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The Effects of Shape Parameterization on the Efficiency of Evolutionary Design Optimization for Viscous Transonic Airfoils

A. Shahrokhi[1], A. Jahangirian[2]

The effect of airfoil shape parameterization on optimum design and its influence on the convergence of the evolutionary optimization process is presented. Three popular airfoil parametric methods including PARSEC, Sobieczky and B-Spline (Bezier curve) are studied and their efficiency and results are compared with those of a new method. The new method takes into consideration the characteristics of viscous transonic flows particularly around the trailing edge. The methods are applied to airfoil shape optimization in turbulent flow conditions of high Reynolds number using Genetic Algorithm. An unstructured grid Navier-Stokes flow solver with a two-equation $K-\epsilon$ turbulence model is used to evaluate the objective function. The original mesh movement strategy (Spring analogy) is modified particularly inside the boundary layer in order to maintain the quality of cells in this area. The aerodynamic characteristics of the optimum airfoil obtained from the proposed parametric method are compared with those from alternative methods. It is concluded that the new method is capable of finding efficient and optimum airfoils in a smaller number of evaluations.

INTRDUCTION

In the recent years, extensive research has been performed in the field of airfoil shape optimization [1-3]. The mathematical representation of airfoil shape is one of the challenging topics for all optimization approaches to provide a wide variety of possible shapes for evaluation. Different methods have been used for airfoil parameterization in aerodynamic shape design. However, most are not suitable for airfoil shape optimization in transonic viscous flow applications. There are two important considerations regarding the choice of a suitable parameterization method. First the flexibility of the method, *i.e.* the parameterization method should have the capability to encode a wide range of geometries with the minimum number of shape variables. Secondly, the robustness of the method in finding the most optimum shapes is important. Although many works have been done on aerodynamic shape optimization using various representation techniques, few efforts have focused on the development of efficient parameterization techniques with suitable design variables. Most of the optimization techniques employ smoothing algorithms based on the polynomials and splines. One of the most popular methods for airfoil representation is the B-spline. Several researchers employed B-spline approach to parameterize the airfoil shapes for the optimization process [4-6].

Another common method for shape parameterization is PARSEC, which has been successfully applied by aerodynamic community for airfoil design optimization [7-9]. It is notable that this technique has been developed to control important aerodynamic features by using the finite number of design parameters. However, PARSEC does not provide enough control over the trailing edge shape where important flow phenomena can occur. One of the alternative ways is adding new design parameters to PARSEC in order to control the trailing edge curvature. Klein and Sobieczky, provide certain improvements to PARSEC through introducing some functions into this method [10].In order to find

1. *PhD Candidate, Dept. of Aerospace Eng., Amir kabir Univ. of Tech., Tehran, Iran, Email: Shahrokhi@aut.ac.ir*
2. *Associate Professor, Dept. of Aerospace Eng., Amirkabir Univ. of Tech., Tehran, Iran, Email: ajahan@aut.ac.ir*

the best possible design variables for airfoil shape optimization, the characteristics of the flow past the airfoil must be considered [8]. On the work by Sobieczky, several mathematical functions useful in generating optimum transonic airfoils are introduced. Increasing the curvature quite close to the trailing edge in this method can reduce the boundary layer de-cambering effect while providing enough flexibility in this part of the airfoil. Utilizing this method, some improvements were obtained [11]. However, the authors experienced shortcomings when applying this method to airfoil shape optimization using Genetic Algorithm. Some modifications proposed by the authors to Sobieczky, together with the new parameterization method was shown to be more appropriate than PARSEC [12]. According to our studies, the B-spline method is the most widely used parameterization technique in design optimization. To complete our previous studies, B-spline method is applied to aerodynamic optimization of transonic airfoils in order to compare it with proposed method.

The main objective of the present work is to study the effect of different airfoil shape parameterization methods in evolutionary shape optimization and to show the superiority of the new method over the alternative methods. The optimization is performed at transonic flow conditions. The numerical solution of Navier-Stokes equations is used for objective function evaluation with Genetic Algorithm as optimizer. Mesh movement is performed using spring analogy. The method developed in this research guaranties the quality of the boundary layer cells during the mesh movement process. The most usual shape parameterization techniques are modeled and applied to the optimization algorithm and the efficiency and characteristics of the optimum shapes are compared with those of our new parameterization.

PARAMETERIZATION TECHNIQUES

a) B-spline Method

In a B-spline representation, the x and y coordinates of the surfaces are written in a parametric form:

$$x(u) = \sum_{i=1}^{n+1} X_i N_{i,k}(u)$$

$$y(u) = \sum_{i=1}^{n+1} Y_i N_{i,k}(u) \qquad n \geq k-1 \tag{1}$$

x and y are the Cartesian coordinates of the surface, $N_{i,k}$ is the ith B-spline basis function of order k. u is the parameter variable and (X_i, Y_i) are the coordinates of the B-spline control points. Therefore B-spline basis functions of an arbitrary dimension n can be evaluated as linear combinations of basis functions of a lower degree. B-spline basis functions are obtained using De boor's relation:

$$N_{i,k}(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_{i,k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1,k-1}(u) \tag{2}$$

where

$$N_{i,1}(u) = \begin{cases} 1, & u_i < u < u_{i+1} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

u_i is the non-decreasing set of real numbers also called the knot sequence. The number of knots is equal to the dimension of the basis function plus the order of the B-spline curve. Since the basis functions are based on knot differences, the shape of the basis functions is only dependent on the knot spacing and not on specific knot values.

