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Time Optimal Closed-Loop Fuzzy-Control Strategy for Nonlinear Lunar Lander Mission

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In this paper a closed-loop time-optimal control strategy for the non-linear lunar lander mission is developed. Generally, determination of closed-loop feedback-control law is not usually feasible for many non-linear dynamic systems. In addition, there exist certain difficulties associated with the numerical determination of open-loop optimal control solution for non-linear systems, such as slow convergence rate and high sensitivity to initial guesstimates. Besides, if one manages to overcome these inherent difficulties, the determined optimal control strategy will be in an open-loop form, and thus, fully dependent on the initial condition. Obviously, in this way perturbations and noise processes will make the optimal trajectory deviate from its ideal predicted values in any actual operating environment. Our study focuses on the planar trajectory and control optimization of a lunar lander spacecraft as a viable example of non-linear dynamic system. A fuzzy algorithm is augmented to our variational formulation of the problem in an attempt to create a closed-loop fuzzy guidance logic. The training process of the fuzzy system is greatly reduced through the introduction of a set of states related non-dimensional variables. Simulation results indicate that the developed methodology can be successfully utilized in other flight scenarios with good robustness to the actuator and measurment system noise.

INTRODUCTION

Optimal control solutions of dynamic systems can be classified into two main categories of open-loop and closed-loop. Open-loop optimal controls are only functions of time and once the system starts from a known initial condition, the predetermined optimal controls activate to take the trajectory toward the final hypersurface with no feedback of states along the way. A task which performs the job well if there is no unmodeled disturbances and/or noise processes. On the other hand closed-loop optimal controls are functions of time and states and in essence posses a sort of inherent robustness against noise and/or undesirable disturbances present in any actual operating environment. Of course, closed-loop optimal control solutions are seldom possible for non-linear dynamic systems, even though they are highly desirable for their robust characteristics. Open-loop optimal control solutions of non-linear dynamic system can be determined either by a dynamic programming approach or through a variational formulation of the optimal control problem [1], [2]. Regardless of the techniques used, both approaches render open-loop solutions. Due to some inherenet difficulties with non-linear dynamic systems, a few attempts have been made toward finding closed-loop optimal feedback control solutions for these systems. Rahbar investigated the possibility of determining a guidance stratrgy for the pursuit problem using neural networks [3]. Dabbous presented a closed-loop optimal control strategy for non-linear regulator problems with the final time specified [4]. Kunisch also presented his approach to non-linear optimal feedback control through numerical solutions of the Hamilton-Bellman-Jacobi equation. In addition, there exist other studies by Naidu on the closed-loop optimal control using singular perturbation methods [5], [6], [7], [8]. Also, Shamma has been able to utilize the algebraic Ricati

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equation to come up with closed-loop control laws for non-linear systems with linear control variables [9]. More recently, Palma has worked on optimal predictive control by discretizing non-linear dynamic systems [10]. Near optimal closed-loop control of reduced non-linear systems is obtained by Lewis using Jacobi equation and neural networks [11]. While with classical methods of control design, complete system information and knowledge of uncertainties are required, fuzzy logic controllers have shown to be more insensitive to these factors and have attracted some attention over the past decade. In fact, it has been shown that these factors have little influence on the performance of the fuzzy control [12]. Steinbauer has utilized fuzzy logic to obtain an open-loop optimal control for a linearized system [13]. Li has also employed fuzzy logic on non-linear systems by first linearizing them in order to come up with optimal controls [14]. Mitsuishi has also researched on the idea of optimal fuzzy control for non-linear system in which the control variable is linearized [15]. Suzuki has fuzzified the performance function and in turn, developed a decision making fuzzy logic to determine the optimal control [16]. Additionally some research on the determination of minimum fuel closed-loop guidance law for a lunar lander has been performed by Ueno and Souza [17], [18], [19].

The present study focuses on a methodology for the determination of an optimal closed-loop control solution for non-linear system using fuzzy logic. The landing mission design of a lunar lander is investigated as an appropriate non-linear system. The time optimal landing mission is to be performed with a thrust limited system with one degree of gimbaled freedom. A set of state dependent non-dimensional variables have been introduced which expedite the fuzzy training process and have proven effective in developing the proposed (FGL). The solution robustness of this time optimal fuzzy closed-loop system is demonstrated against actuation and process noise.

OPTIMAL CONTROL IN DYNAMICAL SYSTEMS

Optimal control problems of dynamic systems can be formulated using calculus of variations [1]. In this regard, one usually assumes a mathematical representation of the system under study as a first order differential equation.

$$\dot{\vec{x}} = a\left(\vec{x}(t), \vec{u}(t), t\right), \quad t_0 \le t \le t_f \tag{1}$$

Where $\vec{x}(t)$ denotes the n-states and $\vec{u}(t)$ is the vector of m-control components. The second step in the formulation of optimal control problem is to introduce an appropriate performance function. A conventional form can be expressed as:

$$J(u) = h\left(\vec{x}(t_f), t_f\right) + \int_{t_0}^{t_f} g\left(\vec{x}(t), \vec{u}(t), t\right) dt \tag{2}$$

Where $h\left(\vec{x}(t_f), t_f\right)$ is the penalty function for the final states at the final time. Additionally there could exist terminal constraints in functional form for final time unspecified situations, presented as:

$$m(\vec{x}(t_f), t_f) = 0 \tag{3}$$

Having officially formulated the problem, the first step toward a variational solution is to determine the system Hamiltonian.

$$H = g\left(\vec{x}(t), \vec{u}(t), t\right) + p^T a\left(\vec{x}(t), \vec{u}(t), t\right) \tag{4}$$

