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Research Note

# Parametric Study of Plastic Load of Cylindrical Shells with Opening Subject to Combined Loading

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*In this paper, the plastic limit load of cylindrical shells with openings subject to combined bending moment and axial force is found through finite element method using ANSYS suite program. The effects of various parameters such as the geometrical ratios of shell and the shape, size, axial and angular position of the opening on the limit load of cylindrical shells under various loading conditions are studied. The plastic limit loads of cylindrical shells with circular openings of different opening ratios and combinations of dimensionless bending moment and axial force are calculated by finite element elastic-plastic method, and their interaction curves are presented.*

## INTRODUCTION

Cylindrical shells are widely used as structural members in aerospace, marine and pressure vessel industries. The operational requirements for provision of openings with various shapes, sizes and positions have made these structures very interesting for research. A review of the literature available on cylindrical shells with openings reveals that the majority of published research works, though limited, are focused on elastic analyses of cylindrical shells. To our knowledge, few works have been published on the limit analysis of cylindrical shells with openings.

Various subjects on the limit analysis of structures and rotationally symmetric plates and shells have been discussed by Hodge [1,2]. Van Dyke [3] found stress field around a circular hole in a long cylindrical shell subject to axial force, internal pressure and torsion based on elastic formulation. Robinson [4] compared various yield surfaces for thin shells. Distribution of membrane stress around an elliptical opening in a cylindrical shell under the effect of torsion was studied by Chou [5]. A lower bound estimation of limit pressure of flush oblique cylindrical branch in a spherical pressure vessel was reported by Robinson & Gill [6]. Foo [7] has found an approximate lower bound limit load of a cylindrical shell with a single circular opening under two separate loading conditions, pure axial force and pure torsion using mathematical nonlinear programming. Alternative approaches calculation of lower bounds using reduced modulus of elasticity method was introduced by Marriot [8]. Mackenzie *et.al.* [9,10] further developed the elastic compensation method for the estimation of the lower and upper bound limit loads. Liu *et.al.* [11] presented a finite element mathematical programming formulation for limit load analysis of defective pipelines under multi-loading system. Plastic limit load of defective pipes subjected to internal pressure and axial tensile force with the help of nonlinear mathematical programming was found by Heitzer [12].

Limit loads of thin pipes with outer and inner surface cracks subject to axial force, internal pressure and bending moment was calculated by Shu [13]. Rahimi [14] has studied the behavior of cylindrical shells under local load applied on a rectangular section of the shell, and found the upper bound to the limit load of the shell with two moment yield surface approximation. Rahimi and Nobahari [15] carried out a parametric study on elastic buckling of cylindrical shells cut out under axial loading. Rahimi *et.al.* [16,17] carried out plastic analysis of cylindrical shells with a circular opening under the effect of end pure bending moment by the analytical method and finite element calculations. Parametric study on the limit load of cylindrical shells

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with cut out under the effect of pure bending moment and pure axial loading was carried out by Rahimi and Poursaeidi [18]. Rahimi and Alashti [19] found the lower bound to plastic limit load of cylindrical shells with openings under the effect of combined bending moment and axial force by nonlinear programming using the Feasible Sequential Quadratic Programming (FSQP) technique. Seshadri & Mangalaramanan [20] carried out the limit analysis on the basis of Mura's variational principle and by introducing an improved elastic compensation method called the $m_\alpha$ method.

In this paper, a limit load analysis of cylindrical shells is carried out using finite element method. Also, the effects of various parameters such as geometrical ratios of the shell and the opening parameters such as shape, opening ratio, longitudinal and angular position of the opening on limit load under combined bending moment and axial loading is studied.

## PLASTIC LIMIT LOAD ANALYSIS

In this section finite element modeling and analysis of the cylindrical shell with openings using ANSYS suite program is discussed. The cylindrical shell is meshed with SHELL93 elements with eight nodes, six degrees of freedom per node and plastic, strain hardening and large deflection capabilities. As the external load could only be applied to nodes within the meshed model, a node is defined at the end of the longitudinal axis of the cylinder. The created node is then connected to the cylinder end circumferential nodes by BEAM4 with two nodes, six degrees of freedom per node with axial tension, compression, torsion and bending capabilities. The modulus of elasticity of BEAM4 is chosen to be higher than that of the shell element SHELL93. The end circumferential nodes are then coupled for rotation in the same direction of application of the end moment to simulate a rigid flange connection. Due to the symmetry of the model with respect to the plane perpendicular to the cylinder axis, only half of the shell was modeled and symmetric boundary condition was imposed at the mid length of the cylinder. A half cylindrical shell with a central opening, meshed with SHELL93 and BEAM4 elements is shown in Figure 1.