Compared to other polynomials, B-spline curves have the advantage of limiting the dimensions of the polynomial to a user-defined level without changing the number of the control points. Figure 1 shows an airfoil created using B-spline curve of order 3 with 21 control points. Research indicates that this is the smallest number of control points necessary to provide the required flexibility in generating predefined shapes

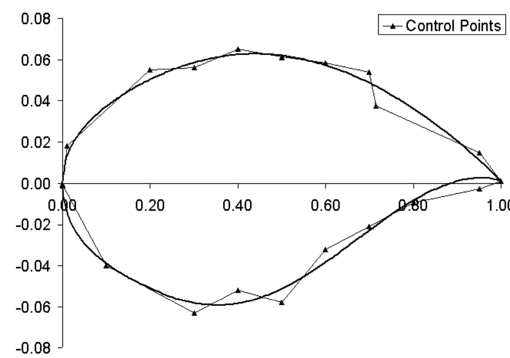

Figure 1. B-spline method for shape parameterization.

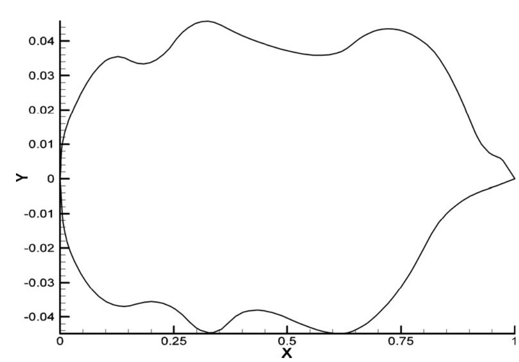

Figure 2. unusual shapes produced by the B-spline method.

as well as optimum configurations [13]. Despite its wide usage in optimization problems, there are several drawbacks in the B-spline method.

Compared to the PARSEC method of parameterization, the number of parameters introduced in the B-spline is greater, which increases the complexity of the optimization process. Another important problem is that this method is not able to control the x location of the related shape. Therefore, the grid points on the new shapes are moved in both x and y directions. The outcome of this problem will be discussed later. The major problem with the B-spline curve is the possibility of producing unusual shapes which cause divergence of the flow solver or might be practically impossible to manufacture. Figure 2 shows an example of an unusual shape created by this method.

b) PARSEC Method

As mentioned above, PARSEC is one of the most usual and most effective methods for airfoil representation in design optimization field. Figure 3 illustrates eleven basic parameters for PARSEC method including the leading edge radius (r_{LE}), the upper and lower crest locations ($X_{UP}, Y_{UP}, X_{LO}, Y_{LO}$), the curvature ($Y_{xxUP}, Y_{xxLO}$), the trailing edge coordinate (Y_{TE}) and direction (α_{TE}), the trailing edge wedge angle (β_{TE}) and the thickness (ΔY_{TE}). A linear combination of shape functions is used to present the airfoil shape in this method:

$$Y_k = \sum_{n=1}^{6} a_{n,k} X_k^{\frac{n-1}{2}} \tag{4}$$

The coefficients a_n are the shape functions, and X and Y are the coordinates of the airfoil.

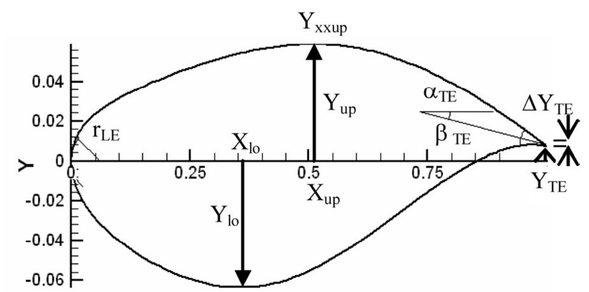

Figure 3. PARSEC method for airfoil parameterization.

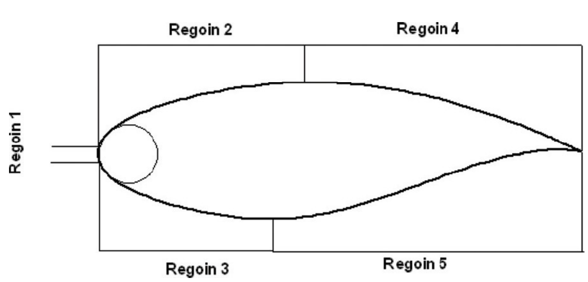

Figure 4. Airfoil divisions in PARSEC.

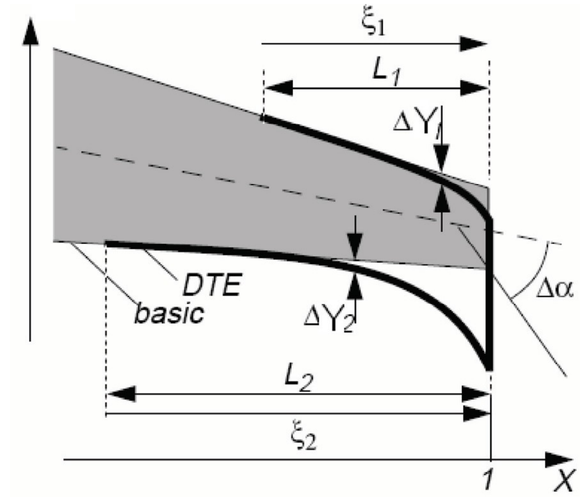

Figure 5. Sobieczky parameters for DTE.