Based on the Hamiltonian, the necessary conditions for an optimal solution are:

i. the state equations,

$$\dot{\vec{x}} = a\left(\vec{x}(t), \vec{u}(t), t\right) \tag{5}$$

ii. the costate equations,

$$\dot{\vec{p}} = -\frac{\partial H}{\partial \vec{x}} \tag{6}$$

iii. and the optimality condition,

$$0 = \frac{\partial H}{\partial \vec{u}} \tag{7}$$

The above equations need to be simultaneously satisfied, considering an appropriate set of initial and boundary conditions given below [2];

iv. initial conditions,

$$\vec{x}_k(t_0) = (\text{is known}), \quad \lambda_k(t_0) = 0. \tag{8}$$

v. terminal condition,

$$\frac{\partial h}{\partial \vec{x}} - \vec{p}\,|_{t_f} = \sum_{i=1}^{k} d_i \frac{\partial m_i}{\partial \vec{x}}, \tag{9}$$

$$\left. H + \frac{\partial h}{\partial t}\right)_{t_f} = \sum_{i=1}^{k} d_i \frac{\partial m_i}{\partial t}, \tag{10}$$

$$m\left(\vec{x}(t_f)\right) = 0. \tag{11}$$

The optimality condition (7) usually allows for optimal determination of the m-control components as functions of states and the costates. The solution of the 2n differential equations (5), (6) are to be considered with the aid of 2n+1+q boundary conditions specified in (8) through (11). Since in most practical applications, the governing equations are non-linear, one does not usually expect to obtain an analytical closed-loop solution. Thus, open loop optimal control solution is sought through numerical techniques [1].

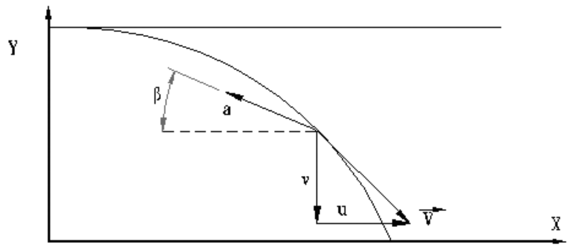

Figure 1. Geometry of the lunar landing problem

ANALYTICAL OPEN-LOOP SOLUTION TO THE LUNAR LANDING PROBLEM

Consider an idealized spacecraft at the orbital of inertial frame (x, y) at $t = 0$, moving under the action of a constant propulsive force making a control angle β with the horizon. Obviously the position and velocity vector of the vehicle will change due to the action of forces acting on it. The objective is to determine the time-optimal control policy of this system for lunar landing from a final target orbit. Based on Figure 1, the governing equations are:

$$\begin{cases} \dfrac{du}{dt} = -a\cos\beta \\ \dfrac{dv}{dt} = a\sin\beta - g \\ \dfrac{dy}{dt} = v \end{cases} \tag{12}$$

with the appropriate boundary conditions:

$$u(t=0) = U_0, \quad v(t=0) = 0, \quad y(t=0) = h, \tag{13}$$

$$u(t=t_f) = 0, \quad v(t=t_f) = 0, \quad y(t=t_f) = 0. \tag{14}$$

For a better physical understanding and reaching an analytical explicit solution, the governing equations and the associated boundary conditions are non-dimensionalized using a set of assumed reference parameters (u^*, v^*, y^*):

$$\bar{u} = \frac{u}{U^*}, \quad \bar{v} = \frac{v}{U^*}, \tag{15}$$

$$\bar{y} = \frac{y}{y^*}, \quad \tau = \frac{t}{t^*}, \tag{16}$$

$$\frac{d}{dt} = \frac{1}{t^*}\frac{d}{d\tau}. \tag{17}$$

For time optimal problems, where t_f is free, one usually utilizes the final time as a referencing condition for non-dimensionalizing. Nevertheless, in this study another approach is followed in order to avoid FGL computational difficulties.

$$U^* = U_0, \quad y^* = h, \quad t^* = \frac{h}{U_0}. \tag{18}$$

Using the above non-dimensional variable equations (15-17), the transformed equations become:

$$\begin{cases} \dfrac{d\bar{u}}{d\tau} = -w_1\cos\beta \\ \dfrac{d\bar{v}}{d\tau} = w_1\sin\beta - w_2 \\ \dfrac{d\bar{y}}{d\tau} = w_3\,\bar{v} \end{cases} \tag{19}$$

where:

$$w_1 = \frac{at^*}{U^*}, \quad w_2 = \frac{g}{a}, \quad w_3 = \frac{U^* t^*}{y^*}, \tag{20}$$

with non-dimensional boundary conditions:

$$\bar{u}(\tau=0) = 1, \quad \bar{v}(\tau=0) = 0, \quad \bar{y}(\tau=0) = 1, \tag{21}$$

$$\bar{u}(\tau=\tau_f) = 0, \quad \bar{v}(\tau=\tau_f) = 0, \quad \bar{y}(\tau=\tau_f) = 0. \tag{22}$$

Since the problem is to determine the control action $\beta = \beta(\tau)$ required for time optimal lunar landing maneuver to a specified orbit, the performance measure is simply:

$$J = \int_0^{\tau_f} d\tau \tag{23}$$

and the corresponding Hamiltonian will be:

$$H = 1 - \lambda_1 w_1 \cos\beta + \lambda_2 (w_1 \sin\beta - w_2) + \lambda_3 w_3 \bar{v} \tag{24}$$

Using the costate equation (6) and the optimality relation (7), one can find an implicit reaction for the optimal control:

$$\tan\beta = \tan\beta_0 + c\,\tau, \tag{25}$$

where:

$$\tan\beta_0 = \frac{C_2}{C_1}, \qquad c = -\frac{C_3}{C_1}. \tag{26}$$

The constants are to be determined using boundary conditions. With veiw of equation (25) and the existing relation between β and τ, the time derivatives appearing in the governing equation can be written with respect to β. This way, β now becomes the independent variable.

$$\begin{cases} \dfrac{d\bar{u}}{d\beta} = -\dfrac{w_1}{c\cos\beta} \\ \dfrac{d\bar{v}}{d\beta} = \dfrac{w_1\sin\beta - w_2}{c\cos^2\beta} \\ \dfrac{d\bar{y}}{d\beta} = \dfrac{w_3\,\bar{v}}{c\cos^2\beta} \end{cases} \tag{27}$$

