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Robust Integral Sliding-Mode Control of an Aerospace Launch Vehicle

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An analysis of on-line autonomous robust tracking controller based on variable structure control is presented for an aerospace launch vehicle. Decentralized sliding-mode controller is designed to achieve the decoupled asymptotic tracking of guidance commands upon plant uncertainties and external disturbances. Development and application of the controller for an aerospace launch vehicle during atmospheric flight in an experimental setting is presented to illustrate the performance of the control algorithm against wind gust and internal dynamics variations. The proposed sliding mode control is compared to non-linear and time-varying gain scheduled autopilot and its superior performance is illustrated by simulation results. Furthermore, the proposed sliding-mode controller is convenient for implementation.

INTRODUCTION

The fundamental purpose of the flight control system, or autopilot, in an Aerospace Launch Vehicle (ALV) is to maintain the vehicle attitude commanded by the guidance section. The autopilot determines the vehicle attitude via an inertial measurement unit (IMU) and commands the appropriate change in the engine trust vector to achieve the commanded attitude from the guidance section. Design of the launch vehicle autopilot must satisfy three main, often conflicting, requirements: stabilize the vehicle, ensure adequate response to guidance commands while minimizing trajectory deviations, and minimize angle of attack in the region of high dynamic pressure to ensure structural integrity of the vehicle (Bletsos, 2004).

The most important problem associated with the attitude control system design for high performance ALV arises because of the non-stationary character of such vehicles, in other words, ALV is a dynamical system which can only be described in mathematical terms by a model which has inaccurate and time-varying parameters (Filho and Hsu, 1986). Therefore, the attitude control systems face time-varying dynamics with uncertain parameters besides non-linearities and different sorts of disturbances. Fixed gain controllers with good performance are impaired by the highly non-stationary nature of the ALV dynamics. This inclines, in general, to a variable performance with steady-state errors. Therefore, robust controllers have been proposed to follow the output resulting from guidance commands through nominal trajectory (Filho, 1995; Malyshev and Krasilshikov, 1996).

In the past few years, there has been an increasing interest within the aerospace control community in exploring the promise of variable structure controls VSC (Vadali, 1986; Dywer and Ramirez, 1988; Chen and Lo, 1993).

The VSC is a non-linear robust control technique that combines and exploits the useful features of different control structures to provide performance and new properties which none of the individual structures can on their own. The VSC theory has provided effective means to design robust state feedback controllers for uncertain dynamic systems. The central feature of VSC is the so-called sliding mode on the switching surface within which the system remains insensitive to internal parameter variations and external disturbances (Utkin, 1977; DeCarlo, et. al., 1988; Hung, et al., 1993).

Sliding-mode control is a particular type of VSC that are characterized by a suite of feedback control law and a decision rule known as switching function. The sliding-mode controllers SMC, is an attractive, robust

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control algorithm because of its inherent insensitivity and robustness to plant uncertainties and external disturbances (Utkin, 1992; Slotine and Li, 1991). Although the technique has good robustness properties, strict enforcement of sliding mode leads to discontinuous control functions and possible control chattering effects (Slotine and Li, 1991) that can be eliminated by continuous approximation of discontinuous control functions (Slotine and Li, 1991; Esfandiari and Khalil, 1991), or by continuous SMC design (Shtessel and Buffington, 1998).

Actually, the ALV system is always modeled ideally and these mathematical descriptions cannot completely describe the ALV motion. Hence, the robust sliding-mode control has been considered as a useful scheme for ALV maneuvers. An example of the application of SMC in the spacecraft was performed by Dywer and Sira-Ramirez (1988). However, with the sliding vector they introduced, complicated algorithms were resulted in their sliding-mode controller. Shtessel, et al., (1998, 2000) performed multiple-time-scale sliding modes for reusable launch vehicles (RLV) control. In this study pitch (longitudinal) channel was selected for study, and this is due to the fact that the major guidance commands for maneuvers are given in longitudinal plane.