In finite element elastic-plastic calculation, the material is assumed to be isotropic, non-hardening and obeying Von Mises yield criterion. The modulus of elasticity, yield stress and the poison's ratio are assumed to be 193E9 N/M$^2$, 350E6 N/M$^2$ and 0.297 respectively. The limit bending moment ($M_P$) and axial load ($N_P$) of a perfect cylindrical shell made of elastic-perfect plastic material are found analytically as follows:

$$M_P = 4R^2 t\sigma_Y \quad (1)$$

$$N_P = 4\pi Rt\sigma_Y \quad (2)$$

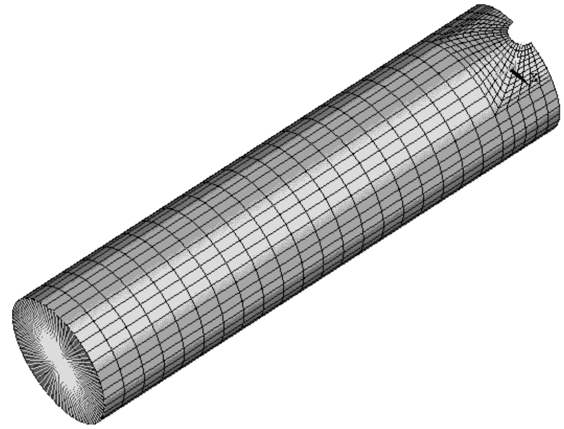

**Figure 1.** A half cylindrical shell meshed with SHELL93 and BEAM4.

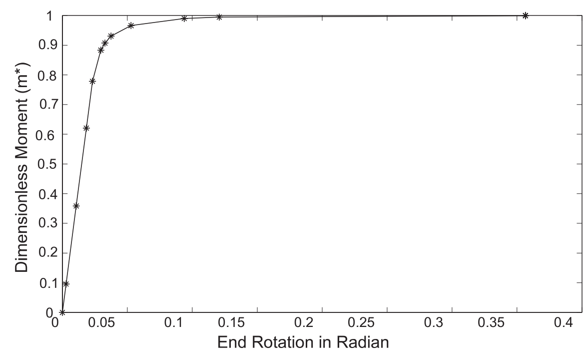


**Figure 2.** Variation of (m*) with cylinder end rotation.

where $R$, $t$ and $\sigma_Y$ are the radius, thickness and the yield stress of the cylindrical shell respectively.

The model is now subjected to an external load ($\bar{L}$) which is a combination of end axial force ($\bar{N}$) and bending moment ($\bar{M}$). The external load ($\bar{L}$) is applied in an incremental manner until, longer due to extensive spread of plastic state in the model and unrestrained plastic flow, the model can no further resist even the slightest increase in the external load defined as the limit load of the model. The dimensionless forms of axial force ($\bar{n}$) and bending moment ($\bar{m}$) are defined as follows:

$$\bar{n} = \frac{\bar{N}}{N_P}, \qquad \bar{m} = \frac{\bar{M}}{M_P} \quad (3)$$

For simplicity, the ratio of dimensionless axial force to bending moment is assumed to be constant for every loading case and the dimensionless form of applied external load ($\bar{l}$) is defined by either $\bar{n}$ or $\bar{m}$ part of the load and the relevant ratio of $\bar{n}$ to $\bar{m}$. In elastic plastic method, the effect of load stepping and mesh density on accuracy of the result was studied and appropriate meshing considering duration and accuracy of calculation was adopted. In order to ensure reasonable accuracy of calculation and required

