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Damage Assessment Using an Inverse Fracture Mechanics Approach

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This paper studies the application of an inverse methodology for problem solving in fracture mechanics using the finite element analysis. The method was applied to both detection of subsurface cracks and the study of propagating cracks. The procedure for detection of subsurface cracks uses a first order optimization analysis coupled with a penalty function to solve for the unknown geometric parameters associated with the internal flaw. The objective function is calculated from normalizing the finite element determined displacements by the prescribed ones at some arbitrary points of the damaged component. The technique was also used for determination of both 1-D and 2-D planar crack growth directions using the well known maximum strain energy release rate criterion. In all cases studied, a good agreement is achieved between the theoretical and/or the experimentally observed crack behavior and the developed technique.

INTRODUCTION

A structure is susceptible to damage during its service life because of extreme events. In many applications, damage in a structural component is defined as the change in structural performance, which can be identified in terms of discrete cracks, voids or a weak zone formation and a consequent stiffness reduction. Sometimes, undetected and unrepaired damage may lead to structural failure demanding costly repair and/or huge loss of lives. Therefore, detecting internal flaws or cracks appearing within a structural component at an early stage is currently classified as an important engineering task. For aged structures, specifically old aircraft structures, it is necessary to test the functionality of the structure under design loads. In these structures, however, the difference between the present and the original stiffness can be considered as a measure of structural/material degradation due to internal damages.

To date, there are only a few articles published in literature on the subject of inverse fracture mechanics. In reference [1], a similar idea but with a different approach is introduced to simulate fatigue crack growth. In this article, it is reported that the beam curvatures would give a fair idea about the location of damage in beams and so, the compatibility equation obtained using Grobner bases can serve as a measure to identify damage from field data of displacement. These data have been used for fatigue crack growth simulation in 3-point bend specimens. A crack identification technique and 1-D self-similar crack growth simulation by inverse fracture mechanics coupled with a nonlinear sequential quadratic programming optimization procedure is introduced in reference [2]. Although the developed technique is able to perform the task, the computational effort is high and therefore, may be inefficient in 2-D planar crack growth simulation problems. An analytical method based on numerical solution of a simultaneous set of non-linear equations is presented in references [3-4] for simulation of planar crack growth in composite materials. The presented technique is able to model general 2-D crack growth, but the technique is tedious and mathematically complicated. Many other methods for modeling 2-D planar crack growth based on moving mesh or node release in finite element (FE) analysis are introduced in literature. All these methods have limitations and so are applicable only in some specific problems. A brief review on these methods can be found in [5].

In this work, an attempt is made to detect

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damage in the form of a discrete flaw, its location and its orientation within a structure and to simulate its growth by numerical means. For this reason, a technique is introduced that searches for the crack geometry and its location by comparing the FE stiffness with the experimental evaluations. The procedure is based on characterizing the defect using several geometric parameters. These parameters are iteratively modified throughout an inverse analysis until a convergent solution is obtained, *i.e.* the numerical stiffness matches the experimental evaluation. For this purpose, an optimization method is used to minimize a cost function in which experimental and numerical displacements at certain points are involved [6].

To exhibit applicability and robustness of the developed technique, three case studies are considered in this paper. These are:

1. Identification of a crack with variable length and orientation in a planar domain.
2. Evaluation of mixed-mode crack growth direction.
3. Growth of a planar crack in a double cantilever beam (DCB) specimen.

For all cases studied, the effectiveness and stability of the technique are observed.

ANALYTICAL MODEL

Inverse problems, contrary to the direct problems, are concerned with determination of inputs such as geometry, material properties, loads, *etc.* from the observed output or responses. Presence of a crack in a body leaves some effects on the body response to the applied loads. So, the main idea in development of an analytical model for crack detection and its growth situation is using these effects in an inverse manner. In the analytical model introduced in this paper, the following assumptions are made:

1. Field measurement of displacements at some reference points is error free.
2. Damage occurs in the form of a discrete crack with no material degradation around the crack tip.
3. Structures behave linearly elastic before and after the damaging event.

The displacement of any structural component is a function of its geometry, material properties and loading distribution. In our inverse model, loading distribution and material properties as well as the outer shape and boundaries of the structure are known quantities. Therefore, the internal geometry of the damaged component, *i.e.* length, location and orientation of a crack, is determined so that the computed displacements might match the experimental data (Figure 1).