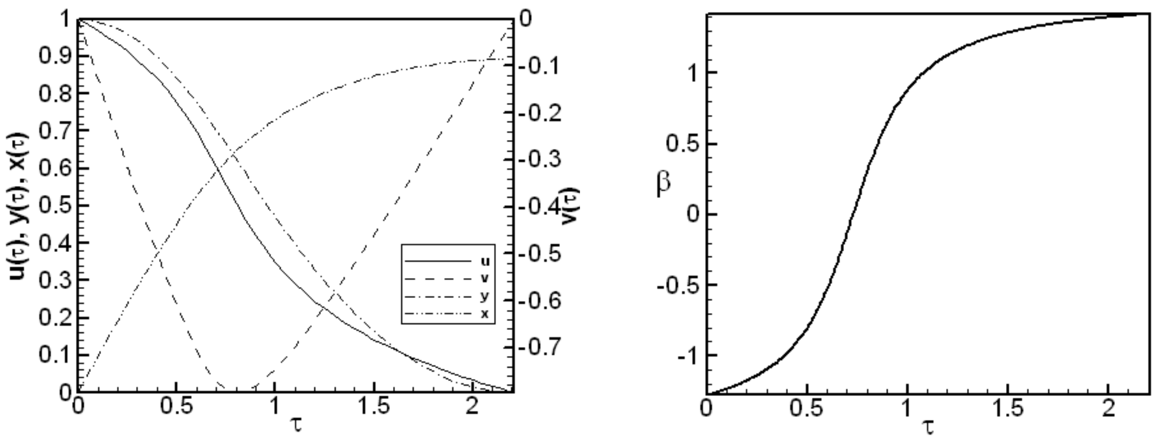

Figure 2. State time histories and minimum-time optimal control for the lunar landing problem

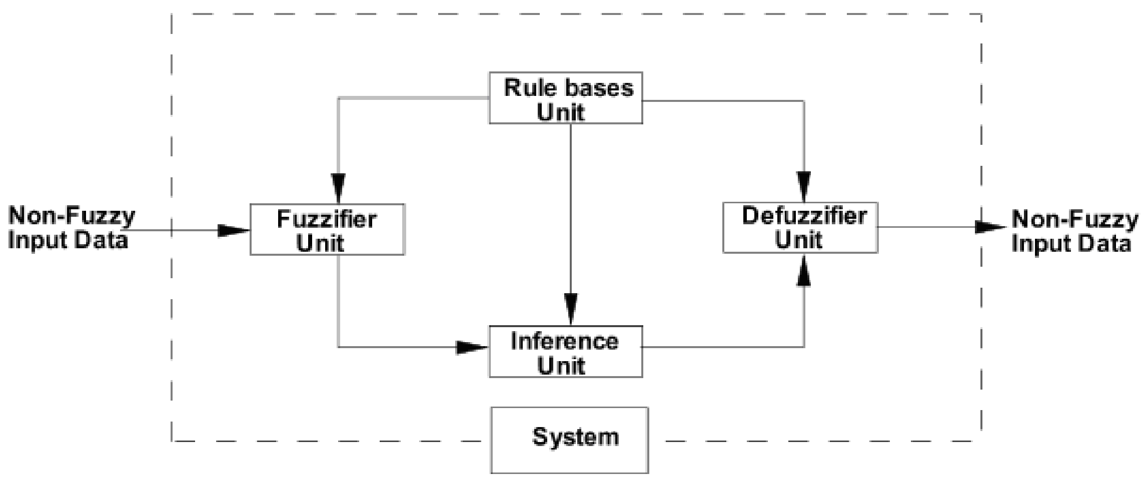

Figure 3. Fuzzy system structure

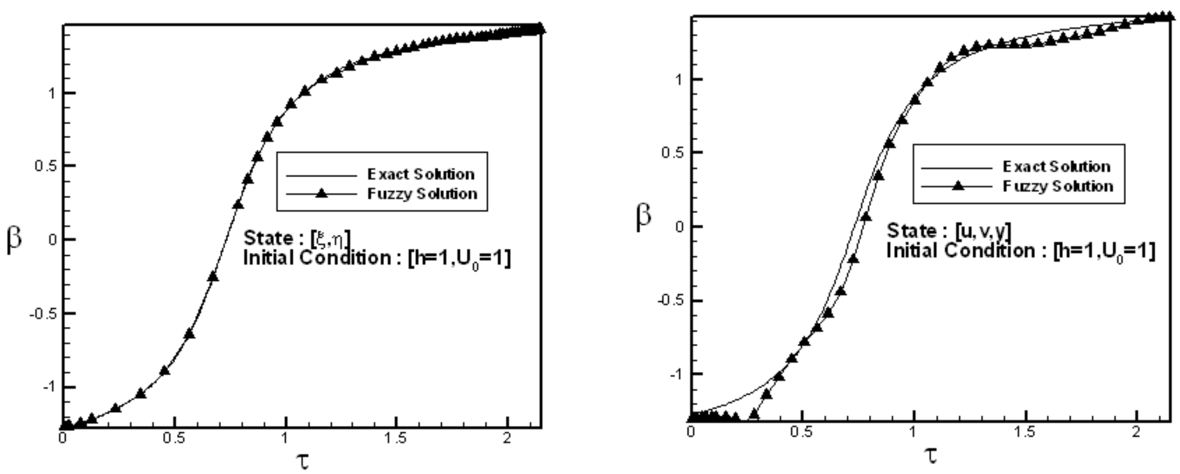

Figure 4. Variation of control parameter(β) for non-dimensional and regular states.

with initial condition;

$$\bar{u}(\beta_0) = 1, \quad \bar{v}(\beta_0) = 0, \quad \bar{y}(\beta_0) = 1. \tag{28}$$

Due to the simpler from of equation (27), it is integrated to yield the results as a function of the control angle, β.

$$\bar{u}(\beta) = -\frac{w_1}{c}\ln\frac{\sec\beta + \tan\beta}{\sec\beta_0 + \tan\beta_0} + 1, \tag{29}$$

$$\bar{v}(\beta) = \frac{w_2\sin(\beta - \beta_0) + w_1(\cos\beta_0 - \cos\beta)}{c\cos\beta\cos\beta_0}, \tag{30}$$

$$\bar{y}(\beta) = \frac{w_1 w_2 w_3}{2c^2}\left[\frac{1}{w_2}\ln\frac{\sec\beta + \tan\beta}{\sec\beta_0 + \tan\beta_0} + \frac{1}{w_2}\tan\beta\left(\frac{1}{\cos\beta} - \frac{2}{\cos\beta_0}\right) + \frac{1}{w_1}\left(\frac{1}{\cos^2\beta_0} - \frac{1}{\cos^2\beta}\right) - \frac{2}{w_1}\tan\beta_0(\tan\beta_0 - \tan\beta) + \frac{1}{w_2}\tan\beta_0\frac{1}{\cos\beta_0}\right] + 1. \tag{31}$$

