JAST, Vol. 2, No. 4, pp 37–43

JAST Journal of Aerospace Science and Technology

Robust Optimal Control of Flexible Spacecraft During Slewing Maneuvers

A. Ebrahimi[1], S. Ali A. Moosavian[2], M. Mirshams[3]

In this paper, slewing maneuver of a flexible spacecraft with a large angle of rotation is considered and, assuming structural frequency uncertainties, a robust minimum-time optimal control law is developed. Considering typical bang-bang control commands with multiple symmetrical switches, a parameter optimization procedure is introduced to find the control forces/torques. The constrained minimization problem is augmented with the robustness constraints, which in turn increases the number of switches in the bang-bang control input to match the total number of the constraint equations. The steps of the solution algorithm to obtain the time optimal control input are discussed next. The developed control law is applied to a given satellite during a slewing maneuver. The simulation results show that the robust control input with just few switching times can significantly lessen the vibrating motion of the flexible appendage in the presence of structural frequency uncertainties, which reveals the merits of the developed control law.

INTRODUTION

Space robotic systems are expected to play an important role in future, e. g. in the servicing, construction, and maintenance of space structures in orbit. Before long, coordinated teams of robots might deploy, transport, and assemble structural modules for a large space structure [1]. In order to control such systems, it is essential to develop a proper kinematics/dynamics model for the system. This has been studied under the assumption of rigid elements [2-4], and elastic elements [5-8]. There also have been various studies on the control problem of such systems with both rigid and flexible elements [9-13].

Due to maneuver time limitations in space, the optimal control with a time minimization constraint is of main concern. It should be noted that high speeds, in turn, may stimulate the system flexible modes which may drastically affect the control system performance. Space projects involving large structures and satellites with antennas or solar panels in general, and robotic manipulators are examples where one should consider achieving rapid maneuvers without stimulating flexible modes [14]. Therefore, the minimum-time optimal control for the rigid mode and n flexible modes has become the focus of several articles [15-19]. Robust time-optimal control problems for slewing spacecraft have recently received attention [20-24]. In all of these published works, the time-optimal controller is obtained by solving the state and co-state equations, considering the Ponteryagins minimum principle.

Spacecraft and satellites in orbit usually operate in the presence of various disturbances, including gravitational torque, aerodynamic torque, radiation torque, and other environmental and nonenvironmental torques. The problem of disturbance rejection is of main concern particularly in the case of Low-Earth-Orbiting satellites that operate in the altitude ranges where their dynamics are substantially affected by most of the preceding disturbances. In addition, as some dynamic parameters of spacecraft are not exactly known, the controller design should take these parametric uncertainties into account. In this paper, a robust minimum-time optimal control law for slewing maneuver of a flexible spacecraft with large angle of rotation is developed. A linear model for a flexible spacecraft with one rigid-body mode and n flexible

1. *Ph.D. Candidate, Research Assistant at Applied Science and Research Association (ASRA).*
2. *Associate Professor, Dept. of Mechanical Eng. K. N. Toosi Univ. of Tech., Tehran, Iran, P.O.Box 16765-3381, Email: Moosavian@kntu.ac.ir.*
3. *Assistant Professor, Dept. of Mechanical Eng., Tehran, Iran, P.O.Box 16765-3381.*

modes is considered. The time-optimal control law for this model is developed, this control law is a typical bang-bang control commands with multiple symmetrical switches. A new approach is expanded for generating robust time-optimal control inputs for the single-axis, rest-to-rest maneuvering problem of flexible spacecraft in the presence of structural frequency uncertainty. A parameter optimization problem, where the objective function to be minimized is the maneuvering time, is formulated with additional constraints for robustness with respect to structural frequency uncertainty. The resulting robustified, time-optimal solution is a multiswitch bang-bang control. The developed control law is applied on a given satellite equipped with on-off reaction jets during a slewing maneuver.

