JAST, Vol. 5, No. 2, pp 51–56
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JAST Journal of Aerospace Science and Technology

Time Optimal Slew Maneuver of Magnetically Actuated LEO Satellites

A. Heydari[1], S. H. Pourtakdoust[2]

Conventional magnetic attitude control methodologies require several orbital periods to accomplish the required attitude maneuvers due to the existence of an uncontrollable axis, namely the local Earth's magnetic field vector. Since in some attitude maneuver missions the elapsed time is of critical importance, those time-consuming controllers are not satisfactory, and we need a much faster controller to achieve the maneuver in a fraction of an orbit. In this research the attitude slew maneuver using magnetic torquers is formulated as a time optimal problem and solved through the calculus of variations. The resulting controller is shown to be very fast in forcing the attitude to converge to the desired condition.

NOMENCLATURE

$\mathbf{B}$	The Earth's magnetic field vector
B	Body frame
$\mathbf{c}$	Unit vector in the direction of nadir
g_n^m, h_n^m	Gaussian coefficients of the IGRF model
$\mathbf{H}$	Positive semi-definite weighting matrix
h	Sampling time
I	Inertial frame
$\mathbf{I}$	Satellite inertia tensor
k	Discretized time index
$\mathbf{m}$	Control magnetic moment
$\mathbf{N}_{net}$	Total torque exerted on the satellite
$\mathbf{N}_{ctrl}$	Control torque
$\mathbf{N}_{gg}$	Gravity gradient torque
$\mathbf{N}_{dist}$	Disturbance torque
$\mathbf{n}$	Rotation axis
P_n^m	Schmidt quasi-normalized associated Legendre function of degree n and order m
$\{q\}$	Four Euler parameters, quaternions
$[\mathbf{q}]$	Last three elements of the quaternions
R_{earth}	The Earth's reference radius
r	Distance from the Earth's center
$\mathbf{u}$	Control vector
V	The Earth's magnetic potential
W_t	Positive scalar penalizing the elapsed time
$\mathbf{x}$	State vector
$\mathbf{x}_0$	Initial state vector
$\mathbf{x}_{des}$	Desired state vector
$\boldsymbol{\delta}_x[k]$	Costate vector
Θ, Φ	Geographic co-latitude and longitude
ϕ, θ, ψ	Euler angles around the roll, pitch & yaw axes
ε	Rotation angle
τ	Positive update step size for the numeric algorithm
μ	The Earth's gravitational constant
$\boldsymbol{\omega}$	Angular velocity of the body frame w.r.t. the inertial frame

1. *MSc. Graduate, Dept. of Aerospace Eng., Sharif Univ. of Tech., Tehran, Iran, Email: heydari.ali@ieee.org*
2. *Professor, Center for Research and Development in Space Sci. and Tech., Dept. of Aerospace Eng., Sharif Univ. of Tech., Tehran, Iran.*

INTRODUCTION

Utilization of magnetic actuators for the purpose of attitude control of small LEO satellites due to light weight, low cost and high reliability has received a lot of interest over the last two decades. Several researchers have focused on the problem of using these actuators as the sole attitude control actuator, and numerous attitude control laws have been developed.

Since magnetic actuators are unable to produce mechanical torque around the local Earth's magnetic field vector, the satellite will not be fully controllable at a fixed point in its orbit; however, because of the satellite motion around the orbit, the direction of the uncontrollable axes is subject to variation, hence, every attitude maneuver can be achieved using magnetic coils/rods as the sole actuators. Unfortunately almost all of the developed magnetic attitude controllers need several orbits to complete a desired maneuver [1-5] which is very time consuming for some satellites.

Since in some satellite missions the elapsed time is critical and of importance, control algorithms capable of performing the required slew maneuver in a fraction of an orbit are of great interest. In [6] the author has used a software package, RIOTS, working based on the direct method of optimization, in order to solve the time optimal formulation of the magnetic attitude control problem and has shown that rest-to-rest attitude maneuvers can be performed in a small fraction of an orbital period.

In this research, the dynamics and kinematics of the satellites are modeled in the state space form and utilized to formulate the optimal control problem subject to the input saturation constraints using calculus of variation. Subsequently the resulting problem is solved through an iterative numeric algorithm for the free final time to be optimized. Finally two slew maneuvers are simulated and the resulting states trajectory and control are presented.