The central part of the model is a first order optimization technique coupled with a standard FE analysis. The first order optimization technique used in this study calculates and makes use of derivative information. The constrained problem statement is transformed into an unconstrained form using penalty functions. Derivatives found for the unconstrained objective function and state variables give the optimum design search direction. Various steepest descend and conjugate direction line searches are performed during each iteration until the convergence is reached.

Different objective functions can be utilized for crack detection and growth analysis. In case of crack detection, as stated above, displacements at some reference points are used and in case of crack propagation, strain energy release rate (G) may be employed. In our inverse analysis, for crack detection problem, the objective function including penalty functions can be assumed as follows:

$$Q(x,q) = \sum_{i=1}^{n} \left(\frac{U_i^*(x)}{U_i(x)} - 1 \right)^2 + P_1(x) + P_2(q), \qquad (1)$$

where U_i^* represents the computed displacement components in each solution iteration, U_i stands for the prescribed displacement components compatible with the U_i^*, n is the number of reference displacement components, and x is the vector of design variables used in the analysis. It is clear that increasing the number n would lead to a more accurate result. P_1 is an exterior penalty function that applies to design variables and P_2 is a penalty function that applies to state variables constraints. In Eq. (1), P_1 and P_2 can be assumed in the form of:

$$P_1(x) = \sum_{i=1}^{n} P_x(x_i),$$

$$P_2(q) = q \left(\sum_{i=1}^{m_1} P_g(g_i) + \sum_{i=1}^{m2} P_h(h_i) + \sum_{i=1}^{m_3} P_w(w_i) \right), \qquad (2)$$

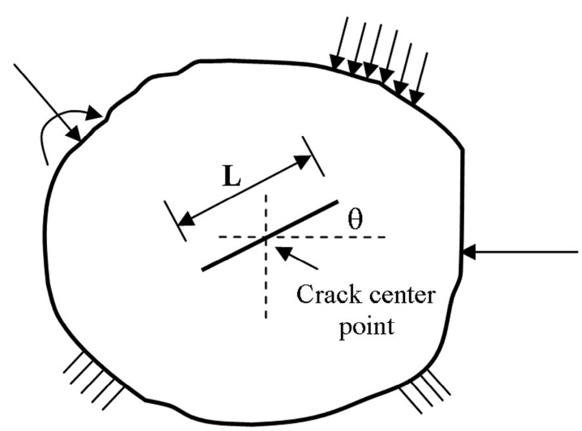

Figure 1. A structural component with an internal crack.

where q is a response surface parameter used for pushing constraints to their limit values as necessary when convergence is achieved. P_g, P_h and P_w are extended interior penalty functions. For example, for state variables constrained by an upper limit, the penalty function is defined as [7]:

$$P_g(g_i) = \left(\frac{g_i}{g_i + \alpha_i}\right)^{2\lambda}, \tag{3}$$

where g_i demonstrates state variables, and α_i represents tolerances associated with each state variable. λ is a controlling exponent and is set to a large number when the constraints are out of the limit and to a small integer number if otherwise. Other P functions in $P_2(q)$ are also defined in a similar manner.

In each solution iteration, for the global minimization of the unconstrained function $Q(x, P)$, a vector of search direction is calculated. For the first iteration, the steepest descent direction is used. Thus, further update of the vector of design variables is performed by the following equation [8]:

$$x_{j+1} = x_j + s_j d_j, \tag{4}$$

where d_j is the search direction vector and s_j is a step length (line search parameter). The adjustment of s_j to reach a global optimization uses a combination of a golden section algorithm and a local quadratic fitting technique [8]. The variation of s_j is limited to:

$$0 \le s_j \le \frac{S_{\max}}{100} S_j^*, \tag{5}$$

where S_j^* is the largest possible step size for the line search of the current iteration and S_{max} is the maximum percent line search step size [7]. The iterative equations used in the analysis for determination of direction vector d_j are [8]:

for the first iteration: $\quad d_j = . -\nabla Q(x_j, q)\big|_{q=1}$,

for the subsequent iterations:

$$d_j = -\nabla Q(x_j, q_k) + r_{j-1} d_{j-1},$$

$$\text{where:} \quad r_{j-1} = \frac{\left[\nabla Q(x_j, q) - \nabla Q(x_{j-1}, q)\right]^{T} \nabla Q(x_j, q)}{\left|\nabla Q(x_{j-1}, q)\right|^2}. \tag{6}$$

