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Three Dimensional Stability Analysis of a Liquid Propellant Combustor

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A theoretical study analyzing three-dimensional combustion acoustic instabilities in a liquid propellant rocket engine combustor has been conducted. A linear theory based on Crocco's pressure sensitive time lag model is used. To apply this theory, the combustor is divided into two main components, including the combustion chamber and the converging part of the nozzle. The assumption of concentrated combustion zone is used and the governing perturbation equations describing oscillations of flow variables are considered. To solve these equations appropriate boundary conditions at both ends of the combustion chamber are required. Combustion zone boundary condition at one end and the nozzle admittance relation at other end are used. To obtain the nozzle admittance the three dimensional flow perturbation equations are solved in the converging part of the nozzle. This approach is capable of predicting acoustic stability behavior of a combustor at a wide range of Mach numbers and frequencies. Also, this analysis enables the rocket engine designer to observe the effects of different parameters such as nozzle entrance Mach number, chamber geometry, nozzle geometry, and gas properties on stability characteristics of an engine combustor. In case of instability observation; one can predict the acoustic mode which causes the instability and achieve an optimum design before conducting any expensive and time consuming experimental tests. This paper presents the stability analysis results and a parametric study of the effect of design parameters on stability characteristics of a typical combustor.

NOMENCLATURE

J_v	Bessel function of the first kind
n	Interaction index
p	Gas pressure
$\bar{q}$	Steady state gas velocity in the axial direction
$S_{(\nu,h)}$	Eigenvalue for a particular mode of oscillation
u	Gas velocity
Φ	Axial velocity perturbation
ϕ	Steady state potential function
ψ	Steady state stream function
γ	Specific heat ratio
μ	Auxiliary function is defined in eq. 5
θ	Variation in the azimuthal direction
ρ	Gas density
τ	Sensitive time lag
ω	Frequency

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INTRODUCTION

Understanding and predicting high frequency combustion instabilities in liquid propellant rocket engines continues to pose significant challenge due to the highly complex and nonlinear nature of turbulent combustion process. This type of instability is considered to be the most destructive, and is usually characterized by well defined frequencies and mode shapes corresponding to the acoustic modes of the chamber. Traditional strategies used to eliminate combustion instability were to increase the damping of the system and/or reduce coupling between unsteady combustion response and periodic flow oscillations. While often effective, these methods usually suffer deficiencies associated with a lack of knowledge concerning fundamental mechanisms and the coupling dynamics leading to combustion instabilities. Difficulties in assessing the impact of various processes arises from the presence of many diverse phenomena such as the hydrodynamics of injection, spray formation processes, transport characteristics of individual droplets, turbulent multiphase flow conditions, and complex chemical reactions in a turbulent environment. Multiple strongly coupled processes with a wide disparity in time and length scales exist in close proximity to one another. Although advances in the field have been made, the largest and most reliable source of information applicable to the design of improved combustion devices to date is the store of experimental data from full scale engine tests. The experimental information available on oscillatory combustion in liquid propellant engines indicates that the oscillations are sustained by a feed back mechanism represented by the interaction between the rate at which propellant is being consumed as well as fluctuations in chamber pressure. This feed back mechanism will result in periodic variations of the heat released in the chamber.

The development of theories for analysis of high frequency instability has been actively pursued for over 50 years [1]. The total effort devoted to stability analysis probably exceeds that applied to performance and compatibility theory by order of magnitude. Most emphasis has centered on two widely accepted approaches, the Sensitive Time Lag Theory and the Droplet Vaporization Model [2]. Unfortunately, only a small fraction of this analysis effort has been openly published and the details of the application of the stability theories to a specific combustor design are rather vague. Among the various theoretical stability approaches, the combustion time lag theory has been used extensively and found to be simple, and realistic [3].

Crocco and Cheng [4] advanced the time lag theory by considering the time lag to be a sum of a constant quantity and a pressure sensitive component. The former is called the insensitive time lag whereas the latter is referred to as the sensitive time lag. Based on this pressure sensitive time lag theory, Crocco and Cheng were able to describe the longitudinal mode acoustic stability using two parameters, namely, the interaction index, n, and the sensitive time lag, τ. These are known as the combustor response parameters. Reardon [5] and Zinn and Savell [6] further extended the application of this theory to three dimensions and the transverse mode stability. See Harrje and Reardon [2] for a comprehensive review of the theory.

To design a robust and reliable engine it is necessary to utilize one of the above stability evaluation models in the engine design cycle. We have chosen the sensitive time lag model to evaluate the stability characteristics of an engine design. The engine design parameters are dictated by the rocket mission, and there are only few parameters that can be adjusted in case of instability observation. In the following sections we shall first discuss the sensitive time lag theory and its application to a specific engine design. The equations governing the thermo-fluid dynamics of a thrust chamber consisting of a combustion chamber and a nozzle are perturbed, linearized, and combined to develop a characteristic equation governing the perturbation characteristics of a thrust chamber. To simplify the mathematical integration procedure, the characteristic perturbation equation can be applied to the combustion chamber and the nozzle separately and solutions matched at the boundary. Hence the nozzle perturbation field is solved and its entrance admittance relation is used as the combustion chamber equation boundary condition.