PROBLEM FORMULATION

Considering a linear model of a flexible spacecraft with one rigid-body mode and n flexible modes during a slewing maneuver, the system can be represented as:

$$M\ddot{q} + Kq = Gu, \tag{1}$$

where M and K are the so-called mass and stiffness matrices, respectively, and G is the control input distribution. The system described by Eq. (1) can be transformed into the decoupled modal equations using the eigenvalue and eigenvector information of the system (for more details see [25]):

$$\ddot{q}_i + \omega_i^2 q_i = \Phi_i u, \qquad i = 1, ..., n \tag{2}$$

where $q_i(t)$ is the i-th modal coordinate, ω_i is the i-th modal frequency (i-th diagonal element of eigenvalues matrix), and scalars Φ_i are defined by:

$$[\ \Phi_1 \quad \Phi_2 \quad \cdots \quad \Phi_n]^T = \Lambda G, \tag{3}$$

where Λ is an $n \times n$ matrix whose columns are the corresponding eigenvectors, and n is the number of modes considered in control design. The control input u(t) is a single bounded one:

$$-u_{\max} \le u(t) \le u_{\max}, \tag{4}$$

where u_{max} is the maximum value of the control input. It is desirable to convey the system described by Eqs. (2) from the initial conditions $q(0) = [0\,0\,0 \cdots 0]^T$, to final conditions $q(t,f) = [\theta_f\,0\,0 \cdots 0]^T$ subjected to the control constraints (4) in minimum time. Therefore, the performance index can be defined as:

$$J = \int_0^{t_f} dt = t_f, \tag{5}$$

where the initial time t_o is set to zero and t_f is the final time of the maneuver.

TIME OPTIMAL CONTROL DESIGN

From the optimal control the ory and Ponteryagins minimum principle, [26], it is known that the solution of the above time optimal control problem is in the form of bang-bang control input with (2n-1) switches. The switching times are distributed symmetrically with respect to $t = t_f/2$. A bang- bang input with (2n-1) switches can be represented as:

$$u(t) = u_{\max} \sum_{j=0}^{2n} b_j \hat{1}(t - t_j), \tag{6}$$

where b_j defines the magnitude coefficient at t_j, $\hat{1}(t)$ defines unit step function, and $t_{2n} = t_f$.

To obtain the switching times t_j, one should obtain the constraints of the problem. Considering the rigid body mode equation with $\omega_1 = 0$, yields:

$$\ddot{q}_1 = \Phi_1 u, \tag{7a}$$

with the following initial conditions:

$$\begin{array}{ll} q_1(0) = 0, & q_1(t_f) = \theta_f, \\ \dot{q}_1(0) = 0, & \dot{q}_1(t_f) = 0. \end{array} \tag{7b}$$

Substituting Eq. (6) into Eq. (7-a) and integrating with respect to tit to time twice, using initial conditions, yields to:

$$\theta_f = \frac{\Phi_1 u_{\max}}{2} \sum_{j=0}^{2n} b_j (t_f - t_j)^2, \tag{8}$$

which describes the constraint for the rigid body motion mode. Next the flexible modes should be considered

$$\ddot{q}_i + \omega_i^2 q_i = \Phi_i u, \qquad i = 2, ..., n \tag{9}$$

where all related initial conditions are set to zero. Substituting Eq. (6) into Eq. (9) and following a similar procedure yields to:

$$q_i(t) = -\frac{\Phi_i u_{\max}}{\omega_i^2} \sum_{j=0}^{2n} b_j \cos\omega_i (t - t_j), \qquad i \ge 2 \tag{10}$$

Substituting $(t - t_j) = (t - t_n) - (t_j - t_n)$, Eq. (10) can be rewritten as:

$$q_i(t) = -\frac{\Phi_i u_{\max}}{\omega_i^2} \left[\cos\omega_i (t - t_n) \sum_{j=0}^{2n} b_j \cos\omega_i (t_j - t_n) + \sin\omega_i (t - t_n) \sum_{j=0}^{2n} b_j \sin\omega_i (t_j - t_n) \right]. \tag{11}$$

Note that the sine function is an odd one and t_j is symmetric about t_n which is equal to t_f /2, hence the

