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Automatic Landing of Small Helicopters on 4DOF Moving Platforms

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In this research, an automatic control system is designed for landing a small helicopter on a 4DOF moving platform. The platform has three translational and one directional degrees of freedom. The controller design approach is based on development of the helicopter nonlinear dynamic model into the SDC (State Dependent Coefficient) form and the real time solving of the State Dependent Riccati Equation (SDRE). To compensate the simplifications and term eliminations necessary to make the SDC form, a nonlinear compensator is added. The performance of the control system in automatic landing is evaluated by computer simulation in different scenarios. The results show satisfactory tracking performance of the controller during landing phase on a moving platform.

NOMENCLATURE

m	Helicopter mass (kg)
u	Helicopter longitudinal velocity in body coordinate (m/s)
v	Helicopter lateral velocity in body coordinate (m/s)
w	Helicopter vertical velocity in body coordinate (m/s)
p	Helicopter roll rate in body coordinate (rad/s)
q	Helicopter pitch rate in body coordinate (rad/s)
r	Helicopter yaw rate in body coordinate (rad/s)
g	Gravity acceleration (m/s^2)
ϕ	Roll angle (rad)
θ	Pitch angle (rad)
ψ	Yaw angle (rad)
I_{xx}, I_{yy}, I_{zz}, I_{xz}	Helicopter mass moment of inertia (kg.m^2)
X_{mr}	Main rotor force in X direction
X_{fus}	Fuselage force in X direction
Y_{mr}	Main rotor force in Y direction (N)
Y_{fus}	Fuselage force in Y direction (N)
Y_{tr}	Tail rotor force in Y direction (N)
Y_{vf}	Vertical fin force in Y direction
Z_{mr}	Main rotor force in Z direction (N)
Z_{fus}	Fuselage force in Z direction (N)
Z_{ht}	Horizontal tail force in Z direction (N)
L_{mr}	Main rotor moment in X direction (N.m)
L_{vf}	Vertical fin moment in X direction (N.m)
L_{tr}	Tail rotor moment in X direction (N.m)
M_{mr}	Main rotor moment in Y direction (N.m)
M_{ht}	Horizontal tail moment in Y direction (N.m)
N_{vf}	Vertical fin moment in Z direction (N.m)
N_{tr}	Tail rotor moment in Z direction (N.m)
Q_e	Engine output torque (N.m)
T	Rotor thrust (N)
C_T	Rotor thrust coefficient
ρ	Air density (kg/m^3)
Ω	Main rotor angular speed (rad/s)
R	Main rotor radius (m)

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λ_0	Main rotor inflow ratio
μ	Advance ratio
μ_x	Longitudinal airflow component
μ_y	Lateral airflow component
μ_z	Normal airflow component
η_w	Coefficient of non-ideal wake contraction
a	Lift curve slop
σ	Solidity ratio
u_{col}	Collective pitch control (rad)
u_{lon}	Longitudinal cyclic pitch control (rad)
u_{lat}	Lateral cyclic pitch control (rad)
u_{tcol}	Tail rotor collective pitch control (rad)
u_{wind}	Wind velocity in X direction (m/s)
v_{wind}	Wind velocity in Y direction (m/s)
w_{wind}	Wind velocity in Z direction (m/s)
a_1	Longitudinal flap angle (rad)
b_1	Lateral flap angle (rad)
τ_e	Effective rotor time constant with the stabilizer bar (s)
$A_{u_{lon}}$	Effective steady state gain for longitudinal cyclic input
$B_{u_{lat}}$	Effective steady state gain for lateral cyclic input
K_μ	Scale of the flap response to speed variation
K_β	Hub torsional stiffness (N.m/rad)
h_{mr}	Vertical distance between hub and c.g. (m)
u^{sd}	SDRE control input
u^c	Compensator control input
u^t	Trim or tracking control input
u^r	Optimal part of SDRE control input
f_{rb}	Rigid body dynamics term in system dynamic
T_u	All control forces (N)
T_d	All drag forces (N)
V_{tip}	The blade tip velocity (m/s)
$\mathbf{R}$	The body to inertial coordinate transformation matrix
ψ	The angular velocity to the Euler angles derivatives transformation matrix

INTRODUCTION

Today; special capabilities of helicopters have made them very useful vehicles in aerial missions over sea and land. Having a highly coupled nonlinear dynamics, guidance, navigation and control of helicopters are challenging tasks for pilots and control engineers. One of these challenging tasks is landing on a moving platform such as a shipboard deck. Many accidents are reported related to this maneuver on a rocking ship. Therefore, an automatic landing system seems to be vital for helicopters in shipboard operations. This problem is even more complicated for small autonomous helicopters. Such helicopters have high thrust to weight ratio, hence they are more agile and less controllable than their full scale counterparts.

Consequently, many researches have been carried out in the field of helicopter modeling. Ref. [1] presents the nonlinear model of a small helicopter. This model is based on the "6DOF rigid body dynamic equations integrated with the helicopter aerodynamics presented in Ref. [2] and the main rotor dynamics presented in Refs. [3,4]". In Ref. [5], the accurate dynamic effects of fly bar is also modeled and implemented into the rest of helicopter mathematical model. In the field of automatic landing of helicopters, the majority of the researches are focused on the method of vision based approach and its associated image processing problems. In Refs. [6,7,8], a detailed description of the subject is presented. In Ref. [9], "Yu" has designed a 3D vision system for estimating the height over the ground in helicopter landing process. "Johnson" has described an algorithm for landing hazard avoidance based on the images taken from a single moving camera in Ref. [10]. In Ref. [11], "Xu" has presented a real-time stereo vision based pose and motion estimation system that can be used for landing an unmanned helicopter on a moving platform.

