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Comparison Between Minimum and near Minimum Time Optimal Control of a Flexible Slewing Spacecraft

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In this paper, minimum and near-minimum time optimal control laws are developed and compared for a rigid space platform with flexible links during an orientating maneuver with a large angle of rotation. The control commands are considered as typical bang-bang with multiple symmetrical switches, the time optimal control solution for the rigid-body mode is obtained as a bang-bang function and applied to the flexible system after smoothening the control inputs to avoid stimulation of the flexible modes. This will also reflect practical limitations in exerting bang-bang actuator forces/torques due to delays and non-zero time constants of existing actuation elements. The smoothness of the input command is obtained by reshaping its profile based on consideration of additional derivative constraints. the optimal control problem is converted into parameter optimization problem. The steps of the solution procedure and numerical algorithm to obtain the time optimal control input are discussed next. The developed control law is applied to a given satellite during a slewing maneuver. The simulation results show that the control input with just a few switching times can significantly lessen the vibrating motion of the flexible appendage, which reveals the merits of the developed control law. The modified realistic optimal input compared to the bang-bang solution goes well with the practical limitations and alleviates the vibrating motion of the flexible appendage, which reveals the merits of the new developed control law.

INTRODUTION

Space robotic systems are expected to play an important role in future, e. g. in servicing, construction, and maintenance of space structures in orbit. Before long, coordinated teams of robots might deploy, transport, and assemble structural modules for a large space structure [1]. In order to control such systems, it is essential to develop proper kinematics/dynamics model for the system. This has been studied under the assumption of rigid elements, [2-4], and elastic elements [5-8]. There have also been various studies on the control problem of such systems with both rigid and flexible elements [9-13].

Due to maneuver time limitations in space, the optimal control with a time minimization constraint is of main concern. It should be noted that high speeds, in turn, may stimulate the system flexible modes, which may drastically affect the control system performance. Space projects involving large structures and satellites with antennas or solar panels in general as well as robotic manipulators are examples where one should consider achieving rapid maneuvers without stimulating flexible modes [14]. Therefore, the minimum-time optimal control for the rigid mode and n flexible modes has become the focus of several articles [15-18]. Robust time-optimal control problems for slewing spacecraft have recently received a lot of attention [19-24]. In this paper, a minimum-time optimal control law for a flexible spacecraft during a slewing maneuver with a large angle of rotation is developed without

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solving the state and co-state equations. The control commands are considered as typical bang- bang with multiple symmetrical switches. The obtained control law for the rigid-body mode is applied to the flexible system after smoothening the control inputs to reflect practical limitations in exerting bang-bang actuator forces/torques. Smoothness of the input command is obtained through reshaping its profile with the first and second time derivative constraints. The steps of the solution procedure and numerical algorithm to obtain the time optimal control input will be discussed next. The developed control laws are applied to a given satellite during a slewing maneuver, where the first five modes are considered in the simulated model, whereas a single torque actuator is located on the central rigid body. The simulation results show that the control input can significantly cope with the end-point motion of the flexible appendage with just a few switching times. the developed realistic optimal input compared to the bang-bang solution goes well with the practical limitations and can successfully control the end-point motion of the flexible appendages.

PROBLEM FORMULATION

Considering a linear model of a flexible spacecraft with one rigid-body mode and n flexible modes during a slewing maneuver, the system can be represented as:

$$M\ddot{q} + Kq = Gu \tag{1}$$

where M and K are the so-called mass and stiffness matrices, respectively, and G represents the control input distribution. The system described by Eqs. (1) can be transformed into the decoupled modal equations using the eigenvalue and eigenvector information:

$$\ddot{q}_i + \omega_i^2 q_i = \Phi_i u \qquad i = 1, \cdots, n \tag{2}$$

where q_i (t) is the i-th modal coordinate, ω_i is the i-th modal frequency (i-th diagonal element of eigenvalue matrix), and scalars ϕ_i are defined by:

$$\begin{bmatrix}\Phi_1 & \Phi_2 & \cdots & \Phi_n\end{bmatrix}^T = \Lambda G \tag{3}$$