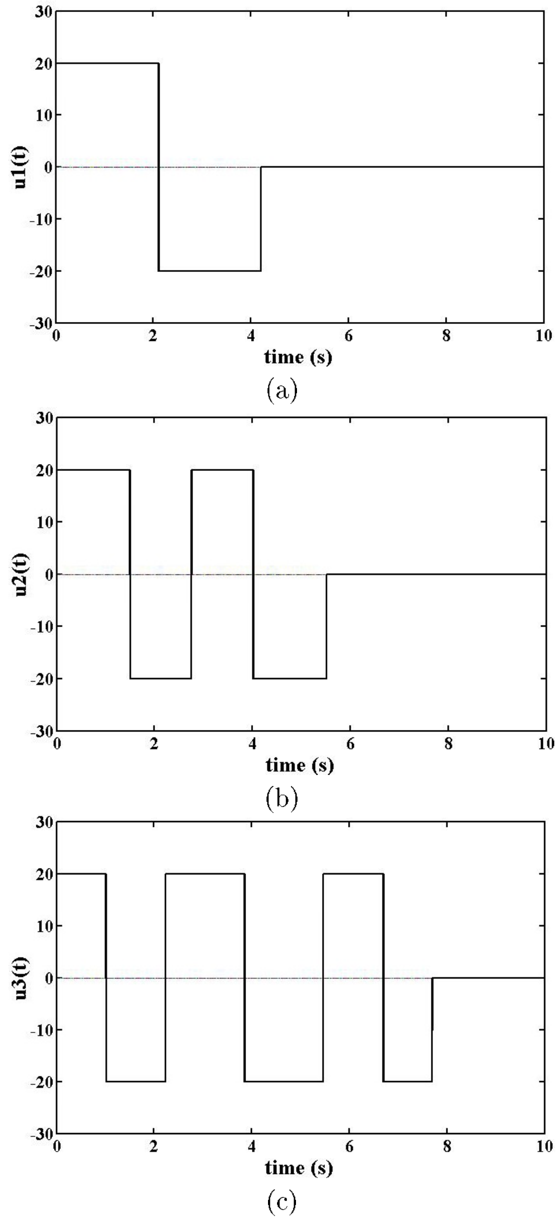

Figure 1. Optimal input (a) $u_1(t)$, (b) $u_2(t)$, and (c) $u_3(t)$.

second term vanishes, and the following holds for any bang-bang input:

$$\sum_{j=0}^{2n} b_j \sin\omega_i(t_j - t_n) = 0. \tag{12}$$

Therefore, to have $q_i(t) = 0$ for $t \geq t_f$, i.e. no residual structural vibration, the following flexible mode constraints are obtained:

$$\sum_{j=0}^{2n} b_j \cos\omega_i(t_j - t_n) = 0, \qquad i \geq 2 \tag{13}$$

To solve the minimum-time optimal control problem, we have to determine (2n-1) unknown switching times such that the final time t_f be minimized. This can be formulated as a constrained parameter optimization problem, i.e. minimization of the performance index of Eq. (5) subjected to the following constraints:

$$f_1(t_1, t_2, \cdots, t_j, \cdots, t_{2n}) = \theta_f - \frac{\Phi_1 u_{\max}}{2} \sum_{j=0}^{2n} b_j (t_f - t_j)^2 = 0, \tag{14a}$$

$$f_i(t_1, t_2, \cdots, t_j, \cdots, t_{2n}) = \sum_{j=0}^{2n} b_j \cos\omega_i(t_j - t_n) = 0, \qquad i = 2, ..., n \tag{14b}$$

To satisfy the necessary and sufficient condition for optimality, the Hamiltonian can be introduced as:

$$H = t_f + \lambda_i f_i, \qquad i = 1, ..., n \tag{15}$$

where λ_i are defined as Lagrange multipliers. Setting up the following equations, a set of 3n equations can be solved to determine 3n unknowns, i.e. (2n-1) switching times, one final time t_f , and n Lagrange multipliers:

$$g_j = \frac{\partial H}{\partial t_j} = 0, \quad j = 1, 2, \cdots, 2n$$

$$g_k = \frac{\partial H}{\partial \lambda_i} = 0, \quad k = 2n + 1, \cdots, 3n \tag{16}$$

This set of equations are often coupled and nonlinear, and can be solved using numerical methods, [27].