It is worth noting that when ill-conditioning is detected or convergence is nearly achieved or constraint satisfaction of critical state variables is too conservative, restarting is employed by setting $r_{j-1} = 0$, forcing the steepest descent iteration. The gradients in Eq. (6) are computed by a standard forward difference technique [9]. The solution iterations are continued until convergence is attained. Convergence criteria can be set to situations when either the difference between two subsequent iteration solutions is less than a tolerance parameter or the difference between the iteration solution and the best solution are less than the defined tolerance. At convergence, one more iteration at the steepest descent direction is used to recheck the solution.

CASE STUDIES

Detection of a crack in a finite membrane

The first case study aims at detection of a crack in a plane strain membrane under unidirectional tension as well as finding its geometrical parameters such as length and orientation. The geometry, material properties and loading condition on the membrane are considered similar to those analyzed in [2] to make a baseline for comparison purposes. Due to the lack of experimental data on this problem, a specific crack is first assumed to exist and the displacements of arbitrary reference points are calculated from the FE analysis. Then, these displacement data are treated as experimental data and subsequently crack is detected by the inverse fracture analysis.

The geometry of the membrane is shown in Figure 2 [2]. The initial crack is assumed to lie parallel to x axis and have a fixed length of 16 mm. The midpoint of the crack is located at point $x = 0.35$ m and $y = 0.6$ m.

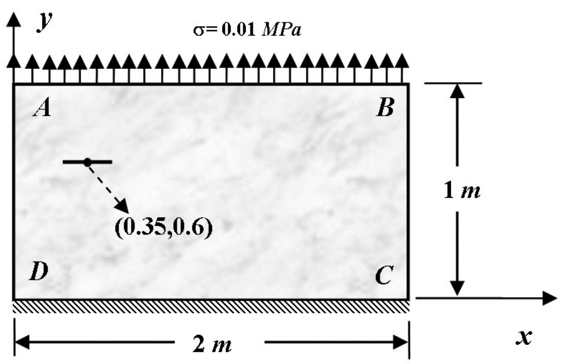

Figure 2. A plane strain membrane with a horizontal crack.

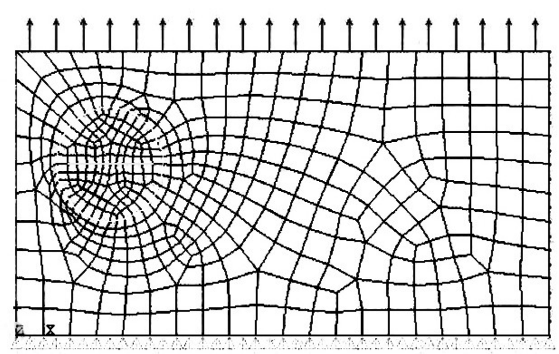

Figure 3. Finite element model of the cracked membrane.

The FE model of the membrane is shown in Figure 3. At five equidistance points along the loaded edge (AB), the FE determined upward displacements are used as reference values. In order to perform iterative FE analysis for each trial crack location and geometry, an automatic mesh generator is required. The automatic mesh generation of the employed software (ANSYS) [7] is facilitated by defining a circular boundary around the crack as well as along the membrane boundary [10]. Solution iterations is started from the trial crack midpoint location at $x = 0.8$ m and $y = 0.55$ m. After 8 iterations, the solution converges to the point (0.351, 0.597) m as the midpoint of the crack. The convergence rate as shown in Figure 4, shows up practically after a five iteration solution is reached, which is much better than 11 iterations reported in [2].