SATELLITE DYNAMICS & KINEMATICS

For the purpose of this research an inertial pointing satellite is selected for the proof of the proposed concept. The satellite dynamics can be represented as [7]:

$$\mathbf{I}\frac{d\boldsymbol{\omega}}{dt} = -\boldsymbol{\omega} \times \mathbf{I}\boldsymbol{\omega} + \mathbf{N}_{net} \tag{1}$$

where all vectors are represented in the body frame. The total torque exerted on the satellite is composed of control, gravity gradient and disturbance torques.

$$\mathbf{N}_{net} = \mathbf{N}_{ctrl} + \mathbf{N}_{gg} + \mathbf{N}_{dist} \tag{2}$$

The control torque is created as the consequence of interaction of the Earth's magnetic field and the magnetic moment created due to the electrical current through the magnetic coils.

$$\mathbf{N}_{ctrl} = \mathbf{m} \times \mathbf{B} \tag{3}$$

The apparent cross product between the two vectors causes the problem of inability to create mechanical torque about the Earth's local magnetic field vector. The gravity gradient torque is [7]:

$$\mathbf{N}_{gg} = \frac{3\mu}{r^3}\left(\mathbf{c} \times \mathbf{I}\,\mathbf{c}\right) \tag{4}$$

where $\mathbf{c}$ is the unit vector in the direction of nadir. In nadir pointing satellites the gravity gradient will be usually exploited as a passive stabilization torque while in the inertial pointing ones, the gravity gradient torque is a disturbing torque with a cyclic nature, and is categorized as the disturbance torques. Hence, the inertia tensor of the satellites will be usually selected in such a way that is proportional to the identity matrix in order for the gravity gradient torque to vanish.

Finally disturbance torque is mainly composed of an aerodynamic drag because of the existence of low density air in LEO orbits, solar radiation torques and residual magnetic moment of the internal electrical equipment [7].

To avoid singularity conditions in modeling the kinematics of the satellites, four Euler parameters known as quaternions denoted by $\{\mathbf{q}\}$ are utilized [7].

Describing the rotation of the body frame with respect to the inertial frame by a unit vector $\mathbf{n}$ representing the rotation axis and a scalar ε representing the rotation angle, the quaternions can be defined as:

$$\{\mathbf{q}\} = \begin{bmatrix} q_0 \\ [\mathbf{q}] \end{bmatrix} = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} \cos(\varepsilon/2) \\ n_1 \sin(\varepsilon/2) \\ n_2 \sin(\varepsilon/2) \\ n_3 \sin(\varepsilon/2) \end{bmatrix} \tag{5}$$

where

$$\mathbf{n} = \begin{bmatrix} n_1 & n_2 & n_3 \end{bmatrix}^T \tag{6}$$

Now, the kinematics equation can be written as [7]:

$$[\dot{\mathbf{q}}] = \frac{1}{2}\boldsymbol{\omega}\, q_0 - \frac{1}{2}\boldsymbol{\omega} \times [\mathbf{q}] \tag{7}$$

$$\dot{q}_0 = -\frac{1}{2}\boldsymbol{\omega}^T [\mathbf{q}] \tag{8}$$

In order to initialize the quaternions based on available initial Euler angles, the following set of equations is utilized.

$$\begin{aligned}
q_0 &= \cos\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\phi}{2}\right) + \sin\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\phi}{2}\right) \\
q_1 &= \cos\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\phi}{2}\right) - \sin\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\phi}{2}\right) \\
q_2 &= \cos\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\phi}{2}\right) + \sin\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\phi}{2}\right) \\
q_3 &= \sin\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\phi}{2}\right) - \cos\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\phi}{2}\right)
\end{aligned} \tag{9}$$

Similarly the Euler angles are related to the quaternions through the following equations.

$$\tan(\phi) = \frac{2(q_2q_3 + q_0q_1)}{q_0^2 - q_1^2 - q_2^2 + q_3^2}$$
$$\sin(\theta) = -2(q_1q_3 - q_0q_2)$$
$$\tan(\psi) = \frac{2(q_1q_2 + q_0q_3)}{q_0^2 + q_1^2 - q_2^2 - q_3^2} \quad (10)$$

OPTIMAL CONTROL FORMULATION

Choosing the four elements of the quaternions and the three elements of the angular velocity vector as the states and the three elements of the magnetic moments as controls and discretizing the resulting time-varying nonlinear system for N time steps with equal sampling time of $h = t_f/N$ will result in the discrete form of the system state equation.

