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Dynamic Stability of Step Beam Carrying Concentrated Masses under Follower Force

M. Sohrabian[1], H. Ahmadian[2]

This paper investigates the dynamical behavior of an assembled beam like structure with tip masses modeling a two stage space rocket structure. The beam like structure is composed of two beams connected together carrying two masses at the free ends to model the payload and the stabilizing fins mass properties. The effect of non-homogeneity of the structure due to different cross section properties at each stage of the rocket and the tip masses strongly affects the dynamical behavior and stability regions of the structure if excited using a follower force. In this study the rocket thrust is considered as follower force as in flexible space structures the thrust direction is affected by the lateral vibrations of the structure. As demonstrated in this study the primary mode of instability in such a structure, i.e. divergence or flutter, depends on the stiffness and mass distributions of the structure. This would enable the designer to alter the mode of instability by modifying these distributions. In inspecting the instability regions of the structure under follower force, and establishing an analytical solution, the Galerkin method is employed and the stability regions of the structure are determined.

INTRODUCTION

In general, forces acting on a structure can be grouped into conservative and non-conservative. The follower force is a typical example of non-conservative force. When a structure is excited under a constant follower force with direction changes according to the deformation of the structure, it may undergo static instability (divergence) whereby transverse natural frequencies merge into zero, or dynamic instability (flutter), where two natural frequencies coincide with each other resulting in the amplitude of vibration growing without bound.

The flexible structure of a rocket deforms under applied engine thrust and consequently the trust direction changes. This phenomenon is modelled using a non-conservative follower force which changes its direction according to the deformation of a space structure. Free-Free beams have been intensively exploited to simulate the stability behaviour of flexible space structures [1-9]. Beal [1] deals with a uniform beam under constant and pulsating thrust including a simplified control system in the model. Park and Mote [2] have studied a similar problem with emphasis on the effects of the location and the inertia of a concentrated mass on the dynamic behaviour of a uniform beam. They also investigated other parameters such as the location of the follower force direction control sensor, the sensor gain and the maximum thrust magnitude allowable for stable planner motion. Wu [3, 4] studied the maximum controlled follower force on a free-free beam carrying a concentrated mass. Parks [5] is concerned with stability of a uniform free-free Timoshenko beam excited by a constant follower force with controlled direction, and discussed the effects of changes in the location of sensor, the sensor gain, and the magnitudes of the rotary inertia and shear deformation parameters of the beam. Kim and Choo [6] have studied dynamic stability of a free-free Timoshenko beam with a concentrated mass under a pulsating follower force and considered the effects of axial location and translation inertia of the concentrated mass. Sohrabian [7] has studied the eigenproperties of a rocket model under conservative force using several methods. Ahmadian and Sohrabian [8] have studied the eigen-values and stability margins of

1. *MSc., Dept. of Mech. Eng., Iran Univ. of Sci. and Tech., Tehran, Iran, Email: Sohrabian@mecheng.iust.ac.ir.*
2. *Professor, Dept. of Mech. Eng., Iran Univ. of Science and Tech., Tehran, Iran.*

a piece-wise uniform free-free beam under follower non-conservative force.

The current paper is concerned with the stability of a stepped beam with concentrated masses at both ends. The added masses and step changes in beam characteristics have a significant influence on the dynamical behaviour of the structure and alter the stability margins of the structure from those obtained by an analysis which had neglected these effects. The assembled structure consists of two different beams carrying two concentrated masses. The following presents the governing equations, the compatibility equations, and boundary conditions required to determine the dynamic behaviour of the structure. The exact solution for the developed governing equations is not available as the coefficients of these equations vary along the structure axis. An approximate solution for the set of developed model is presented using the Galerkin method. The solution is obtained using a linear combination of trial functions where weight of each function in the solution is specified by minimizing the residue of the governing equations. A parametric stability analysis study is provided using the obtained solution and the results are discussed in this paper.