The subscript k changes from 1 to 2 in order to consider the length of the upper or lower surfaces. In order to obtain the shape functions, the airfoil is divided into five regions on the upper and lower surfaces as illustrated in Figure 4. Region one is a very small bow of a circle of radius (r_{LE}) near the leading edge. Regions two and three are limited to the X_{UP} and X_{LO} respectively. a_n coefficients for these regions are obtained using the parameters in each region. Regions four and five include the rest of the airfoil on the upper and lower surfaces. Similar to regions two and three, a_n coefficients for these regions are obtained using related parameters.

Using PARSEC parameters mentioned above, one can effectively control the maximum curvature of the upper and lower surfaces and their location, which are very useful in reducing the shock wave strength or delaying its occurrence. However, at the ending part of the airfoil, PARSEC fits a smooth curve between the maximum thickness point and the trailing edge, which in turn disables the necessary changes in the curvature close to the trailing edge. Therefore, in spite of its benefits in controlling the important parameters on the upper and lower surfaces, PARSEC does not provide enough control over the trailing edge shape where important flow phenomena can occur.

c) The Modified Sobieczky Method

One of the techniques in removing the disadvantages of the PARSEC parametric method is proposed by Sobieczky for the trailing edge modeling [11]. The practical consequence of using this method is concave surface shaping with curvature increasing towards the trailing edge at both the upper and lower surfaces. Such airfoils are known as Divergent Trailing Edge (DTE). This method is mainly based on the viscous flow control near the trailing edge, which strongly influences the aerodynamic efficiency. Figure 5 illustrates the Sobieczky parameters for the trailing edge modeling.

In their simplest form, the parameters $\Delta\alpha, L_1, L_2$ that control the increment in the trailing edge thickness (ΔY) are added to make the airfoil surface a divergent trailing edge. The parameter $\Delta\alpha$ controls the camber added to the upper and lower surfaces, which create a DTE. L_k is the chord length measured from the trailing edge which is modified in the Sobieczky method. The function considered for ΔY is:

$$\Delta Y_k = \frac{L_k.\tan\Delta\alpha}{\mu.n}[1-\mu.\xi_k^n-(1-\xi^n)^\mu] \tag{5}$$

ξ_k is the x-coordinate variable. Parameters and variables of this method are illustrated in Figure 5. The subscript k changes from 1 to 2 in order to consider the length on the upper and lower surfaces. The shaded region in this figure is the original airfoil generated by the PARSEC method. Different values are possible for parameters μ and n. In the present study, the values considered are 1.3 and 6, respectively.

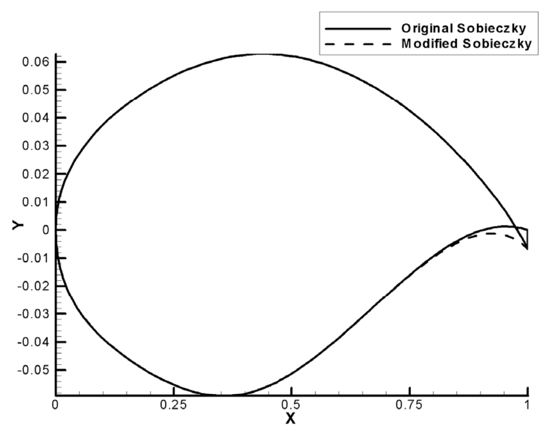

Figure 6. Original and modified Sobieczky method for DTE.

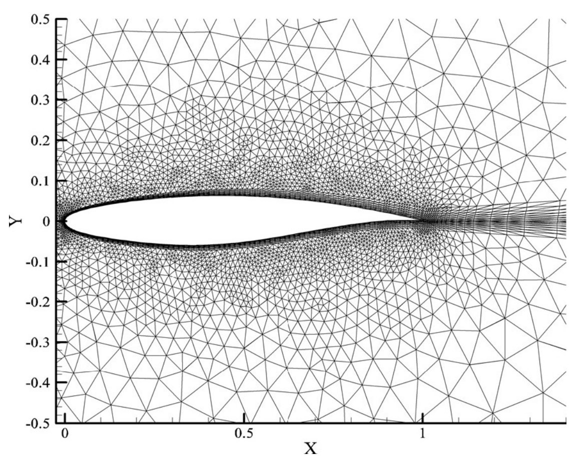

Figure 7. Unstructured viscous grids around RAE 2822 airfoil.

In the present investigation, airfoil shapes are represented by a combination of PARSEC for the main part of the airfoil and Sobieczky method for the trailing edge modeling. Trailing edge coordinate (Y_{TE}) and the thickness parameters of the PARSEC method are considered zero; thus, they can be omitted from the list of design variables. Consequently, the total number of design variables is increased to 12, including the leading edge radius, the upper and the lower crest location, the curvature, the trailing edge direction, the wedge angle from the PARSEC method, and $\Delta\alpha_{TE}$ as well as L_k from the Sobieczky method. Approximating curvature quite close to the trailing edge can create a flow in the vicinity of the trailing edge, which has a favorable pressure gradient on the airfoil surface. This pressure distribution compensates for the probable decrease in the lift caused by the decrease in the upper surface camber. The application of this method to the airfoil shape optimization is carried out by authors in reference [7]. Despite improving the characteristics of the final optimum shapes, the Sobieczky method may lead to an overlap of the upper and lower surfaces. To overcome this problem, a modified Sobieczky method for the trailing edge is proposed. The original and modified Sobieczky methods are shown in Figure 6. As illustrated in this figure, there is a rather sharp change in the trailing edge lower surface camber, which causes the overlap of the upper and lower surfaces. However, in the modified Sobieczky, the lower surface is gradually adapted so that it terminates at the ending part of the upper surface.