$$\mathbf{x}[k+1] = \mathbf{f}(\mathbf{x}[k], \mathbf{u}[k], k, h), \qquad k = 0, 1, ..., N-1 \quad (11)$$

with

$$\mathbf{x}[0] = \mathbf{x}_0 \quad (12)$$

where $\mathbf{f}, \mathbf{x}$ and $\mathbf{u}$ are:

$$\mathbf{f} = h\begin{bmatrix} (-\boldsymbol{\omega}_x q_1 - \boldsymbol{\omega}_y q_2 - \boldsymbol{\omega}_z q_3)/2 \\ (\boldsymbol{\omega}_x q_0 + \boldsymbol{\omega}_z q_2 - \boldsymbol{\omega}_y q_3)/2 \\ (\boldsymbol{\omega}_y q_0 - \boldsymbol{\omega}_z q_1 + \boldsymbol{\omega}_x q_3)/2 \\ (\boldsymbol{\omega}_z q_0 + \boldsymbol{\omega}_y q_1 - \boldsymbol{\omega}_x q_2)/2 \\ (\boldsymbol{\omega}_y \omega_z (\mathbf{I}_{yy} - \mathbf{I}_{zz}) + \mathbf{m}_y \mathbf{B}_z - \mathbf{m}_z \mathbf{B}_y)/\mathbf{I}_{xx} \\ (\boldsymbol{\omega}_x \boldsymbol{\omega}_z (\mathbf{I}_{zz} - \mathbf{I}_{xx}) + \mathbf{m}_z \mathbf{B}_x - \mathbf{m}_x \mathbf{B}_z)/\mathbf{I}_{yy} \\ (\boldsymbol{\omega}_x \boldsymbol{\omega}_y (\mathbf{I}_{xx} - \mathbf{I}_{yy}) + \mathbf{m}_x \mathbf{B}_y - \mathbf{m}_y \mathbf{B}_x)/\mathbf{I}_{zz} \end{bmatrix} + \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \\ \boldsymbol{\omega}_x \\ \boldsymbol{\omega}_y \\ \boldsymbol{\omega}_z \end{bmatrix} \quad (13)$$

$$\mathbf{x} = \begin{bmatrix} q_0 & q_1 & q_2 & q_3 & \boldsymbol{\omega}_x & \boldsymbol{\omega}_y & \boldsymbol{\omega}_z \end{bmatrix}^T \quad (14)$$

$$\mathbf{u} = \begin{bmatrix} \mathbf{m}_x & \mathbf{m}_y & \mathbf{m}_z \end{bmatrix}^T \quad (15)$$

Moreover, $\mathbf{I}_{xx}$,$\mathbf{I}_{yy}$ and $\mathbf{I}_{zz}$ are the moments of inertia about the three principle axes, and the elements of vectors $\boldsymbol{\omega}$, $\mathbf{B}$ and $\mathbf{m}$ are denoted by subscripts x, y and z.

As can be seen through the state equation, the system is time varying and of course nonlinear. Its time dependency is because of the change of the Earth's magnetic field vector, which is due to the satellite motion around the orbit.

The problem at hand is a time-optimal terminal-control one, therefore a suitable cost function can be:

$$J = (\mathbf{x}[N] - \mathbf{x}_{des})^T \mathbf{H}(\mathbf{x}[N] - \mathbf{x}_{des}) + \sum_{k=0}^{N-1} W_t h \quad (16)$$

where $\mathbf{x}_{des}$ is the desired final states of the satellite (attitude + rates), $\mathbf{H}$ is a 7×7 positive semi definite weighting matrix penalizing the terminal errors in the states and W_t is a positive scalar weight penalizing the total elapsed time.

Adjoining the system Eq. (11) to the cost function (16) as an equality constraint using Lagrangian multipliers (costates) $\boldsymbol{\delta}_x[k]$, produces the augmented cost function.

$$J_{aug} = (\mathbf{x}[N] - \mathbf{x}_{des})^T \mathbf{H}(\mathbf{x}[N] - \mathbf{x}_{des}) + \sum_{k=0}^{N-1}\left\{W_t h + \boldsymbol{\delta}_x^T[k+1](\mathbf{f}(\mathbf{x}[k], \mathbf{u}[k], k, h) - \mathbf{x}[k+1])\right\} \quad (17)$$

In order to find the minimal of the augmented cost function (17), a mathematical approach is followed where its derivatives with respect to all independent variables are set to zero to derive the required optimality conditions.