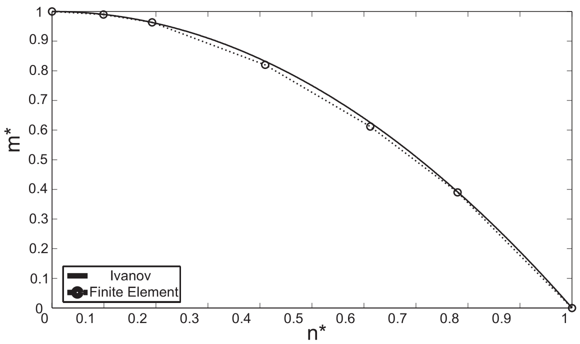


**Figure 3.** Yield surfaces defined by Ivanov and finite element results.

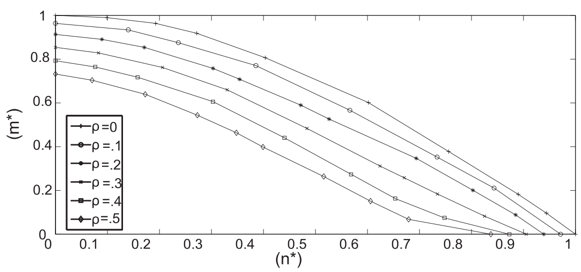


**Figure 4.** $(m^* - n^*)$ curves for a cylindrical shell with different opening ratios.

convergence, the nonlinear analysis is carried out with minimum, normal and maximum number of sub-steps chosen equal to 50, 500 and 5000 with a total number of 1150 shell elements. Variation of dimensionless bending moment and the end rotation of the shell in radians for a perfect cylinder subject to pure bending moment is shown in Figure 2. The length, radius and thickness of the shell are assumed to be equal to 500, 31 and 1.5 mm respectively. As it was expected, bending moment initially varies linearly with the end rotation of the shell and as the load increases, slope of the curve reduces until it becomes flat reaching the asymptotic value of the load called the limit load.

In order to verify finite element modeling and correctness of results, two pure loading cases of an axial force and a bending moment applied separately at the end node of a perfect cylinder are considered, and the elastic strain and stress distributions in the shell are compared with analytical results. The limit load bending moment and axial force are calculated by elastic plastic finite element method and the results are compared with analytical results given in Eqs. (1) and (2) respectively. In all the above cases, the finite element results were found to be in a very good agreement with the analytical results.

It is also interesting to note that the limit load of a perfect cylinder without defect for different combined loading cases are found by elastic plastic finite element method and compared with higher order yield surface approximations for thin shells *i.e.* Ivanov's approximation defined as follows [6]:

$$Y = q_{nn} + .5q_{mm} + \sqrt{(.25q_{mm}^2 + q_{nm}^2)} - 1 \leq 0 \qquad (4)$$

where:

$$q_{nn} = n_x^2 + n_\varphi^2 - n_x n_\varphi + 3n_{x\varphi}^2$$

$$q_{mm} = m_x^2 + m_\varphi^2 - m_x m_\varphi + 3m_{x\varphi}^2$$

$$q_{nm} = n_x m_x - .5n_x m_\varphi - .5n_\varphi m_x + n_\varphi m_\varphi + 3n_{x\varphi} m_{x\varphi} \qquad (5)$$

in which $n_i$ & $m_i$ are the dimensionless axial and bending stress resultants in the $i$ direction.

As it is shown in Figure 3, the results obtained by elastic plastic finite element method are in very good agreement with Ivanov's yield surface approximation.

## INTERACTION CURVE OF LIMIT BENDING MOMENT AND AXIAL FORCE

Using the finite element elastic-plastic calculation, the interaction curves of dimensionless limit bending moment $(m^*)$ and axial force $(n^*)$ of a cylindrical shell with central circular opening is found. The axis passing through the center of the opening is assumed to coincide with the y axis of the global Cartesian coordinate system. The cylindrical shell is assumed to have length, radius and thickness the same as in the previous section with the opening ratio $(\rho)$, the ratio of opening radius to cylinder radius, equal to 0, 0.1, 0.2, 0.3, 0.4 and 0.5. The results of limit load calculation for various combinations of dimensionless bending moment $(\bar{m})$ and axial forces $(\bar{n})$ are shown in Table 1 and Figure 4.