where Λ is an $n \times n$ matrix whose columns are the corresponding eigenvectors, and n is the number of modes considered in the control design. The control input u (t) is a single bounded one:

$$-u_{\max} \leq u(t) \leq u_{\max} \tag{4}$$

where u_{max} is the maximum value of control input. It is desirable to convey the system described by Eqs. (2) from the initial conditions $q(0) =$, to final conditions $q(t_f)$ = subjected to the control constraints (4) in minimum time. Therefore, the performance index can be defined as:

$$J = \int_0^{t_f} dt = t_f \tag{5}$$

where the initial time t_o is set equal to zero and t_f is the final time of the maneuver.

TIME OPTIMAL CONTROL DESIGN

From the optimal control theory and Ponteryagins minimum principle,[25], it is known that the solution of the above time optimal control problem is in the form of bang-bang control input with (2n-1) switches symmetric about $t = t_f/2$. A bang-bang input with (2n-1) switches can be represented as:

$$u(t) = u_{\max} \sum_{j=0}^{2n} b_j \hat{1}(t - t_j) \tag{6}$$

where b_j defines the magnitude coefficient at t_i, $\hat{1}(t)$ defines unit step function, and $t_{2n} = t_f$. To obtain the switching times t_j, one should obtain the constrains of the problem. Considering the rigid body mode equation with $\omega_1 = 0$ yields:

$$\ddot{q}_1 = \Phi_1 u \tag{7a}$$

with the following initial conditions:

$$q_1(0) = 0, \quad q_1(t_f) = \theta_f$$
$$\dot{q}_1(0) = 0, \quad \dot{q}_1(t_f) = 0 \tag{7b}$$

Substituting Eq. (6) into Eq. (7-a) and integrating with respect to time twice, using initial conditions, we obtain:

$$\theta_f = \frac{\Phi_1 u_{\max}}{2} \sum_{j=0}^{2n} b_j (t_f - t_j)^2 \tag{8}$$

which describes the constraint for the rigid body motion mode. Next the flexible modes should be considered:

$$\ddot{q}_i + \omega_i^2 q_i = \Phi_i u \quad i = 2, \cdots, n \tag{9}$$

where all related initial conditions are set equal to zero. Substituting Eq. (6) into Eq. (9), and following a similar procedure we obtain:

$$q_i(t) = -\frac{\Phi_i u_{\max}}{\omega_i^2} \sum_{j=0}^{2n} b_j \cos\omega_i(t - t_j) \qquad i \geq 2 \tag{10}$$

This equation can be rewritten as:

$$q_i(t) = -\frac{\Phi_i u_{\max}}{\omega_i^2} \left[\cos\omega_i(t - t_n) \sum_{j=0}^{2n} b_j \cos\omega_i(t_j - t_n) + \sin\omega_i(t - t_n) \sum_{j=0}^{2n} b_j \sin\omega_i(t_j - t_n)\right] \tag{11}$$

Note that the sine function is an odd one and t_j is symmetric about t_n which is equal to $t_f/2$, hence the second term vanishes, and the following holds for any bang-bang input:

$$\sum_{j=0}^{2n} b_j \sin\omega_i(t_j - t_n) = 0 \qquad (12)$$

therefore, to have $q_i(t) = 0$ for $t \geq t_f$, i.e. no residual structural vibration, the following flexible mode constraints is obtained:

$$\sum_{j=0}^{2n} b_j \cos\omega_i(t_j - t_n) = 0 \qquad i \geq 2 \qquad (13)$$

This equation gives the necessary constraints for solving this problem.