$$\frac{\partial J_{aug}}{\partial \mathbf{x}[k]} = \boldsymbol{\delta}_x^T[k+1]\frac{\partial \mathbf{f}(\mathbf{x}[k], \mathbf{u}[k], k, h)}{\partial \mathbf{x}[k]} - \boldsymbol{\delta}_x^T[k] = \mathbf{0} \qquad k = 0, 1, ..., N-1 \quad (18)$$

$$\frac{\partial J_{aug}}{\partial \boldsymbol{\delta}_x[k+1]} = (\mathbf{f}(\mathbf{x}[k], \mathbf{u}[k], k, h) - \mathbf{x}[k+1])^T = \mathbf{0} \qquad k = 0, 1, ..., N-1 \quad (19)$$

$$\frac{\partial J_{aug}}{\partial \mathbf{u}[k]} = \boldsymbol{\delta}_x^T[k+1]\frac{\partial \mathbf{f}(\mathbf{x}[k], \mathbf{u}[k], k, h)}{\partial \mathbf{u}[k]} = \mathbf{0} \qquad k = 0, 1, ..., N-1 \quad (20)$$

$$\frac{\partial J_{aug}}{\partial h} = \sum_{k=0}^{N-1}\left\{W_t + \boldsymbol{\delta}_x^T[k+1]\frac{\partial \mathbf{f}(\mathbf{x}[k], \mathbf{u}[k], k, h)}{\partial h}\right\} = 0 \quad (21)$$

$$\frac{\partial J_{aug}}{\partial \mathbf{x}[N]} = 2\mathbf{H}(\mathbf{x}[N] - \mathbf{x}_{des}) - \boldsymbol{\delta}_x[N] = \mathbf{0} \quad (22)$$

Re-arranging the results yields:

$$\boldsymbol{\delta}_x[k] = \left(\frac{\partial \mathbf{f}(\mathbf{x}[k], \mathbf{u}[k], k, h)}{\partial \mathbf{x}[k]}\right)^T \boldsymbol{\delta}_x[k+1] \qquad k = 0, 1, ..., N-1 \quad (23)$$

$$\mathbf{x}[k+1] = \mathbf{f}(\mathbf{x}[k], \mathbf{u}[k], k, h), \quad k = 0, 1, ..., N-1 \quad (24)$$

$$\left(\frac{\partial \mathbf{f}\left(\mathbf{x}[k], \mathbf{u}[k], k, h\right)}{\partial \mathbf{u}[k]}\right)^T \boldsymbol{\delta}_x[k+1] = \mathbf{0}$$

$$k = 0, 1, ..., N-1 \qquad (25)$$

$$\sum_{k=0}^{N-1} \left\{ W_t + \left(\frac{\partial \mathbf{f}\left(\mathbf{x}[k], \mathbf{u}[k], k, h\right)}{\partial h}\right)^T \boldsymbol{\delta}_x[k+1] \right\} = 0 \qquad (26)$$

$$\boldsymbol{\delta}_x[N] = 2\mathbf{H}\left(\mathbf{x}[N] - \mathbf{x}_{des}\right) \qquad (27)$$

Eq. (23) and (24) are the costate and the state equations respectively. Eq. (25) and (26) are the resulting optimality conditions. The free final time problem is formulated using a fixed number of time steps N and the variable sampling time h. Hence h must be optimized as a parameter as well using the optimality condition (26).

NUMERIC ALGORITHM

The two point boundary value problem attained thus far can be solved using iterative methods such as the first order gradient as outlined below.

Choose an initial guess on the control history and the sampling time and follow the three step algorithm given next.

1. Use the system Eq. (11) and the initial condition (12) to propagate the states through time and store the resulting state trajectory.
2. Use the costate Eq. (23) and the final condition (27), back-propagate the costates through time and store the resulting costate time history.
3. Update the guess on the control history and the sampling time according to the following rules:

$$\Delta\mathbf{u}[k] = -\tau\left(\frac{\partial \mathbf{f}\left(\mathbf{x}[k], \mathbf{u}[k], k, h\right)}{\partial \mathbf{u}[k]}\right)^T \boldsymbol{\delta}_x[k+1] \qquad (28)$$

$$\mathbf{u}[k] \leftarrow \mathbf{u}[k] + \Delta\mathbf{u}[k] \qquad (29)$$

$$\Delta h = -\tau \sum_{k=0}^{N-1} \left\{ W_t + \left(\frac{\partial \mathbf{f}\left(\mathbf{x}[k], \mathbf{u}[k], k, h\right)}{\partial h}\right)^T \boldsymbol{\delta}_x[k+1] \right\} \qquad (30)$$

$$h \leftarrow h + \Delta h \qquad (31)$$

The above algorithm should be iterated until the control history and the sampling time converge to their optimal values. Selecting a suitable update step size τ causes the number of iterations needed for convergence to decrease substantially and this selection is usually attempted either through a search algorithm or through trial and error.