d) New Parameterization Method

After using the above method in some optimization problems, we found that some modifications can still be made to obtain an optimum shape. One more important problem associated with the Sobieczky method is that it mainly tends to pull the trailing edge downward in order to increase the curvature at the rear part of the airfoil. However, this change may increase the upper surface adverse pressure gradient in the viscous flow. On the other hand, considering the negative values for $\Delta\alpha_{TE}$ will make the trailing edge shape worse due to formation of negative curvature over the airfoil. One way to reduce the pressure drag in the transonic flow is to flatten the upper surface of the airfoil, which creates a weaker shock wave on the airfoil. Therefore, the Sobieczky formulation is changed to create a smoother upper surface. The following function is then proposed for ΔY instead of Eq. (5):

$$\Delta Y = \frac{\tan\Delta\alpha}{\mu.n}[1+\eta.\xi^n-(1-\xi^n)^\mu] \tag{6}$$

where η is set to 0.8, and n is equal to 6. It would also be beneficial that the entire (upper and lower) surface be exposed to the above equation, in which case no L

parameter would be used in this method. Therefore, the total number of design variables is reduced to 10. The new formulation provides a smoother upper surface by pulling the trailing edge upwards. However, the changes in the curvature of the lower surface are smaller in comparison to the Sobieczky method, which may produce smaller lift. To provide a better pressure distribution for the lower surface, the changes of ΔY in the lower surface are computed using the original Sobieczky method as proposed by the authors [12].

EVOLUTIONARY AERODYNAMIC OPTIMIZATION USING GA

Among the optimization algorithms , gradient-based methods are well-known techniques. They seek the optimum by calculating the local gradient information. Although these methods are superior to non-gradient-based techniques in a local search, the optimum obtained from such methods may not be a global one, especially in aerodynamic design problems. Alternatively, Genetic Algorithms are more likely to find a global optimum and are therefore attractive for aerodynamic design optimization where the objective functions are nonlinear. More information about GA can be found in [14], [15]. In the present study, simple Genetic algorithm is applied to the optimization of a viscous transonic airfoil. Thus, fitness, chromosomes and genes correspond to the objective function, design candidates as well as design variables, respectively. Design variables are the parameters introduced by the parameterization technique. Simple one-point crossover operator is used with an 80% probability of combination since the use of smaller values was observed to deteriorate the GA performance [16]. The mutation probability is set to 10%, which then adds a random disturbance to the parameter for about 15% of the design space defined for each chromosome's gen.

In this work, the tournament operator [15] is used with an elitist strategy, where the best and the second best chromosomes in each generation are directly transferred into the next generation.

NUMERICAL EVALUATION

The airfoil shapes that are generated by the Genetic Algorithm are evaluated based on the numerical simulation of turbulent viscous flows governed by Reynolds-average Navier-Stokes equations. Since most of the computational time required for the optimization process is consumed by the flow solver, CFD solver, which drives the optimization process, must possess a high efficiency and convergence rate. To achieve the above goals, a dual-time implicit method is used in the present work. This method follows the work of Jahangirian and Hadidoolabi for unstructured grids [17], and has the advantage of higher time step value

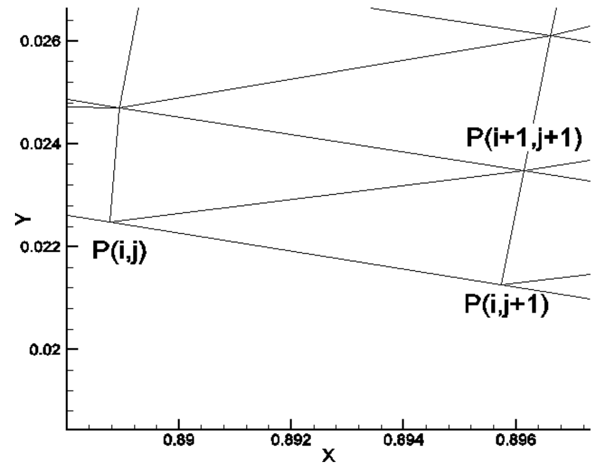

Figure 8. Boundary layer cells.

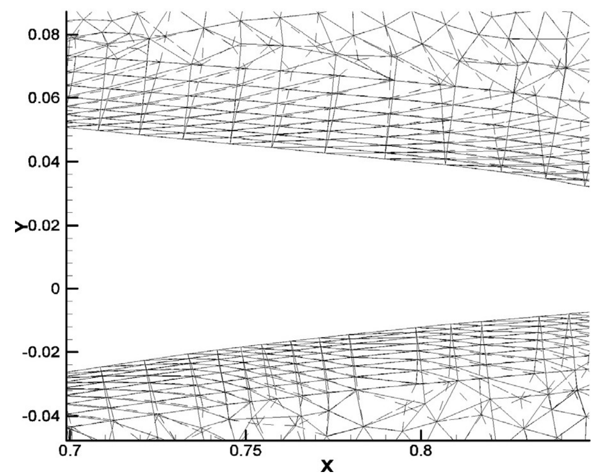

Figure 9. Improvement in boundary layer cells.

of implicit methods as well as convergence acceleration tools of explicit techniques. Further details of the method can be obtained from the above reference.