SLIDING MODE CONTROLLER

An n^{th} order non-linear dynamic system can be given as:

$$\frac{d^n x(t)}{dt^n} = f(x,t) + b(x,t).u(t) + d(x,t) \tag{1}$$

where $x(t) = (x, x, ..., x^{(n-1)})$ is the state vector, $u(t)$ is the input vector, $d(x,t)$ is a disturbance with known upper bound and $f(x,t)$ and $b(x,t)$ are nonlinear functions determining the system characteristics. The sliding surface $s(x,t) = 0$ with initial conditions $e(0) = 0$ can be defined as:

$$s(x,t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} e \tag{2}$$

where λ is strictly positive real constant determining the slope of the sliding surfaces and e is the tracking error as

$$e = x - x_d = [e\ \dot{e}\ ...\ e^{(n-1)}] \tag{3}$$

where x_d is the desired trajectory. In this study, second order systems are considered. For $n = 2$, sliding surface can be written as:

$$s = \dot{e} + \lambda e \tag{4}$$

which is a linear function in terms of error. A homogeneous differential equation that has a unique solution $e = 0$ could be obtained by setting $s = 0$. Thus, the error will asymptotically reach zero with an appropriate control law that could keep the trajectory on the sliding surface. Lyapunov direct method could be used to obtain the control law that would maintain this goal candidate function is defined as:

$$V = \frac{1}{2} s^T s \tag{5}$$

with $V(0) = 0$ and $V(s) > 0$ for $s > 0$ (Slotine and Le 1991). Therefore, an efficient condition for stability of the system can be gives as:

$$\dot{V} = \frac{1}{2}\frac{d}{dt} s^2 \leq -\eta |s| \tag{6}$$

where η is a strictly positive real constant. Obtaining Eq. (6) means that the system is stable and controlled such that the state always moves toward and reaches the sliding surface. Therefore, Eq. (6) is called as the reaching condition for the sliding surface. Replacement of Eq. (4) in the equation of reaching condition results in an expression for obtaining a control input as

$$s.(f + b.u - \ddot{x} + \lambda e) \leq -\eta |s| \tag{7}$$

Therefore, a control input satisfying the reaching condition can be chosen as

$$u = \frac{-\hat{f} + \ddot{x} + \lambda e}{b} - \rho\, \text{sign}(s) \hat{=} u_{eq} + u_{disc} \tag{8}$$

where $\hat{f}$ is the estimated state equation, ρ is a strictly positive real constant with a lower bound depending on the estimated system parameters. u_{eq} is the continuous term that is known as equivalent control based on estimated system parameters and that compensates the estimated undesirable dynamics of the system. u_{disc} is the discontinuous control law that requires infinite switching at the intersection of error state trajectory and sliding surface. In this way, the trajectory is forced to move always towards the sliding surface.

THE EQUATIONS OF MOTION FOR A LAUNCH VEHICLE

The governing equations of motion for an ALV can be derived from Newton's second law of motion, which states that the summation of all external forces acting on a body should be equal to time rate of its momentum change, and the summation of the external moments acting on a body must be equal to the time rate change of its moment of momentum (angular momentum). Considering rigid airframe for an ALV, the 6-DOF equations of motions can be obtained as follows

(Blakelock, 1991).

$$
\begin{aligned}
F_x &= m(\dot{U} + qW - rV), \\
F_y &= m(\dot{V} + rU - pW), \\
F_z &= m(\dot{W} + pV - qU), \\
M_x &= I_x \dot{p}, \\
M_y &= I_y \dot{q} + (I_x - I_y)pr, \\
M_z &= I_z \dot{r} + (I_y - I_x)pq.
\end{aligned} \tag{9}
$$

Although the strength of sliding-mode controller lies in its ability to handle non-linearities in the control dynamics, the control design for a linear ALV is being considered in this paper, due to the fact that the ALV attitude control systems are commonly designed using linearized equations of motion. This is mainly because the nominal trajectory of the system is intended to maintain the ALV at near-zero angle of attack. This is normally attempted by programming the pitch attitude or pitch rate to yield a zero-g trajectory (Blaklock, 1991). Therefore, the assumption of near-zero angle of attack for the equilibrium condition is quite valid, and any changes in angle of attack can be considered perturbations from the equilibrium conditions. Thus, considering small perturbations, linearized equations of motion can be obtained as below:

$$\dot{v}_z = Z_V v_z + Z_q q + Z_\theta \theta + Z_{\delta e}\delta_e, \tag{10}$$

$$\dot{q} = M_{VZ} v_z + M_q q + M_{\delta e}\delta_e \tag{11}$$

$$\dot{v}_y = Z_V v_y + Z_r r + Z_\theta \theta + Z_{\delta r}\delta_r, \tag{12}$$

$$\dot{r} = M_{VY} v_y + M_y r + M_{\delta r}\delta_r, \tag{13}$$

$$\dot{p} = M_p p + M_{\delta a}\delta_a \tag{14}$$

where Z, M represent dynamic coefficients and the control force is provided by the deflection of trust vector shown by δ.