Elsewhere in Ref. [12], "Oh" has described how a tether can be used for a more secure landing in rough weather. It has been shown that the tether tension can be used to make coupling between the translational and rotational motion in order to augment the position controllability of the helicopter. In the other researches, Refs. [13,14,15], the adaptive control of the helicopter is the main issue of concern. Different approaches have been taken to design and implement adaptive control laws including the intelligent control approaches such as fuzzy logic and artificial neural network (Refs. [16,17]). In [18], "Bogdanov" has used SDRE (State Dependent Riccati Equation) method to control a small helicopter. This method has been previously introduced by "Cloutier" in Refs. [19,20,21]. These papers are the main sources of SDRE and its developments. In this approach, which in fact belongs to the Model Based Control family, the nonlinear dynamic model is converted to a pseudo linear model and then the Riccati equation is solved. SDRE is a suboptimal method which can easily be used for extremely nonlinear systems. Also the issues of stability, controllability and observability of the SDRE method are the subject of many other recent works including Refs. [22,23,24].

In the present research, the SDRE method is used to design a nonlinear control system for the automatic landing of small helicopters on a 4DOF moving platform. Here, the rotor flapping dynamics is also considered. Neglecting the flapping dynamics in [18] has resulted in an inefficient attitude tracking performance by helicopters, which is an important issue in landing problems.

HELICOPTER MODEL

Based on the selected controller design approach, SDRE, which is a model based method, an analytical helicopter model is required. Assuming rigid body dynamics, the 6DOF equations of motion of helicopter can be written as follows (Figure (1)):

$$\dot{u} = vr - wq - g\sin\theta + \frac{X_{mr} + X_{fus}}{m}$$
$$\dot{v} = wp - ur + g\sin\phi\cos\theta + \frac{Y_{mr} + Y_{fus} + Y_{tr} + Y_{vf}}{m}$$
$$\dot{w} = vr - wq + g\cos\phi\cos\theta + \frac{Z_{mr} + Z_{fus} + Z_{ht}}{m}$$
$$\dot{p} = \frac{qr\left(I_{yy} - I_{zz}\right)}{I_{xx}} + \frac{L_{mr} + L_{vf} + L_{tr}}{I_{xx}}$$
$$\dot{q} = \frac{pr\left(I_{zz} - I_{xx}\right)}{I_{yy}} + \frac{M_{mr} + M_{ht}}{I_{yy}}$$
$$\dot{r} = \frac{pq\left(I_{xx} - I_{yy}\right)}{I_{zz}} + \frac{-Q_e + N_{vf} + N_{tr}}{I_{zz}} \quad (1)$$

In this work, the cross product moment of inertia I_{xz} is neglected. This assumption is based on the relative magnitude of the helicopter cross product and axial mass moments of inertia, especially in unmanned helicopters, of Ref. [1].

The modeling of helicopter aerodynamics needs a complete modeling of the induced flow distribution on its rotor blades, Refs. [2,3,4]. Any simplification may result in an unsatisfactory dynamic prediction. However, using the system identification techniques in Ref. [1], the defects of the simplified induced flow distribution have been compensated for. In this reference, the rotor thrust is calculated from the solution of the following equations derived based on the momentum theory.

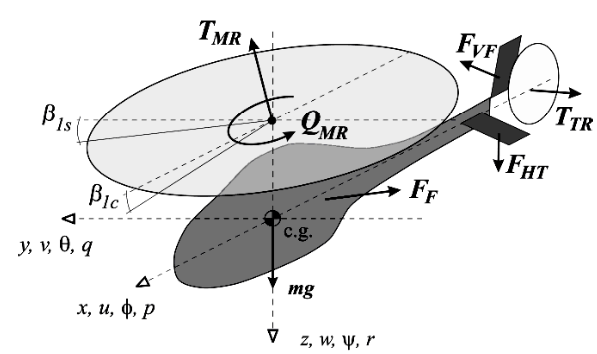

Figure 1. Helicopter forces and moments.

$$C_T = \frac{T}{\rho(\Omega R)^2 \pi R^2}$$
$$\lambda_0 = \frac{C_T}{2\eta_w\sqrt{\mu^2 + \left(\lambda_0 - \mu_z\right)^2}}$$
$$C_T = \frac{a\sigma}{2}\left(u_{col}\left(\frac{1}{3} + \frac{\mu^2}{2}\right) + \frac{\mu_z - \lambda_0}{2}\right) \quad (2)$$

In Equation (2), the solidity ratio, σ, is equal to the quotient $\frac{2chord}{\pi R}$, and the speed ratios are calculated as follows:

$$\mu = \frac{\sqrt{\left(u - u_{wind}\right)^2 + \left(v - v_{wind}\right)^2}}{\Omega R}$$
$$\mu_u = \frac{u - u_{wind}}{\Omega R}$$
$$\mu_y = \frac{v - v_{wind}}{\Omega R}$$
$$\mu_z = \frac{w - w_{wind}}{\Omega R} \quad (3)$$

Also the lateral and longitudinal flapping equations are given as follows:

$$\dot{b}_1 = -p - \frac{b_1}{\tau_e} + \frac{1}{\tau_e}\frac{\partial b_1}{\partial \mu_y}\frac{v - v_w}{\Omega R} + \frac{B_{u_{lat}}}{\tau_e}u_{lat}$$
$$\dot{a}_1 = -q - \frac{a_1}{\tau_e} + \frac{1}{\tau_e}\left(\frac{\partial a_1}{\partial \mu_x}\frac{u - u_w}{\Omega R} + \frac{\partial a_1}{\partial \mu_z}\frac{w - w_w}{\Omega R}\right) + \frac{A_{u_{lon}}}{\tau_e}u_{lon} \quad (4)$$

In addition, flapping derivatives are determined as:

$$\frac{\partial a_1}{\partial \mu_x} = 2K_\mu\left(\frac{4u_{col}}{3} - \lambda_0\right)$$
$$\frac{\partial a_1}{\partial \mu_z} = K_\mu\frac{16\mu^2}{\left(1 - \mu^2/2\right)\left(8|\mu| + a\sigma\right)}sgn\ \mu \approx K_\mu\frac{16\mu^2}{8|\mu| + a\sigma}sgn\ \mu \quad (5)$$

Having had the flapping angles, the rotor forces and moments can be calculated as follows:

$$L_{mr} = \left(K_\beta + Th_{mr}\right)b_1$$
$$M_{mr} = \left(K_\beta + Th_{mr}\right)a_1$$
$$X_{mr} = -T_{mr}a_1, \quad Y_{mr} = T_{mr}b_1, \quad Z_{mr} = -T_{mr}. \quad (6)$$

Tail rotor thrust is calculated with the same method as used for the main rotor thrust with its own characteristic parameters. Other forces and moments are drag based forces determined as mentioned in Ref. [1].