GRID GENERATION AND MESH MOVEMENT STRATEGY

The computational field is discretized utilizing the triangular unstructured grids. A successive refinement method is used for unstructured grid generation [18]. Figure 7 shows the generated grid that contains 10651 triangular cells around an RAE 2822 airfoil. In the present work, the primary mesh generated around the initial airfoil is moved to be fitted to the new generated airfoil using spring analogy. This provides an automatic and efficient mesh movement tool that has to be called some hundreds of time during a single optimization process.

All parameterization techniques employed in this research, are able to provide the Y coordinates for the predefined X positions. Therefore, considering a fixed location for X coordinate of the surface grid points during the optimization process, the points only need to move in the Y direction. However, according to Eq. (1) for the B-spline method, both X and Y coordinates of the airfoil are dependent on parameter variable u. Therefore, more precise employment of the spring analogy is demanded when the B-spline method is implemented firstly because the grids should move in both X and Y directions, and secondly because the grid movements in both X and Y directions, do not guarantee the quality of the cells in the boundary layer. The normal cell edges should be moved in a way that they remain vertical to the solid boundary. Therefore, after the grids are moved using the spring analogy in X and Y directions, the boundary layer cells are checked to see if they satisfy the following condition:

$$P(i,j)\,P(i+1,j) \bullet P(i+1,j)\,P(i+1,j+1) \leq 10^{-5} \tag{7}$$

where $P(i,j)$, $P(i+1,j)$ and $P(i+1,j+1)$ are three points representing the boundary layer cell shown in Figure 8. If the points do not satisfy this condition, then $P(i+1,j+1)$ is moved on the line passing through points $P(i,j+1)$ and $P(i+2,j+1)$ to satisfy the orthogonal condition.

This condition checks for all boundary layer cells, beginning from the closest cells to the solid boundary. The improvement in the boundary cells through this method is shown in Figure 9.

RESULTS

Numerical experiments are carried out in two parts. In the first part the efficiency of the new parameterization method is investigated through a simple geometric inverse design while in the second part, the method is applied to the aerodynamic optimization of airfoils using CFD as an evaluation tool.

a) Geometric Reconstruction

Since a full process of optimization problem with CFD evaluation is very time consuming, some preliminary studies are carried out in order to show the efficiency of the new airfoil parameterization method over some other existing methods. The objective function is considered as follows:

$$F = \frac{\sum_{i=1}^{n_p} (Y_i - Y_{ti})^2}{2n_p} \tag{8}$$

where Y_i and Y_{ti} are the design and target coordinates of surface points with X_i fix coordinates. Utilizing this simple objective function, there is no need to use the costly CFD flow solver, so the inverse design is quite fast.

The capabilities of the four parameterization methods *i.e.* B-spline, PARSEC, Sobieczky and the proposed method are reconsidered in geometric reconstruction.

The initial and the target airfoils are NACA0012 and K-1 transonic airfoils respectively (Figure 10). All GA parameters are set equal for all three parameterization methods. Target and design airfoils using the

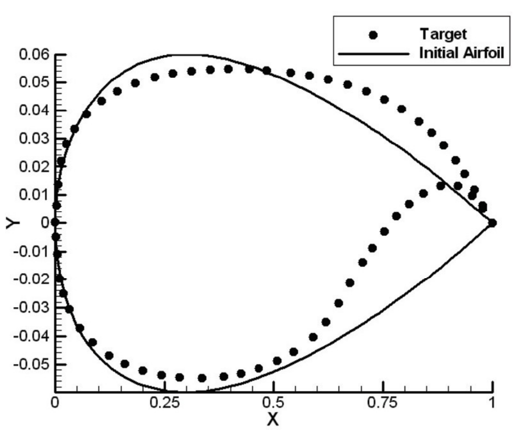

Figure 10. Initial and target airfoils.

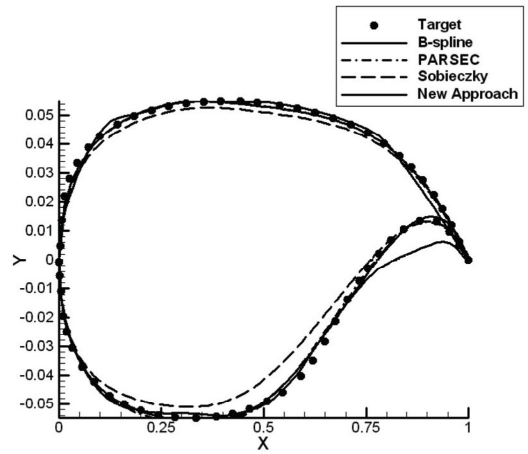

Figure 11. Target and design airfoils using different parameterization methods.

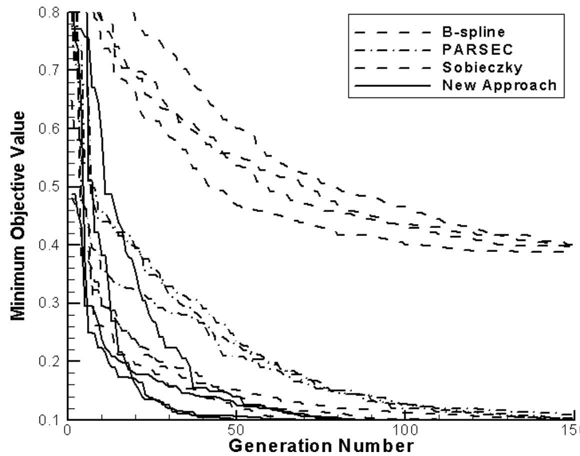

Figure 12. Convergence history of the maximum objective value.

