly increases, but not significantly since the variation of the angle ply has a more dominant effect than reduction in thickness. However, for larger ply

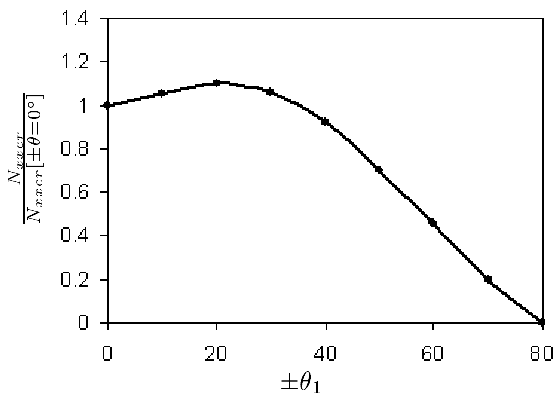

Figure 2. Axial compressive buckling load of laminated conical shells ($\alpha=45°$) with $t_1=2\,mm$, $N_{xxcr}[\pm\theta=0°]=29.130\,kN/m$.

angles the reduction in thickness is the dominant effect, and the buckling load decreases. Moreover, in those cases the volume of the shell decreases and at the same time the buckling load increases, leading to an optimum design configuration.

To summarize, both the buckling load (the constraint), and the weight of the laminated conical shell (the objective function) are strong functions of the fiber configuration (the design variables). Also the fiber orientation and the thickness of the conical shell are strong functions of the shell coordinates. This complicates the optimization procedure significantly. Furthermore, since the structural analysis is based on variable stiffness coefficients, high computational cost is involved in calculating the buckling load.

OPTIMIZATION PROCEDURE

Before starting with the optimization procedure, a preliminary investigation was made to determine the characteristic behavior of the buckling load with regard to the volume as a function of the ply orientation. In Figure 3, the axial compressive buckling loads are plotted against the angle–ply ($\pm\theta_1$) for CFRP laminated conical shells ($\alpha = 45°$, $R_1 = 0.2\,m$, $L = 0.4\,m$, $SS4$), all having the same volume ($V = 680\,cm^3$). To obtain the same volume for all shells, each configuration has a different thickness at the small end of shell. It appears that for a specific volume of shell, one can gain a 30% increase in buckling load by changing the ply orientation. Furthermore, the buckling curve in Figure 3 is smooth and convex in the neighborhood of the maximum buckling load, and has only one maximum. For a different shell volume, 80% smaller than the previous one, the axial compressive buckling load was also calculated and is plotted by the dashed line in Figure 3. It is seen that the buckling loads have the same characteristic behavior at both volumes. Furthermore, the maximum buckling loads of both shells occur at the same ply angle ($\pm\theta_1 = 50°$). In Figure 4, buckling loads of the same shell with cone semi–vertex of $\alpha = 15°, 30°, 45°$ and $60°$ are plotted against ply - angle ($\pm\theta_1$). It is seen that all buckling load curves have the same characteristic behavior, the curves are smooth and convex in the neighborhood of the maximum buckling load. The relative difference between the maximum and minimum buckling loads decreases as the cone semi – vertex angle increases. Furthermore, as the cone semi – vertex angle increases, the optimum ply – angle also increases.

As was shown previously, the volume of the cone is calculated by a simple equation, Eq. (4). However, calculation of the buckling load involves high computational cost. A change in volume is made by varying the ply orientation and/ or the ply thickness. Every alternative demands calculation of the buckling load (since it is not known if the current configuration satisfies the buckling load constraint). Therefore several calculations of buckling load are required for each change in volume. Calculation of the buckling load is the most expensive in this problem. Thus, the main goal of the optimization procedure is to reduce the number of buckling load calculations and at the same time to obtain the global extreme.

In the present work, the process is divided into two main stages. First the maximum buckling load is found for some volume that satisfies the buckling load constraint in an initial ply configuration, using an adaptive penalty function technique. Thus, an alternative optimization problem is solved; maximization of the buckling loads for a certain volume. This is to say the buckling load is calculated only once for

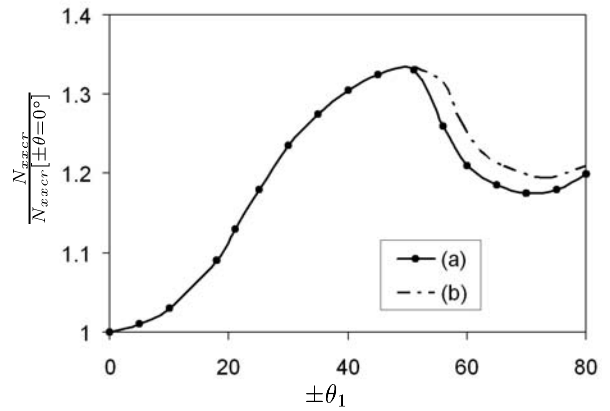

Figure 3. Axial compressive buckling load of laminated conical shells ($\alpha = 45°$) with constant volume.
(a) $V = 544\,cm^3$: $N_{xxcr}[\pm\theta = 0°] = 7.670\,kN/m$,
(b) $V = 680\,cm^3$: $N_{xxcr}[\pm\theta = 0°] = 12.493\,kN/m$.