STATEMENT OF THE PROBLEM

The typical beam like space structure whose dynamic response is investigated in this paper is shown in Figure 1. The structure consists of two jointed beams carrying two concentrated masses at the free ends and excited at one end using a time invariant follower force. The force direction is normal to the end of the second beam cross section.

One global x-y axis at the tip of the first beam as shown in Figure 1 and two local axes at the beginning of each beam are defined ($x_1 - y_1$ and $x_2 - y_2$). The Euler-Bernoulli beam theory is employed to model the two beams with the length of L_i, flexural rigidity of EI_i, linear mass density of ρA_i and mass and moment of inertia of M_i and $I'i$ for the concentrated masses at the end of each beam, where index i refers to the section number. The equations of motion for the beams according to Euler-Bernoulli beam theory are:

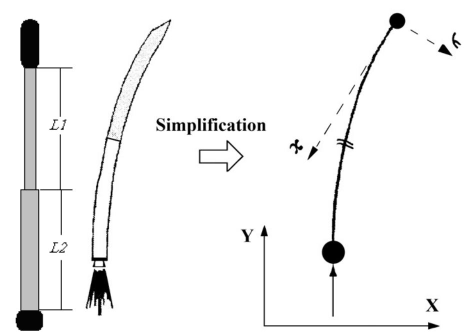


Figure 1. The compound beam like structures.

$$EI_1\frac{\partial^4 w_1}{\partial x_1^4}+\frac{\partial}{\partial x_1}\left(p_1(x)\frac{\partial w_1}{\partial x_1}\right)+\rho A_1\frac{\partial^2 w_1}{\partial t^2}=0, \quad 0<x<L_1$$

$$EI_2\frac{\partial^4 w_2}{\partial x_2^4}+\frac{\partial}{\partial x_2}\left(p_2(x_2)\frac{\partial w_2}{\partial x_2}\right)+\rho A_2\frac{\partial^2 w_2}{\partial t^2}=0, \quad L_1<x<L \qquad (1)$$

In the governing equations the axial force distributions along the structure, $p_i(x)$, $i=1, 2$ are defined as follows:

$$p_1(x)=\frac{T_0\times(M_1+\rho A_1 x)}{\rho A_1 L_1+\rho A_2 L_2+M_1+M_2}, \quad 0\leqslant x\leqslant L_1$$

$$p_2(x)=\frac{T_0\times[M_1+\rho A_1 L_1+\rho A_2(x-L_1)]}{\rho A_1 L_1+\rho A_2 L_2+M_1+M_2}, \quad L_1\leqslant x\leqslant L. \qquad (2)$$

where M_1 and M_2 are the end masses. In order to solve the set of governing equations defined in Eq. (1), one needs to specify the boundary conditions of the structure and its compatibility requirements. The boundary conditions are:

at $x=0$:

$$\begin{cases} EI_1\frac{\partial^2 w_1}{\partial x_1^2}=J_1\ddot{\theta}_1=J_1\frac{\partial^3 w_1}{\partial t^2\partial x_1}=-\omega^2 J_1\frac{\partial w_1}{\partial x_1} \\ EI_1\frac{\partial^3 w_1}{\partial x_1^3}=-M_1\frac{\partial^2 w_1}{\partial t^2}=M_1\omega^2 w \end{cases} \qquad (3a)$$

at $x=L_1+L_2$:

$$\begin{cases} EI_2\frac{\partial^2 w_2}{\partial x_2^2}=J_2\,\ddot{\theta}_2=\;J_2\,\frac{\partial^3 w_2}{\partial t^2\partial x_2}=\omega^2 J_2\,.\,\frac{\partial w_2}{\partial x_2} \\ EI_2\frac{\partial^3 w_2}{\partial x_2^3}=-M_2\frac{\partial^2 w_2}{\partial t^2}=-M_2\,\omega^2\,w_2 \end{cases} \qquad (3b)$$

where J_1 and J_2 are moments of inertia of the end masses.