CONTROLLER DESIGN

The SDRE approach is based on the minimization of an objective function $J(x)$ for an input affine system.

$$J = \int_{t_0}^{\infty} (x^T Q(x) x + u^T R(x) u)\, dt$$

$$\dot{x} = f(x) + g(x)u \tag{7}$$

where x and u are the system state and input vectors respectively, and $Q(x)$ and $R(x)$ are positive definite state dependent matrices. In addition, $f(x)$ must be a continuously differentiable function of x and $f(\bar{0}) = \bar{0}$.

The SDRE approach for obtaining a suboptimal solution to the problem given in Eq. (7) can be stated as follows:

- Use direct parameterization to express the nonlinear problem in the state dependent coefficient (SDC) form.

$$\dot{x} = A(x)x + B(x)u \tag{8}$$

- Solving Riccati equation for $(A(x), B(x))$:

$$A^T(x)P + PA(x) - PB(x)R^{-1}(x) B^T(x)P + Q(x) = \bar{0} \tag{9}$$

- Using Riccati solution to obtain the suboptimal control feedback:

$$u = -R^{-1}(x)B^T(x)P(x)x \tag{10}$$

As is shown in Refs. [22,23,24], the stability, controllability and observability of this method are guaranteed. To use the SDRE method, the helicopter nonlinear dynamic model must be converted to the SDC form (Eq. (8)), satisfying the condition $f(\bar{0}) = \bar{0}$. However, in its present form, it is not possible to do such a conversion. In general, if each mathematical function $f(x) \in C^1$ is written in the following form:

$$f(x) = A(x)x + \Delta f(x) \tag{11}$$

then the first term, $A(x)x$, which includes all the terms that satisfy the condition $\lim_{x\to 0} \frac{f_1(x)}{x} \leq M < \infty$, can be used in SDC form. Examples of such functions contain $f_1(x) = ax^n$, with $A(x) = ax^{n-1}$, or $f_1(x) = \sin x$, which yields $A(x) = \frac{\sin x}{x}$. However, the second term, $\Delta f(x)$, cannot be used in SDC form (Eq. (8)) because $\lim_{x\to 0} \frac{\Delta f(x)}{x} = \mp\infty$. To solve this problem, reference [18] has added a nonlinear compensator to the control inputs as mentioned in the following.

Let's consider that the helicopter nonlinear mathematical model is given by:

$$\dot{x} = f(x, w, u) \tag{12}$$

where w is the wind velocity. If the model can be converted to the following form:

$$f(x, w, u) = A(x, w)x + B(x, w)u + \Delta f(x, w, u) \tag{13}$$

then, as previously mentioned, it will be possible to exclude $\Delta f(x, w, u)$ by considering the compensator u^c which satisfies the Eq. (14).

$$f(x, w, u^{sd} + u^c) = A(x, w)x + B(x, w)u^{sd}$$

$$u^{sd} = u^t + u^r \tag{14}$$

In Eq. (14), u^r is the control input obtained from the Riccati Eq. (10), and u^t is the trim or tracking control input. The compensator introduced here is shown in Figure (2).

Development of the helicopter SDC form

The state equations of the helicopter nonlinear dynamics can be written as follows:

$$\dot{x} = f_{rb}(x) + T_u(x, w, u) + T_d(x, w)$$

$$x = [x, v, w, p, q, r, \phi, \psi, x, y, z, a, b]^T \tag{15}$$

where the helicopter state vector, x, is defined by fourteen components and it is assumed that all of these state variables are measurable.

The main rotor thrust is numerically calculated from Eq. (2). Bearing in mind that most of the control forces are produced by the main rotor thrust, therefore, it is necessary to linearize the thrust equation about the main rotor collective input (u^0_{col}) in order to be used in conversion to the SDC form.

$$\begin{aligned} T_{mr} &= T(x, w, u^0_{col}) + \frac{\partial T_{mr}(x, w, u^0_{col})}{\partial u_{col}} (u_{col} - u^0_{col}) + \mathcal{O}^2 \\ &= \frac{\partial T_{mr}(x, w, u^0_{col})}{\partial u_{col}} u_{col} + \Delta T_{mr}(x, w, u^0_{col}) + \mathcal{O}^2 \end{aligned} \tag{16}$$

where in this equation:

$$\frac{\partial T_{mr}}{\partial u_{col}} = \frac{\alpha_{mr}\sigma}{4} \left[\frac{2}{3} + \mu^2 - \frac{C_T \alpha_{mr}\sigma\lambda_0 (2/3 + \mu^2)}{4C_T^2 + C_T\alpha_{mr}\sigma\lambda_0 - 16\lambda_0^3\eta_w^2(\mu_z - \lambda_0)} \right] \times \rho(\Omega R)^2 \pi R^2 \tag{17}$$

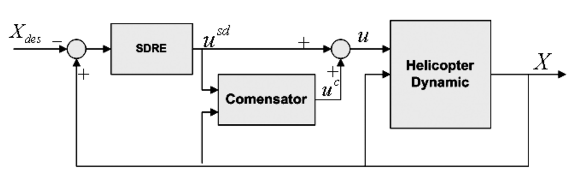

Figure 2. Controllers scheme.