**Table 1.** Variation of $(m^*)$ and $(n^*)$ for cylindrical shell with different opening ratios.

| $\rho$ | $\bar{n}/\bar{m} = 0$ $m^*$ | $\bar{n}/\bar{m} = 0.5$ $m^*$ | $\bar{n}/\bar{m} = 1$ $m^*$ | $\bar{m}/\bar{n} = 0.5$ $n^*$ | $\bar{m}/\bar{n} = 0$ $n^*$ |
|---|---|---|---|---|---|
| 0 | 1 | .814 | .611 | .768 | 1 |
| 0.1 | .963 | .771 | .566 | .734 | .971 |
| 0.2 | .914 | .717 | .526 | .694 | .939 |
| 0.3 | .854 | .663 | .484 | .624 | .905 |
| 0.4 | .793 | .606 | .44 | .56 | .873 |
| 0.5 | .732 | .544 | .399 | .519 | .837 |

The $(m^* - n^*)$ curve presented is accurate enough to represent the variation of dimensionless limit bending moment and axial force for different opening ratios which could be approximated by a third degree polynomial as shown in Eq. (6).

$$m^* = \sum_{i=0}^{3}[C_i(n^*)^i] \tag{6}$$

In which parameter $C_i$ for different opening ratios are shown in Table 2.

Comparison of these results with the results reported in [11] reveals that the general pattern and shapes of the interaction curves of dimensionless limit bending moment and axial force are similar. However, it should be noted that the objection of the study in [11] was estimation of the plastic limit load of defective pipes with part thorough slots whereas in this paper the plastic limit load of cylindrical shells with (thickness through) openings are found, hence the limit loads obtained are not compared.

## PARAMETRIC STUDY OF LIMIT LOAD OF CYLINDRICAL SHELLS

In this section, the effect of various parameters including the geometrical parameters of the shell and the opening such as the shape, ratio, longitudinal and angular position of the opening on the limit load of cylindrical shells under combined bending moment and axial force is studied. It is assumed that the axial load is always tensile and applied in the direction of the main axis of the shell, *i.e.*, positive $z$ in global Cartesian coordinate system and the bending moment is applied in the positive direction of $x$ global coordinate as shown in Figure 1. If the axis passing through the opening center coincides with the $y$ axis, then the opening will always be in tensile stress zone due to both the bending moment and axial force. As this angle increases and passes a specific value, which depends on the opening ratio, then some parts or the whole opening will be in compressive stress zone due to the bending moment. This compressive stress partially compensates for the tensile stress effect of the axial force and in turn will increase the limit load of the shell.

### Cylinder Radius to Length Ratio

A parametric study of limit load of cylindrical shells within a wide range of radius to half length ratio ($\gamma$) is carried out. In this study, shells with different lengths of 1200, 500 and 250 mm and fixed radius and thickness of 31 and 1.5 mm respectively are considered. Variation of the limit load with different opening ratios ($\rho$) of 0, 0.1, 0.2, 0.3, 0.4 and 0.5 for two loading cases of pure bending moment and combined loading $(\bar{n}/\bar{m}) = 1$ are calculated, which are shown in Figures 5 and 6, respectively.

**Table 2.** Coefficients of third degree polynomial fit for $(m^* - n^*)$ curves.

| $\rho$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|---|
| $C_0$ | 0.288 | 0.358 | 0.369 | 1.09 | 1.83 | 2.12 |
| $C_1$ | -1.35 | -1.29 | -1.15 | -1.96 | -2.63 | -2.74 |
| $C_2$ | 0.059 | 0.078 | -.216 | -0.06 | 0.001 | 0.066 |
| $C_3$ | 1 | 0.965 | 0.917 | 0.851 | 0.788 | 0.727 |