Next we shall present the stability analysis results and a parametric study of the effect of design parameters; such as entrance Mach number, non-dimensional chamber length, or acoustic mode shapes on stability characteristics of a typical engine. Finally stability boundaries for a sample operating engine are determined and presented.

STABILITY PREDICTION

Theoretical Modeling

There are still no comprehensive models that accurately represent the major aero-thermo-chemical processes taking place in a combustor. These processes include the fuel and oxidizer injection, atomization, vaporization, mixing, chemical reactions, and their response to the chamber flow field oscillations. Hence analysis of the complete unsteady behavior of the combustion process taking place in a liquid propellant rocket combustion chamber is rather out of question. Crocco's time lag theory concentrates all of the unsteady combustion processes into an infinitesimally narrow flame region that can be treated as a surface of discontinuity in an otherwise non-chemically reacting combustor flow field. This surface of discontinuity can be ideally placed on the combustion chamber's injector face. Based on this model, analysis is restricted to the study of interaction between the simplified combustion flame region and acoustic wave processes that result in oscillatory operation of the combustor. If the amplitude of the oscillations is small, then a linear analysis is an appropriate mathematical approximation. The combustion chamber acoustic perturbations solution must satisfy proper boundary conditions representing the unsteady combustion process at the injector face of the chamber and the nozzle acoustic admittance matching at the nozzle entrance. Since the nozzle transverse dimensions are of the same order of magnitude as the longitudinal dimension of the combustion chamber, the assumption that the nozzle response is quasi-steady cannot be used. In fact the short nozzle approximation assumption [4] must be replaced by an appropriate acoustic admittance matching boundary condition at the nozzle entrance. Therefore, to obtain governing equations for the acoustic perturbations in the combustion chamber and the nozzle, the non-reacting, single phase Euler equations are used. To develop the appropriate linear form of the conservation equations the dependent variables p, ρ, etc. are represented as a sum of the steady and oscillatory components; thus,

$$p = \bar{p} + p', \quad \rho = \bar{\rho} + \rho'$$

Here the overbar indicates a steady state variable whereas a prime superscript denotes an oscillatory variable. The steady state solution of the flow through the combustion chamber and the nozzle is known and given by the combustor designer. Subtraction of the steady state equations from the complete set leaves the equations governing the oscillatory field. These are linearized in the usual way by neglecting products of the perturbation variables. In addition, all dependent variable are nondimensionalized with respect to corresponding steady state stagnation quantities, with the exception of the velocity components which are nondimensionalized with respect to steady state stagnation speed of sound, $\bar{c}_0^*$. Also, lengths are nondimensionalized with the nozzle throat radius, r_{th}^*, time and frequency with $\bar{c}_0^*/r_{th}^*$. Here * superscript denotes a dimensional quantity. The nondimensional, linearized perturbation equations can be presented in the following form:

$$\text{Continuity} \quad : \frac{\partial \rho'}{\partial t} + \nabla.(\rho'\vec{\bar{u}} + \bar{\rho}\vec{u}') = 0$$

$$\text{Momentum} : \begin{array}{l} \bar{\rho}\dfrac{\partial \vec{u}'}{\partial t} + \bar{\rho}\nabla(\vec{\bar{u}}.\vec{u}') \\ + \dfrac{1}{2}\rho'\nabla\bar{u}^2 + \dfrac{1}{\gamma}\nabla p' = 0 \end{array}$$

$$\text{Energy} \quad : p' = \bar{c}^2\rho'$$

where ρ, p, and u are density, pressure, and velocity respectively. γ is the specific heat ratio. To simplify the mathematical integration of these equations it is conventional to consider three dimensional oscillations in the nozzle and those in the combustion chamber separately. Therefore to analyze the oscillatory behavior of a combustion chamber this set of equations must be first solved for the nozzle geometry to obtain the nozzle admittance at the nozzle-chamber boundary. Next the same equations are solved for the chamber geometry with appropriate boundary conditions. Appropriate steady state solutions must be provided for the nozzle and the combustion chamber.

Since the flow Mach number at the nozzle throat is sonic and no finite amplitude perturbations can propagate upstream of the nozzle throat, only the converging section of the nozzle geometry is considered for the stability analysis. Due to the geometry of the converging section of the nozzle it is convenient to analyze the behavior of the solution in a (ϕ, ψ, θ) coordinate system, where ϕ represents the steady state potential function, ψ the steady state stream function, and θ the variation in the azimuthal direction. The steady state nozzle flow is considered to be one dimensional. In the above coordinate system the steady state parameters are independent of the ψ and θ directions and only depend on ϕ. The nozzle solution (admittance relation) then serves as the boundary condition for the chamber acoustic field. We shall discuss the nozzle admittance relation in the next section, followed by the combustion chamber response evaluation.