REALISTIC NEAR MINIMUM TIME OPTIMAL CONTROL DESIGN

According to Eq.(6), the optimal control input for rigid-body mode will be obtained as:

$$u_1(t) = u_{\max}\left[1(t) - 2[1(t-t_1)] + 1(t-t_f)\right] \qquad (14)$$

In this Section, this control input profile is approximated by a smooth and continuous profile throughout the entire maneuver, with the saturation limits of $\pm u_{max}$. Furthermore, this will reflect practical limitations in exerting bang-bang actuator forces/torques in reality, due to delays and non-zero time constants of existing actuation elements. Therefore, a realistic optimal (near-minimum time) control law is found that eliminates the jump-discontinuities of the input torque in order to reduce structural vibrations. By this near optimal approach, we can "tune" the control profile in such a way that systematically trades off residual vibration with the maneuver time. To this end, the time derivative constrains of the control input can be used. In the following, two cases are considered as employing the first and second derivatives for reshaping the control input profile.

Case I. First derivative constraint of control input

The bang-bang input for rigid body mode obtained in the previous section is shown in Figure (1a). An approximated control input that is smoother than the bang-bang input is shown in Figure (1c). This control

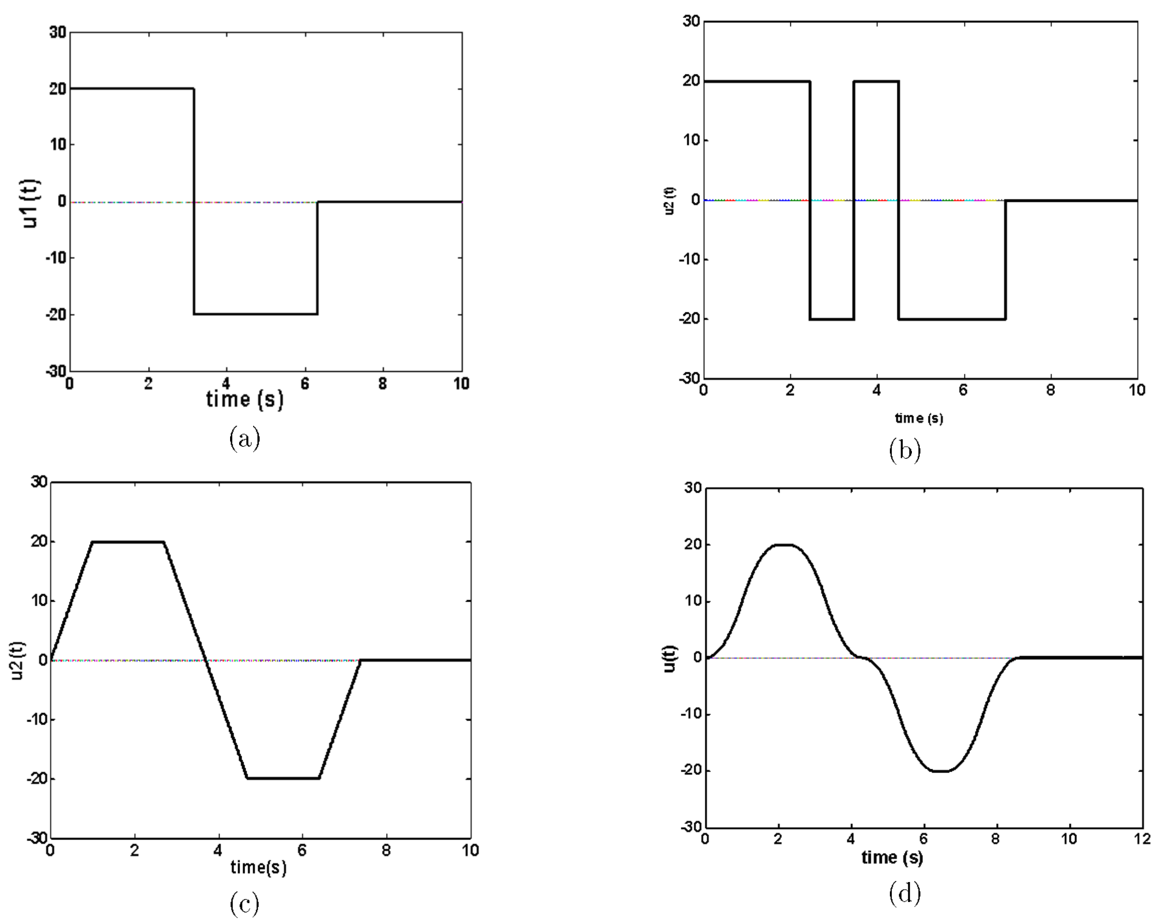
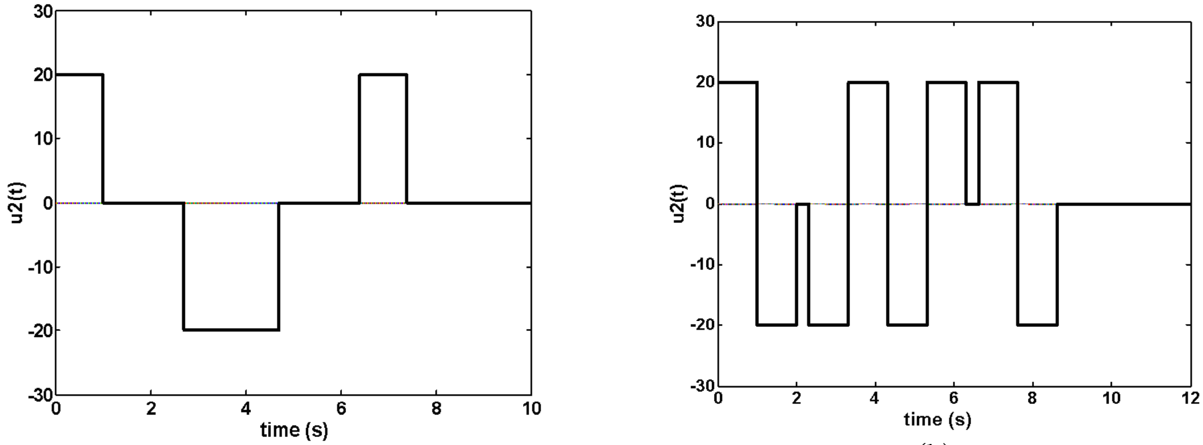