Now, by replacing the equations associated with the main rotor thrust and other terms related to x and u into the Eq. (13), the following matrices are produced:

$$\mathbf{A}(x,u)=\begin{bmatrix}\mathbf{A}_1^{3\times3} & \mathbf{A}_2^{3\times3} & \mathbf{A}_3^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{A}_4^{3\times2}\\ \mathbf{A}_5^{3\times3} & \mathbf{A}_6^{3\times3} & \mathbf{0} & \mathbf{0} & \mathbf{A}_7^{3\times2}\\ \mathbf{0}^{3\times3} & \mathbf{\Psi} & \mathbf{0} & \mathbf{0} & \mathbf{0}^{3\times2}\\ \mathbf{R} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}^{3\times2}\\ \mathbf{A}_8^{2\times3} & \mathbf{A}_9^{2\times3} & \mathbf{0}^{2\times3} & \mathbf{0}^{2\times3} & \mathbf{A}_{10}^{2\times2}\end{bmatrix}$$

$$\mathbf{B}(x,w)=\begin{bmatrix}\mathbf{B}_1^{6\times4}\\ \mathbf{0}^{6\times4}\\ \mathbf{B}_2^{2\times4}\end{bmatrix}$$

$$\mathbf{A_1}=\begin{bmatrix}\frac{X^u_{fus}}{m} & r & -q\\ -r & \frac{\left(Y^v_{fus}+Y^v_{vf}+Y^v_{tr}\right)}{m} & p\\ q & -p & \frac{\left(Z^w_{fus}+Z^w_{ht}\right)}{m}\end{bmatrix}$$

$$\mathbf{A_2}=\begin{bmatrix}0 & 0 & 0\\ Y^p_{tr}/m & 0 & \left(Y^r_{tr}+Y^r_{vf}\right)/m\\ 0 & Z^q_{ht}/m & 0\end{bmatrix}$$

$$\mathbf{A_3}=\begin{bmatrix}0 & -g\frac{\sin\theta}{\theta} & 0\\ g\cos\theta\,\frac{\sin\phi}{\phi} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\mathbf{A_4}=\begin{bmatrix}-\triangle T_{mr} & 0\\ 0 & \triangle T_{mr}\\ 0 & 0\end{bmatrix}$$

$$\mathbf{A_5}=\begin{bmatrix}0 & \left(L^v_{vf}+L^v_{tr}\right)/I_{xx} & 0\\ 0 & 0 & M^w_{ht}/I_{yy}\\ 0 & \left(N^v_{vf}+N^v_{tr}\right)/I_{zz} & 0\end{bmatrix}$$

$$\mathbf{A_6}=\begin{bmatrix}\frac{L^p_{tr}}{I_{xx}} & \frac{I_{yy}-I_{zz}}{2I_{xx}}r & \frac{I_{yy}-I_{zz}}{2I_{xx}}q+\frac{L^r_{vf}+L^r_{tr}}{I_{xx}}\\ \frac{I_{zz}-I_{xx}}{2I_{yy}}r & \frac{M^q_{ht}}{I_{yy}} & \frac{I_{zz}-I_{xx}}{2I_{yy}}p\\ \frac{I_{xx}-I_{yy}}{2I_{zz}}q+\frac{N^p_{tr}}{I_{zz}} & \frac{I_{zz}-I_{xx}}{2I_{yy}}p & \frac{N^r_{tr}+N^r_{vf}}{I_{zz}}\end{bmatrix}$$

$$\mathbf{A_7}=\begin{bmatrix}0 & h_{mr}\triangle T_{mr}+K_\beta\\ h_{mr}\triangle T_{mr}+K_\beta & 0\\ 0 & 0\end{bmatrix}$$

$$\mathbf{A_8}=\begin{bmatrix}\frac{1}{T_eV_{tip}}\frac{\partial a}{\partial\mu_x} & 0 & \frac{1}{T_eV_{tip}}\frac{\partial a}{\partial\mu_z}\\ 0 & \frac{1}{T_eV_{tip}}\frac{\partial b}{\partial\mu_y} & 0\end{bmatrix}$$

$$\mathbf{A_9}=\begin{bmatrix}0 & -1 & 0\\ -1 & 0 & 0\end{bmatrix},\ \mathbf{A_{10}}=\begin{bmatrix}-1/T_e & 0\\ 0 & -1/T_e\end{bmatrix}$$

$$\mathbf{B_1}=\begin{bmatrix}-\frac{a}{m}\frac{\partial T_{mr}}{\partial u_{col}} & 0 & 0 & 0\\ \frac{b}{m}\frac{\partial T_{mr}}{\partial u_{col}} & 0 & 0 & \frac{Y^{u_tcol}_{tr}}{m}\\ -\frac{1}{m}\frac{\partial T_{mr}}{\partial u_{col}} & 0 & 0 & 0\\ \frac{b\ h_{mr}}{I_{xx}}\frac{\partial T_{mr}}{\partial u_{col}} & 0 & 0 & \frac{L^{u_tcol}_{tr}}{I_{xx}}\\ \frac{a\ h_{mr}}{I_{yy}}\frac{\partial T_{mr}}{\partial u_{col}} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{N^{u_tcol}_{tr}}{I_{zz}}\end{bmatrix}$$

$$\mathbf{B_2}=\begin{bmatrix}0 & \frac{A_{u_{lon}}}{T_e} & 0 & 0\\ 0 & 0 & \frac{B_{u_{lat}}}{T_e} & 0\end{bmatrix} \tag{18}$$

where, $\mathbf{R}(\phi,\theta,\psi)$ is the body to inertial coordinate transformation matrix and $\boldsymbol{\psi}(\phi,\theta,\psi)$ is the angular velocity to the Euler angles derivatives transformation matrix.