Obviously for explicit results, it is necessary to specify the values of c, β_0 and τ_0. This can be accomplished through using the known terminal condition and solving a set of non-linear algebraic equations.

$$\bar{u}(\beta_f) = 0, \quad \bar{v}(\beta_f) = 0, \quad \bar{y}(\beta_f) = 0,$$

$$\beta_f = \tan^{-1}(\tan\beta_0 + c\,\tau_f). \tag{32}$$

Thus, for a set of assumed values of the parameteres (U_0, h, a, g) the required unknown parameteres $(\beta_0, \beta_f, \tau_f, c)$ can be determined from equation (32). for example, if $U_0 = h = a = 1$ and $g = \frac{1}{3}$, the optimal control and state trajectories are computed and depicted in Figure 2.

$$t_f = 2.2113, \tag{33}$$

$$\beta_0 = -1.2748, \tag{34}$$

$$c = -4.5, \tag{35}$$

$$\beta_f = 1.422, \tag{36}$$

$$\bar{u}(\beta) = -0.2222\ln(\sec\beta + \tan\beta) + 0.5770, \tag{37}$$

$$\bar{v}(\beta) = 0.07408\,\frac{3 - \sin\beta - 13.57\cos\beta}{\cos\beta}, \tag{38}$$

$$\bar{y}(\beta) = 0.02469\tan\beta\sec^2\beta + 0.02469\ln(\sec\beta + \tan\beta) - 0.008231\sec^2\beta - 0.2233\tan\beta + 0.6890, \tag{39}$$

$$\bar{x}(\beta) = \frac{0.5595 + 0.5595\cos 2\beta}{\cos 2\beta + 1} - \frac{0.02469\sin 2\beta\ln\left(\frac{3+4\sin\beta - \cos 2\beta}{\cos 2\beta + 1}\right)}{\cos 2\beta + 1} + \frac{0.09878\cos\beta + 0.1282\sin 2\beta}{\cos 2\beta + 1}, \tag{40}$$

$$\beta = \tan^{-1}(6.671 - 4.5\,\tau). \tag{41}$$

Note that the results are identical with the analytical results of reference [20] and that the solution parameters $\bar{u}$, $\bar{v}$, $\bar{y}$ and $\bar{x}$ have been expressed in terms of β, where β is related to τ. This control law is of course in the open-loop form.

FUZZY GUIDANCE LAW

A fuzzy system consists of four main units which include a rule bases unit, a fuzzifier unit, an inference unit and a defuzzification unit. In the fuzzifier unit, the input signals are transformed into linguistic variables or so called fuzzy variables. Subsequently, the fuzzy variables enter the decision making unit that utilizes the assigned bases to generate a fuzzy output. The fuzzy output is next converted to a regular output in the defuzzification unit. The following steps are followed in determining the fuzzy guidance law. First the input-output quantities (non-dimensionalized variables and control action) are specified, either through analytical solution techniques or numerical methods. For this purpose ξ, η are taken as the input variables and β is considered as the output variable of our fuzzy system. Second, the types of membership function presenting the input-output fuzzy sets are selected. In this regard, the gauss function has been used for the orbital problem. Next the decision making rule bases relating the system input to the output are chosen and saved in the appropriate form (usually tabulated). These rules are usually qualitative expressions, such as: very large, very small etc. The value of the membership functions for conditional parts of the rule bases are subsequently determined in a sequential manner discussed below. The fuzzy output of the system is next passed through a defuzzifier unit making it in a non-fuzzy form. Even though various methods of defuzzification exist, the area center techniques is more conventionally used for this purpose and is utilized in this study. Figure 3 shows a schematic diagram of the fuzzy system. The utilized fuzzy system follows the TSK (Takagi-Sugeno-Kang) approach in order to come up with the fuzzy guidance law [12]. In this system the fuzzy rules are simply defined as:

$$\text{if} \quad \xi_1 = A \quad \text{and} \quad \eta_1 = B \quad \text{then} \quad z = f(\xi_1, \eta_1) \tag{42}$$

where A,B are antecedents of the fuzzy law and f is an explicit function in an obverse of the fyzzy law. In this study, f is a first order polynomial and, therefore, the resulting fuzzy system is a first order TSK model. Accordingly, the final output of the fuzzy system is determined using the following relation:

$$\text{Final Output} = \frac{\sum_{i=1}^{N} w_i z_i}{\sum_{i=1}^{N} w_i} \tag{43}$$

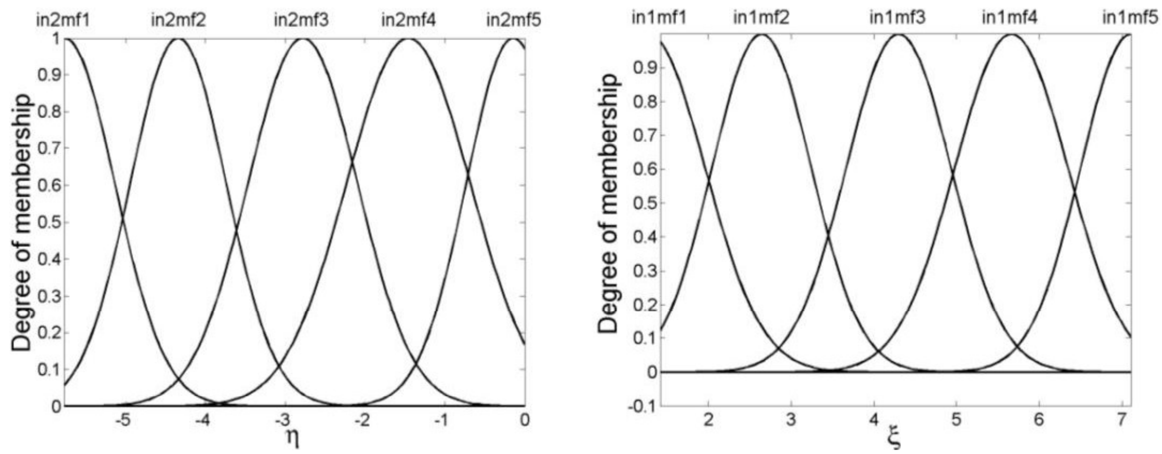

Figure 5. Behavior of the membership function with respect to ξ, and η.