Figure 1. Control input profiles: (a)Bang-Bang, rigid body mode solution, $u_1(t)$, (b) Bang-Bang, first flexible mode solution, $u_2(t)$, (c) Case I, $u_3(t)$, (d) Case II, $u_4(t)$.



(a) (b)

Figure 2. Control input derivatives: (a) First rate of $u_3(t)$, (b) Second rate of $u_4(t)$.

input is obtained by adding another state variable to the first time optimal control problem which describes the first time derivative of control input, along with an additional constraint that confines the magnitude of this derivative to a given value. Consequently, the degree of smoothness of the generated control input is controlled by choosing an appropriate value for the maximum value of the input first time derivative. Considering the new Hamiltonian for the three state variables, and using Ponteryagin's minimum principle, as will be discussed in next section, the modified control input is obtained as:

$$u_3(t) = a u_{\max} \sum_{j=0}^{5} b_{1j}(t-t_j)1(t-t_j) \tag{15}$$

where $b_1=[1,-1,-1,1,1,-1]$, 1(t-tj) defines the unit step function, t0=0, t5=tf , and "a" is the slope of the inclined lines that is the maximum value of the input first time rate, and controls the smoothness of the modified input $u_2(t)$. The rate of this control input is shown in Figure (2a), which certainly satisfies the given limits.

Case II. Second derivative constraint of control input

To make the control input smoother than the one computed in the previous case, one could add a fourth state variable to the previous time optimal control problem which describes the second time derivative of control input, along with an additional constraint that confines the magnitude of the second rate to a given value, Figure (1d). Following a similar procedure as described above, the modified control input in this case is obtained as:

$$u_4 = \frac{a u_{\max}}{2} \sum_{j=0}^{10} b_{2j}(t-t_j)^2 1(t-t_j) \tag{16}$$

where b_2=[1,-1,-1,1,1,-1,-1,1,1,-1], 1(t-tj) defines the unit step function, t0=0, t5=tf , and "a" is the maximum value of the input second time rate. The second rate of $u_3(t)$ is shown in Figure (2b).