The compatibility equations of the structure at the interface of the two beams are:

at $x=L_1$:

$$\begin{cases} w_1\,|_{x_1=L_1}=w_2\,|_{x_2=0} \\ \left.\frac{\partial w_1}{\partial x_1}\right|_{x_1=L_1}=\left.\frac{\partial w_2}{\partial x_2}\right|_{x_2=0} \\ EI_1\frac{\partial^2 w_1}{\partial x_1^2}\,|_{x_1=L_1}=EI_2\frac{\partial^2 w_2}{\partial x_2^2}\,|_{x_2=0} \\ EI_1\frac{\partial^3 w_1}{\partial x_1^3}\,|_{x_1=L_1}=EI_2\frac{\partial^3 w_2}{\partial x_2^3}\,|_{x_2=0} \end{cases} \qquad (4)$$

This leads to four compatibility requirements. As mentioned earlier in the paper one can not obtain an exact solution for the set of partial differential equations defined in (1) that could satisfy the eight compatibility requirements and boundary conditions. An approximate solution for the defined problem is presented in the following section.

APPROXIMATE SOLUTION USING GALERKIN METHOD

The set of partial differential equations defined in Eq. (1) have an exact closed form solution when the follower force is set to zero. The modes of the structure with no force excitations ψ_i, $i = 1, 2, ...$ can be used to form the trial functions in reconstructing the modes of the structure when it is excited under a follower force excitations:

$$\phi^{(n)}(x) = \sum_{j=1}^{n} a_j\, \psi_j(x) \tag{5}$$

Inserting the assumed solution (5) into the partial differential Eq. (1), one obtains the following vector of residues:

$$R\left(\phi^{(n)}(x), x\right) = \sum_{j=1}^{n} a_j \left[\frac{d^4\psi_j}{dx^4} + \frac{1}{EI}\frac{\partial}{\partial x_1}(p(x)\frac{d\psi_j}{dx}) - \bar{\lambda}^4\psi_j\right] \tag{6}$$

where $\bar{\lambda}^4 = \frac{\rho A\omega^2}{EI}$. In Eq. (6) the actual eigen-value λ is replaced by the approximate estimate of it namely

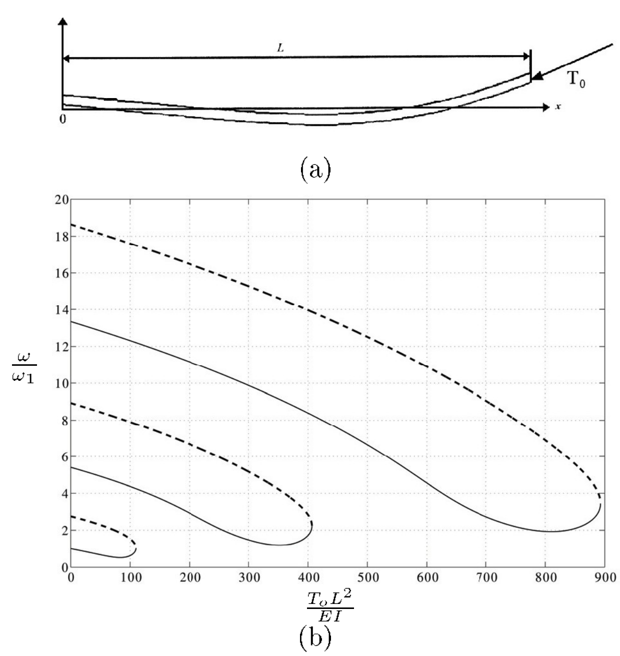


Figure 2. (a) The deformed uniform beam under follower force, (b) Changes in natural frequencies at different loading levels for a uniform beam.