Eqs. (10) and (11) describe longitudinal equations of motions for ALV. Variation of longitudinal dynamic coefficients and ALV parameters during atmospheric part of powered flight using 6-DOF implementation are obtained as shown in Figure 1.

Note 1. For confidentiality reasons, flight time and parameters given are unit-less.

The servo dynamics describing the trust vector deflection is:

$$[TF]_{servo} = \frac{\delta}{\delta_c} = \frac{1}{0.1s + 1} \tag{15}$$

with a rate limit of $\left|\frac{d}{dt}\delta\right| < 25$ deg / sec. Reference signal of the control system is pitch rate, so that a rate gyro is used to measure pitch rate which has dynamics described as follows:

$$[TF]_{gyro} = \frac{(80\pi)^2}{s^2 + 40\pi s + (80\pi)^2} \tag{16}$$

SMC CONTROLLER SETUP

In this section, we present the design procedure for sliding-mode ALV pitch channel controller, and simulation results.

Pitch program of an ALV is provided by guidance system. Some guidance systems provide only a pitch program while some other also require that the control system be capable of accepting a commanded pitch rate.

The motion of tracking error dynamics is constrained by proper control action against the sliding surface of the form

$$s = \dot{\theta}_e + K_1\theta_e + K_2 \int_0^t \theta_e \mathrm{d}\tau \tag{17}$$

where $\theta_e = \theta_c - \theta$ and $K1, K_2 = Const.$ are chosen so that the output tracking error θ_e exhibits a desired linear asymptotic behaviour on the sliding surface $s = 0$. In this paper, we will consider $K_1 = K_2 = 1$. The objective of the SMC is to generate the control input δ_e necessary to cause the vehicle to track the commanded angular rate q_c. In other words, the control law δ_e is designed to provide asymptotic convergence of the system to the sliding surface $s = 0$. Dynamics of the sliding surface are described as

$$\dot{s} = \dot{q}_c - M_{VZ}v_z - M_q q - M_{\delta e}\delta_e + K_1\dot{\theta}_e + K_2\theta_e \tag{18}$$

The SMC design is initiated by choosing a candidate Lyapunov function of the form

$$V = \frac{1}{2}S^T S > 0 \tag{19}$$

whose derivative is shown as:

$$
\begin{aligned}
\dot{V} = s^T\dot{s} = s^T[\dot{q}_c - M_{VZ}v_z - M_q q - M_{\delta e}\delta_e \\
+ K_1\dot{\theta}_e + K_2\theta_e]
\end{aligned}
$$

To ensure asymptotic stability of the origin of the system in Eq. (18), the following derivative inequality of the candidate Lyapunov function is enforced (Decarlo, et al., 1988; Utkin, 1992; Hung et al., 1993; Slotine and Li, 1991):