PARAMETER OPTIMIZATION PROBLEM

To solve the minimum and near minimum time optimal control problem, we have to determine (2n-1) unknown switching times such that the final time t_f is minimized. This can be formulated as a constrained parameter optimization problem, i.e. minimization of the performance index of Eq. (5) subjected to the following constraints:

$$f_1(t_1, t_2, \cdots, t_j, \cdots, t_{2n}) = \theta_f - \frac{\Phi_1 u_{\max}}{2} \sum_{j=0}^{2n} b_j (t_f - t_j)^2 = 0 \tag{17a}$$

$$f_i(t_1, t_2, \cdots, t_j, \cdots, t_{2n}) = \sum_{j=0}^{2n} b_j \cos\omega_i (t_j - t_n) = 0 \qquad i = 2, ..., n \tag{17b}$$

To satisfy the necessary and sufficient condition for optimality, the Hamiltonian can be introduced as:

$$H = t_f + \lambda_i f_i \qquad i = 1, ..., n \tag{18}$$

where λ_i are defined as Lagrange multiplier. Setting up the following equations, a set of 3n equations, can be solved to determine 3n unknowns, i.e. (2n-1) switching times, one final time t_f, and n Lagrange multipliers:

$$g_j = \frac{\partial H}{\partial t_j} = 0 \quad j = 1, 2, \cdots, 2n$$
$$g_k = \frac{\partial H}{\partial \lambda_i} = 0 \quad k = 2n+1, \cdots, 3n \tag{19}$$

This set of equations are often coupled and nonlinear, which can be solved employing numerical methods[26].