Finally in order to take the control saturation problem into account, in the control update process, one can simply restrict/saturate the control values in the desired admissible interval.

SIMULATIONS

To simulate the proposed magnetic attitude controller, one needs a precise model for the Earth's magnetic field. In this research the spherical harmonics model with coefficients set IGRF2000 of degree 13 is utilized [8]. In the spherical harmonics model, the geomagnetic field vector is approximated as follows:

$$\mathbf{B} = -\nabla V \qquad (32)$$

where the Earth's magnetic potential V is:

$$V\left(r, \Theta, \Phi\right) = R_{earth} \sum_{n=1}^{k} \left\{ \left(\frac{R_{earth}}{r}\right)^{n+1} \sum_{m=0}^{n} \left(g_n^m \cos\left(m\Phi\right) + h_n^m \sin\left(m\Phi\right)\right) P_n^m\left(\cos\left(\Theta\right)\right) \right\} \qquad (33)$$

In this form, R_{earth} is the Earth's reference radius (6371.2 km), r, Θ and Φ are the distance from the Earth's center in km, the co-latitude ($\Theta = 90 - lattitude$), and the longitude respectively. g_m^n and h_m^n are Gaussian coefficients provided by International Association of Geomagnetism and Aeronomy (IAGA).

Table 1. Characteristics of satellite A & its orbit.

Parameter	Value
Orbit Apogee Altitude	850 km
Orbit Perigee Altitude	450 km
Inclination	96.1°
Argument of Perigee	0
Right Ascension of the Asc. Node	105.2
Eccentricity	0.028599
Orbital Period	5855 s
Satellite Inertia Tensor	Diag([2.5,2.5,2.5]) kg.m^2
Actuators Saturation Limits	11.5 A.m^2

Table 2. Maneuver specifications for satellite A.

Parameter	Value
Initial Euler Angles	[40,-40,50] deg.
Initial Angular Velocities	[0,0,0] rad/s
Desired Euler Angles	[0,0,0] deg.
Desired Angular Velocities	[0,0,0] rad/s
Satellite's Position	The Perigee

The set of these coefficients in the model is called International Geomagnetic Reference Field (IGRF) and every 5 years, IAGA updates the set of coefficients and calls the updated set "IGRF" followed by the year of revision, *i.e.* IGRF2000. Finally P_n^m is Schmidt quasi-normalized associated Legendre function of degree n and order m. Additional information about this model can be found in [8].

The iterative algorithm for two different satellites in different orbits and maneuvers has been solved and the results are shown below.

Satellite A:
The characteristics of the satellite subject to this simulation and its orbit are mentioned in Table 1.

The weighting matrix **H** is selected such that it penalizes the final rates error (with unit of rad/s) with factor of 1000 and penalizes the final quaternion error with a factor of 0.01. The elapsed time penalizing weight W_t is selected equal to 2×10^{-7}, and the number of time steps (N) is selected to be equal to 100.

Simulation has been performed for a rest-to-rest maneuver with specifications mentioned in Table 2. The optimal states are depicted in Figures 1 and 2, and the optimal control is shown in Figure 3. To have an intuition of the attitude changes during the maneuver, the Euler angles trajectory is depicted in Figure 4.

As can be seen in Figure 3, the optimal control is in the form of bang-bang and the optimal time-to-go is about 240 s. Since the orbital period is equal to 5855 s, the maneuver is accomplished only in a fraction of an orbit, namely less than 5 percent of the orbital period.

The final attitude error is less than 1 deg. and the error rate is less than 1×10^{-5} rad/s. The balance between the rates error and the attitude error is tuned by the elements of the weighting matrix **H**. For the purpose of decreasing the total error, one should decrease the value of W_t as compared to the value of the elements of matrix **H**.