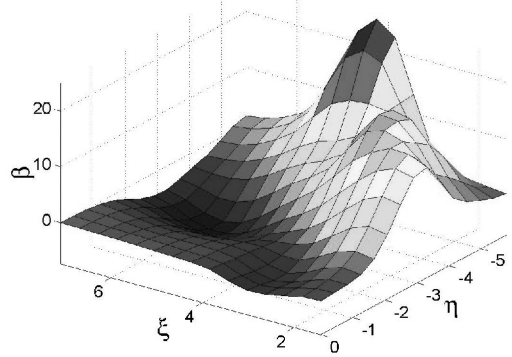

Figure 6. 3-D plot of the generated FGL output

where w_i are the explicit output weighting functions determined from an "AND" rule among the input membership functions.

APPLICATION OF FGL

For fuzzy application, one can generate the required non-dimensional variables through grouping the state variables. Keep in mind that the number of the non-dimensional variables will be at most n-1 due to PI-Bukinkham theorem. For the lunar landing from the orbit problem with $(\bar{u}, \bar{v}, \bar{y})$ as the state variables, the corresponding non-dimensionalized variables will be:

$$\xi = \frac{\sqrt{2ay^*}}{u^*}\frac{1-\sqrt{y^*}}{1-u^*}, \tag{44}$$

$$\eta = \frac{v^*}{1-u^*}. \tag{45}$$

It is worth nothing that having a minimum number of non-dimensional variables will expedite the training process of the fuzzy system and increase the accuracy of the FGL as well. For the problem at hand, the closed-loop FGL with states as $(\bar{u}, \bar{v}, \bar{y})$ is constructed with 343 fuzzy laws and 734 nodes for a deviation of $\sqrt{\delta^2 U_0 + \delta^2 h} = 0.42$ from the base condition $(U_0, h) = (1, 1)$, while for the same condition, the closed-loop FGL for the non-dimensional variables (ξ, η) is realized for only 25 fuzzy laws and 75 nodes. Variation of the control variable (β) with non-dimensional time parameter (τ) is depicted in Figure 4 for both cases, ie. When FL is constructed using the regular states (u, v, y) and when FGL is constructed using the non-dimensional state variables. Both results have been compared with the available exact solution. It is interesting to see that the control history behaves much better, and more closely follows the exact solution result for the non-dimensional states based FGL than for the regular states based FGL. This observation verifies the previous statements made regarding a considerable reduction in the fuzzy laws when utilizing the non-dimensional variables for training purposes. Obviously, this fact will also save a great deal of time in coming up with a more accurate FGL. Another effective utilization of this approach can be realized with training around several scenarios. This approach will subsequently allow the resulting FGL to produce more accurate, close-to-exact results in a wider flight spectrum. As a demonstration, the non-dimensional fuzzy system is considered for four terminal flight scenarios tabulated below. Obviously, the goal is to generate an optimal FGL utilizing a minimum number of flight scenarios for training purposes. Figure 5 shows the typical behavior

Table 1: Fore Starting Flight Scenarios

	scenarios			
h	1	0.7	1	0.7
U_0	1	0.7	0.7	1

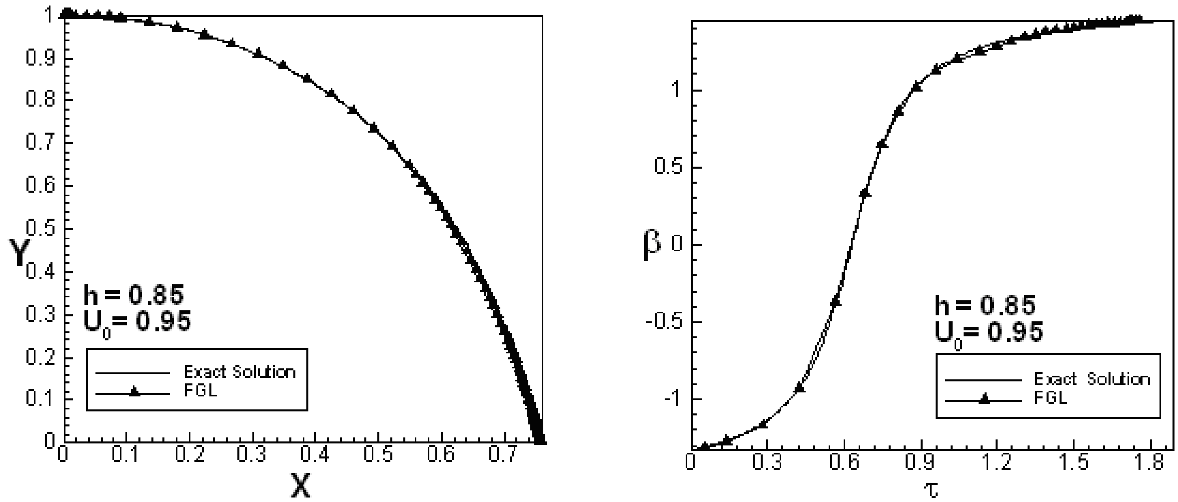

Figure 8. Optimal state-space trajectory and variation of control variable (β) with non-dimensional time