ROBUST TIME OPTIMAL CONTROL DESIGN

Constraint equations (13) can be represented as:

$$f(T, P) = 0, \tag{17}$$

where T represents a set of switching times and P consists of flexible mode frequencies that are considered uncertain. Expanding f(T,P) about its nominal value P^0 the following can be obtained:

$$f(T, P) = f(T, P^0) + \frac{\partial f}{\partial P}\Big|_{(T,P^0)} (P - P^0) + \cdots \tag{18}$$

Then, the set of switching times T can be redesigned to satisfy the following two sets of constraints:

$$f(T, P^0) = 0, \tag{19a}$$

$$\left.\frac{\partial f}{\partial P}\right|_{(T,P^0)} = 0, \tag{19b}$$

where the second constraint is called the first-order robustness constraint because it limits the amplitude of

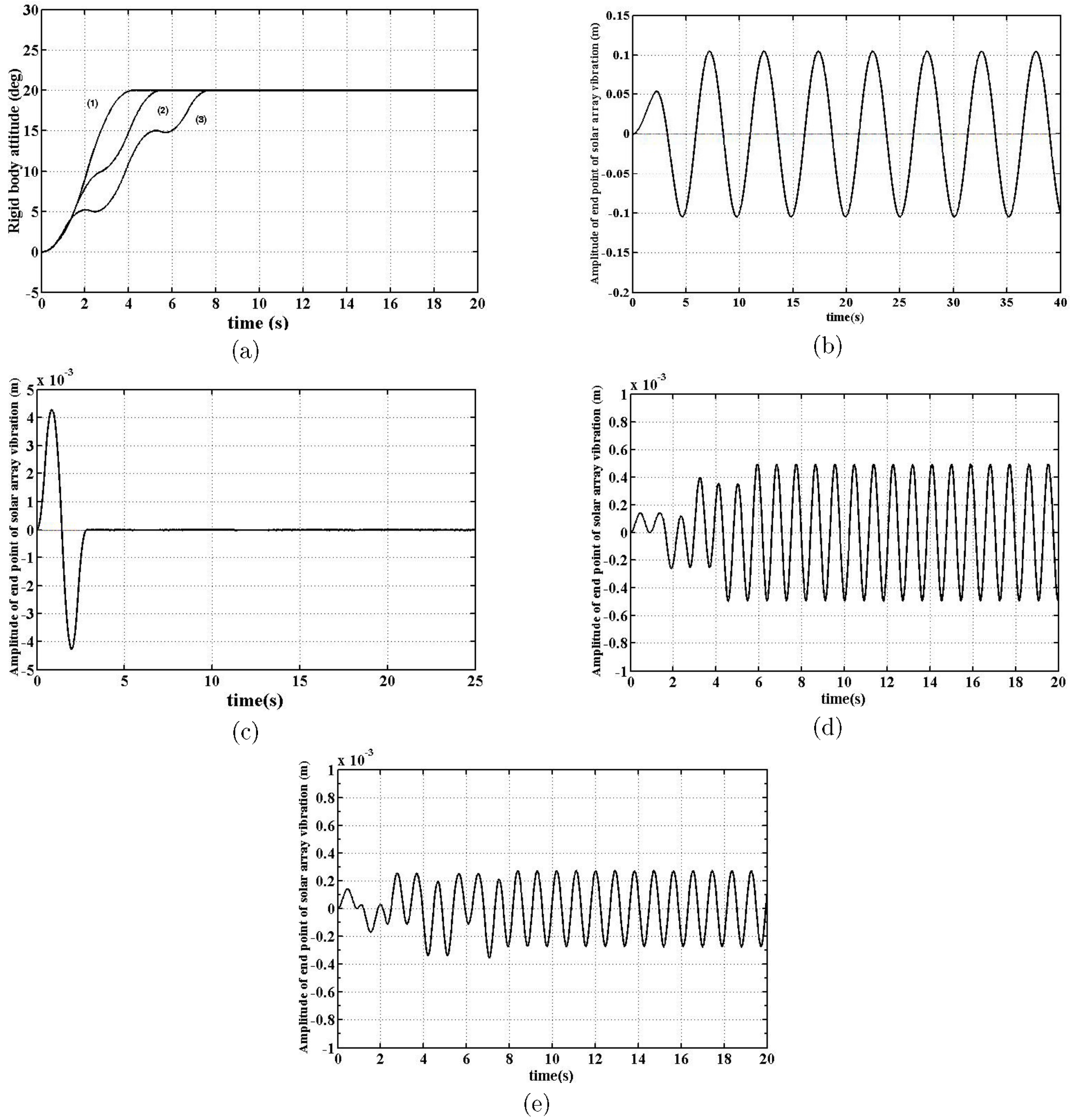

Figure 2. (a) Central rigid body responses to inputs $u_1(t)$, $u_2(t)$ and input $u_3(t)$; (b) Flexible appendage response due to $u_1(t)$; (c) Flexible appendage response due to $u_2(t)$; (d) The second flexible mode response due to $u_2(t)$; (e) The second flexible mode response due to robust input $u_3(t)$.