Having determined the state and input matrices $(A(x,w),B(x,w))$, the Riccati Eq. (9) can be solved. There are two groups of approaches to solving this equation, direct and iterative. Schur-decomposition is a direct approach and Kleinman, Quasi-Newton or Newton-Kleinman methods are iterative approaches (Refs. [25,26,27,28]). However, by a suitable first initial guess, the Kleinman method will need less computational time than the other approaches, Ref. [29]. Here, the last computed dynamic costate value of the Riccati equation, P, is considered as the best initial guess for each iteration period. This comes from the continuity property of the solution, Ref. [18]. Having determined the value of P by solving the Riccati equation, the control input can be computed using Eq. (10).

Compensator design

By substitution of Eq. (15) into Eq. (14), the following is achieved:

$$f_{rb}(x)+T_u\left(x,w,u^{sd}+u^c\right)+T_d(x,w)= A(x,w)\,x+B(x,w)\,u^{sd} \tag{19}$$

Collecting all the independent terms of u^c, the vector $D(x,w,u^{sd})$ can be defined as:

$$D\left(x,w,u^{sd}\right)=A(x,w)\,x+B(x,w)\,u^{sd}-f_{rb}(x)-T_d(x,w) \tag{20}$$

Therefore, this equation can be written as:

$$T_u\left(x,w,u^{sd}+u^c\right)=\ D\left(x,w,u^{sd}\right) \tag{21}$$

where the flapping components in $f_{rb}(x)$ and $T_d(x,w)$ are defined as:

$$\begin{bmatrix}f_{rb13}\\ f_{rb14}\end{bmatrix}=\begin{bmatrix}-q-\frac{a}{T_e}\\ -p-\frac{b}{T_e}\end{bmatrix}$$

$$\begin{bmatrix}T_{d13}\\ T_{d14}\end{bmatrix}=\begin{bmatrix}\frac{\partial a}{\partial\mu_x}\frac{u-u_w}{V_{tip}}+\frac{\partial a}{\partial\mu_z}\frac{w-w_w}{V_{tip}}\\ \frac{\partial b}{\partial\mu_y}\frac{v-v_w}{V_{tip}}\end{bmatrix} \tag{22}$$

If the operator $T_u\left(x,w,u^{sd}+u^c\right)$ is control-invertible, then the compensator output can be computed as $u^c=T_u^{-1}(x,w,D)-u^{sd}$.

$$T_u\left(x,w,u^{sd}+u^c\right)=\left[\frac{X_{mr}}{m}\ \frac{Y_{mr}+Y_{tr}}{m}\ \frac{Z_{mr}}{m}\ \frac{L_{mr}+L_{tr}}{I_{xx}}\ \frac{M_{mr}}{I_{yy}}\ \frac{N_{tr}-Q_e}{I_{zz}}\ 0^{1\times6}\ \frac{A_{u_{lon}}}{T_e}u_{lon}\ \frac{B_{u_{lat}}}{T_e}u_{lat}\right]^T \tag{23}$$

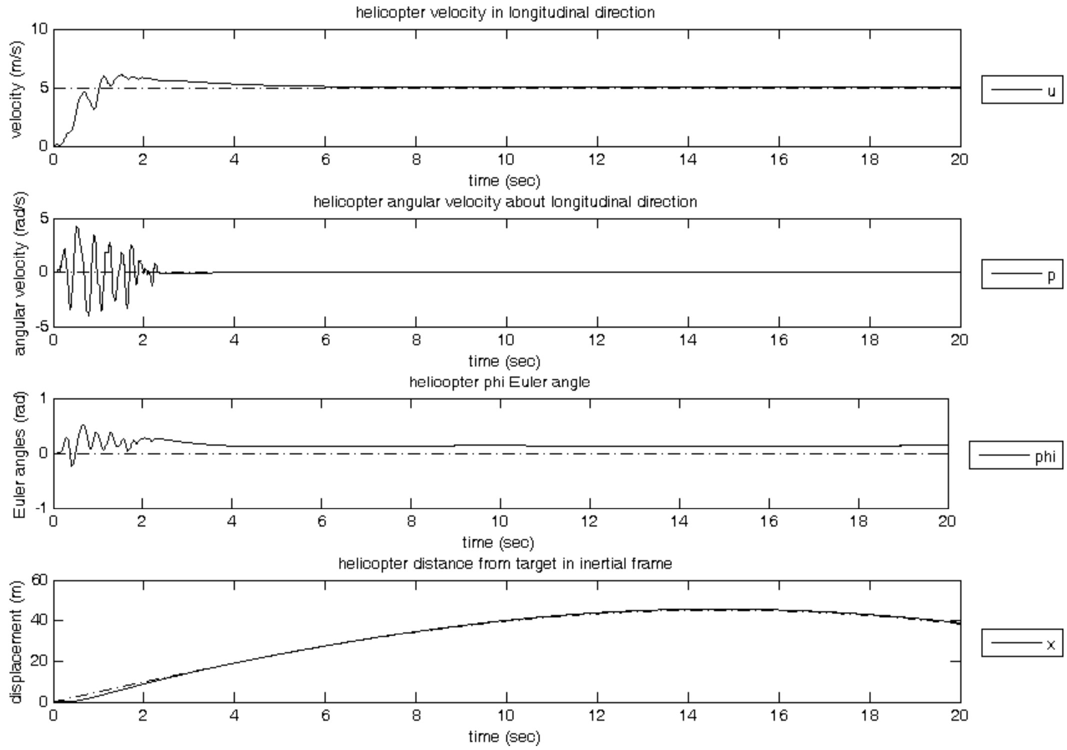

Figure 3. Longitudinal translational and angular velocity, ϕ and distance in X direction of inertial coordinates in the first simulation.