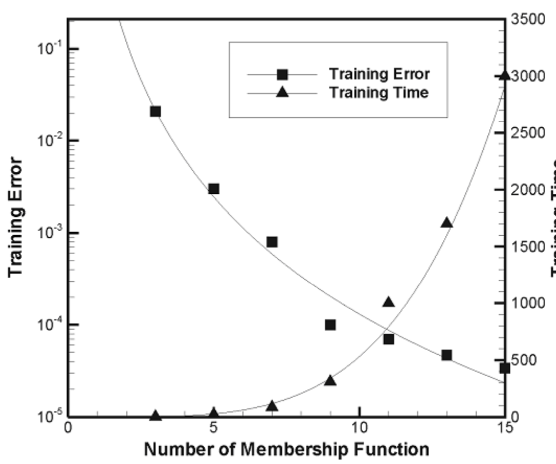

Figure 7. Variation of fuzzy system training error and time with membership function on a pentium 1.8GHZ computer

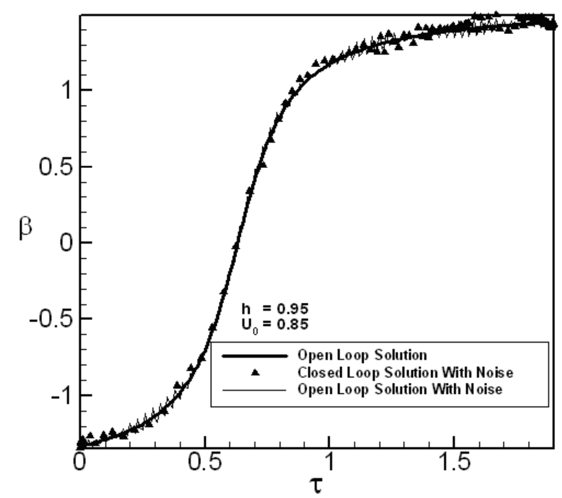

Figure 9. Variation of the control variable (β) with non-dimensional time using sinusoidal noise on the actuation system

of the membership function with respect to the two selected non-dimensional parameters ξ, η. Also a 3-D graph of the resulting FGL output is plotted in Figure 6. In addition , it is known that the fuzzy training error decreases with the increasing number of membership function toward a fixed constant in an asymptotic fashion. This behavior is shown in Figure 7 for the system under study. Based on this observation, nine membership functions are considered appropriate for this lunar landing from the orbit problem. The developed FGL is next applied to an out-of trained flight scenario, namely for $(U_0, h) = (0.95, 0.85)$. The optimal fuzzy results of this case are compared with corresponding exact solutions in Figure 8. Also as can be seen in Table 2, there exists an excellent agreement between the exact optimal control solution (OCS) and fuzzy optimal control (FOC) solution for the terminal parameters.

PERFORMANCE OF FGL IN NOISY ENVIROMENTS

There are several sources of disturbances in the operating environment which could downgrade the performance of our optimal FGL. To investigate robustness potentials of the proposed FGL, the system at a non-trained flight scenario considered in the previous section ($U_0 = 0.95, h = 0.85$) is analyzed under the influence of state feedback (measurement) and the actuator noise. The actuator noise is simulated using the following relations [21]:

$$n^a(t) = \epsilon_1 \sin(\omega_1 t) + \epsilon_2 \cos(\omega_2 t), \tag{46}$$

$$\beta_n = \beta(\xi, \eta) + n^a(t), \quad \omega_1, \omega_2 >> 1 \tag{47}$$

The noise parameters are chosen as $\omega_1 = 200$, $\omega_2 = 215$, $\epsilon_1 = 0.03$ and $\epsilon_2 = 0.035$ and the problem is

Table 2: Different scenarios

Scenario Inputs	Solution Method	t_f	U_0	h	v_f	x_f
$U_0 = 1$	FOC[†]	2.21130	0	0	$1.08e-4$	0.89260
$h = 1$	OCS[††]	2.211364	0	0	0	0.89264
$U_0 = 0.7$	FOC	1.84280	0.001	-0.0005	$3.2e-5$	0.84630
$h = 0.8$	OCS	1.842321	0	0	0	0.84627
$U_0 = 0.9$	FOC	2.0571	0.003	-0.0114	0.0141	0.6312
$h = 0.7$	OCS	2.05327	0	0	0	0.6281

[†] FOC: Fuzzy Optimal Control
[††] OCS: Optimal Control Solution

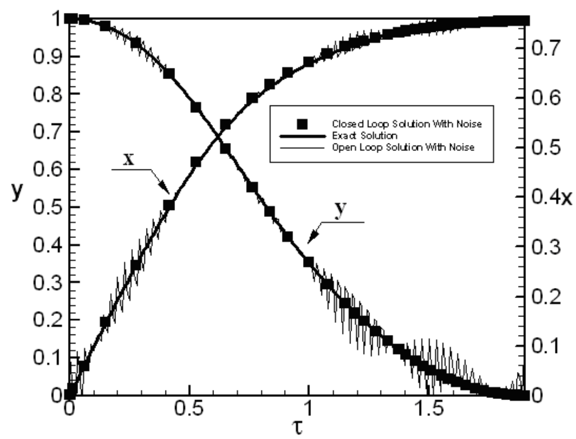

Figure 10. Variation of the trajectory $(\bar{x}, \bar{y})$ with non-dimensional time using sinusoidal noise on the actuation system

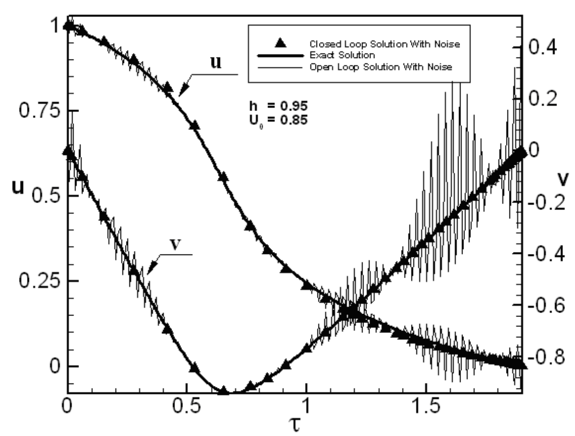

Figure 11. Variation of the trajectory $(\bar{u}, \bar{v})$ with non-dimensional time using sinusoidal noise on the actuation system