Nozzle Admittance Determination

After mathematical manipulations of the linearized mass, momentum and energy perturbations equations a three dimensional unsteady equation is obtained for the velocity potential function; Φ'. Harmonic time dependence assumption and separation of variables are applied to the velocity potential function to obtain:

$$u' = \nabla\Phi' = \nabla(\Psi\Theta\Phi e^{i\omega t}) \tag{1}$$

Here Ψ, and Θ are transverse modes of oscillatory velocity potential function, Φ', whose spatial variations are given as:

$$\Psi(\psi) = J_\nu(S_{(\nu,h)}(\frac{\psi}{\psi_{wall}})^{1/2}) \tag{2}$$

where J_v denotes a Bessel function of the first kind and ψ_{wall} is the steady state stream function at nozzle walls, and

$$\Theta(\theta) = \begin{cases} \cos(\nu\theta) & \text{for standing wave} \\ \exp(i\nu\theta) & \text{for travelling wave} \end{cases} \tag{3}$$

To extract the equation governing the behavior of the longitudinal component of the velocity potential function we shall substitute the above solutions into the velocity potential function three dimensional equation. The following equation governing the behavior of the axial velocity perturbation, $\Phi(\phi)$, is obtained.

$$\begin{aligned} &\bar{q}^2(\bar{c}^2 - \bar{q}^2)\frac{d^2\Phi}{d\phi^2} - \bar{q}^2(\frac{1}{\bar{c}^2}\frac{d\bar{q}^2}{d\phi} + 2i\omega)\frac{d\Phi}{d\phi} \\ &+ (\omega^2 - i\omega\frac{\gamma-1}{2}\frac{\bar{q}^2}{\bar{c}^2}\frac{d\bar{q}^2}{d\phi} - S_{(\nu,h)}^2\bar{\rho}\bar{q}\bar{c}^2)\Phi = 0 \end{aligned} \tag{4}$$

where $\bar{q}$ is the steady state gas velocity in the axial, ϕ direction, and $S_{(\nu,h)}$ denotes the eigenvalue for a particular mode of oscillation. Indices ν and h are running integers with zero starting values. They indicate tangential and radial modes of oscillation, respectively.

The special case of $S_{0,0} = 0$ corresponds to the pure longitudinal mode. Profiles of the Mach number and steady state velocity in the nozzle are obtained using the gas dynamics relation and the geometry of nozzle.

Since we are interested in the prediction of overall stability characteristics and not in the details of fluctuating flow field, an auxiliary function is defined as follows:

$$\mu = \frac{\frac{d\Phi}{d\phi}}{\Phi} \tag{5}$$

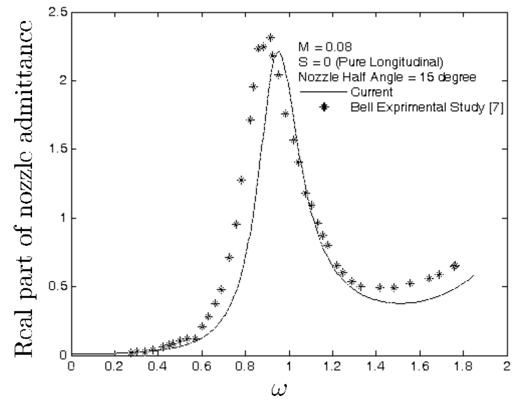
This change of variable reduces equation (4) to the following nonlinear Riccati type equation for μ

$$\bar{q}^2(\bar{c}^2 - \bar{q}^2)(\frac{d\mu}{d\phi} + \mu^2) - \bar{q}^2(\frac{1}{\bar{c}^2}\frac{d\bar{q}^2}{d\phi} + 2i\omega)\mu$$
$$+ (\omega^2 - i\omega\frac{\gamma-1}{2}\frac{\bar{q}^2}{\bar{c}^2}\frac{d\bar{q}^2}{d\phi} - S^2_{(\nu,h)}\bar{\rho}\bar{q}\bar{c}^2) = 0 \tag{6}$$

This equation is a complex Riccati type, and can only be solved by numerical integration. Due to singular behavior at the nozzle throat (where $\bar{q} = \bar{c}$) the numerical integration must start at this point. The difficulties associated with the initiation of the numerical integration are circumvented using a Taylor series expansion for μ around the point $\phi = 0$ as well as using the resultant series to evaluate μ at the short distance upstream of the throat. The resulting value is then used as an initial value for the numerical integration. A fourth order Runge-Kutta method is then used for integration of the equation (6). The nozzle admittance relation is defined as the ratio of the axial velocity perturbation to the pressure perturbation. This is presented in the following form:

$$\alpha = -\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left[\frac{\bar{q}}{\bar{c}^{\frac{2}{\gamma-1}}}\right] \tag{7}$$

A computer program has been developed based on the above solution methodology for the determination of acoustic admittance of any nozzle geometry. The program was validated using a conical nozzle, with a circular arc section with radius of curvature of one at the throat, inflow Mach number of 0.08, and nozzle half angle (δ) of 30°. Results were compared with the experimental results presented by Bell [7] for the pure longitudinal ($S_{(0,0)} = 0$) and the pure tangential ($S_{(1,0)} = 1.84129$) modes. Figure 1 compares the present results with the experimental data.