$\bar{\lambda}$. The residue can be defined in terms of excitation follower force as:

$$R(\phi^{(n)}(x), x) = \begin{cases} \sum_{j=1}^{n} a_j \left\{\frac{d^4\psi_j}{dx_1^4} + \frac{1}{EI_1}\frac{\partial}{\partial x_1}(p_1(x)\frac{d\psi_j}{dx_1}) - \bar{\lambda}_1\psi_j\right\} \\ \qquad 0 < x < L_1 \\ \left\{\frac{d^4\psi_j}{dx_2^4} + \frac{1}{EI_2}\frac{d}{dx_2}\left[p_2(x)\frac{d\psi_j}{dx_2}\right] - \bar{\lambda}_2\psi_j\right\} \\ \qquad L_1 < x < L_1 + L_2 \end{cases}$$

Next, one may minimize the residue $R(\phi^{(n)}(x), x)$ by projecting it into the trial functions ψ_j, $j = 1, 2, \ldots, n$ over the considered domain and setting the results to zero.

$$\int_0^{L_2} \psi_i\, R(\phi^{(n)}(x), x)dx = 0 \qquad i = 1, 2, .., n$$

This results in the coefficients a_j, $j = 1, 2, \ldots, n$ as follows:

$$\sum_{j=1}^{n} a_j \left\{\int_0^{L_1} \psi_i(x)\left[\frac{d^4\psi_j}{dx^4} + \frac{1}{EI_1}\frac{\partial}{\partial x}\left(p_1(x)\frac{d\psi_j}{dx}\right) - \bar{\lambda}_1^4\,\psi_j\right] dx_1 + \int_{L_1}^{L} \psi_i(x)\left[\frac{d^4\psi_j}{dx_2^4} + \frac{1}{EI_2}\frac{\partial}{\partial x_2}\left[p_2(x)\frac{d\psi_j}{dx_2}\right] - \bar{\lambda}_2^4\,\psi_j\right] dx_2\right\} = 0 \tag{7}$$

Simplifying Eq. (7), one obtains:

$$\{[\, k_{ij} + k'_{ij}\,] + [g_{ij} + g'_{ij}] + \bar{\lambda}^4(m_{ij} + m'_{ij})\}\,\{a\} = 0. \tag{8}$$

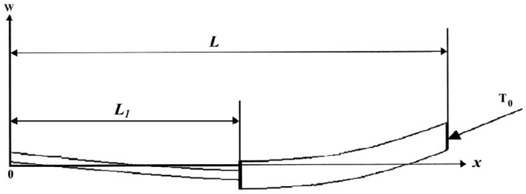
The expressions used in Eq. (8) are as follows:

$$\begin{cases} k_{ij} = \int_0^{L_1} \psi_i \frac{d^4\psi_j}{dx^4} dx_1 = \lambda_j^4 \int_0^{L_1} \psi_i\psi_j dx_1 \\ m_{ij} = \int_0^{L_1} \psi_i\psi_j dx_1 \\ g_{ij} = \left(\psi_i \left.\frac{d\psi_j}{dx_1}\right|_{x_1=L_1} - \int_0^{L_1} \frac{d\psi_i}{dx_1}\frac{d\psi_j}{dx_1} dx_1\right)\frac{p(L_1)}{EI_1} \end{cases}$$