residual structural vibrations caused by uncertainties of P. By taking the derivative of Eq. (10-b) with respect to ω_i, for each flexible mode it can be understood that:

$$\frac{dq_i(t)}{d\omega_i} = \frac{u_{\max}\Phi_i}{\omega_i^2}\cos\omega_i(t-\frac{t_f}{2})\sum_{j=0}^{2n}(t_j-\frac{t_f}{2})b_j\sin\omega_i(t_j-\frac{t_f}{2}). \tag{20}$$

Letting $dq_i(t)/d\omega_i = 0$ for all $t \geq t_f$ yields to:

$$\sum_{j=0}^{2n}(t_j-\frac{t_f}{2})b_j\sin\omega_i(t_j-\frac{t_f}{2}) = 0, \qquad i = 2,\cdots,n \tag{21}$$

which describes the first-order robustness constraints. If these robustness constraints are in the constrained minimization problem formulation described by Eqs.(14), the number of switches in the bang-bang control input must be increased to match the number of the constraint equations. Adding one robustness constraint will require two more switches. Then robust time optimal control problem can be formulated as a constrained parameter optimization problem, i.e. minimization of the performance index of Eq. (5) subjected to the constraints set (14) and the new additional set of constraints (21), i.e., the previous ones plus robustness constraints.

THE SOLUTION ALGORITHM

Collecting g_i functions defined by Eq. (16) in a $(3n \times 1)$ vector as:

$$g = [g_1 g_2 \cdots g_{2n} \cdots g_{3n}]^T, \tag{22}$$

the steps of the solution procedure and numerical algorithm to obtain the time optimal control and robust time optimal control inputs are given below:

1. Determine the numbers of flexible modes (n).
2. Define the bang-bang input with (2n-1) switches.
3. Apply this control input function for rigid body mode and flexible modes to obtain constraints of problem with performance index, $J=t_f$. At this stage, the time optimal control problem is converted into a parameter optimization problem. Parameters of this problem can be reduced by consideration of time symmetry property of the bang- bang input function.
4. Form vector g, using Eqs. (14)- (22).
5. Form a $(3n \times 1)$ vector h for unknowns:

$$\mathbf{h} = [t_1, \cdots, t_{2n}, \lambda_1, \cdots, \lambda_n]^T \tag{23}$$

6. Assume some starting values for h as h_0.
7. Calculate the $3n \times 3n$ Jacobian matrix:

$$J = \frac{\partial g}{\partial h} = \begin{bmatrix} \frac{\partial g_1}{\partial t_1} & \cdots & \frac{\partial g_1}{\partial t_{2n}} & \frac{\partial g}{\partial \lambda_1} & \cdots & \frac{\partial g_1}{\partial \lambda_n} \\ \frac{\partial g_2}{\partial t_1} & \cdots & \frac{\partial g_2}{\partial t_{2n}} & \frac{\partial g_2}{\partial \lambda_1} & \cdots & \frac{\partial g_2}{\partial \lambda_n} \\ \vdots & & & & & \\ \frac{\partial g_{3n}}{\partial t_1} & \cdots & \frac{\partial g_{3n}}{\partial t_{2n}} & \frac{\partial g_{3n}}{\partial \lambda_1} & \cdots & \frac{\partial g_{3n}}{\partial \lambda_n} \end{bmatrix} \tag{24}$$

   where $J_{ij}=\partial g_i/\partial h_j$ are calculated using the current values of h.
8. Calculate the step direction as:

$$\Delta = J^{-1} g \tag{25}$$

9. Update the unknown variables:

$$h = h_c - \Delta \tag{26}$$

   where h_c denotes the current value of h.
10. Repeat steps 7-9 until:

$$||g|| \leq \varepsilon \tag{27}$$

   where refers to $||\cdots||$ Euclidean norm, and ε is a chosen accuracy threshold.
11. The unknown variables are obtained as:

$$h = h_c \tag{28}$$

Next, to illustrate the developed optimal control law and described numerical procedure, the slewing maneuver of a given satellite is simulated.