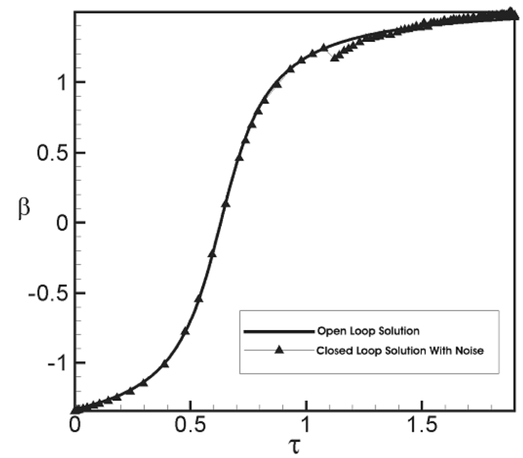

Figure 12. Variation of the control variable (β) with non-dimensional time using sinusoidal noise on the measurement system

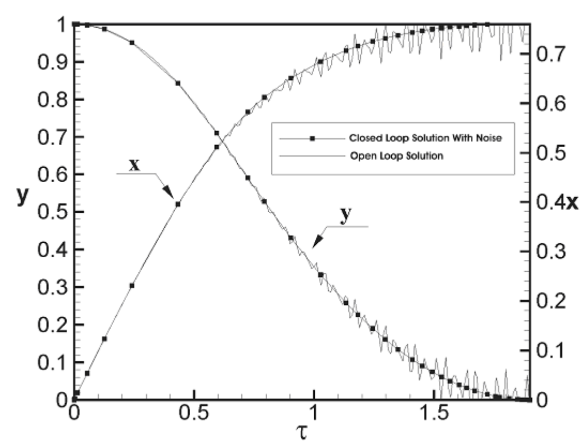

Figure 13. Variation of the trajectory $(\bar{x}, \bar{y})$ with non-dimensional time using sinusoidal noise on the measurement system

solved using the previously trained FGL. The lander states trajectory and control history are depicted in Figures 9, 10 and 11. One can easily verify from the results that the modeled actuation noise has no effect on the performance of FGL, while the system behaves in an oscillatory fashion when it not closed with FGL.

The state (measurement) noise modeled similar to the control actuation noise is taken as:

$$n^s(t) = \epsilon_1 \sin(\omega_1 t) + \epsilon_2 \cos(\omega_2 t), \tag{48}$$

$$\bar{u}_n = \bar{u} + n^s(t), \quad \bar{v}_n = \bar{v} + n^s(t), \tag{49}$$

with the following set of parameters $\omega_1 = 200$, $\omega_2 = 150$, $\epsilon_1 = 0.01$ and $\epsilon_2 = 0.015$. The results pertinent to this case are shown in Figures 12, 13 and 14 for open optimal and closed FGL solutions. Again it is observed that the performance of the open-loop optimal control is deteriorated as compared with the result of the closed FGL. Furthermore, the closed optimal FGL is analyzed with assumed asymmetric random noise on the control actuation taken as:

$$n^a(t) = \epsilon_1[\text{RANDOM} + \epsilon_1], \qquad (50)$$

$$\beta_n = \beta_n + n^a(t), \qquad (51)$$

where RANDOM denotes a random number from a normal distribution with $\mu = 0$, $\sigma = 1$. The results of this case are shown in Figures 15, 16 and 17, where the noise parameters of relation (50) are taken as $\epsilon_1 = \frac{1}{20}$, $\epsilon_2 = 0$. It is observed that the closed FGL performance

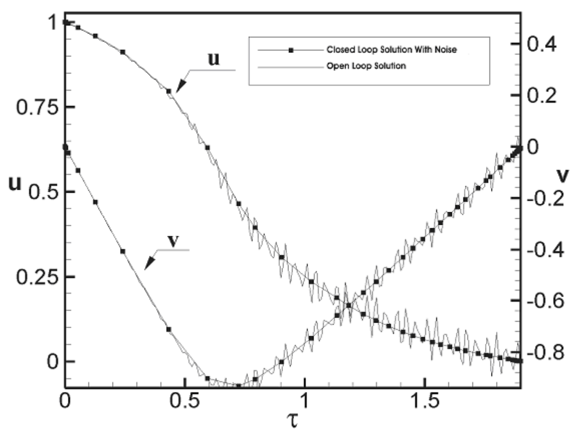

Figure 14. Variation of the trajectory $(\bar{u}, \bar{v})$ with non-dimensional time using sinusoidal noise on the measurement system.

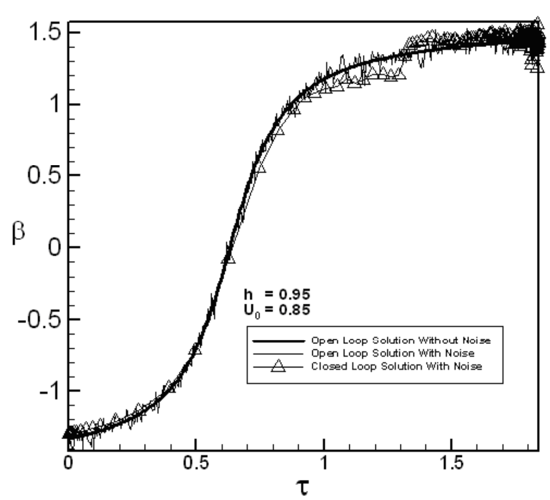

Figure 15. Variation of the control variable (β) with non-dimensional time using random noise on the actuation system.

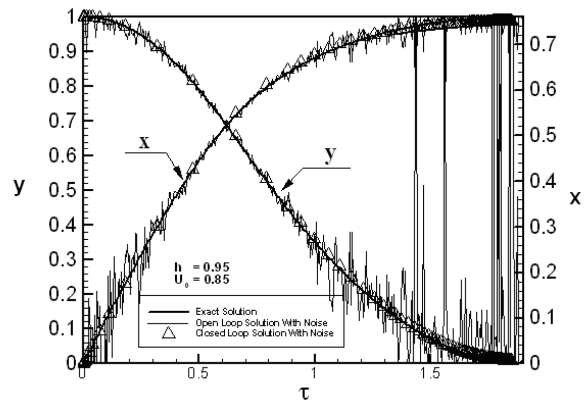

Figure 16. Variation of the trajectory $(\bar{x}, \bar{y})$ with non-dimensional time using random noise on the actuation system.