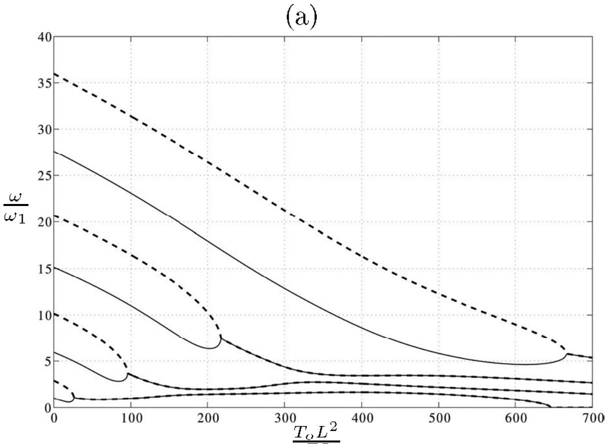
$$\begin{cases} k'_{ij} = \int_{L_1}^{L} \psi_i \frac{d^4\psi_j}{dx_2^4} dx_2 \\ m'_{ij} = \int_{L_1}^{L} \psi_i\,\psi_j\,dx_2 \\ g'_{ij} = \frac{1}{EI_2}\left[\left(\psi_i|_{L_2} .T \left.\frac{d\psi_j}{dx_2}\right|_L - \psi_i|_0 .p(L_1) \left.\frac{d\psi_j}{dx_2}\right|_{L_1}\right) \right. \\ \qquad \left. - \int_{L_1}^{L} \left[p(x).\frac{\partial\psi_j}{\partial x_2}\right]\frac{\partial\psi_i}{\partial x_2} dx_2\right] \end{cases} \tag{9}$$

The obtained eigen-values from Eq. (8) are approximates of the eigen-values of the original problem.

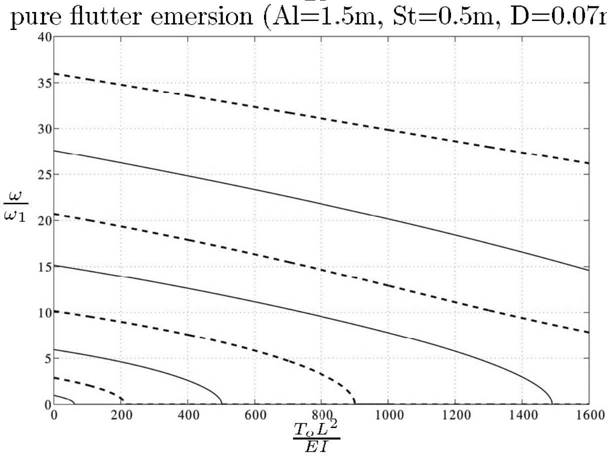
The following section presents numerical case studies. In these studies the stability of the uniform and piece-wise uniform beam-like structure with and without concentrated masses is investigated and their effects on the instability modes are generated.



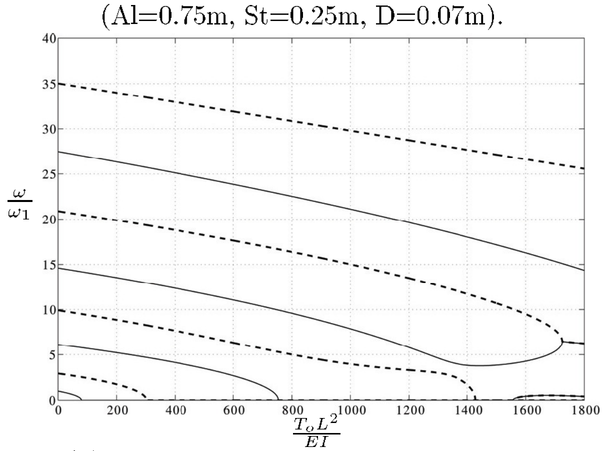
(a)



(b) pure flutter emersion (Al=1.5m, St=0.5m, D=0.07m).



(c) pure divergence emersion (Al=0.75m, St=0.25m, D=0.07m).



(d) both flutter and divergence emersion (Al=1m, St=0.5m, D=0.07m).

Figure 3. (a) The deformed piece-wise beam, (b), (c) and (d) Instability regions of piece-wise beam under follower force caused by non-homogeneity property of the beam.

NUMERICAL CASE STUDY

In the numerical case study initially the stability of a uniform beam with no attached masses under follower force is considered. The same problem is considered in Ref [1] and the obtained results can be validated using Ref [1] monographs.