To find out more about the crack detection capability of the technique, a more general problem for this plane strain membrane is considered. In this problem, both crack length and orientation as well as its location are involved in the analysis. The loading distribution is also considered to linearly vary traction form 0 at point B to 10 MPa at point A. Again, a crack with a specific length and orientation was assumed to exist at a pre-defined position and the FE determined displacements were used as experimental

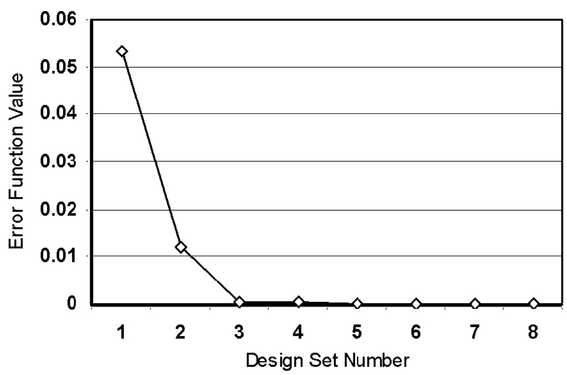

Figure 4. The convergence rate of the objective function in case of the horizontal crack detection.

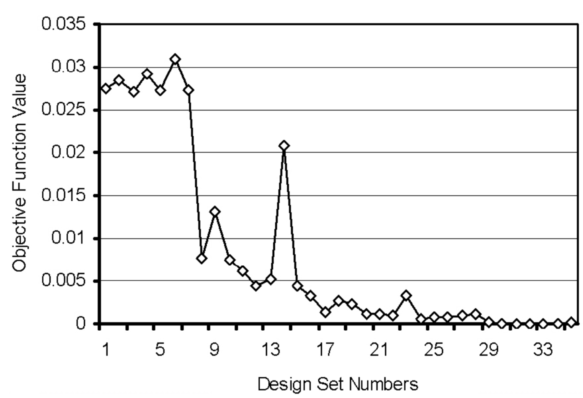

Figure 5. The convergence rate of the objective function in case of the angle crack detection.

data and the inverse problem was solved. The analysis results together with the assumed initial values appear in Table 1. The convergence rate of the solution process is also depicted in Figure 5. The obtained results show a good stability and robustness of the solution scheme.

Detection of Crack Growth Direction

One of the old problems in fracture mechanics open to further research is prediction of crack growth direction in a mixed-mode loading case. Prediction of the angle at which a crack grows under a certain growth criterion can be stated as an inverse relation. Many criteria have been developed for crack growth under mixed-mode loading. (A brief review of these criteria can be found in [11-12]). One of the most popular growth criteria that is compatible well with experiments is based on total strain energy release rate (G_T). According to this criterion, cracks grow in a direction of maximum G_T (Eq. (7)) [13].

$$G_T = G_I + G_{II}, \tag{7}$$

where G_I and G_{II} are partitioned strain energy release rates associated with modes I and II, respectively. In our proposed model, the objective function for the inverse problem can be defined as:

$$f = \left| \left(\frac{G_T}{G_c} \right)^2 - 1 \right|, \tag{8}$$

where G_c is the critical mixed-mode I/II strain energy release rate which is considered to be solely a function of material properties. It should be noted that, in some materials, G_c depends on mixed mode ratio as well as the material properties. In such cases, the proposed model employing the objective function of Eq. (8) is also applicable. If the calculation of crack growth direction is intended, the objective function $f = 1/G_T$ may also be used. But, using the Eq. (8) gives the direction of crack growth at the onset of growth as well as the loads required to start growth.

The calculation of G is performed using the modified virtual crack closure method [3-5, 14-15]. In this method, force components generated at nodes ahead of the crack tip are multiplied by displacement components of similar nodes behind the crack tip to produce the energy required to close the crack to a value of the crack tip element length along the crack (Δa). This energy, after proper scaling, is assumed to be equal to the energy required to extend the crack with the same value, Δa. In this research, a computer code has been developed and attached to the FE software (ANSYS) as a post-processor to calculate the required G for each trial crack geometry and location.

Mixed-mode Crack Growth Initiated from the Corner of a U-shape Component

The first example for the evaluation of crack growth direction is a cracked U-shape machine element made of steel ($E = 200GPa, \nu = 0.3$) subjected to tensile load [16]. The component geometry and its FE mesh are shown in Figure 6.