Comparing these results to the time-to-go of periodic finite horizon controller or infinite horizon controller suggested in [2] and also those of the sliding mode controller suggested in [3], where two or more orbits are required to complete a similar attitude maneuver, shows that the time optimal controller has considerable benefits in time critical maneuvers.

Table 3. Maneuver Required Time.

Satellite Position	Req. Time
Passing through the perigee	240 s
Passing above the north pole	228 s
Passing through the apogee	319 s
Passing below the south pole	214 s

Table 4. Characteristics of satellite A & its orbit.

Parameter	Value
Orbit Apogee Altitude	630 km
Orbit Perigee Altitude	630 km
Inclination	45°
Argument of Perigee	0
Right Ascension of the Asc. Node	0
Eccentricity	0
Orbital Period	5829 s
Satellite Inertia Tensor	Diag([1,1,1]) kg.m^2
Actuators Saturation Limits	10 A.m^2

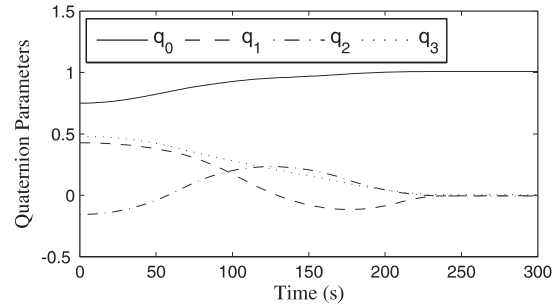

Figure 1. Quaternions propagation (simulation A).

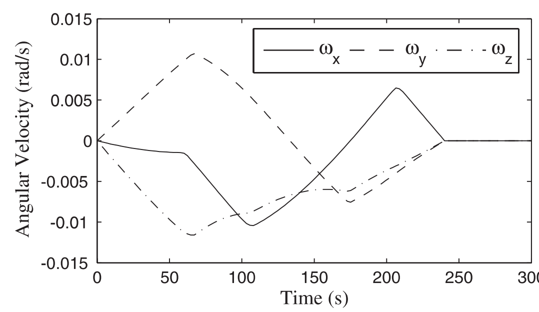

Figure 2. Angular rates history (simulation A).

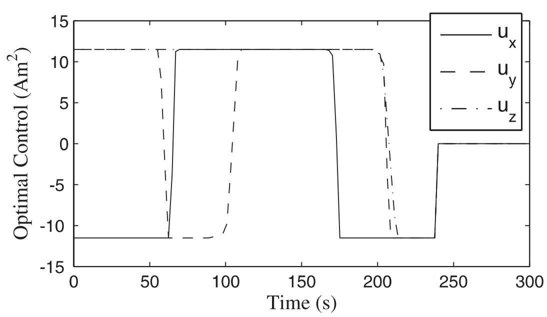

Figure 3. Magnetic control history (simulation A).

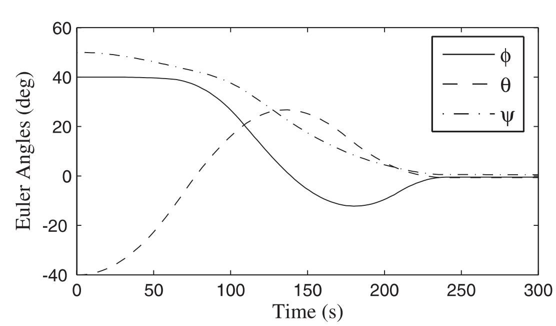

Figure 4. Euler angles' trajectory (simulation A).

Table 5. Maneuver specifications for satellite A.

Parameter	Value
Initial Euler Angles	[30,40,50] deg.
Initial Angular Velocities	[0,0,0] rad/s
Desired Euler Angles	[0,0,0] deg.
Desired Angular Velocities	[0,0,0] rad/s
Satellite's Position	The Perigee

To check the ability of the controller in different orbital positions, the algorithm has been solved for three other orbital positions for the same satellite and the same maneuver. The results are summarized in Table 3. As mentioned in this table, the controller has satisfactory performance in all four positions.

Satellite B:
In this simulation another satellite within another orbit is selected to check the controller's performance. The satellite and its orbital characteristics are mentioned in Table 4 and its maneuver conditions are mentioned in Table 5.

Using the same weights of the simulation A, the solution of the algorithm is depicted in Figures 5 to 8. The optimal time-to-go for this satellite and this maneuver is 234 s, *i.e.* 4% of its orbital period.