Similarly the inputs u_{lon}^c and u_{lat}^c can be calculated from components 13 and 14 of the vector T_u given in Eq. (23) respectively. However, there are six equations available for determination of u_{col}^c and u_{tcol}^c, from which the sensitive mode equation is preferred, the translational motion in the body Z direction is chosen for u_{tcol}^c and the angular motion about the body Z direction is selected for u_{tcol}^c:

$$u_{col}^c = T_{mr}^{-1}\left(x, w, mD\left(3\right)\right) - u_{col}^{sd}$$

$$u_{tcol}^c = T_{mr}^{-1}\left(x, w, \frac{I_{zz}D\left(6\right) + Q_e\left(x, w, u_{col}\right)}{I_{tr}}\right) - u_{tcol}^{sd}$$

$$u_{lon}^c = \frac{D\left(13\right)T_e}{A_{u_{lon}}} - u_{lon}^{sd}$$

$$u_{lat}^c = \frac{D\left(14\right)T_e}{B_{u_{lat}}} - u_{lat}^{sd} \tag{24}$$

By calculating u^c, the cycle in Figure (2) is completed.

SIMULATION

Two scenarios are considered to show the controller performance. In the first scenario, the landing of helicopter on a ship platform with only three translational motions is studied. In the second scenario, an angular directional motion is added. The study has been carried out using a helicopter/platform computer simulation program developed based on the presented mathematical formulation.

First scenario: In this scenario, the platform is considered to have translational velocities of 5 m/s in longitudinal and 1 m/s in lateral direction. At time $t_0 = 0$, the helicopter hovers in 5 meters height above the ship. The sinusoidal motion in Z direction with periodic velocity $w = \sin\frac{2\pi}{T}t$ is also considered, where $T = 10\,\mathrm{s}$. Running the simulation, the helicopter starts to track the shipboard deck.

The simulation results are shown in Figures (3-5). As can be seen, the helicopter tracks the platform very well after seven seconds. Landing is performed as soon as the position misalignment gets zero which takes place after seven seconds. The steady error observed in the bank angle, ϕ, (Figure (3), third graph), is a result of the pendulum motion of the helicopter body relative to the thrust rotor. This property causes the bank angle to return to its trim value when the control efforts are ended.

Second scenario: In this scenario, the ship is considered to have a 5 m/s longitudinal velocity and at the same time rotate with 0.1 rad/s yaw rate and have a 0.5 m/s side slip velocity. A sinusoidal vertical motion $w = \sin\frac{2\pi}{T}t$, where $T = 10\,\mathrm{s}$ is also considered for the ship. At $t = 0\,\mathrm{s}$, the helicopter is hovering in 5 meters height above the ship. The results of this simulation are given in Figures (6-8). In these graphs, the helicopter and platform motions are shown by solid and

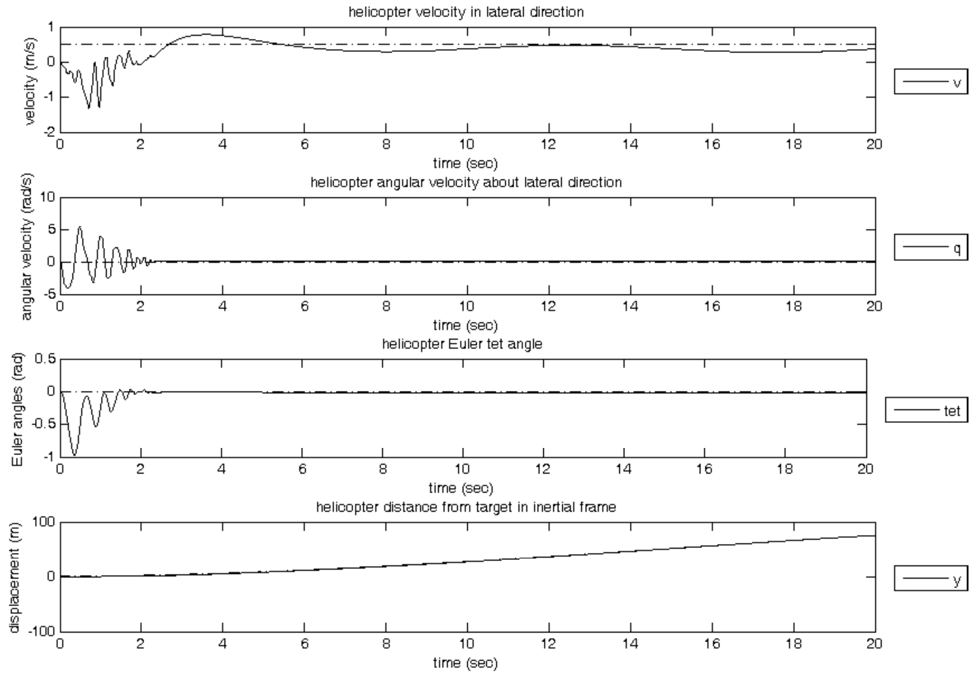

Figure 4. Lateral translational and angular velocity, θ and distance in Y direction of inertial coordinates in the first simulation.