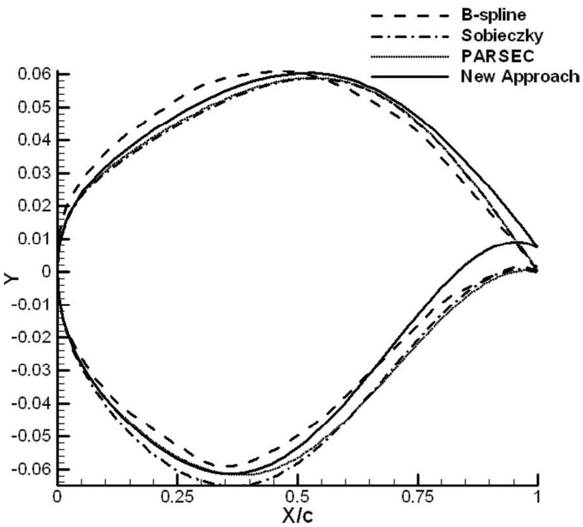

Figure 13. Optimum airfoil shapes using different parameterization methods.

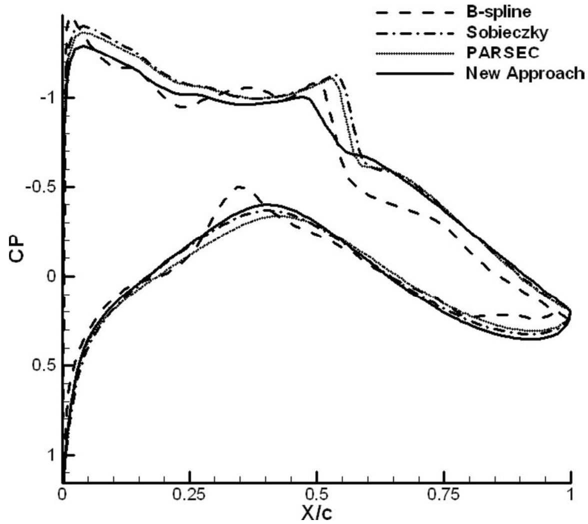

Figure 14. Pressure distribution of optimum shapes using different parameterization methods.

above four methods are shown in Figure 11. Although all methods are able to produce the target airfoil, the convergence rates are considerably different.

The convergence histories of the four methods are shown in Figure 12. To ensure that the results are not influenced by the random initial population or random nature of the GA, the convergence histories of different parametric shape methods are repeated using different random seed numbers. According to this figure, the best convergence rate is obtained by new approach and the lowest convergence rate is related to PARSEC. The B-spline method has failed to converge into the optimum shape even after 150 generations. The new method is approximately converged after 50 generations while the convergence results are obtained after about 70 and 150 generations using PARSEC and Sobieczky methods respectively. This is equivalent to 67% and 28% reduction in the computational time when using the new method compared with PARSEC and Sobieczky, respectively.

b) Aerodynamic Design Optimization

To show the performance of the aforementioned parameterization methods in the aerodynamic design optimization, the optimization results using B-spline, PARSEC, Sobieczky and the proposed approach are compared. The optimization is carried out at a transonic Mach number of 0.75, and the fully turbulent flow of Re=6.5 million. The incidence angle is 2.79 degrees. The objective function is C_l/C_d, and RAE-2822 airfoil is considered as the initial airfoil.

Figures 13 and 14 illustrate the optimum shapes and their corresponding pressure distributions obtained for different parameterization methods. Figure 14 indicates a rather strong shock wave over the optimum shape of the B-spline method, while the optimum shape of the new approach is almost shock free. The optimization results of PARSEC and Sobieczky include a weak shock over the airfoil.

The history of the maximum objective functions is shown in Figure 15 for different parameterization methods. Regarding the maximum fitness value, the Sobieczky's gives nearly the same results as PARSEC

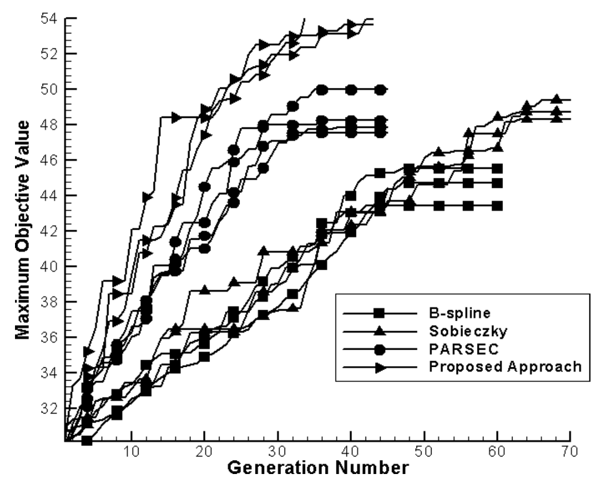

Figure 15. Convergence history of the maximum objective value.