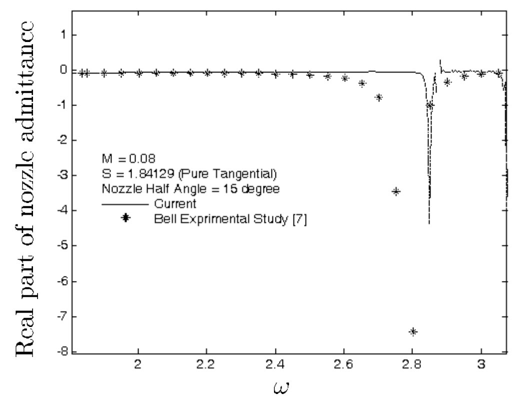


Figure 1. Comparison with Bell's experimental data, a) pure longitudinal mode, b) pure tangential mode

Predicted values of the nozzle admittance are plotted versus frequency for the first pure longitudinal and first pure tangential modes. The predicted values and experimental data compare well in the case of the pure longitudinal mode. In the case of the pure tangential mode, there is less correspondence, but the predicted values capture the trend of the experimental data accurately. These results indicate that the values of the real part of the admittance are always positive for the longitudinal mode and negative for the transverse mode. This implies that for the case of pure longitudinal mode the nozzle has the stabilizing effect on chamber, i.e., it dampens the fluctuations of the chamber flow by absorbing energy from the combustion chamber in the form of a flux of acoustic energy. Physically, this implies that the gases in the combustion chamber are working on the gases in the nozzle [8].

On the contrary, in the case of the pure transverse mode the nozzle flow fluctuations pump work (acoustic energy) into the combustion chamber. Thus, in the case of first tangential mode the nozzle has a destabilizing effect on the combustion chamber flow fluctuations.

Although we have only presented results for the pure longitudinal and tangential modes, the procedure is general to any mode, depending on the specific value of $S_{(\nu,h)}$ (see references 1, 2, and 4). For the mixed acoustic modes, the nozzle effect depends on the real part of its admittance. If the admittance value was small (in order of mean flow Mach number), the nozzle would probably not have a significant impact on stability [1].

The nozzle admittance prediction method presented here is a robust method that works well for a large range of Mach number values. The nozzle entrance Mach number values ranges from the highest of about 0.6 to the lowest of about 0.05. We shall present results for Mach numbers up to 0.6 in the following sections. It is in the low range where other formulations result in non-converging solutions. For example, Crocco and Sirignano [9] have used a formulation that is not suitable for application to low Mach number values. The minimum value of the Mach number in the plots presented by them is limited to 0.294. Figure 2 shows the comparison of the results obtained by the present method to the procedure proposed by Crocco and Sirignano for the Mach number value of 0.05. Results obtained by the use of the Crocco and Sirignano's method are indicated by the dash line and the present results are indicated by the solid line. Their method results in an oscillatory solution.

Combustion Chamber Analysis

In order to study the flow acoustic perturbations in the combustion chamber, it is convenient to use the same coordinate system in which the nozzle admittance relation is expressed. The governing equations are similar to those presented for the nozzle. The only difference is that due to simple chamber geometry the function $\Phi(\phi)$ is controlled by a simpler differential

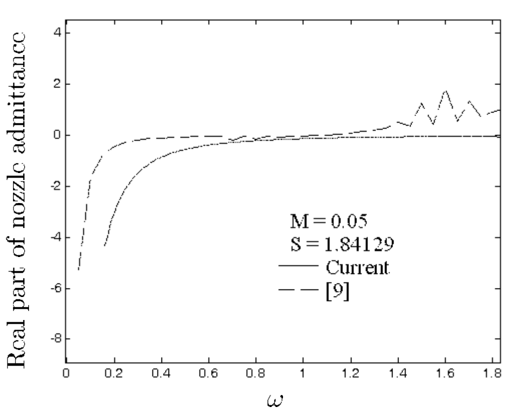


Figure 2. Comparison with results of Crocco and Sirignano [9]

equation, i.e.:

$$\bar{q}^2(\bar{c}^2 - \bar{q}^2)\frac{d^2\Phi}{d\phi^2} - 2i\omega\bar{q}^2\frac{d\Phi}{d\phi} + (\omega^2 - S^2_{(\nu,h)})\Phi = 0 \quad (8)$$

This simple form is a consequence of the fact that in the combustion chamber, $\bar{q}$ is constant. Here dependent variables are nondimensionalized with respect to their corresponding steady state quantities and lengths with chamber radius.

Due to its constant coefficients, equation (8) can be directly integrated. Determination of integration constants requires the application of the boundary conditions. The nozzle admittance matching condition is used as the combustion chamber outflow boundary, and the concentrated combustion zone condition at the injector surface. This is done along with the use of the Crocco's time lag theory as will be shown below. The concentrated combustion zone is a discontinuity of flow velocity, not a discontinuity of pressure, density, or temperature. Hence this boundary condition consists of two parts: the steady state values, as well as the small perturbations of pressure and density, are continuous at any instant across the concentrated combustion zone. On the other hand, the fractional increase of the difference of mass flow rates across the concentrated combustion zone is equal to the fractional increase of the burning rate at the concentrated combustion zone.

Based on the Crocco's time lag theory, the linear stability limits are presented on a (n, τ) coordinate system. The (n, τ) correlation can be obtained by the use of the above boundary condition. Details of derivation of these relations are provided by Crocco and Chang [4].

$$\begin{cases} n &= \frac{(H_r-1)^2+H_i^2}{2\gamma(1-H_r)} \\ \tau &= \frac{1}{\omega}\,\mathrm{Sin}^{-1}(\frac{-H_i}{n\gamma}) \end{cases} \quad \text{(9-a, b)}$$

Here H is a complex function of the steady state flow parameters, nozzle admittance, chamber geometry, frequency, and the eigenvalue for a particular mode of oscillation, $S_{(\nu,h)}$. The H_r and H_i are the real and imaginary parts of the complex function H.