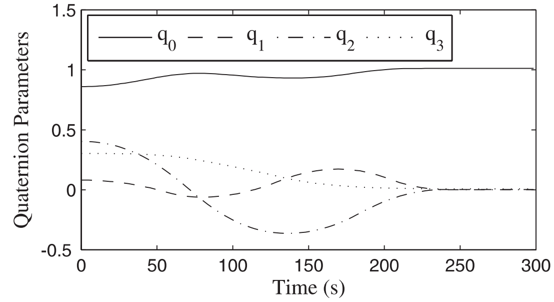

Figure 5. Quaternions propagation (simulation B).

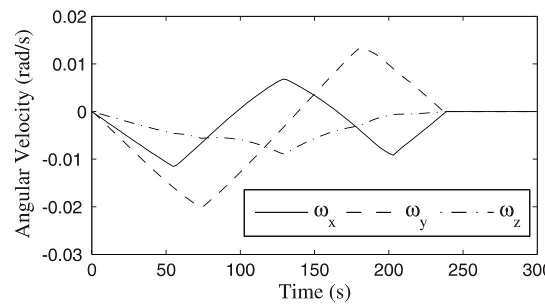

Figure 6. Angular rates history (simulation B).

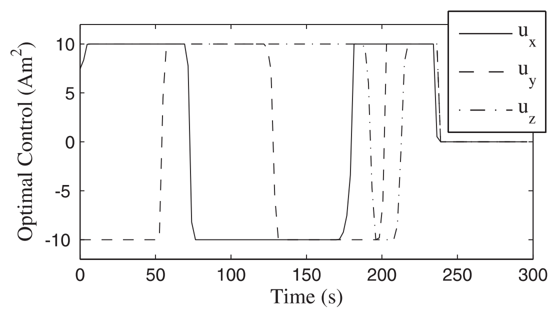

Figure 7. Magnetic control history (simulation B).

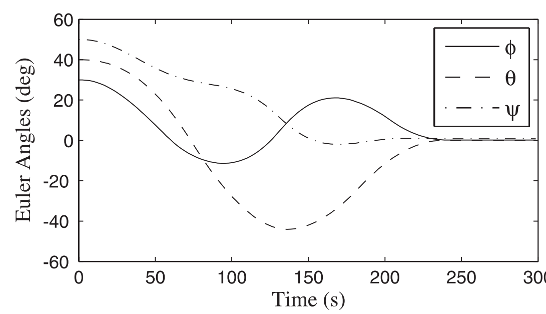

Figure 8. Euler angles trajectory (simulation B).

CONCLUSIONS

Unlike the common notion of time consuming nature of magnetic attitude control methods, the time optimal controller is able to complete the attitude slew maneuvers in a small fraction of an orbital period. Hence using time optimal attitude control for satellites with time critical missions is a desirable choice. Of course, a drawback currently under consideration for this controller is its open loop nature; thus presenting it in a closed loop form through a hybrid approach is of considerable value.

REFERENCES

1. Arduini C., and Baiocco P., "Active Magnetic Damping Attitude Control for Gravity Gradient Stabilized Spacecraft", *Journal of Guidance, Control, and Dynamics*, **20(1)**, PP 117-122(1997).
2. Wisniewski R., "Linear Time Varying Approach to Satellite Attitude Control Using Only Electromagnetic Actuation", *Proceedings of the AIAA Guidance, Navigation and Control Conference*, **23**, PP 243-251(1997).
3. Wisniewski R., "Sliding Mode Attitude Control for Magnetic Actuated Satellite", *Proceedings of the14th IFAC Symposium on Automatic Control in Aerospace*, (1998).
4. Psiaki M. L., "Magnetic Torquer Attitude Control via Asymptotic Periodic Linear Quadratic Regulation", *Journal of Guidance, Control, and Dynamics*, **24(2)**, PP 386-394(2001).
5. Lovera M., Astolfi A., "Global Magnetic Attitude Control of Inertially Pointing Spacecraft", *Journal of Guidance, Control and Dynamics*, **28(5)**, PP 1065-1067(2005).
6. Liang J., "Optimal Magnetic Attitude Control of Small Spacecraft", PhD Thesis, Utah State University, Logan , (2005).
7. Wertz J. R., *Spacecraft Attitude Determination and Control*, Kluwer Academic, (1978).
8. Davis J., "Mathematical Modeling of Earth's Magnetic Field", *Technical Note*, Virginia Tech., Blacksburg, (2004).