SIMULATIONS

The system parameters and maneuver specifications are listed in Table 1. To see the inherent behavior of the system, the first five modes are retained in the developed model in the simulation routine prepared in MATLAB environment, in which a single torque actuator is located on the rigid central body to control the maneuver. The task is to control the satellite orientation during a rest-to-rest maneuver in minimum time. Table 2 shows the natural frequencies ω_i, in radian per second, and the components of ϕ_i in Eq. (2), for the first five modes.

For the first trial, only the rigid body mode is considered, i.e. n=1, and so there exists just one switching time. The control torque will be defined as:

$$u_1(t) = u_{max}\{\hat{1}(t) - 2[\hat{1}(t-t_1) + \hat{1}(t-t_f)]\}. \tag{29}$$

By applying the presented algorithm, the middle and final time, t_1 and t_f , are obtained as shown in Table 3. The input amplitude is given as u_{max}=20N.m as shown in Figure (1a), and the attitude of the central rigid body due to this input torque varies according to curve (1) in Figure (2a). To find the vibration of the end point of appendages (solar panels), due to this input torque, if we solve the equation:

$$\ddot{y}_2 + \omega_2^2 y_2 = \Phi_2 u_1, \tag{30}$$

and transform the solution back to the physical coordinates, the end point vibration will be obtained as Figure (2b). As seen, the amplitude is considerably large and may cause significant damage to the spacecraft. Therefore, at least the first flexible mode, i.e. n=2, should be considered.

To consider the first flexible mode (n=2) in order to apply the control input $u_2(t)$, three switching times will be introduced as:

$$u_2(t) = u_{\max}\{\hat{1}(t) - 2[\hat{1}(t-t_1)] + 2[\hat{1}(t-t_2)] - 2[\hat{1}(t-t_3)] + \hat{1}(t-t_f)\}. \tag{31}$$

According to the presented algorithm, the switching and final times are obtained as shown in Table 3,

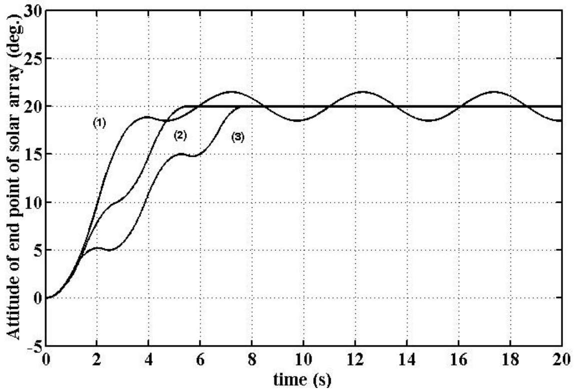

Figure 3. Attitude of end point of solar array for input $u_1(t)$ $u_2(t)$ and robustifed $u_3(t)$.

Table 1. System Parameters and Maneuver Specifications

Central body inertia	I_1	132 Kgm^2
	I_2	77 Kgm^2
	I_3	135 Kgm^2
Solar panels Length	L	4m
Solar panels Thickness	t	0.02 m
Solar panels width	w	0.50 m
Solar panels material stiffeness	EI	20.10 Nm^2
Solar panels material density	ρ	0.81 Kg/m^2
Maximum torque avilable	u	20 N.m
Total mass of spacecraft	M	800 Kg
Total slewing angle		20 deg

and the control input is illustrated in Figure (1b).

The response of the central rigid body and the flexible appendage are shown as Curve (2) in Figure (2a), and Figure (2c), respectively. As shown in Figure (2c), by applying $u_2(t)$, the vibration of the appendage in its first flexible mode does completely vanish, however it seems that the second flexible (n=3) mode is excited. Therefore, to investigate this, the amplitude of vibrations for the second flexible mode is shown in Figure (2d). As seen in the figure, the amplitude is about 0.5 mm which is reasonably small. This will be substantially reduced by considering the robust control input u_3 (t).