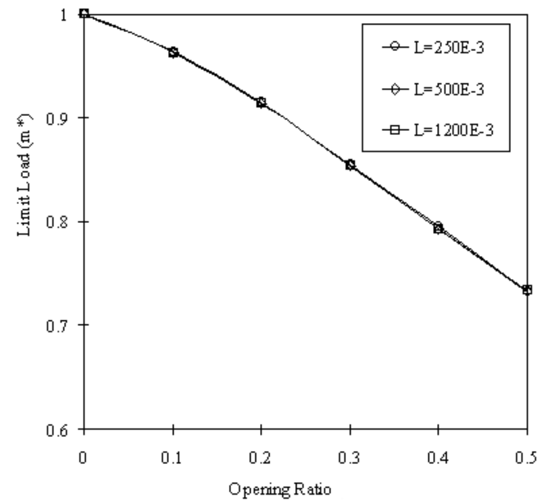


**Figure 5.** Variation of $(m^*)$ with $\rho$, fixed $R\&t$, different $L$, $(\bar{n}/\bar{m}) = 0$.

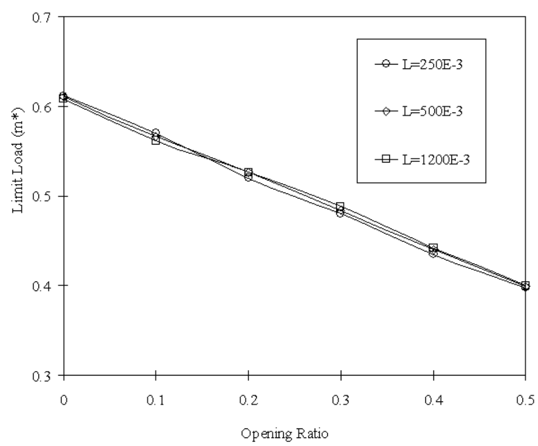


**Figure 6.** Variation of $(m^*)$ with $\rho$, fixed $R\&t$, different $L$, $(\bar{n}/\bar{m}) = 1$.

As it is shown in Figures 5 and 6, in case of pure bending moment, the limit load for a wide range of radius to half length ratio, *i.e.* $(0.05 \leq \gamma \leq .25)$, is very close to each other, with a very small difference of nearly 0.3 % . However, in the case of combined loading $(\bar{n}/\bar{m}) = 1$, the difference is higher and reaches 1.5 % of the limit load.

### Angular Position of the Opening

In this research, the plastic limit load of cylindrical shells with in the same range of opening ratios and different angular positions of 0, 30, 60, 90, 120, 150 and 180 degrees with respect to the global y axis for five loading cases of dimensionless ratios of the axial load to the bending moment equal to $(\bar{n}/\bar{m}) = 0, 0.5, 1$ and $(\bar{m}/\bar{n}) = 0, 0.5$ respectively are considered. Limit load results for the pure bending moment loading are shown in Figure 7. The results clearly indicate that the variation of limit load is generally symmetric with respect to the opening angular position of 90 degrees, and only few points deviate from the symmetry with a maximum deviation value of 0.1%. The maximum limit load occurs at 90 degrees, where the opening is in neutral zone, and the minimum limit load occurs at 0 and 180 degrees where the opening is either in tension or compression zone. It should also be noted that by changing the state of stress of the opening from tensile to compressive, there will be no change in the limit load.

Variation of the limit load for two loading cases of $(\bar{n}/\bar{m}) = 0.5, 1$, are shown in Figures 8 and 9, respectively. By comparing the results obtained for the aforementioned loading cases, it can be concluded that, in contrast to the pure bending moment and mainly