Due to the complicated form of the $n(\omega)$ and $\tau(\omega)$, a computer program has been developed for numerical evaluation of these parameters. The computer program has been divided into two main parts. In the first part, the differential equations describing the unsteady flow perturbations in the subsonic portion of the nozzle were integrated numerically. Due to its singularity, the nozzle throat was chosen as the initial point from where the integration proceeded in the direction of the combustion chamber. The integration was terminated when the local Mach number in the nozzle was equal to Mach number of the mean flow inside the combustion chamber. The results of the numerical integration then is fed into the second part of the computer program where $n(\omega)$ and $\tau(\omega)$ were calculated. In order to run this program it is necessary to specify the ratio of the specific heats, the frequency of the oscillation, the eigenvalue for a particular mode of oscillation ($S_{(\nu,h)}$), the length of the combustion chamber, the Mach number of the steady flow at the nozzle entrance, and the geometry of the nozzle. Specification of the nozzle shape is equivalent to the specification of the steady state velocity distribution in the subsonic portion of the nozzle which can be obtained by numerical integration of appropriate gas dynamics relations. The unstable range of τ, for a specific Mach number, when the combustion is concentrated at the injector face, has been determined for different values of n for transverse and mix modes. Figure 3 compares results of the present work with that of Zinn and Savell [6].

RESULTS AND DISSCUTION

In this section, we shall consider the effects of a combustor geometrical and physical parameters on its stability curves. The complicated form of the stability equations makes it practically impossible to determine the effect of each parameter on the stability boundary by mere consideration of the analytical form of these equations. To study the effects of different parameters, we have conducted a parametric study through obtaining linear stability limits for several sample cases which are characterized by a different set of parameters. These stability limit solutions are used to investigate the effect of the Mach number of the mean flow, the combustion chamber length to diameter ratio, and the frequency of the oscillation upon combustion instability limits. A few representative results are presented here. Each case requires specification of the combustor parameters including the specific gas constants ratio, γ, the Mach number at the nozzle entrance, the eigenvalue for a particular mode of oscillation $S_{\nu,h}$, length and diameter of the combustion chamber, and geometry of the converging part of the nozzle. Given these

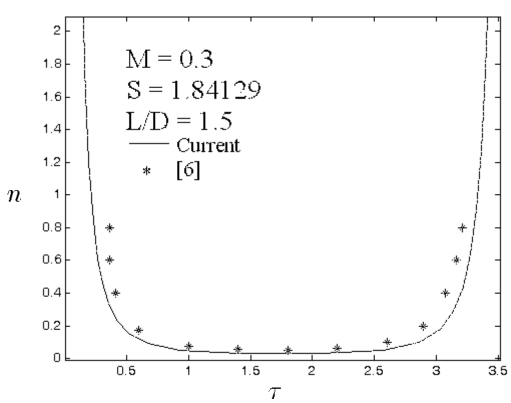


Figure 3. Neutral stability limits comparison

parameters, equations (9a, b) are solved to obtain n and τ as functions of the nondimensionalized frequency, ω. Next, we plot n versus τ to obtain stability curves for the range of frequencies considered in the computations. In cases considered here, the value of the specific gas constants ratio is set to 1.14, the nozzle half angle is 30 degrees, the nozzle throat radius is 0.1 of the chamber diameter, and other parameters are indicated in the relevant results figure. Also, the "Mach number" in the chamber is assumed equal to the "entrance Mach number".

Figure 4 shows the plot of n versus ω and Figure 5 shows the neutral stability curves in terms of n versus τ for a wide range of frequencies. The appearance of multiple loops in Figure 5 is associated with the presence of multiple instability regimes in the range of frequencies considered.

Conventionally, the stability curve is given for a single mode of oscillation. However, the solution method presented here enables us to determine the stability curves for multiple modes of oscillations. Figures 4 and 5 are for the case of $S_{1,0} = 1.84129$.

Each loop in Figure 5 corresponds to a spe-

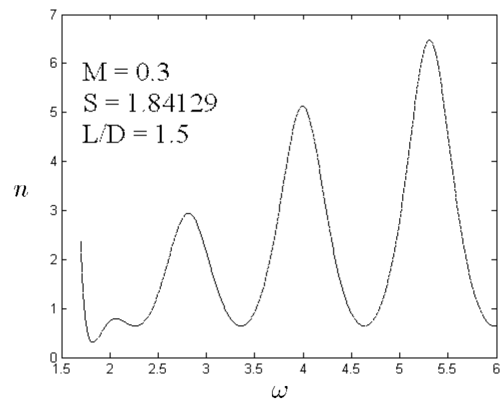


Figure 4. Variations of interaction index vs. nondimentional frequency

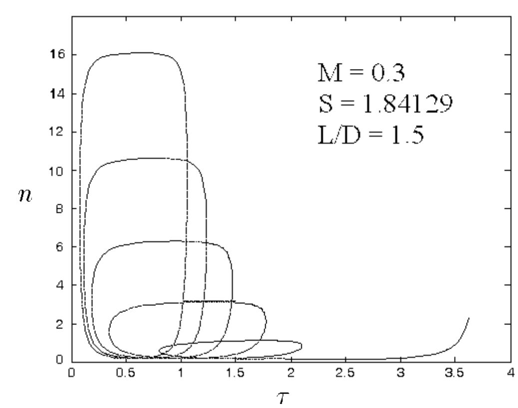


Figure 5. Neutral stability limits at a wide range of nondimentional frequencies

cific mode of oscillation, including the first pure tangential mode and four consecutive mixed tangential-longitudinal modes.