Table 2. Flexible Modes Specifications

i	ω_i	Φ_i
1	0	0.0628
2	1.2355	-0.0328
3	6.9311	0.0092
4	19.3320	0.0043
5	38.2100	-0.0026

Table 3. Switching and final maneuver times (sec.)

Times for $u_1(t)$	Times for $u_2(t)$	Times for $u_3(t)$
t_1=2.104	t_1=1.498	t_1=1.012
t_f=4.208	t_2=2.755	t_2= 2.235
	t_3=4.012	t_3=3.851
	t_f=5.509	t_4=5.467
		t_5=6.690
		t_f=7.702

The robust control input u_3 (t) can be defined by adding first-order robustness criterion described in Eq. (21), as follows:

$$u_3(t) = u_{\max}\{\hat{1}(t) - 2[\hat{1}(t-t_1)] + 2[\hat{1}(t-t_2)] - 2[\hat{1}(t-t_3)] + 2[\hat{1}(t-t_4)] - 2[\hat{1}(t-t_5)] + \hat{1}(t-t_6)\} \quad (32)$$

Following the presented algorithms, switching and final times for this case will be obtained as shown in table 3. The control profile $u_3(t)$, and obtained responses are illustrated in Figure 1-2.

Finally, the end point motion of the appendage is compared for the three cases in Figure 3. It should be noted that the vibration amplitude has been decreased by 99% due to u_3 (t), compared to that of $u_1(t)$, whereas the maneuver time has increased by 83%.

CONCLUSIONS

Focusing on the slewing maneuver of a flexible spacecraft with large angle of rotation, and assuming structural frequency uncertainties a robust minimum-time optimal control law was developed in this paper. Employing the assumed modes method for the flexible appendage, and the Euler-Bernoulli beam assumption, the system dynamics were modeled. Considering typical bang-bang control commands with multiple symmetrical switches, a parameter optimization procedure was introduced to find the control forces/torques. Augmenting the constrained minimization problem with the robustness constraints, the number of switches in the bang-bang control input was increased to match the total number of the constraint equations. The steps of the solution algorithm to obtain the time optimal control input were then discussed. The developed control law was applied to a given satellite during a slewing maneuver, where the first five modes were considered in the simulated model of the system. The simulation results accentuated that the robust control input, exerted by a single torque actuator located on the rigid central body, with just few switching times significantly diminishes the vibrating motion of the flexible appendage.

REFERENCES

1. Jacobsen, S., Lee, C., Zhu, C., and Dubowsky, S., "Planning of Safe Kinematic Trajectories for Free Flying Robots Approaching an Uncontrolled Spinning Satellite ", *20002*, Proc. Of ASME 2002 Design Engineering Technical Conferences, Montreal, Canada, .
2. Vafa, Z. and Dubowsky, S., " On The Dynamics of Manipulators in Space Using The Virtual Manipulator

Approach", *Proc. of IEEE Int. Conf. on Robotics and Automation*, PP 579-585(1987).