Moreover the same properties for a piece-wise beam are assumed and mass and moment of inertia of the concentrated masses are set to zero. This assumption will change the model to a uniform one (Ref [1] case) which can utilize the one to validate the current model. The stability analysis is performed by increasing the follower force from zero to a large value. The results, demonstrated in Figure (2-b), show that flutter occurs by merging the first and second modes as demonstrated in Ref [1]. It should be noticed that in the current graphs, the behavior of frequency changes, should be discussed only up to the instability points whether flutter or divergence point.

Next, for the aim of inspecting the non-homogeneity effects on the instability regions, the stability problem is investigated for the same beam when two beams have different cross section properties but no masses are attached to the structure. This provides the effects of non-homogeneity on the structural stability properties.

The chosen step beam is made of an aluminium-steel beam with various lengths. Both beams have the same cross sections of diameter 0.07m. As is shown, changing non-homogeneity parameters will alter the instability modes. The natural frequencies of the structure under follower force are obtained using the Galerkin weighted residual method as explained in the previous section. The results are shown in Figures (3-b) to (3-d) for different amplitudes of follower forces.

Figure (3-b) describes the instability plots for a step beam with aluminum length of 1.5m, steel length of 0.5m and the same diameter of 0.07m for both beams. For this non-homogeneity parameter values, instabilities occurs in pure flutter modes like the previous uniform beam.

Figure (3-c) shows the instability plots for a step beam with aluminum length of 0.75m, steel length of 0.25m, and the same diameter for both beams. The instability regions for this example of non-homogenous beam under follower force appear like the beam under constant directional force (dealt by Ref [7]) as pure divergence. In some examples of non-homogeneous beam (like the current one) no flutter occurs for the structure under the non-conservative follower force.

Figure (3-d) describes the instability plots for a step beam with aluminum length of 1m, steel length of 0.5m and the same diameter for both beams. As it can be seen on this example of non uniform beam, the first four modes of instability in the piece-wise beam are divergence. Flutter occurs in the structure when the fifth and sixth modes merge at high amplitudes of follower force.

The instability plot for this example shows occurrence of flutter and divergence simultaneously.

As it can be noticed, the instability regions and critical points are altered from the case where the uniform beam was the case of study. Non-homogeneity can make the instability modes completely different from a uniform beam under follower force.

As one realizes, the results are completely different from the pervious homogenous case, and the non-homogeneity severely affects the dynamical behaviour of the system by changing the instability regions or even the instability modes. Unlike the uniform beam, appearance of pure flutter is not the only prediction for non-homogenous beam instability.

As the third case, the stability is investigated for the same beam but made of only steel with two masses attached to the free ends (to realize the concentrated masses effects on the structural stability). The concentrated masses are assumed to be equal to

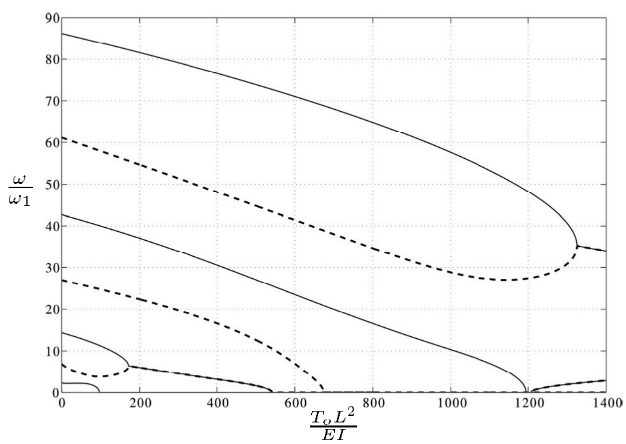


Figure 4. Changes in natural frequencies of a uniform beam carrying concentrated masses under follower force.