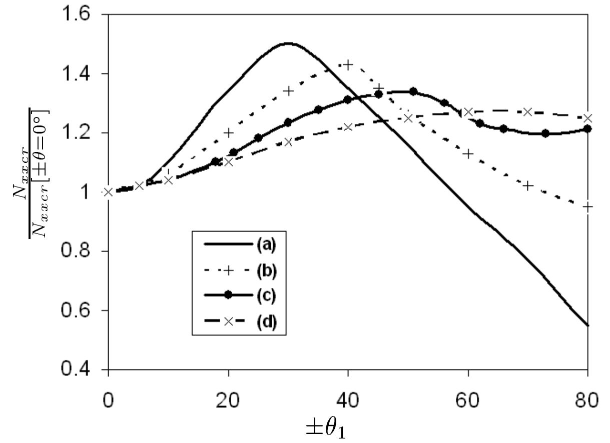

Figure 4. Axial compressive buckling load of laminated conical shell with constant volume ($V = 680cm^3$).
(a) $\alpha = 15°$: $N_{xxcr}[\pm\theta_1 = 0°] = 39.927\,kN/m$,
(b) $\alpha = 30°$: $N_{xxcr}[\pm\theta_1 = 0°] = 21.437\,kN/m$,
(c) $\alpha = 45°$: $N_{xxcr}[\pm\theta_1 = 0°] = 21.437\,kN/m$,
(d) $\alpha = 60°$: $N_{xxcr}[\pm\theta_1 = 0°] = 7.384\,kN/m$.

each ply configuration with the given volume, and expensive and unnecessary calculations of the buckling load are avoided. For each configuration, the thickness is determined by the volume constraint. After the maximum buckling load, as described above, has been found, the thickness of the layers is then decreased to reduce both the volume of the shell and the buckling load.

Adaptive Penalty Function Technique

The selection of the function used for the penalty method has a major influence on the accuracy of the approximation.

$$\begin{cases} \min f(x) = f(\theta_1, t_1) = -p_{cr}(\theta_1, t_1) \\ \text{subject to:} \\ g(x) = g(\theta_1, t_1) = V(\theta_1, t_1) - \nu^* \leq 0 \end{cases} \quad (16)$$

$$\phi(x, r) = f(x) + r \log\left[-g(x)\right] \quad (17)$$

$$\begin{cases} A : \frac{\partial\phi(x,r)}{\partial\theta_1} = \frac{\partial\phi((\theta_1,t_1),r)}{\partial\theta_1} = 0 \\ B : \frac{\partial\phi(x,r)}{\partial t_1} = \frac{\partial\phi((\theta_1,t_1),r)}{\partial t_1} = 0 \end{cases} \quad (18)$$

It is worth mentioning that, in order to show the superiority of the adaptive penalty function with respect to the existing approaches as cited in reference [13], Table 2 compares the computational cost among them.

CONCLUSION

In this study, an optimization procedure for filament - wound, laminated conical shells under buckling constraint is proposed. In the case of a laminated conical shell, the buckling load and the weight of the shell are functions of the ply configurations, which in turn are functions of the shell coordinates. This complicates the optimization procedure significantly; first, the correlation between the volume and the buckling load has to be taken into consideration, and second high computational cost involved in the calculation of the buckling load has to be reduced. Thus, based on the characteristic buckling behavior of a laminated conical shell, a special optimization procedure has been developed.

Table 1. Optimum configuration for maximum buckling load subject to specific volume.

r	θ_1^*	t_1^*	ϕ^*	f^*	p_{cr}
10^0	26.5241	0.7862	-22.6855	-24.6623	24.6623
10^{-1}	27.3641	0.8653	-25.57199	-27.49679	27.49679
10^{-2}	30.02361	0.9786	-32.53048	-32.54895	32.54895
10^{-3}	32.08145	1.12481	-38.92684	-38.92823	38.92823
10^{-4}	33.23421	1.44736	-51.29407	-51.29417	51.29417
10^{-5}	34.6152	1.4986	-54.70307	-54.70308	54.70308
Ref.(13)	35 (deg)	1.531 (mm)	-	-	55.647 (kN/m)

Table 2. Computational cost.

α	Number of calculation cycles			
	Response surface (Ref.13)	Hook and Jeev's method (Ref.13)	Steepest ascent (Ref.13)	Penalty function (current study)
30	26	12	12	6

Table 3. Optimum configuration for minimizing the volume with p^* as a buckling load constraint.

$\alpha\,(deg)$	p^* (kN/m)	$\theta_1\,(deg)$	$t_1\,(mm)$	$V\,(cm^3)$	p_{cr} (kN/m)
30	45	34.6152	1.250	168.370	45.6284

From the results presented, the following conclusions can be drawn:

- The characteristic buckling behavior of a laminated conical shell with respect to change in ply orientation for different volumes (indirectly thickness) is similar.
- Fiber orientation has a significant influence on the buckling load. The relative difference between the largest and the smallest axial compressive buckling loads can reach beyond 30% just by changing the ply orientation.
- The volume of the shell (indirectly called the thickness) has a major effect on the buckling load, a slight reduction in volume leading to significant reduction in buckling load.
- An adaptive penalty function technique is an effective procedure for optimization of laminated conical shells for buckling; it gives low cost with rapid convergence to the global optimum.
- The optimum fiber orientation for a laminated conical shell under axial compression increases as the cone semi – vertex increases. Usually the outer ply angle is larger than the inner.

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