3. Moosavian, S. Ali A. and Papadopoulos, E., "On the Kinematics of Multiple Manipulator Space Free-Flyers ", *Journal of Robotic Systems*, **15**(4), PP 207-216(1998).
4. Moosavian, S. Ali A. and Papadopoulos, E., " Explicit Dynamics of Space Free-Flyers with Multiple Manipulators via SPACEMAPL", *Journal of Advanced Robotics*, **18**(1), (2004).
5. Baillieul, J. and Levi, M., "Rotational Elastic Dynamics", *Journal of Physica*, **27D**, PP 43-62(1987).
6. Mah, H. W., Modi, V. J., Morita, Y., and Yokota, H., "Dynamics During Slewing and Translational Maneuvers of the Space Station Based MRMS", *Journal of the Astronautical Space Sciences*, **34**(8), PP 557-579(1990).
7. Cyril, X., Angeles, J., and Misra, A., "Dynamics of Flexible Multibody Mechanical Systems", *Transactions of the CSME*, **15**(3), PP 235-256(1991).
8. Kuang, J. L., Kim, B. J., Lee, H. W., and Sung D. K., "The Attitude Stability Analysis of a Rigid Body with Multi-Elastic Appendages and Multi-Liquid-Filled Cavities Using The Chetaev Method", *Journal of the Astronautical Space Sciences*, **15**(1), PP 209-220(1998).
9. Joshi, S. M., Alberts, T. E., and Kakad, Y. P., "Advances in Dynamics and Control of Flexible Spacecraft and Space-Based Manipulators ", *DCS, ASME*, NY, (1990).
10. Carusone, J., Buchan, K. S., and D[Eleuterio, G. M. T., "Experiments in End-Effector Tracking Control for Structurally Flexible Space Manipulators", *IEEE Transactions on Robotics and Automation*, **9**(5), PP 553-560(1993).
11. Papadopoulos, E. and Moosavian, S. Ali A., "Dynamics & Control of Multi-arm Space Robots During Chase & Capture Operations", Proc. Of IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, Munich, Germany, (1994).
12. Papadopoulos, E. and Moosavian, S. Ali A., "Dynamics & Control of Space Free-Flyers with Multiple Arms ", *Journal of Advanced Robotics*, **9**(6), PP 603-624(1995).
13. Moosavian, S. Ali. A. and Rastegari, R., "Force Tracking in Multiple Impedance Control of Space Free-Flyers", Proc. of the IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, Takamatsu, Japan., (2000).
14. Scrivener, S. and Thompson, R. C., "Survey of Time-Optimal Attitude Maneuvers", *Journal of Guidance, Control, and Dynamics*, **17**(2), PP 225-233(1994).
15. Meirovitch, L. and Sharony, Y., "Accommodation of Kinematic Disturbances During Minimum- Time Maneuvers of Flexible Spacecraft", *Journal of Guidance, Control, and Dynamics*, **14**(2), PP 268-277(1991).
16. Bobrow, J. E., Dubowsky, S., and Gibson J. S., "Time- Optimal Control of Robotic Manipulators A Long Specified Paths", *The International Journal of Robotics Research*, **4**(3), PP 244-258(1985).
17. Singh, G., Kabama, P. T., McClamroch, "Planar Time-Optimal Rest-to-Rest Slewing Maneuvers of Flexible Spacecraft", *Journal of Guidance, Control, and Dynamics*, **12**(1), PP 71-81(1989).
1993
18. Barbieri, E. and Ozgunar, U., "A New Minimum-Time Control Law for a One- Mode Model of a Flexible Slewing Structure", *IEEE Transactions on Automatic Control*, **38**(1), PP 142-146.
19. Bassam A. A., "Optimal Near-Minimum-Time Control Design for Flexible Structures", *Journal of Guidance, Control, and Dynamics*, **25**(4), PP 618-625(2002).
20. Fontes, F. A., and Magni, L., "Min-Max Model Predictive Control of Nonlinear Systems Using Discontinuous Feedbacks", *IEEE Transactions on Automatic Control*, **48**(10), PP 1750-1755(2003).
21. Boskovic, J. D., Ming, L. S., and Mehra, R. K., "Robust Adaptive Variable Structure Control of Spacecraft Under Control Input Saturation", *Journal of Guidance, Control, and Dynamics*, **24**(1), PP 14-22(2001).
22. Singh, S. N., "Rotational Maneuver of Nonlinear Uncertain Elastic Spacecraft", *IEEE Transactions on Aerospace and Electronic Systems*, **24**(2), PP 114-123(1988).
23. Liu, Q., and Wie, B., "Robust Time-Optimal Control of Uncertain Flexible Spacecraft", *Journal of Guidance, Control, and Dynamics*, **15**(3), PP 597-604(1992).
24. Baldeli, D. H., Mazzaro, M. C. , and Pena, S. S., "Robust Identification of Lightly Damped Flexible Structures by Means of Orthonormal Bases", *IEEE Transactions on Control Systems Technology*, **9**(5), PP 696-707(2001).
25. Ebrahimi, A., Moosavian, S. Ali A., and Mirshams, M., "Minimum-Time Optimal Control of Flexible Spacecraft for Rotational Maneuvering", Proc. Of IEEE Int. Conf. on Control Applications, Taipei, Taiwan, PP 961-966(2004).
26. Kirk, D. E., "Optimal Control Theory, an Introduction", *Prentice- Hall, Englewood Cliffs*, NI, (1970).
27. Gerald, C. F. and Wheatley, P. O., *Applied Numerical Analysis*, 6th Edition, Addison- WesleyEd., (1999).