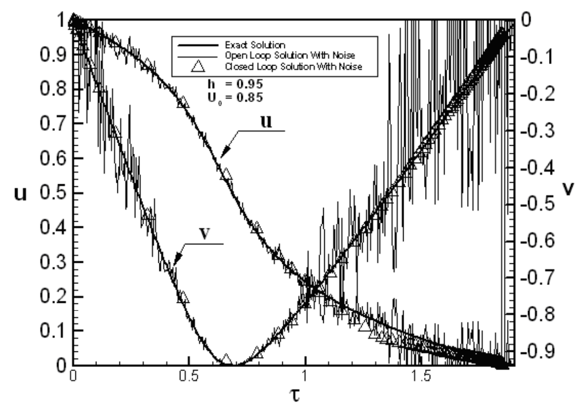

Figure 17. Variation of the trajectory $(\bar{u}, \bar{v})$ with non-dimensional time using random noise on the actuation system.

is superior to the open-loop optimal solution and in close agreement with the exact solution.

CONCLUSION

This study is focused on the determination of an optimal feedback control strategy for the non-linear problem of lunar lander mission. Due to inherent complexities associated with its variational formulation, a closed-loop time optimal solution is not feasible. However, the desired task is achieved using a fuzzy system in an attempt to generate a closed-loop optimal fuzzy guidance law (FGL). The fuzzy training process is performed on a series of numerically determined open-loop time optimal solutions that utilize non-dimensional state variables as opposed to regular states. This approach is proven to increase the accuracy as well as the performance of the fuzzy training process. The potential of the FGL is demonstrated by producing optimal missions for the non-trained scenarios polluted with stochastic actuator and process noise.

REFERENCES

1. Bryson, A. and Ho, Y. C., "Applied Optimal Control",

Hemisphere, New York, (1975).

2. Dabbous, T.E. and Ahmed, N.U., "Nonlinear Optimal Feedback Regulation of Satellite Angular Momenta", *IEEE Trans Aerosp ElecSys*, **18(1)**, PP 2-10(1982).

3. Kirk, D. E., "Optimal Control Theory", Englewoods Cliffs, NJ: Prentice-Hall, (1970).

4. Kunisch,K., Volkwein, S. And Xie, L., "Nonlinear Feedback Design for the Optimal Control of Evolution Problems", Mathematik, Heinrichstrasse 36, A-8010 Graz, Austria, (2003).

5. Lewis, F. L., and Syrmos, V. L., "Optimal Control", John Wiley & Sons Inc., New York , (1995).

6. Lewis, F. L., Abu-Khalaf, M., "Nearly Optimal State Feedback Control of Constrained Nonlinear, Systems Using a Neural Networks HJB Approach", *IFAC Annual Reviews in Control*, **28**, PP 239-251(2004).

7. Li, X., Wang, H.O., "A Course in Fuzzy Systems and Control", Prentice Hall, Englewood Cliffs, (1994).

8. Li, J., Wang, H. O., Bushnell, L. and Hong, Y., "A Fuzzy Logic Approach to Optimal Control of Nonlinear Systems", *International Journal of Fuzzy Systems*, **2**(3), (2000).

9. Lee, A. Y. and Smyth, P., "Synthesis of Minimum-Time Feedback Laws for Dynamic Systems Using Neural Networks", *Journal of Guidance, Control, And Dynamics*, **17**(4), (1993).

10. Mitsuishi, T., Kawabe, J., Wasaki, K. Kawamoto, P and Shidama, Y., "Membership Function in Space and Its Application to Fuzzy Optimal Control", 7th IFAC Symposium AIRTC'98, Session H-2 , (1998).

11. Naidu, D. S. and Price, D.B., "Time Scale Analysis of a Closed-Loop Discreete Optimal Control System", Proceedings of the AIAA Guidance, Navigation, and Control Conference, Williams Burg, VA, (1986).

12. Naidu, D. S. and Price, D.B., "Time Scale Synthesis of a Closed-Loop Discrete Optimal Control System", *AIAA Journal of Guidance, Control, and Dynamics*, **10**(5), PP 417-421(1987).

13. Rajagopalan, P. K. and Naidu, D.S., "Singular Perturbation Analysis of a Closed-Loop Fixed-End-Point Optimal Control Problem ", *EE Proceedings-D: Control Theory and Applications*, **127**, PP 194-203(1980).

14. Subbaram, D., Naidu, "Singular Perturbations and Time Scales in Control Theory and Applications an Overview ", *Dynamics of Continuous,Discrete and Impulsive Systems Series B: Applications Algorithms*, **9**, PP 233-278(2002).

15. D. Palma F. and Magni, L., "On Optimality of Nonlinear Model Predictive Control", 16th International Symposium on Mathematical Theory of Networks and Systems (MTNS 2004), Katholieke Universiteit Leuven, Belgium, (2004).

16. Rahbar N., Bahrami, M., "Short Communication Synthesis of Optimal Feedback Guidance Law for Homing Missiles Using Neural Networks ", *Optimal Control Applications Methods J.*, **21**, PP 137-142(2000).

17. Shamma, J.S., Cloutier, J.R., " Existence of SDRE Stabilizing Feedback", *Automatic Control, IEEE*, **48**(3), PP 513-517(2003).

18. Steinbauer, P., Valasek, M., "Fuzzy Optimal Control of Mechanical Systems ", *Inteligent Systems for Manufacturing. Multi-Agent System and Virtual Organization* , Dordrocht: Kluwer Academic Publishers, PP 491-498(1998).

19. D'Souza , C.N., "An Optimal Guidance Law for Planetary Landing", Proceedings of the AIAA Guidance, Navigation, and Control Conference, New Orleans, LA. , (1997).

20. Suzuki, S. and Yoshizawa, T., "Near-Minimum Fuel Guidance Law of a Lunar Landing Module ", *IFAC Automatic Control in Aerospace*, Seoul, Korea., (1994).

21. Ueno S., Yamaguchi, Y., "3-Dimensional Near-Minimum Fuel Guidance Law of a Lunar Landing Module", *AIAA-99-3983*, (1999).