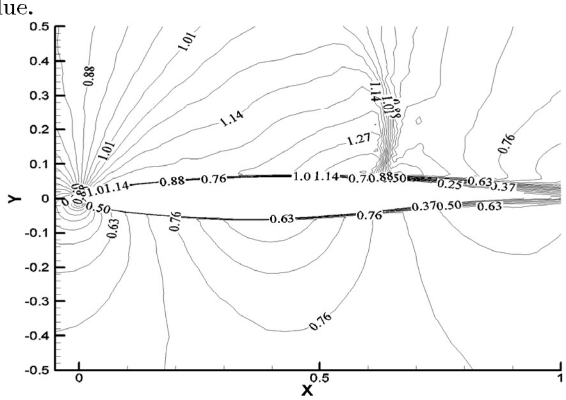

Figure 16. Mach number contours for the initial airfoil.

while the proposed method shows more than 10% improvement in the final fitness value.

The level of maximum objective function value for the B-spline optimum shape is less than that for other methods in all generations. It is evident from the figure that the Sobieczky's and the B-spline need at least 80% more computational time than the new method for achieving the same convergence level.

Having known that a greater portion (about 99%) of the total computational time required in such an evolutionary optimization problem is consumed in evaluations could represent the scale of the CPU fitness function evaluation. This means that nearly a 10% reduction in computational time is obtained when the proposed method is used for shape parameterization instead of PARSEC. This conclusion would be more important when turbulent viscous flow solution is considered since each CFD fitness evaluation takes about 15-20 minutes on a current PC.

Figures 16 and 17 illustrate the Mach contours for the initial and optimum shapes, respectively, using different airfoil parameterization methods. Figure 16 indicates a strong shock wave over the initial airfoil, which is the main flow feature in this case. However, this strong shock wave is weakened when applying the different parameterization methods to the optimization process. According to Figure 17(d), the shock is totally damped when using the proposed parameterization approach.

The values of the lift and drag coefficients and the objective functions for optimum shapes are shown in Table (1) for each parameterization method. The maximum objective function is obtained using the new parameterization technique. The maximum value of lift coefficient is obtained through the Sobieczky method. This is mainly due to the decreased curvature on the upper surface of the airfoil in the new parameterization technique. The minimum value of the drag coefficient achieved through the new method is about 14% less than that of Sobieczky and PARSEC.

Table 1. Lift and drag coefficients for optimum airfoils using different airfoil parameterizations.

Parameterization Method	C_l	C_d	C_l / C_d
B-spline	0.669	0.0158	42.34
PARSEC	0.775	0.0162	47.83
Sobieczky	0.795	0.0161	49.37
Our Proposed Method	0.736	0.0138	53.33