$$\dot{V} = -\rho\, s^T \mathrm{sign}(s) \qquad \rho > 0 \tag{20}$$

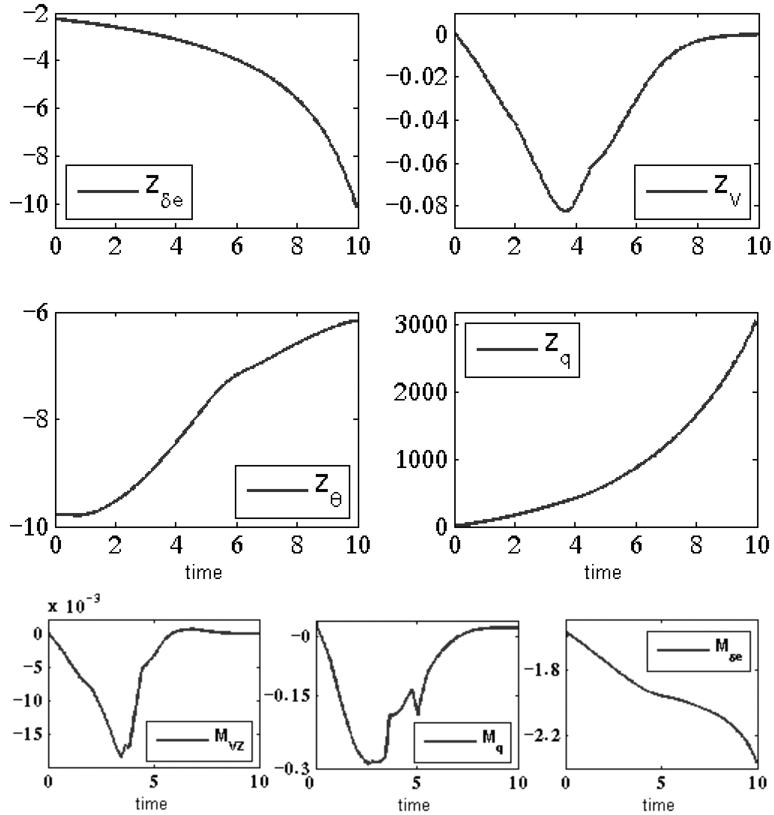


Figure 1. Longitudinal dynamic coefficients.

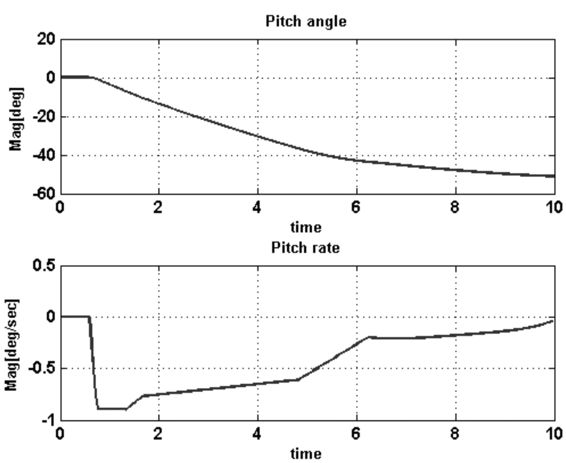


Figure 2. Desired pitch angle and pitch rate to be followed.

Considering Eq. (20), the required control δ_e to ensure asymptotic stability is defined as:

$$\delta_e = M_{\delta e}^{-1}[\dot{q}_c + K_1\dot{\theta}_e + K_2\theta_e - M_{VZ}v_z - M_q q + \rho\,\mathrm{sign}(s)] \quad (21)$$

The achieved controller in Eq. (21) is discontinuous; as a result, the control actuation will chatter during system operation on the sliding surface Eq. (17). Practically, this is an unwanted effect. Moreover, a discontinuous profile cannot be accurately tracked. To solve this problem, the discontinuous term sign(s) in Eq. (21) is replaced by the sat$(\frac{s}{\epsilon})$, where ϵ is a real small constant (Utkin, 1992; Slotine and Li, 1991).

SIMULATION RESULTS

(Comparison with Non-Linear Time-Varying Gain Scheduled Autopilot)

Using the above data we can now proceed with simulations and compare the results with non-linear time-varying gain scheduled autopilot. The input to the controller is pitch rate program illustrated in Figure 2 which has been previously designed off-line. Simulation results for nominal trajectory following in absence of any disturbance are shown in Figure 3. It can be seen clearly that the command signal is followed by sliding-mode very closely with slight errors in comparison to that of gain scheduled autopilot in both the pitch angle and the pitch program. Note that the design procedure for the gain scheduled (GS) autopilot, which is addressed for comparison with sliding mode, is introduced in Appendix A.

To demonstrate the robustness of the proposed sliding-mode control, the control algorithm was executed in the presence of a sample powerful gust. The examination of simulation results (Figure 4(a)) reveals that the proposed sliding-mode is quite robust to exerted wind disturbances and is able to reject the disturbance caused by wind. The wind profile is depicted in Figure 4(b).