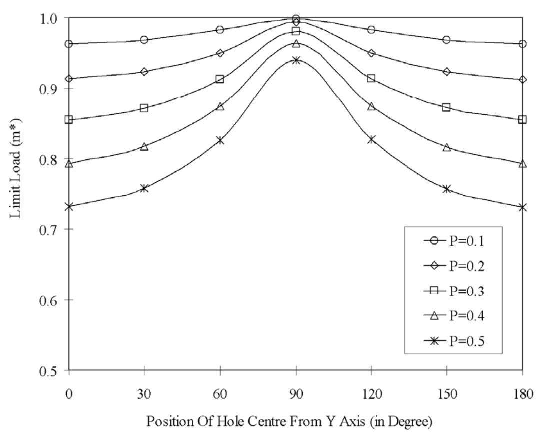


**Figure 7.** Variation of $(m^*)$ with an opening angular position, $(\bar{n}/\bar{m}) = 0$.

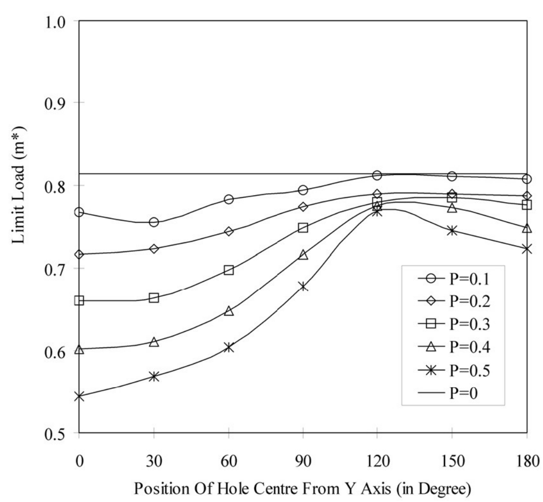


**Figure 8.** Variation of $(m^*)$ with an opening angular position, $(\bar{n}/\bar{m}) = 0.5$.

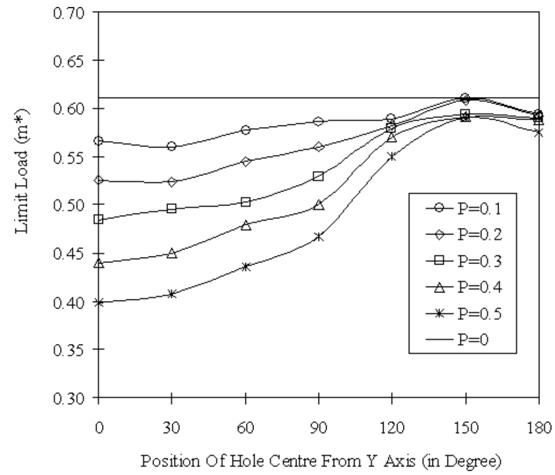


**Figure 9.** Variation of $(m^*)$ with an opening angular position, $(\bar{n}/\bar{m}) = 1$.

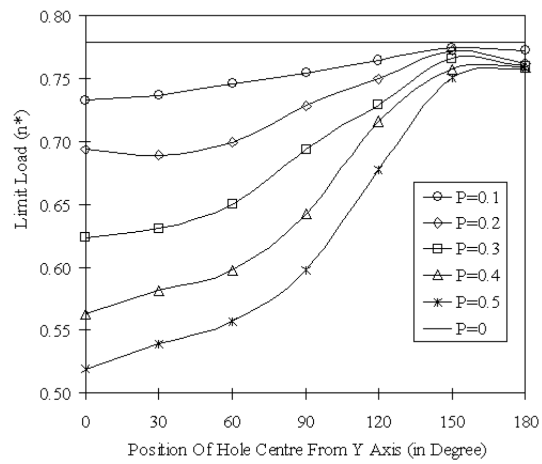


**Figure 10.** Variation of $(n^*)$ with an opening angular position, $(\bar{m}/\bar{n}) = 0.5$.

due to the presence of axial force which induces tensile stress, the limit load is highly affected by the angular position of the opening. When the axis passing through the opening center coincides with the $y$ coordinate, the effect of both bending moment and axial force would be tensile which reduces the limit load.

In the case of small opening ratios within the range of $0 \leq \rho \leq 0.2$, deviation of the opening center from the $y$ axis of less than 30 degrees would result in situations where some of the points on the boundary of the opening perpendicular to the cylinder axis will have the highest stress concentration due to axial loading. This increase in tensile stress will in turn compensate for reduction in tensile stress resultant from the bending moment due to deviation of the opening center, which causes reduction of limit load. Further increase in the deviation angle from $y$ axis causes increase in limit load until it reaches the maximum value. By comparing the curves shown in Figures 8 and 9, it is found that for the second case of loading $(\bar{n}/\bar{m}) = 0.5$, the maximum limit load occurs at around 120 degrees, while for the third loading case $(\bar{n}/\bar{m}) = 1$, this angle reaches around 150 degrees.

In the case of pure axial loading, $(\bar{m}/\bar{n}) = 0$, the angular position of the opening has no effect on the limit load of cylindrical shell. Variations of the limit load for the fifth loading case, $(\bar{m}/\bar{n}) = 0.5$, are shown in Figure 10. The general trend of the variations in the limit load against the angular position of the opening seems to be the same as that in the previous two loading cases. The maximum limit load occurs at angles around 150 degrees. By increasing the deviation angle from 150 to 180 degrees, the limit load is reduced in a manner similar to the second and third loading cases.