Figure 4 is used to identify the frequency of each mode of oscillation. The curve presented in this figure resembles a sinusoidal curve. The first minimum on the curve correspond to the frequency of the first pure tangential mode. Subsequent minimums at higher values of the frequency are associated with the mixed first tangential-longitudinal acoustic modes, $S_{1,0} = 1.84129$.

Figures 6 and 7 are extracted from Figures 4 and 5 for a limited range of the frequencies and present the frequency range and stability curve for the first pure tangential mode only.

Parametric study

Figures 8a and 8b present the effects of variations of the Mach number at the nozzle entrance upon the stability limits for the first pure tangential and longitudinal modes, respectively. These figures indicate that increasing the Mach number at the nozzle entrance causes widening of the (n, τ) neutral stability curve, increasing the possibility of an unstable behavior of the

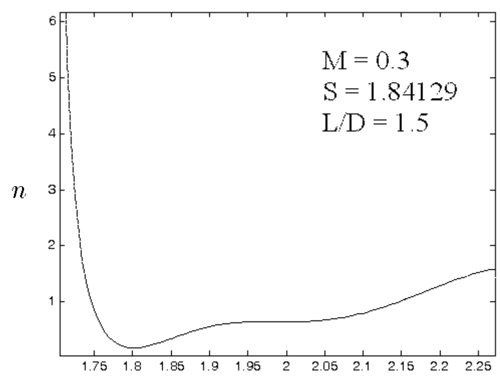


Figure 6. Interaction index vs. nondimentional frequency for first tangential mode

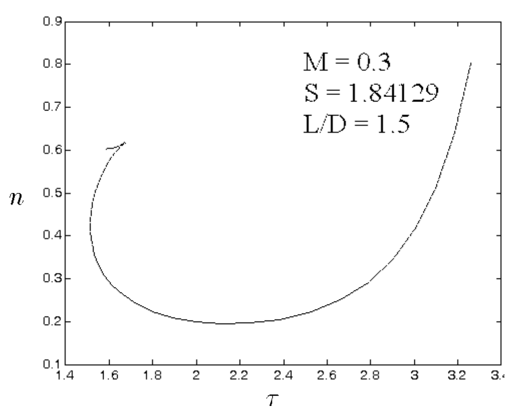


Figure 7. Neutral stability limit for the first tangential mode

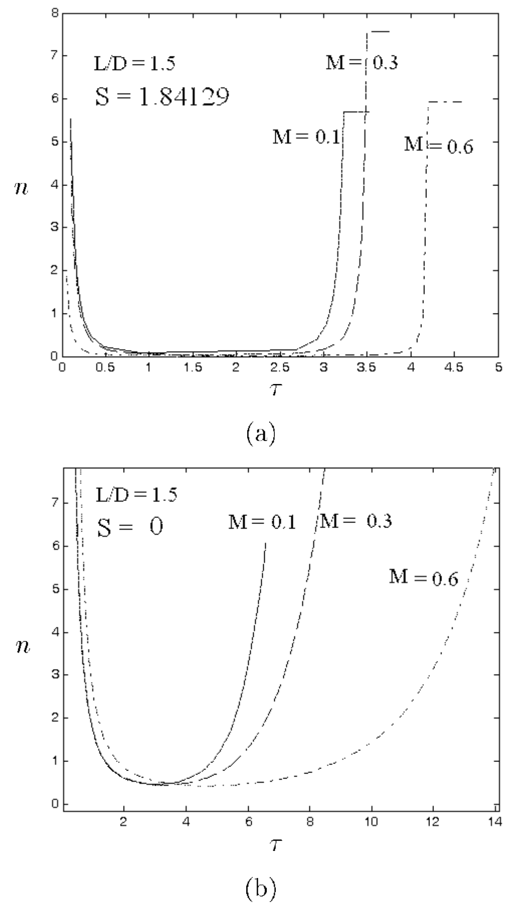


Figure 8. Effect of the Mach number on the neutral stability curve; a) tangential mode, b) longitudinal mode

combustor. Hence, increasing the Mach number at the nozzle entrance has a destabilizing effect. These results are in complete agreement with the results presented by Reardon [5]. Studies by Zinn and Savell [6] on the other hand have indicated that the increase of the Mach number in the range of 0.1 to 0.4 has a stabilizing effect; however, further increase from 0.4 to 0.6 results in a destabilizing effect. Figure 9 indicates that increasing the Mach number at the nozzle entrance can also shift the frequency associated with a particular mode of oscillation. This is a secondary effect and may not be of much use in the combustor design.

The dependence of the neutral stability curves on the combustion chamber L/D for low and high values of the Mach number is presented in Figure 10. It is observed that increase of the combustion chamber L/D causes a widening of the neutral stability curves and hence has a destabilizing effect. This is in agreement with Zinn and Savell [6] results.