The performance of sliding-mode control under sever uncertainties was also verified by altering dynamic coefficients with +%50 uncertainty in Z_θ and -%25 uncertainty in M_{VZ}. The results are shown in Figure 5 and 6, respectively. As illustrated in Figure 5 and 6, due to the robustness property of sliding-mode, the control system was able to compensate for uncertainty very well. Thus, sliding-mode provides much superior performance as compared to a popular method of non-linear and time varying control, the GS designed control law. The GS control elaborated in Appendix A, has been first reported by Filho and Hsu, (1986).

CONCLUSION

The design of variable structure based robust sliding-mode controller through the atmospheric flight of an ALV is considered in this paper. The designed controller can compensate for dynamical changes in the system during flight time, without any need for the pertaining and/or off-line computations. The closed-loop system performance using sliding-mode was compared with that of non-linear and time-varying gain scheduled controller and efficient results were obtained against typical wind disturbances, in addition to parameter uncertainties. The proposed sliding-mode controller provided much superior performance as compared to a popular method of non-linear and time varying control, GS designed control law. However, the chosen configuration of sliding-mode in this case has, as shown in Figure 3, already resulted in a much smaller control effort without any need for its further improvement.

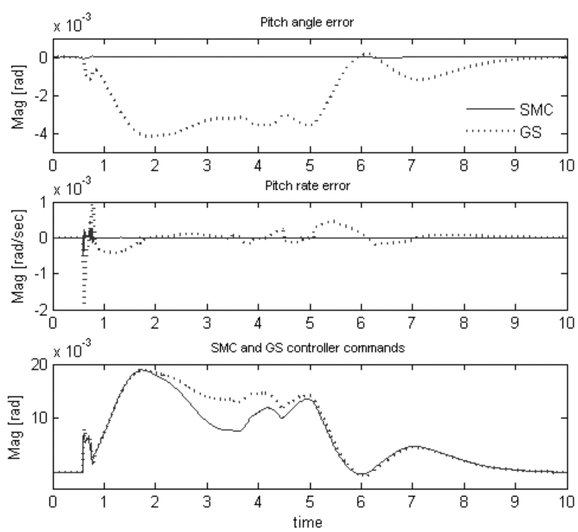
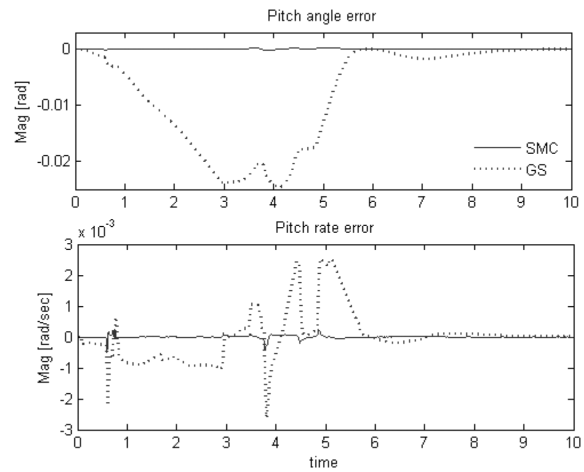
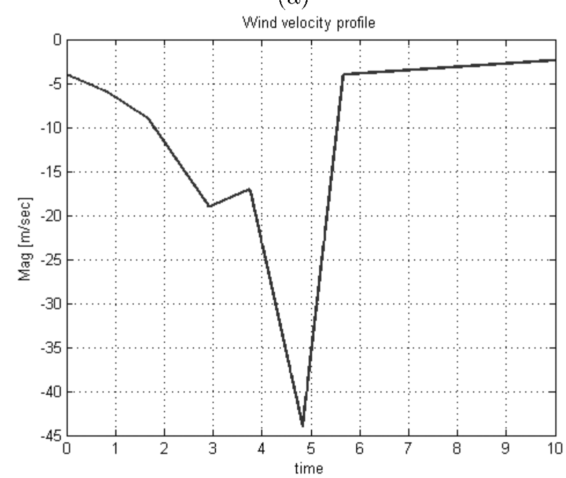


Figure 3. Pitch angle, pitch rate, realization error and controller command for designed SMC and gain scheduled autopilot.



(a)



(b)

Figure 4. (a) Pitch angle, pitch rate and realization error for designed SMC and gain scheduled autopilot in presence of gust. (b) Sample wind velocity profile.

Furthermore, the proposed control scheme was simple and convenient for implementation.