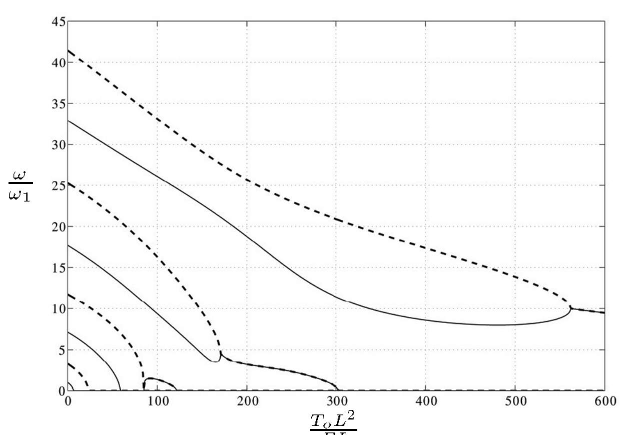


Figure 5. Instability plot of a two stage (St-Al) beam under follower force.

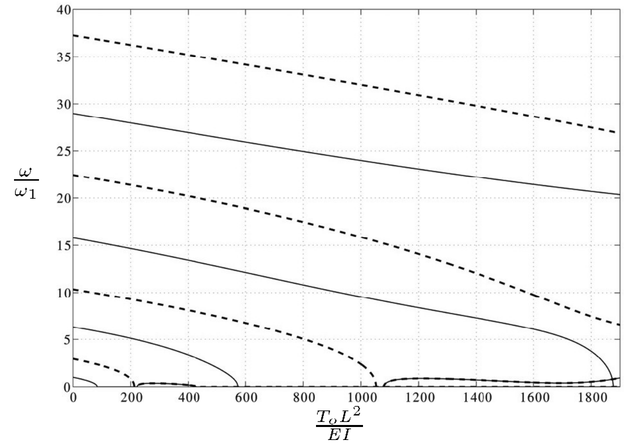


Figure 6. Instability plot of a two stage (Al-St) beam under follower force.

$m = 5$ kg and $J = 3$ kg.m^2 at both sides. The same procedure as in the previous case studies is performed using Galerkin weighted residual method to obtain the natural frequencies of the structure under follower force. Again as can be seen in Figure 4, dynamic instabilities for the structure under different levels of follower force are not purely flutter (like the uniform beam), but presence of divergence on the inspected range. Similar to the latter non-homogenous case, the instability modes occur as flutter, divergence or both of them simultaneously (pure flutter and pure divergence appearance are not brought here). It is observed that presence of concentrated masses, like material non-homogeneity, severely affects the dynamical behaviour of the system.

In the last case study a step beam with two sections and concentrated masses is considered. This free-free beam under follower force can model a rocket under its thrust. The first section is made of steel with the length of 0.5m and the second section is aluminium of length 1m, and the concentrated masses are similar with the properties of $m = 5$ kg, $J = 0.003$ kg.m^2. The same procedure is preceded and dynamic instability graphs for the system under follower force are shown in Figure 5. The plot says both divergence and flutter instabilities are observed for this example.

As another example, lets swap the material properties of each of the sections of the latter model. Therefore, the first section is made of aluminium with the length of 0.5m and the second section is steel of length 1m; the other parameters are identical to the latter example. Figure 6 describes the instability regions of the current case. For the taken bound, it can be seen that the structure face to only the divergence instabilities.

So, non-homogeneity and concentrated masses have profound effects on the dynamical behaviour of the structure as well as its level of critical force, and change the instability modes of the structure from flutter to divergence and vice versa.

CONCLUSION

This paper investigates the dynamical behaviour of a free-free step-beam structure carrying concentrated masses under constant follower force. The problem leads to a set of partial differential equations with variable coefficients. As the exact solution for the problem is not available, an approximate solution is presented using Galerkin weighted residual method. The selected trial functions were chosen as the shape functions of the same system when the level of follower force is set to zero. It is demonstrated that the non-homogeneity and concentrated masses have a profound effect on the dynamical behaviour of the system. Using numerical examples, it is shown that the added masses and heterogeneity of the beam change the instability mode of the structure from flutter to divergence. The level of critical force also changes significantly.

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