Figure 11 shows the effect of the L/D on the interaction index. As in figure 10b, it indicates that for low Mach number values, the minimum value of

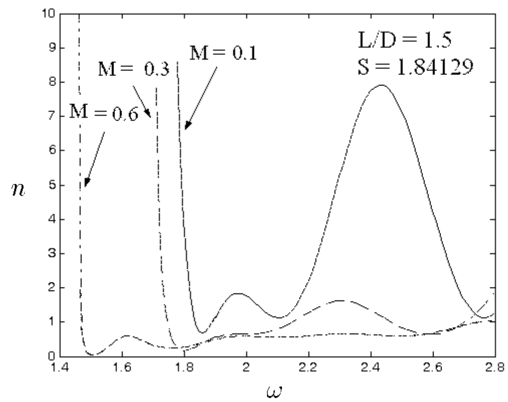
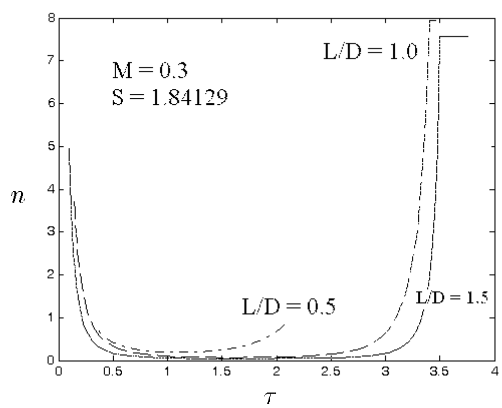
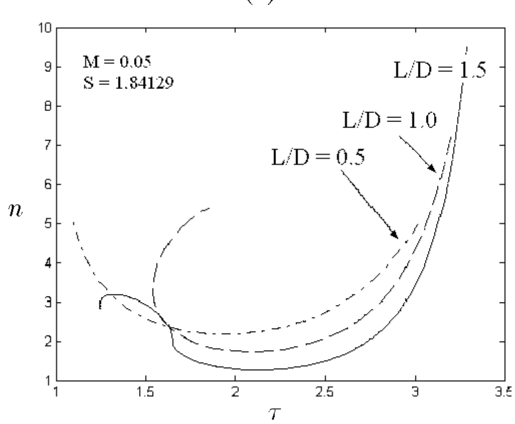


Figure 9. Effect of the Mach number on the interaction index



(a)



(b)

Figure 10. Effect of chamber L/D on the neutral stability curve; a) Mach number 0.3, b) Mach number 0.05

the n decreases as the L/D increases. To justify this observation, a relation for minimum value of the n from Reference [10] has been considered.

$$n_{\min} = \frac{1}{2\gamma}(1 - \frac{\mu_i}{\omega}) \tag{10}$$

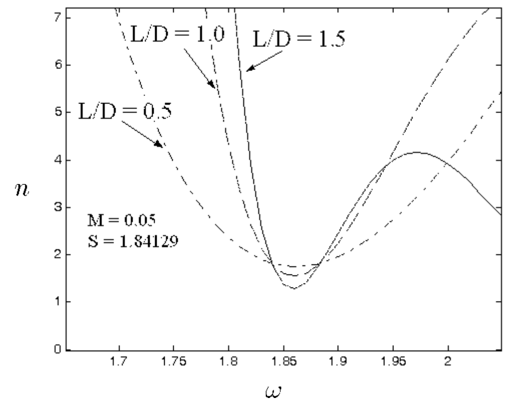


Figure 11. Effect of the L/D on the interaction index

Here μ_i is the imaginary part of the μ. Our calculations indicate that the value of μ_i is greater than zero. Due to the fact that the value of γ and frequency are positive, decreasing the frequency will decrease the minimum value of n. On the other hand, since in this model the combustion plane is concentrated at the injector face, the increased combustor length has no effect on the combustion process and only decreases the frequency of the combustor, which means decreasing the minimum value of n. Therefore, increasing the combustor length results in a larger unstable region.

CASE STUDY

Application of Time Lag Theory

A preliminary design of a combustor must satisfy both performance and stability goals. The combustor design includes specification of combustion chamber dimensions and core elements size, number and layout of injectors, and specification of the nominal operating pressure schedules. The preliminary sizing is accomplished using a combination of empirical correlations and analytical relations.

Since these models are interactive, the optimization of a combustor design is normally achieved through an iterative design and analysis procedure during which a series of tradeoffs are made to achieve a balance design which best meets the overall requirements. Stability analysis is performed for a combustor with a given set of parameters describing the characteristics of the combustor design. Our goal in this study has been to prepare an analysis tool which can predict the stability map of a specific combustor.

We have chosen a thrust chamber that consists of a circular cylinder of 513 mm diameter and 152 mm long, joined to an exhaust nozzle with a conical subsonic portion 279 mm long and a cone half angle of 18.6°. The nozzle concentration ratio is 1.58, so that the chamber Mach number is about 0.249. It is to operate at a chamber pressure of 217.7 bar using LOX/CH_4 propellants. As this is a hydrocarbon fueled rocket engine and the significant portion of the lifetime of the droplet is spent in the vaporization mode, the Crocco's time lag theory is applicable to this case.

Injectors are shear coaxial non-impinging type. Combustion chamber gas temperature is 3749 degree Kelvin. Gas constant is 389.3 and specific heat ratio is 1.14. Given this information, we can determine the stability boundary in the (n- τ) diagram and evaluate stability criterion of the motor.