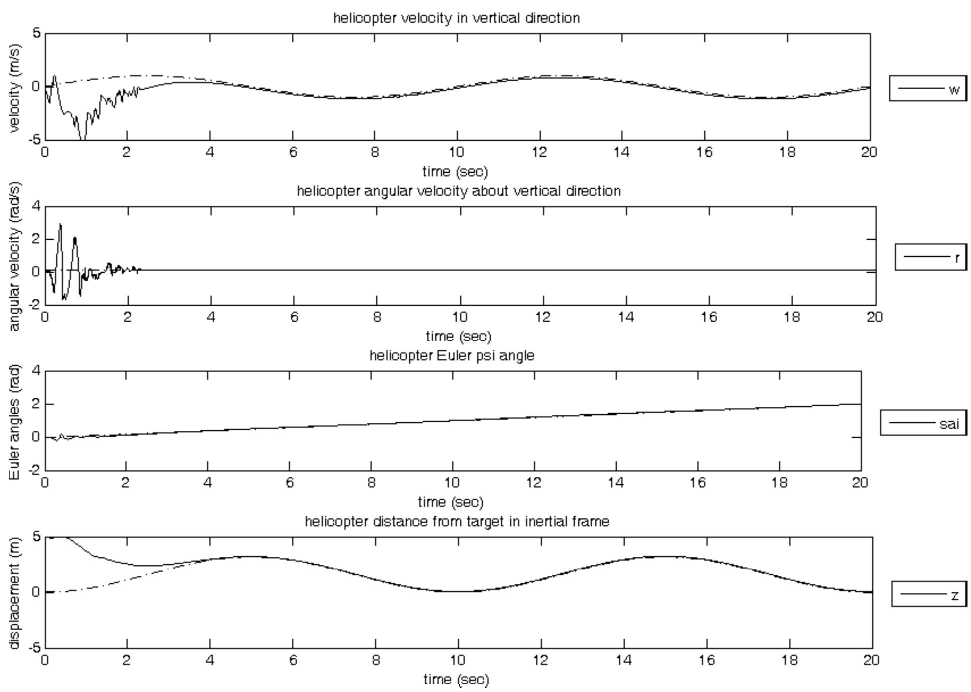

Figure 5. Vertical translational and angular velocity, ψ and distance in Z direction of inertial coordinates in the first simulation.

dashed lines respectively. The simulation shows that the helicopter tracks both directional and translational motion of the platform quite satisfactorily.

CONCLUSION

For automatic landing, a helicopter must be able to track the translational and rotational motion of the target platform accurately. In marine vehicles the main dynamic modes include three translational and one directional motion. This is due to the fact that in most conditions the vehicle situation can be determined by these four modes. In addition, small helicopters have four controls including two cyclic and two collective pitch inputs. Therefore, the desired SDRE controller for automatic landing is designed based on the four

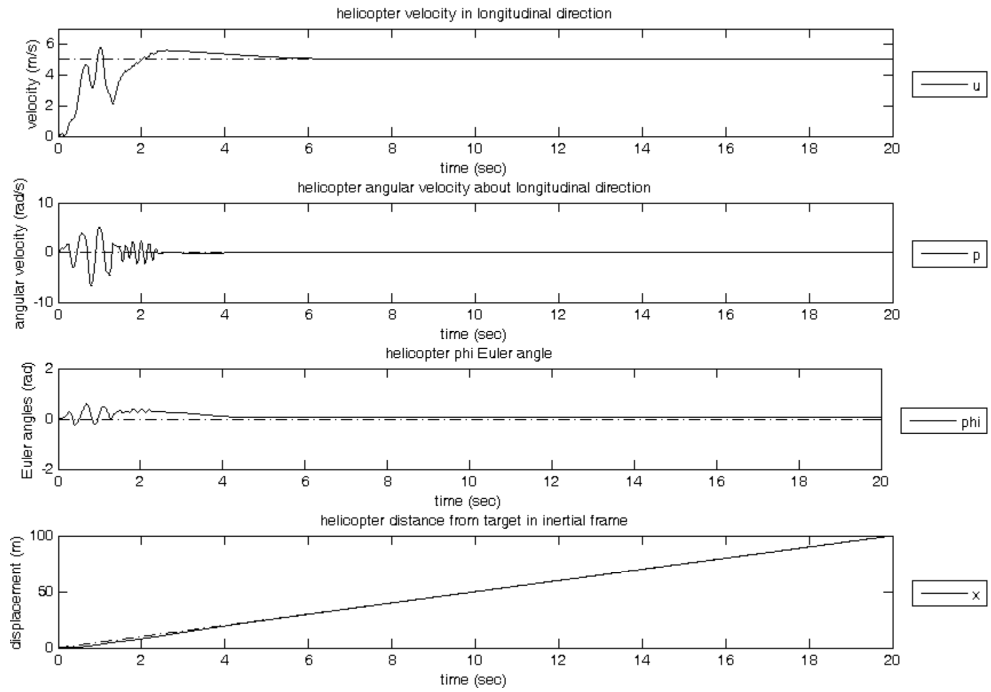

Figure 6. Longitudinal translational and angular velocity, ϕ and distance in X direction of inertial coordinates in the second simulation.