APPENDIX A

Block diagram of a typical classic ALV attitude control system is shown in Figure 7. In the scheme, pitch rate is considered as input of the system. However, the system is time varying, and transfer function cannot be defined in the usual way. Thus, the frozen pole approximation is used (Filho and Hsu, 1986; Malyshev and Krasilshikov, 1996) which assumes that the coefficients of vehicle model have constant values during a certain interval of time. In this way, a transfer function can be obtained. Using well-known methods of classical control theory, the control law may be obtained as follows:

$$G(s) = \frac{K_c(T_c s + 1)}{s} \qquad (A.1)$$

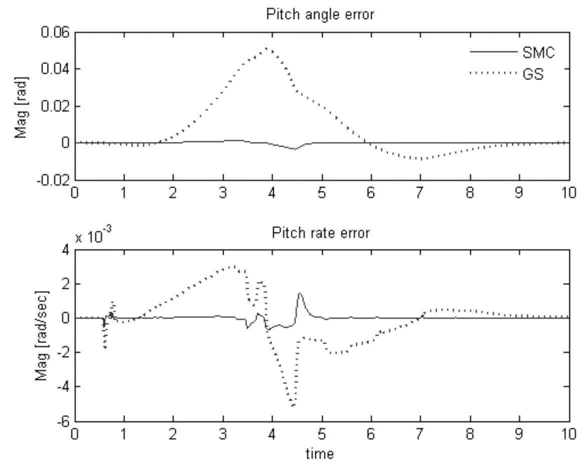


Figure 5. Pitch angle. Pitch rate realization error for designed SMC and gain scheduled autopilot with +%50 uncertainty in Z_θ.

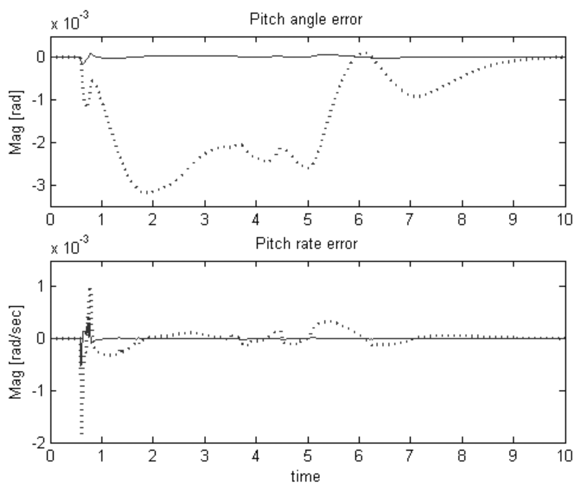


Figure 6. Pitch angle. Pitch rate realization error for designed SMC and gain scheduled autopilot with -%25 uncertainty in M_{VZ}.

A single fourth-order vector matrix equation defining the system is obtained from (10), (11), and (A.1). Defining the state vector by:

$$x = [v_z \;\; q \;\; \theta \;\; e] \qquad (A.2)$$

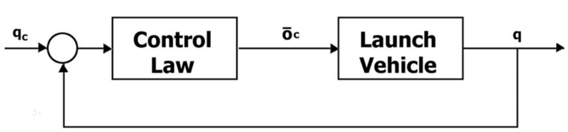


Figure 7. Classic pitch channel control scheme.

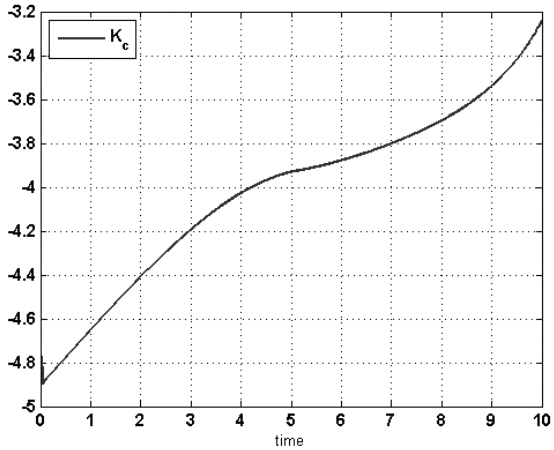


Figure 8. The scheduled gain, K_c, for classic pitch channel autopilot.