The experimental and numerical studies of the above component as reported in [16] show that crack extension started at an angle of about 24° from the tip of a small horizontal initial crack. Further crack extension takes place along a curved path with the crack tip angle approaching 0°. In this mixed-mode crack growth study, only qualitative results were intended. Thus, the value of G_c was set to unity and then crack tip angle at growth was determined using Eq. (8) as the objective function. The upward displacement applied to the component holes was constant during the analysis. Our inverse analysis converges (after 13 iterations) to the value of 25.66° for the crack growth angle at the start (Figure 7). The solution convergence rate is rapid enough and accuracy of the result is acceptable. Further crack extensions give a growth path similar to the experimental observations shown in Figure 8.

Growth of an Oblique Crack from the Edge of a Plate under Uniaxial Tension

In this problem, a square steel plate ($E = 200GPa, \nu = 0.3$) containing an edge oblique crack under uniform uniaxial tension is considered. The length to width ratio (L/W) of the plate is 1.0 and the crack length to width ratio (a/W) is 0.2. Angle of the crack with respect to the plate edge is 67.5°. The results of an analytic solution for a similar case are depicted in Figure 9, from which the angle of crack growth initiated from an oblique edge crack in a plate under uniform tension can be determined [17]. In this specific case β is 22.5° and, accordingly based on Figure 9, the angle of crack growth (θ) would be about 23° to 24°. In our inverse analysis, the objective function for this case can be adopted as:

$$f = \frac{1}{G_T}. \quad (9)$$

Finite element model of the plate using 3-node triangular elements and 6-node triangular singular elements [7] at the crack tip region is exhibited in Figure 10. The inverse analysis for this problem converged after 14 iterations with a fair rate to a value of 22.67°, which is very near to the value determined in Figure 9.

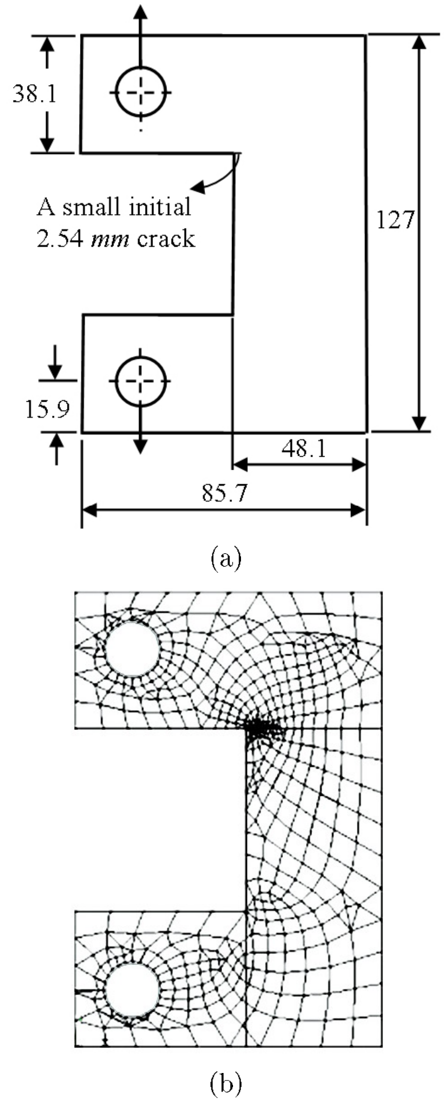

Figure 6. (a) The U-shape component containing a small crack at the corner (b) its FE mesh (units in millimeter).

An Interface Crack Growth in a Double Cantilever Beam Specimen

Double cantilever beam (DCB) specimen, as shown in Figure 11, is a popular mode-I fracture specimen for

Table 1. The initial and the converged parameters of an angle crack in the finite membrane.

	x-coordinate(m)	y-coordinate(m)	Length(m)	Orientation*(deg.)
The assumed crack geometry	0.5	0.7	0.15	100
The initial values given to the inverse analysis	1.5	0.3	0.05	145
The converged values	0.51	0.71	0.15	98.79

*) The orientation angle is defined counterclockwise with respect to the x axis.

testing interlaminar fracture toughness of composites [18]. It is also used widely for testing adhesives. In this case study, a DCB specimen made of aluminum with a fracture toughness of 15000 J/m^2 is considered. The interface crack is opened symmetrically by application of vertical displacements to the loading edges.