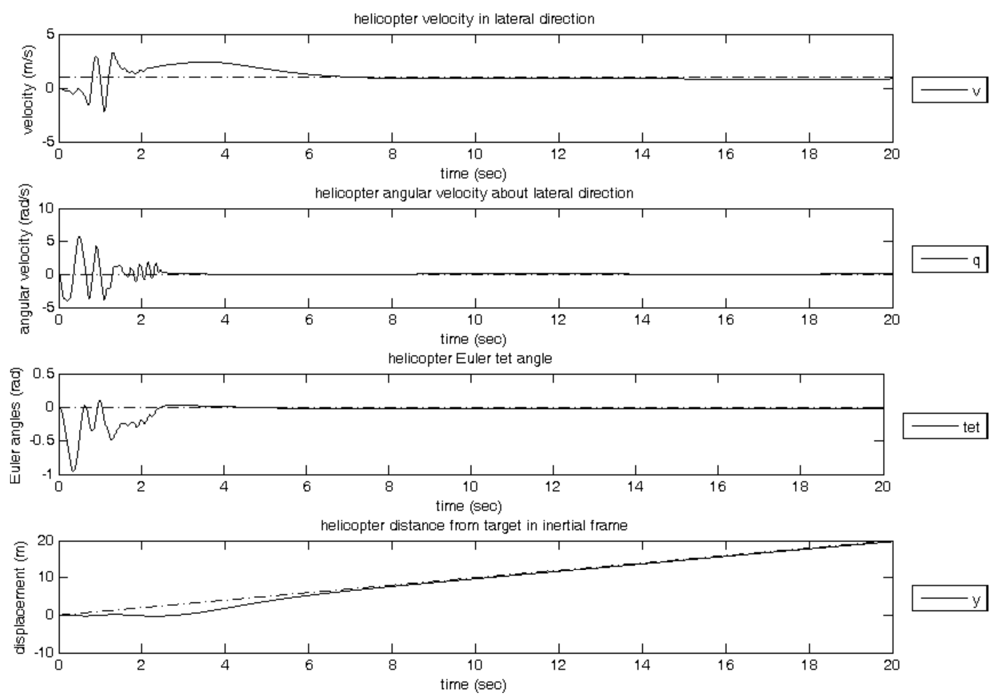

Figure 7. Lateral translational and angular velocity, θ and distance in Y direction of inertial coordinates in the second simulation.

degrees of freedom assumption and its associated four input controllers. The simulation results show that the helicopter is able to track the ship position, velocity and directional angle and damp other angle deviations and angular rates. This confirms the satisfactory performance of the designed controller.

Using the position and angle feedbacks in controller design, the steady errors of the angular rates and velocities approach zero. This is mainly because the mentioned feedbacks play the roles of integral feedbacks. On the other hand, in positions and angles tracking, the steady errors are not exactly zero because there aren't any integral feedbacks for these states. However, it can be made desirably small by setting

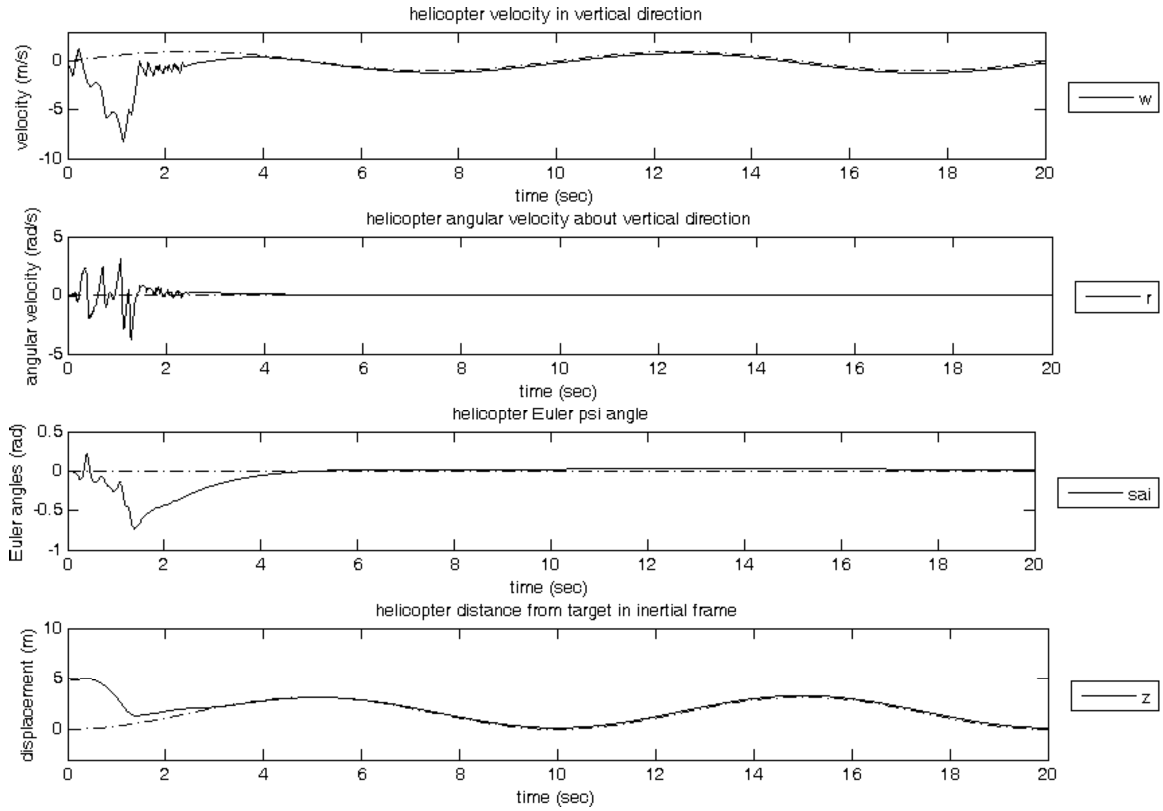

Figure 8. Vertical translational and angular velocity, ψ and distance in Z direction of inertial coordinates in the second simulation.

an objective function states gain matrix (Eq. (7)). Absence of integral feedbacks in positions and angles tracking results in overshoot reducing, which is in fact one of the most important requirements for automatic landing especially in vertical direction. This is obvious from graphs given in Figures (3-8), which show the helicopter displacements from target in different axes. This fact is more observable in Figures (5) and (8) in which the vertical displacement of the helicopter from the platform is shown. As it is shown, the settling times are about 7 seconds in both scenarios. Short settling time and low overshoot make this method suitable for the helicopter automatic landing.

In second and third graphs of Figures (3,4) and (6,7), the angular behavior of helicopter in longitudinal and lateral directions is an indication of oscillatory motion which is damp and made stable by the designed controller. As shown in the figures, the maximum roll angle amplitude is about 30 degrees.

In this work, all the equations are solved analytically except the Riccati equation. The number of the iterations needed to solve the Riccati equation by Kleinman method is less than five. This is a very good agreement with the requirement of the real time implementation of the designed controller.

After converting the complicated models into the simple SDC form, application of this method is straightforward. Also using a compensator prevents decreasing the controller performance in different conditions. All of these have made it a suitable method of controller design for complicated systems like helicopters.

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