Evaluation of Combustor Response Parameters

In order to apply the sensitive time lag theory to the stability prediction of a particular thrust chamber the values of combustor response parameters; the sensitive time lag (τ) and the pressure interaction index (n), must be specified. These are based on empirical correlations for a specific engine design [2]. There is a certain amount of uncertainty introduced into the n and τ values by these correlations. This limits the accuracy of the stability analysis and can introduce up to $\pm 25\%$ error into the specified values of n and τ.

In this study we have considered a combustor with coaxial injectors for bipropellants. The sensitive time lag is presented in terms of combustor characteristics such as the injector diameter d_i, the injector velocity ratio VR, which is the ratio of velocity of the outer annular stream to the velocity of the central stream, the chamber reduced pressure p_r, which is the chamber pressure divided by the controlling propellant critical pressure, and the nozzle entrance Mach number M_e. The sensitive time lag values have been correlated by the equation

$$\frac{\tau M_e^{1/7}\beta_p}{\beta_{VR}} = 0.076\,\text{milli sec} \tag{11}$$

where the pressure dependence factor β_p is given by

$$\beta_p = \begin{cases} p_r^{1/3} & p_r < 1 \\ 1.0 & p_r \geq 1 \end{cases} \tag{12}$$

and the velocity dependent factor, β_{VR}, an empirical function of the injector velocity ratio and injection angle, is constant and equal to 0.9 for non-impinging injectors [2].

The pressure interaction index for a combustor with coaxial injectors appears to be essentially constant, independent of the velocity ratio, or chamber pressure at about 1.56 [11]. The coaxial injector elements are arranged symmetrically to minimize transverse velocity oscillation effects.

According to the above information n is varied from 1.5 to 1.6 and τ is obtained from equation (11). The value of τ calculated from this equation for the combustor under consideration is equal to 0.0834 millisecond. Since the stability diagrams are in the nondimensionalized format, the value of τ must be nondimensionalized with the sound speed in the combustion chamber and the chamber radius, (C/R). The nondimensionalized value of τ is 0.4203. As the chamber parameters such as the gas temperature may vary, we consider a bounded range for the value of τ between 0.4 and 0.44, that is $0.42 \pm 5\%$. Figure 12 illustrates the stability boundary and the value of n and τ for the designed combustor.

According to Figure 12 it is predicted that the motor will experience instability in second mix mode (first tangential and first longitudinal).

The stability analysis tool presented here helps the designer to modify his design to prevent any predicted instabilities. There are a number of parameters which can be changed, for example injector type or pattern, fuel type, or geometrical parameters. However, the parameter study presented in the previous section indicates that the chamber length and Mach number are dominant parameters that can affect the combustor stability boundary. The designer can also add baffles to prevent selected instabilities. The baffle design and geometry selection is strongly dependent on the mode which is the source of instability [12].

CONCLUSION

We have presented results of a stability analysis and a parametric study of the effect of design parameters on stability characteristics of a typical combustor. It has been shown that the values of the real part of the nozzle admittance are always positive for pure longitudinal modes and negative for pure transverse modes. This means that in the case of a pure longitudinal mode the nozzle has a stabilizing effect on the chamber, i.e., it dampens fluctuations of the chamber flow by absorbing energy from the combustion chamber in the form of a flux of acoustic energy. On the contrary, in the case of a pure transverse mode the nozzle flow fluctuations pump work (acoustic energy) into the combustion chamber. Thus, in the case of a pure

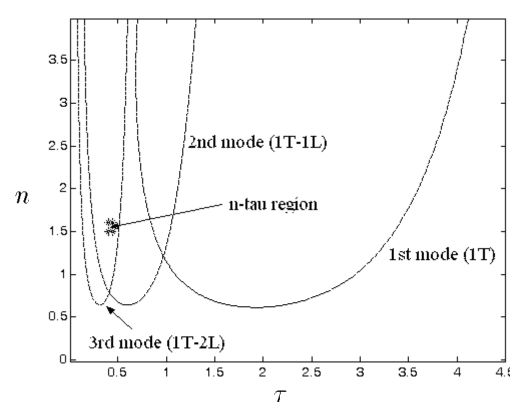


Figure 12. Stability diagram for the chosen combustor

tangential mode the nozzle has a destabilizing effect on the combustion chamber flow fluctuations. Next, we have determined the stability curves for multiple modes of oscillations and identified the frequency of each mode. Our analysis indicates that increasing the Mach number at the nozzle entrance causes widening of the (n, τ) neutral stability curve, increasing the possibility of an unstable behavior of the combustor. We have also observed that increase of the combustion chamber L/D decreases the minimum value of the n and results in a widening of the neutral stability curves, and hence has a destabilizing effect.

The purpose of this work has been to develop a tool that can provide two distinct facilities for a liquid rocket designer. First, a designer can observe the effects of different parameters such as nozzle entrance Mach number, chamber geometry, nozzle geometry, gas property, etc. on the stability characteristics of a motor. This can help the designer to choose the best way for optimization of a design. Secondly, this software can predict the stability of a designed motor and, in case of instability; it can predict the acoustic mode which causes the instability. By means of this tool a designer can achieve an optimum design before conducting time and money consuming experimental tests. In fact this software is an appropriate bed to conduct studies about the combustor high frequency acoustic stability and evaluate the effects of each component on governing mode shapes. One can also model the combustion process and study the coupling of combustion with acoustics.

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