The inverse analysis can be used for simulation of interface crack growth in this specimen. First, a constant displacement is applied to the loading edges and then location of crack tip front is searched so that the G_I distribution at all points along the crack front approaches the interface fracture toughness (G_{Ic}). In this problem, a crack front exists rather than a crack tip. Therefore, the adopted objective function must be able to take into account G at several points along the crack front. Thus, the objective function given in Eq. (8) is generalized as follows:

$$f = \sum_{i=1}^{n} \left| \left(\frac{(G_I)_i}{G_{Ic}} \right)^2 - 1 \right|. \qquad (10)$$

Where n is the number of selected points along the crack front. FE model for one arm of the DCB specimen was produced using 8-node plate elements (Figure 12a). Out of plane 7.5 mm displacement was applied to the arm and G_I distribution was calculated using the virtual crack closure method that is extended for plate elements [12, 15].

The inverse analysis was applied to the model and converged to the solution after 29 iterations with a fair rate as shown in Figure 13. Each element along

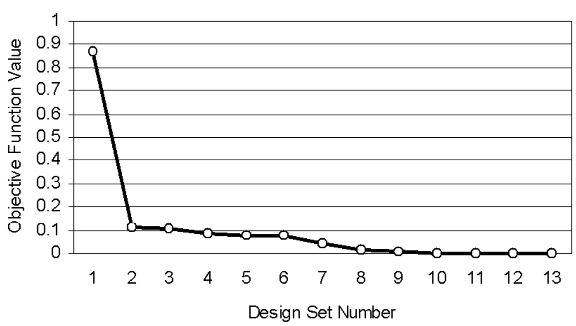

Figure 7. Solution convergence rate of the crack growth angle from the corner.

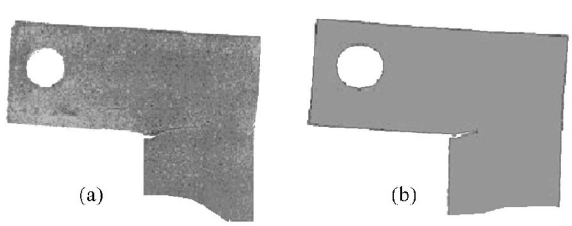

Figure 8. path of the crack growth initiated from corner of a U-shape component (a) Experimental observation [16] (b) Proposed inverse fracture analysis.

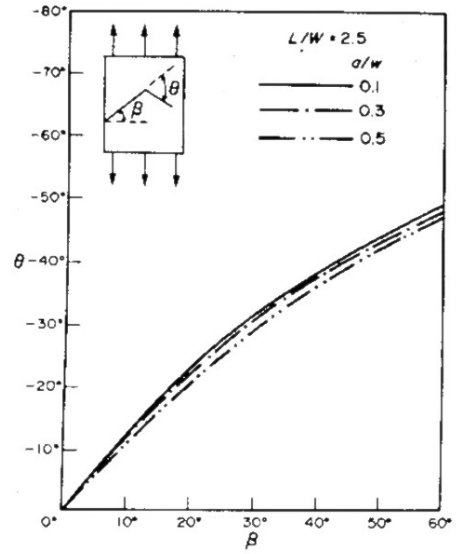

Figure 9. Angle of crack growth from an oblique edge crack [17].

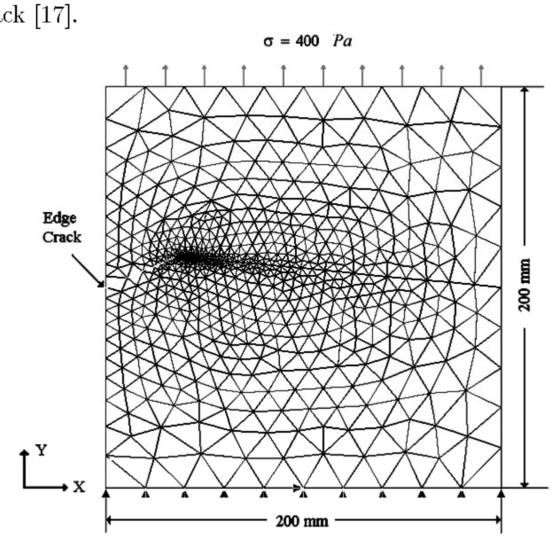

Figure 10. FE mesh of the square plate containing an edge crack.