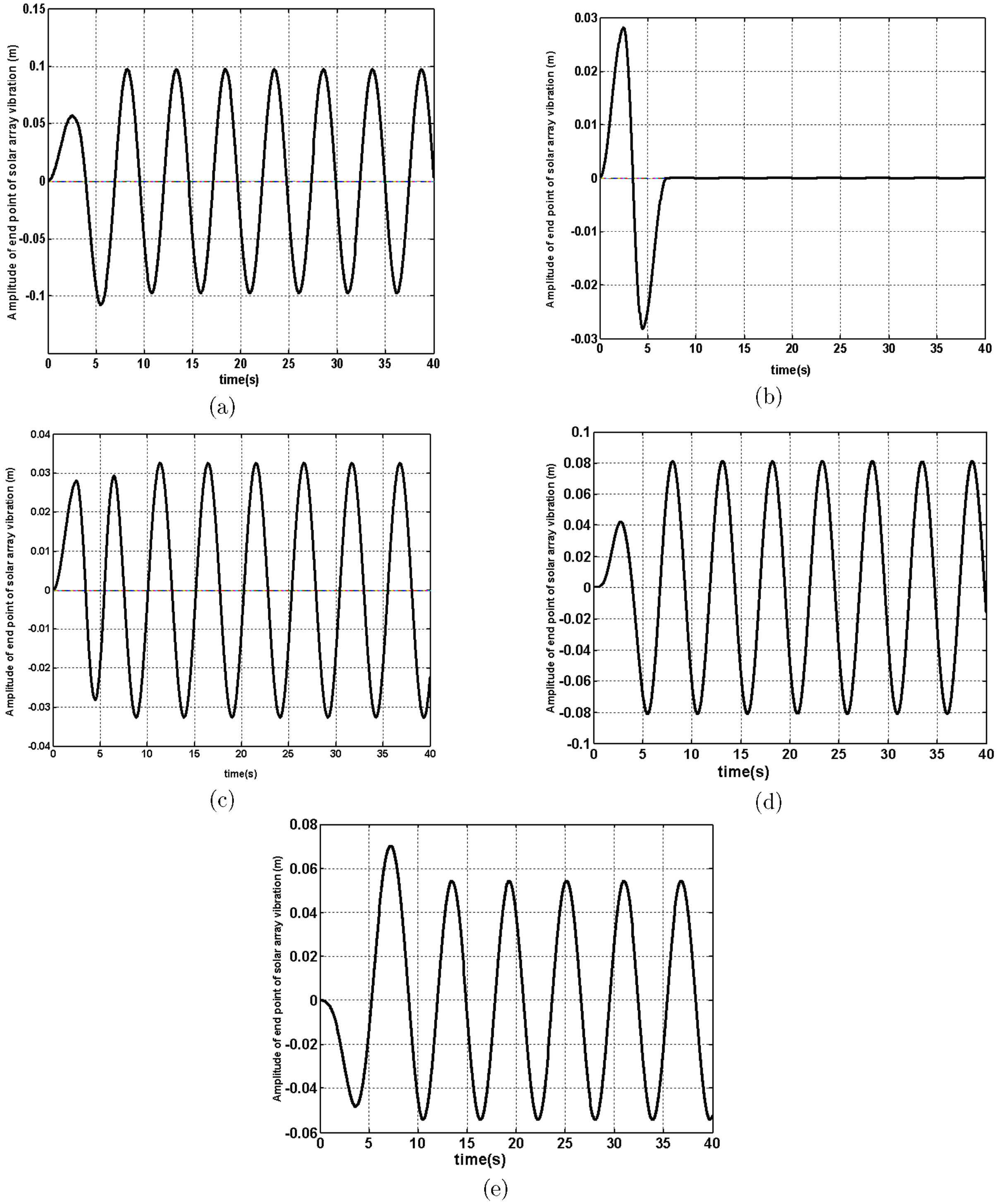


Figure 3. Mode responses: (a) First flexible mode response to $u_1(t)$, (b) First flexible mode response to $u_2(t)$, (c) Second flexible mode response to $u_2(t)$, (d) First flexible mode response to $u_3(t)$, (e) First flexible mode response to $u_4(t)$.

SOLUTION ALGORITHM

Collecting g_i functions defined by Eq. (19) in a $(3n\times 1)$ vector as:

$$\mathbf{g} = [g_1\, g_2\, \cdots\, g_{2n}\, \cdots\, g_{3n}]^T \tag{20}$$

the steps of the solution procedure and numerical algorithm to obtain the time optimal control and robust time optimal control inputs are given below.

1. Determine the numbers of flexible modes (n).
2. Define the bang-bang input with (2n-1) switches.

3. Apply this control input function for rigid body mode and flexible modes to obtain constraints of problem with performance index, J=t_f. At this step, the time optimal control problem is converted into parameter optimization problem. Parameters of this problem can be reduced by consideration of time symmetry property of bang- bang input function.
4. Form vector **g**, using Eqs. (17)- (20).
5. Form a $(3n \times 1)$ vector **h** for unknowns:

$$\mathbf{h} = [t_1, \cdots, t_{2n}, \lambda_1, \cdots, \lambda_n]^T \tag{21}$$

6. Assume some starting values for **h** as $\mathbf{h_o}$.
7. Calculate the $(3n \times 3n)$ Jacobian matrix:

$$J = \frac{\partial g}{\partial h} = \begin{bmatrix} \frac{\partial g_1}{\partial t_1} & \cdots & \frac{\partial g_1}{\partial t_{2n}} & \frac{\partial g_1}{\partial \lambda_1} & \cdots & \frac{\partial g_1}{\partial \lambda_n} \\ \frac{\partial g_2}{\partial t_1} & \cdots & \frac{\partial g_2}{\partial t_{2n}} & \frac{\partial g_2}{\partial \lambda_1} & \cdots & \frac{\partial g_2}{\partial \lambda_n} \\ \vdots & & & & & \\ \frac{\partial g_{3n}}{\partial t_1} & \cdots & \frac{\partial g_{3n}}{\partial t_{2n}} & \frac{\partial g_{3n}}{\partial \lambda_1} & \cdots & \frac{\partial g_{3n}}{\partial \lambda_n} \end{bmatrix} \tag{22}$$

where $J_{ij} = \partial g_i / \partial h_j$ are calculated using the current values of **h**.

$$\mathbf{\Delta} = \mathbf{J}^{-1}\mathbf{g} \tag{23}$$

8. Update the unknown variables

$$\mathbf{h} = \mathbf{h_c} - \mathbf{\Delta} \tag{24}$$

where $\mathbf{h_c}$ denotes the current value of **h**.
9. Repeat Steps 7-9 until:

$$||g|| \leq \epsilon \tag{25}$$

where ||...|| refers to Euclidean norm, and ϵ is a chosen threshold.
10. The unknown variables are obtained as

$$\mathbf{h} = \mathbf{h_c} \tag{26}$$

Next, to illustrate the developed optimal control law and described numerical procedure, the slewing maneuver of a given satellite is simulated.