where $e = q_c - q$, we obtain the state-space equations:

$$\dot{x} = \begin{bmatrix} Z_V & Z_q - Z_{\delta e} K_c T_c & Z_\theta & Z_{\delta e} K_c \\ M_{VZ} & M_q - M_{\delta e} K_c T_c & 0 & M_{\delta e} K_c \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} Z_{\delta e} K_c T_c \\ M_{\delta e} K_c T_c \\ 0 \\ 1 \end{bmatrix} \qquad (A.3)$$

Eq. (A.3) shows that the term $M_{\delta e} K_c$ plays a main role in non-stationary characteristics of the ALV system. To reach an adequate performance with law sensitivity to wind disturbances, it is proposed to use gain scheduling for K_c. Knowing that the term $M_{\delta e}$ inversely depends on the longitudinal moment of inertial I_y (which experiences more than 30% reduction during atmospheric flight), its variation significantly alters the longitudinal dynamics of the vehicle. Therefore, constant value of the gain K_c obviously cannot satisfy the requirements of system performance. Thus, the gain K_c is computed to compensate for variation of $1/I_y$ or $M_{\delta e}$ during atmospheric flight as depicted in Fig. 8. As a result, the product $M_{\delta e} K_c$ will stay roughly constant; hence the designed controller will satisfy classical design criteria as stated below:

- Rise time = 0.3 sec;
- Overshoot = 30%;
- Settling time = 3 sec.

The mentioned procedure for gain scheduling essentially improves non-stationary behaviour of ALV in pitch channel (Filho and Hsu, 1986).

REFERENCES

1. Blakelock, J. H., *Automatic Control of Aircraft and Missiles*, Wiley, (1991).

2. Bletsos, N. A., "Launch Vehicle Guidance, Navigation, and Control", *Crosslink* , **5**, PP 30-33(2004).
3. Decarlo, R. A., Zak S. H., and Matthews G. P., "Variable Structure Control of Nonlinear Multivariable Systems: A Tutorial", *IEEE Proceedings*, **26**, PP 212-232(1988).
4. Dywer, T. A. W. III and Sira-Ramirez H., "Variable-Structure Control of Spacecraft Attitude Manoeuvres", *Journal of Guidance, Control, and Dynamic*, **11**, PP 262-270(1988).
5. Esfandiari, F. and Khalil H. K., "Stability Analysis of a Continuous Implementation of Variable Structure Control", *IEEE Transaction on Automatic Control*, **36**, PP 616-619(1991).
6. Filho, W. C. L., "Application of Adaptive Control Sounding Rocket Attitude", Proceedings of International Conference on Intelligent Autonomous Control in Aerospace, Beijing, China, (1995).
7. Filho, W. C. L. and Hsu L., "Adaptive Control of Missile Attitude", IFAC Adaptive systems in control and signal processing, Lund, Sweden, (1995).
8. Hung, J. Y., Gao W., and Hung J. C., "Variable Structure Control: A Survey", *IEEE Transactions on Industrial Electronics*, **40**, PP 2-21(1993).
9. Malyshev, V. V. and Krasilshikov M. N., "Aerospace Vehicle Navigation and Control", FAPESP Publication, Sao-Paulo, Brazil, (1996).
10. Shtessel, Y. and Buffington J., "Finite-Reaching-Time Continuous Sliding Mode Controller for MIMO Nonlinear Systems", Proceedings of the Conference on Decision and Control, **2**, PP 1934-1935(1998).
11. Shtessel, Y., Hall C., and Jackson M., "Reusable Lunch Vehicle Control in Multiple-Time-Scale Sliding Modes", *Journal of Guidance, Control, and Dynamic*, **23**, PP 1013-1020(2000).
12. Slotine. J.-J. and Li W., *Applied Nonlinear Control*, Prentice-Hall, NJ., PP 276-309(1991).
13. Utkin, V. I., "Variable Structure Systems with Sliding Modes", *IEEE Transaction on Automatic Control*, **22**, PP 212-222(1977).
14. Utkin, V. I., *Sliding Modes in Control and Optimization*, Springer-Verlag, Berlin, PP 111-130(1992).
15. Vadali, S. R., "Variable-Structure Control of Spacecraft Large Angle Maneuvers", *Journal of Guidance, Control, and Dynamic*, **9(**235-23), (1986).