In order to have a better illustration and comparison of the limit load variations with the angular position of the opening, the limit load variation for four loading cases of $(\bar{n}/\bar{m}) = 0, 0.5, 1, 2$ with constant opening ratios of 0.1 and 0.5 are shown in Figures 11 and 12, respectively. As shown in these figures, in the case of the pure bending moment, the percentage of variation of the limit load with angular position of the opening increases considerably by increasing the opening ratio ($\rho$); however, this rate of variation reduces when increasing the ratio of axial loading to the bending moment in case of combined loading. Furthermore, it is found that the difference in the results of the limit loads for two loading cases of $(\bar{n}/\bar{m}) = 0, 0.5$ reduces for angular positions greater than 90 degrees and the results approach each other for angles of around 150 degrees or bigger.

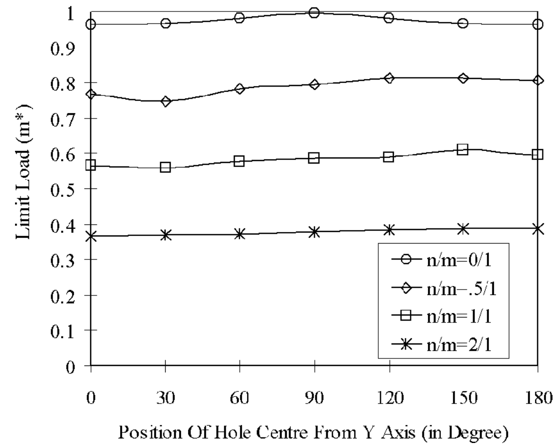


**Figure 11.** Variation of $(m^*)$ with an opening angular position, $\rho = 0.1$.

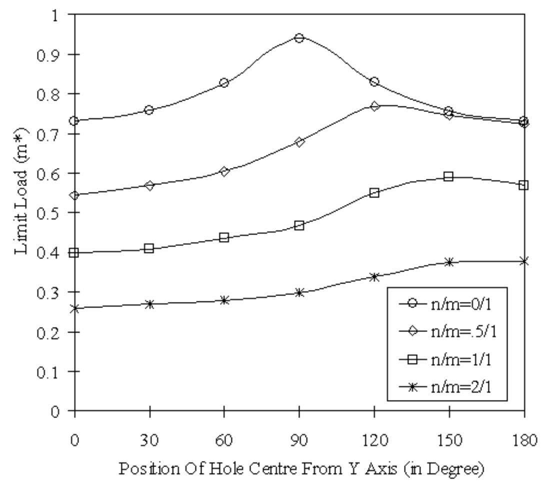


**Figure 12.** Variation of $(m^*)$ with an opening angular position, $\rho = 0.5$.

## Longitudinal Position of the Opening

The effect of the longitudinal position of the circular opening on the limit load of the cylindrical shell under five different loading conditions of $(\bar{n}/\bar{m}) = 0, 0.5, 1$ and $(\bar{m}/\bar{n}) = 0, 0.5$ with a constant opening ratio of $\rho = 0.4$ is studied through the finite element elastic plastic method. Four different longitudinal positions of the opening centre from the rigid end of the shell, equal to 0.125, 0.25, 0.375 and 0.5 times the cylinder length are considered. The limit loads calculated for four different longitudinal positions are presented in Table 3. Comparison of the results shown in Table 3 indicates that reduction of distance of the opening center from the rigid end will increase the limit load of the cylindrical shells although this increase is not significant and is limited to around 0.5 % of the limit load.

## Elliptical Opening and Axes Ratio

The plastic limit loads of cylindrical shells with elliptical hole for two loading conditions, *i.e.* pure bending moment and combined loading $(\bar{n}/\bar{m} = 1)$, are calculated through the elastic plastic finite element method. In this comparison, the major axis of the ellipse is assumed to be always parallel to the longitudinal axis of the shell, the opening ratio ($\rho$) represents the ratio of semi minor axis of the opening to the radius of the cylindrical shell. Elliptical openings with major

**Table 3.** Variation of $(m^*)$ or $(n^*)$ with the longitudinal position of the opening.

| Axial Position | $\bar{n}/\bar{m} = 0$ $m^*$ | $\bar{n}/\bar{m} = 0.5$ $m^*$ | $\bar{n}/\bar{m} = 1$ $m^*$ | $\bar{m}/\bar{n} = 0$ $n^*$ | $\bar{m}/\bar{n} = 0.5$ $n^*$ |
|---|---|---|---|---|---|
| 0.125 | 0.795 | 0.610 | 0.442 | 0.877 | 0.5605 |
| 0.25 | 0.7945 | 0.6075 | 0.4408 | 0.876 | 0.5603 |
| 0.375 | 0.7935 | 0.6068 | 0.4405 | 0.875 | 0.56 |
| 0.5 | 0.793 | 0.606 | 0.44 | 0.873 | 0.56 |

to minor axes ratio equal to 1, 2, 3, 4 and 5 are considered, and the limit load results for two different loading conditions, ($\bar{n}/\bar{m} = 0, 1$) are shown in Table 4.