the front can be moved along direction x until G_I distribution at all points along the front satisfy the growth criterion, *i.e.* the objective function reaches within the defined accuracy to the global minimum of 0. The initial G_I distribution for straight crack front is minimum at specimen edges and maximum at the centerline of the specimen, which is in agreement with findings reported in [3-5]. At convergence, distribution of G_I along the crack front is uniform as shown in Figure 14. It has to be noted that to prevent the zig-zag pattern of crack front that may be produced in each trial solution, the crack front elements are adjusted to a smooth cubic spline curve which is passed through the nodes at the crack front.

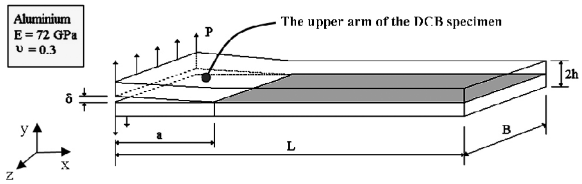

Figure 11. An aluminum DCB specimen (L = 150 mm, B = 20 mm, a = 50 mm, h = 2 mm and δ=15 mm).

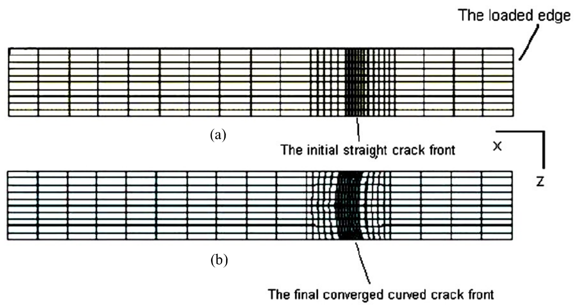

Figure 12. FE mesh for the upper arm of the DCB specimen (a) Initial straight crack front (b) Final converged crack front.

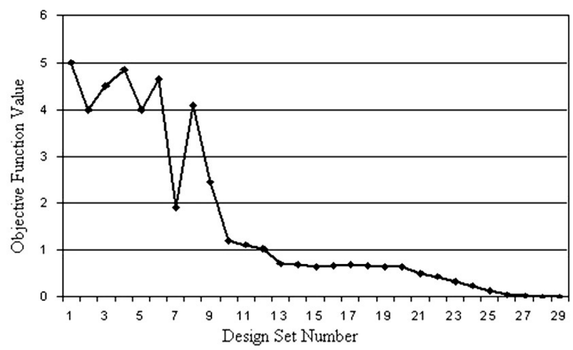

Figure 13. Solution convergence rate for the DCB specimen.

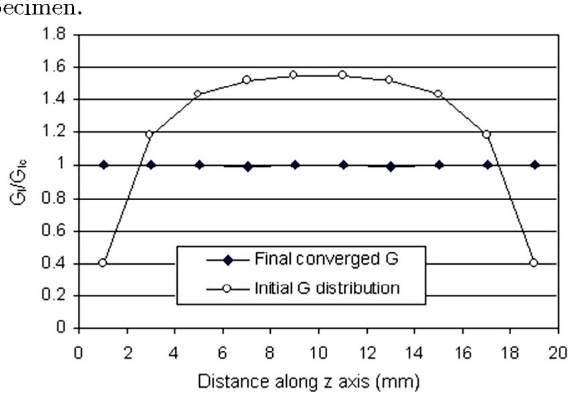

Figure 14. G_I distribution along the straight and curved crack fronts.

CONCLUDING REMARKS

In this paper a numerical procedure is introduced that can be known as an inverse fracture mechanic. This method which incorporates finite element method coupled with an optimization technique can be used to solve many engineering problems such as crack detection and 1-D/2-D planar crack growth simulation. It is shown that this technique is able to find crack geometry parameters such as length and orientation as well as its location within a 2-D body provided that some reference point displacements are known. It is seen that the adopted optimization technique needs less computational effort than the sequential quadratic programming method previously used for the inverse fracture analysis. It is also shown that the method has a good stability for crack growth simulation and so, this technique can be introduced as an effective method for crack growth modeling purposes.

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