SIMULATIONS

The system parameters and maneuver specifications are listed in Table 1. To see the inherent behavior of the system, the first five modes are retained in the developed model in the simulation routine prepared in MATLAB environment, in which a single torque actuator is located on the rigid central body to control the maneuver. The task is to control the satellite orientation during a rest-to-rest maneuver in minimum time. Table 2 shows the natural frequencies ω_i, in radian per second , and the components of ϕ_i in Eq. (2) , for the first five modes. For the first trial, just the rigid body mode is considered, i.e. n=1, and so there exists just one switching time and the control torque will be defined as Eq.(17). By applying the presented algorithm, the middle and final time, t_1 and t_f , are obtained as shown in Table 3. The input amplitude is given u_{max}=20N.m as shown in Figure (1a). To find the vibration of the end point of appendages (solar panels), due to this input torque, if we solve the equation:

$$\ddot{y_2} + \omega_2^2 y_2 = \Phi_2 u_1 \tag{27}$$

and transform the solution back to the physical coordinate, the end point vibration will be obtained as Figure (3a). As seen, the amplitude is considerably large and may cause significant damage to the spacecraft. Therefore, at least the first flexible mode, i.e. n=2, should be considered. To consider the first flexible mode (n=2), in order to apply the control input $u_2(t)$, three switching times will be introduced as:

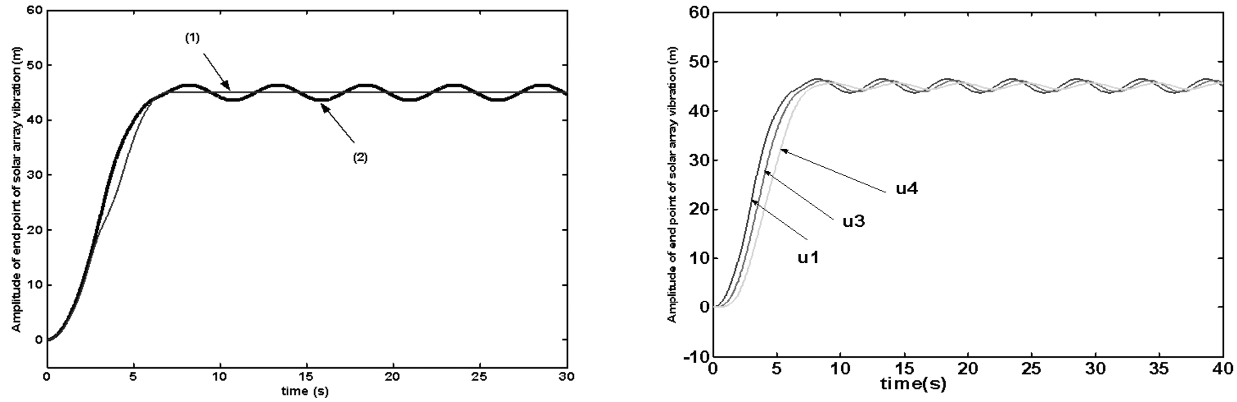
$$u_2(t) = u_{\max}\{\hat{1}(t) - 2[\hat{1}(t-t_1)] + 2[\hat{1}(t-t_2)] - 2[\hat{1}(t-t_3)] + \hat{1}(t-t_4)\} \tag{28}$$

According to the presented algorithm, the switching and final times are obtained as shown in Table 3, and the control input is illustrated in Figure (1b).