As shown in Table 4, the limit load variation of cylindrical shell due to elliptical opening's axes ratio is not substantial, and is limited to approximately 1.5% and 1.8% for two loading cases of ($\bar{n}/\bar{m} = 0, 1$), respectively. By comparing the effect of opening ratio on the limit load of cylindrical shells, it can be concluded that changes in the size of the opening in the direction perpendicular to the main axis of the cylinder have a strong effect on the limit load, but changes in the opening size in the direction parallel to the main axis of the cylinder have a much smaller effect as reported in [11].

## CONCLUSION

In this paper, a parametric study of the limit load of cylindrical shells with circular and elliptical openings for various geometrical parameters have been carried out and the results are presented in the relevant Tables and Figures. The main conclusions from the obtained results are as follows:

1. The opening dimension in the circumferential direction greatly affects the limit load of cylindrical shell subject to combined axial load and bending moment. On the contrary, the opening dimension in the longitudinal direction has a slight effect on the limit load.
2. The parametric study of the limit load of the cylindrical shell with different radius to half length ratios ($\gamma$) shows that in the case of the pure bending moment loading, variation of the limit load for a wide range of ($\gamma$) , ($0.05 \leq \gamma \leq .25$), is very close to each other with a maximum deviation of 0.3 % but for combined loading such as ($\bar{n}/\bar{m} = 1$), this deviation is greater and reaches nearly 1.5 %.
3. The angular position of the opening has a strong effect on the limit load of the cylindrical shell. In the case of pure bending, the limit load is minimum when the opening changes 0 or 180 degrees on the $y$ axis, where the opening is in the maximum tensile or compressive stress zone. The limit load will be maximal when the opening creates a 90 degree angle on the y axis, where the opening is in the neutral zone of the cylinder.
4. In the case of combined loadings, due to the increased effect of the axial load, the limit load curve versus angular position of the opening is not symmetric with respect to the 90 degree angle. By increasing the axial load ratio, the angular position of the opening for maximum limit load shifts from 90 degrees to 120 degrees in the case of ($\bar{n}/\bar{m} = 0.5$), and further to 150 degrees in the case of ($\bar{n}/\bar{m} = 1$). In the case of pure axial loading, there is no variation of limit load when varying the angular position of the opening.
5. By shifting the longitudinal position of the opening from the center towards the end of the cylinder, the limit load of the cylindrical shell is expected to be increased. However, such increase is not considerable, and is not more than 1 % of the limit load.

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**Table 4.** Variation of ($m^*$) for cylindrical shell with elliptical opening, ($\bar{n}/\bar{m} = 0$) and ($\bar{n}/\bar{m} = 1$).

| Axes Ratio | $\bar{n}/\bar{m} = 0$ | | | | $\bar{n}/\bar{m} = 1$ | | | |
|---|---|---|---|---|---|---|---|---|
| | $\rho = 0.1$ | $\rho = 0.2$ | $\rho = 0.3$ | $\rho = 0.4$ | $\rho = 0.1$ | $\rho = 0.2$ | $\rho = 0.3$ | $\rho = 0.4$ |
| 1 | 0.963 | 0.914 | 0.854 | 0.793 | 0.566 | 0.526 | 0.484 | 0.440 |
| 2 | 0.962 | 0.908 | 0.8495 | 0.788 | 0.560 | 0.5258 | 0.4835 | 0.440 |
| 3 | 0.9607 | 0.906 | 0.846 | 0.786 | 0.559 | 0.525 | 0.483 | 0.439 |
| 4 | 0.958 | 0.902 | 0.8456 | 0.785 | 0.557 | 0.525 | 0.4825 | 0.438 |
| 5 | 0.956 | 0.901 | 0.842 | 0.785 | 0.556 | 0.5245 | 0.480 | 0.437 |

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