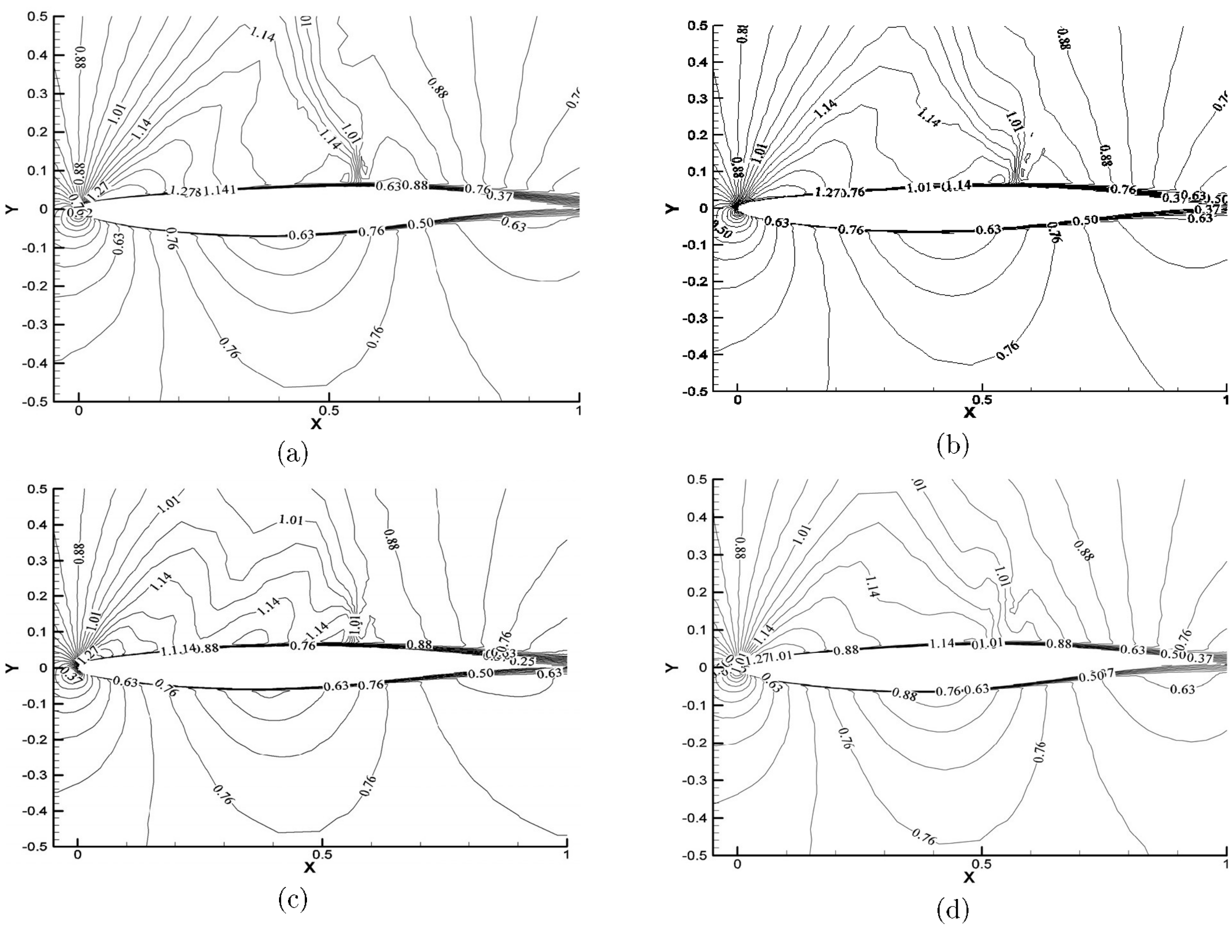

Figure 17. Mach contours for optimum shape using a) B-spline b) PARSEC c) Sobieczky.

CONCLUSIONS

Three common shape representation methods *i.e.* B-spline, PARSEC and Sobieczky together with our new approach along with their applications in the aerodynamic optimization of airfoil were investigated. The characteristics of a suitable parameterization technique were introduced. The parameterization methods were applied to the optimization problem of a viscous transonic airfoil to create the maximum C_l/C_d at specified flow conditions. It was concluded that the new method is able to create optimum shapes in a shorter computational time.

REFERENCES

1. Chiba, K., Obayashi, S., Nakahashi, K., "High-Fidelity Multidisciplinary Design Optimization of Aerostructural Wing Shape for Regional Jet", *23rd AIAA Applied Aerodynamics Conference*, Toronto, Ontario, Canada, (2005).
2. Hazra, S.B., Schulz, V., Brezillon, J., Gauger, N.R., "Aerodynamic Shape Optimization Using Pseudo-Timestepping", *Journal of Computational Physics*, **204**, PP 46-64(2005).
3. Quagliarella, D., Iannelli, P., Vitagliano, P., Chinnici, G., "Aerodynamic Shape Design Using Hybrid Evolutionary Computation and Fitness Approximation", *AIAA 1ST Intelligent Systems Technical Conference*, Chicago, Illnois, (2004).
4. Anderson, K.W., Venkatakrishnan, V., "Aerodynamic Design Optimization on Unstructured Grid with a Continuous Adjoint Formulation", *AIAA Paper 97-0643*, (1997).
5. Anderson, K.W., Bonhaus, D.L., "Airfoil Design on Unstructured Grids for Turbulent Flows", *AIAA Journal*, **37**(2), PP 185-191(1999).
6. Obayashi, Sh., Nakahashi, K., Oyama, A., Yoshino, N., "Design Optimization of Supersonic Wing Using Evolutionary Algorithms", *European Congress on Computational Methods in Applied Science and Engineering (ECCOMAS)*, (1998).
7. Shahrokhi, A., Jahangirian, A., Fouladi, N., "Navier-Stokes Optimization Using GA and a Flexible Parametric Airfoil Method", *ERCOFTAC conference on design optimization: Methods and Application*, Gran Canaria, Spain, (2006).
8. Oyama, A., Obayashi, Sh., Nakahashi, K., "Real Coded Adaptive Range Genetic Algorithm and its application to Aerodynamic Design", *Lecture Notes in Computer Science*, **1917**, PP 712-721(2000).
9. Oyama, A., Obayashi, Sh., Nakahashi, K., Nakamura, T., "Aerodynamic Optimization of Transonic Wing Design Based on Evolutionary Algorithm", *Third International Conference on Non Linear Problems in Aviation and Aerospace Methods and Soft Ware*, Daytona Beach, FL, USA, (2000).
10. Klein, M., Sobieczky, H., "Sensitivity of Aerodynamic Optimization to Parameterized Target Function", *Proc. Int. Symp. On Inverse Problems in Engineering Mechanics (ISIP 2001)*, Nagano, Japan, (2001).
11. Sobieczky, H., "Parametric Airfoils and Wings", *Notes on Numerical Fluid Mechanics*, PP 71-88(1998).
12. Shahrokhi, A., Jahangirian, A., "Airfoil Shape Parameterization for Optimum Navier-Stokes Design with Genetic Algorithm", *Aerospace Science and Technology*, **11**(6), PP 443-450(2007).
13. Castonguay, P., Nadarajah, S.K., "Effect of Shape Parameterization on Aerodynamic Shape Optimization", *45th AIAA Aerospace Sciences Meeting and Exhibit*, Reno, Nevada, (2007).
14. Deb, K., *Multi-objective Optimization Using Evolutionary Algorithms*, John Wiley & Sons, (2001).
15. Goldberg, D.E., *Genetic Algorithm in Search, Optimization and Machine Learning*, Massachusetts, Addison-Wesley, (1989).
16. Tes, D. and Chan, Y. Y., "Multi-point Design of Airfoil by Genetic Algorithm", *8th Annual Conference of the CFD Society of Canada*, Montreal, Canada, (2000).
17. Jahangirian, A., Hadidoolabi, M., "Unstructured Moving Grids for Implicit Calculation of Unsteady Compressible Viscous Flows", *International Journal for Numerical Methods in Fluids*, **47**(10-11), PP 1107-1113(2005).
18. Jahangirian, A., Johnston, L.J., "Automatic Generation of Adaptive Unstructured Grids for Viscous Flow Applications", *5th International Conference on Numerical Grid Generation in CFD*, Mississippi State University, (1996).