The response of the flexible appendage is shown in Figure (3b). As shown in Figure (3b), by applying $u_2(t)$, the vibration of the appendage in its first flexible mode does completely vanish; however, it seems that the second flexible(n=3) mode is excited. Therefore, to investigate this, the amplitude of vibrations for the second flexible mode is shown in Figure (3c). As seen in the figure, the amplitude is about 32 mm, which is reasonably small. Application results of realistic modified control input $u_3(t)$ and $u_4(t)$, obtained in the case I and II, are shown in the Figure (3d-e). Comparison of vibrations of the endpoints of appendages show

Table 1. System Parameters and Maneuver Specifications

Central body inertia	I_1	132 Kgm^2
	I_2	77 Kgm^2
	I_3	135 Kgm^2
Solar panels Length	L	4m
Solar panels Thickness	t	0.02 m
Solar panels Width	w	0.50 m
Solar panels material stiffeness	EI	20.10 Nm^2
Solar panels material density	ρ	0.81 Kg/m^2
Maximum torque avilable	u	20 N.m
Total mass of spacecraft	M	800 Kg
Total slewing angle	θ_f	45 deg



(a) (b)

Figure 4. Attitude of end point of solar array for: (a) input $u_1(t)$ and $u_2(t)$, (b) $u_3(t)$, $u_4(t)$.

Table 2. Flexible Modes Specifications.

i	ω_i	ϕ_i
1	0	0.0628
2	1.2355	-0.0328
3	6.9311	0.0092
4	19.3320	0.0043
5	38.2100	-0.0026

Table 3. Switching and final maneuver times (sec.).

$u_1(t)$	$u_2(t)$	$u_3(t)$	$u_4(t)$
t_1=3.155	t_1=2.446	t_1=1.000	t_1=1.000
t_f=6.311	t_2=3.474	t_2= 2.690	t_2= 2.000
	t_3=4.502	t_3=4.694	t_3=2.310
	t_f=6.9486	t_4=6.389	t_4=3.310
		t_f=7.389	t_5=4.310
			t_6=5.310
			t_7=6.310
			t_8=6.620
			t_9=7.620
			t_f=8.620

that by application of $u_3(t)$, the amplitude of vibration compared to that of $u_1(t)$ has reduced by 2 cm. For more reduction, one can increase the value of "a" that is taken equal to one so far , but this results in a trade off between maneuver time and amplitude of the vibration. Application of $u_4(t)$ is a better approach for vibration suppression and maintaining the maneuver time near its minimum value. To this end, switching times and the final maneuver time are obtained as shown in Table 3.

The attitudes of the appendages are illustrated in Figure (4a) for $u_1(t)$ and $u_2(t)$, and in Figure (4b) for $u_3(t)$ and $u_4(t)$, and the vibration of the endpoint of appendages are shown in Fig. (3d-e). As seen, the amplitude has reduced to 5.3 cm, which shows a drastic suppression of the endpoint vibration under application of $u_4(t)$. Comparing the maneuver duration in these cases, it can be seen that application of $u_3(t)$ results in 2.309s increase of maneuver time where tf=6.311s is the minimum duration obtained for $u_1(t)$, while reduces the amplitude of appendages vibration by 55%. It should be noted that the vibration amplitude has been decreased by 99% due to $u_2(t)$, compared to that of $u_1(t)$, whereas the maneuver time has increased by 31%.

CONCLUSIONS

In this paper, minimum and a near-minimum-time optimal control law for a rigid space platform with flexible links during an orientating maneuver with large angle of rotation was developed. The time optimal control solution for the rigid-body mode was obtained as a bang-bang function, and applied to the flexible system before and after smoothening the control inputs to reflect practical limitations and flexible modes ignoring in exerting bang-bang actuator forces/torques. The first flexible mode consideration reduced the amplitude of vibration very well. The modified control input was obtained by adding additional state variables to the original time optimal control problem to describe derivatives of control input, along with additional constraints that confine the magnitude of the derivatives to given values. The steps of the solution algorithm as converting of time optimal control problem to parameter optimization problem to obtain the switching and final times of control profiles were discussed next. The developed control law was applied to a given satellite consisting of two elastic panels during a slewing maneuver.

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